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INVESTIGATION OF SPHERICAL TEARING MODE

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1. Introduction

The purpose of NASA Contract NASW–4651 ("Investigation of Spherical Tearing Mode") was to better understand tearing and reconnection in genuinely three-dimensional configurations. We have identified an equilibrium model that should contain the required features. Three papers have been written, and a fourth is in preparation. They are listed in the Bibliography.

Briefly, these papers reached the following conclusions. The first paper, dealing with Vlasov-Maxwell equilibria, showed that an magnetohydrodynamic (MHD) approximation displays all the useful information on the configuration. If MHD force balance is obtained, no new information is gained by studying particle orbits. Further, the electric field can not be evaluated from the charge density, but must be obtained from Ohm's law. The second paper in our series, regarding a screened dipole, extended some previous work on a paraboloidal model of the magnetopause. The third paper was concerned with the nature of reconnection. The conclusion was that reconnection of magnetic field lines can best be treated as a topological property of the field lines. The dynamics of reconnection depends on the medium in which it occurs, but the location of reconnection depends on the topology of the field lines. In the fourth paper, it is pointed out that short closed field lines are essential for the appearance of tearing modes in toroidal systems. Thus the classical formulation of tearing modes is not applicable in the magnetosphere. In this paper we propose an alternate configuration that is strongly analogous to the low order rational surface of toroidal systems, but is quite different in detail. Thus it should yield an alternative tearing mode.

This is not a review, but only a description of what we did. Thus the only items in the Bibliography are papers that we have written. This report is organized around these papers, with a section for each. We have taken this opportunity to give some background material that was omitted in the papers in order to make them more publishable. For reasons to be discussed further in Section 5 of this report, we are convinced that the magnetopause remains close to a configuration in which there is a closed surface that has a vanishing normal magnetic field. This is a central feature of our work. Section 2 contains information on Vlasov-Maxwell equilibria that is useful for establishing force balance normal to the magnetopause. In Section 3 the field of a dipole screened in a paraboloidal magnetopause is discussed. Section 4 details the problems of fitting classical tearing mode theory into a magnetosphere, and a solution of these problems is introduced in Section 5. Section 6 contains a simple conclusion.

2. Vlasov-Maxwell Equilibria

In this section we discuss the first paper in our bibliography "One-dimensional Vlasov-Maxwell equilibria." The main purpose of this paper was to demonstrate that an MHD fluid model is adequate to describe all that can be usefully known about the detailed structure of the magnetopause. This is in direct distinction to the universal use of kinetic models for this purpose. Here we sketch out the logical outline of the argument.

The first point of this paper is that the usual Vlasov equation is Newtonian while the full Maxwell equations are relativistic. Thus it is not obvious that the Vlasov equations are sufficiently accurate from a relativistic viewpoint that their solution can be used to evaluate the currents and charges needed in the Maxwell equations. Indeed, when the electric field is sufficiently small that the $\boldsymbol{E} \times \boldsymbol{B}$ velocity is much less than the velocity of light, the net charge density is sufficiently small that relativistic corrections are significant. In particular, consider two frames with relative velocity $\boldsymbol{U}$ so that the electric fields $\boldsymbol{E}$ and $\boldsymbol{E}'$, and charge densities $\boldsymbol{\sigma}$ and $\boldsymbol{\sigma}'$, in the two frames are related by

$$\boldsymbol{E}' = \boldsymbol{E} - \boldsymbol{U} \times \boldsymbol{B} \quad , \qquad \boldsymbol{\sigma}' = \boldsymbol{\sigma} + \boldsymbol{U} \cdot \boldsymbol{j}/c^2 \quad .$$

The modification of the electric field by the velocity $\boldsymbol{U}$ is clearly significant for reasonable values of $\boldsymbol{U}$, and thus the modification of the charge density is both significant and non-Newtonian. It follows that the Maxwell's equation

$$\nabla \cdot \boldsymbol{E} = \mu_0 \, c^2 \, \boldsymbol{\sigma} \quad ,$$

can not be used to find the electric field in the magnetopause, because it is not possible to evaluate the charge density to sufficient accuracy.

The next point is that, since the charge density is extremely small, it is appropriate to drop the displacement current from the Faraday equation. Then the electric current must be given by $(1/\mu_0)\nabla \times \boldsymbol{B}$. That is, the current depends only on the magnetic configuration and not on particle motions. This is true even of the current flowing parallel to the magnetic field. To make this result acceptable, it is necessary to show that the charged particle drifts in a given magnetic field necessarily add up to yield a current appropriate for the given magnetic configuration. To accomplish this, consider the momentum equation for each particle. If none of the particles is being perpetually accelerated in the given electromagnetic field, summing momentum balance over all the particles yields

$$\nabla \cdot \Pi - \boldsymbol{j} \times \boldsymbol{B} = 0 \quad ,$$

where $\boldsymbol{j}$ is the sum of the contributions of each particle to the electric current, and Π is the total particle stress. The first term is essentially the pressure gradient, with corrections. Note that the stress is much easier to calculate than the current since each term is positive and there are no cancellations. The current calculated from this momentum equation is equal to the perpendicular component of $(1/\mu_0)\nabla \times \boldsymbol{B}$ if the stress Π satisfies

$$\nabla \cdot \left[\Pi + \frac{1}{\mu_0}\left(\frac{1}{2}\, B^2 \boldsymbol{I} - \boldsymbol{B}\boldsymbol{B}\right)\right] = 0 \quad ,$$

where $\boldsymbol{I}$ is the unit tensor. Thus, to a good approximation, if $p + B^2/2$ is constant across the magnetopause, the currents flowing within the magnetopause are necessarily consistent with its magnetic structure.

Thus far we have shown that the current can be calculated from Maxwell's equations, but the electric field can not. The logical place to find an equation for the electric field is Ohm's law, which can be expressed in the form

$$\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B} = \text{electric field in the frame moving with velocity } \boldsymbol{v} \quad ,$$

and this velocity can conveniently taken to be the fluid velocity. To evaluate the fluid velocity it is necessary to find its source of momentum. That is outside the scope of this report.

There is great freedom in choosing particle distributions that are consistent with a prescribed pressure profile. There may be few energetic particles, or many slow particles, and as long as the pressure is appropriate for force balance, kinetic theory considerations will be satisfied. Kinetic theory by itself provides no guidance to the structure of the magnetopause. In the model treated in this report, the source of particles is obvious. It will be seen in Section 5 below that the magnetic lines in the magnetopause all connect to the cusps, where the magnetic field strength vanishes. Clearly these are convenient entry points for whatever particles are in the neighborhood.

Altogether MHD provides all the useful information concerning the structure of the magnetopause that can be derived theoretically. Formulations of the BGK type are elegant, but their results are quite sensitive to arbitrary assumptions. Given that MHD considerations are of dominant interest, the significant problem is to find the magnetic line structure. This is investigated in the next sections.

3. The Field of a Screened Dipole

Here we discuss the third paper in our bibliography ("The field of a screened magnetic dipole"). As discussed in the Introduction, and for reasons developed in Section 5, we believe that the magnetosphere is nearly closed. Thus it makes sense to consider closed magnetospheres as equilibria, and treat their opening as a perturbation or instability. Since the location of the magnetopause determines the location of the currents, and the magnetopause currents affect the location of the magnetopause, a truly global self-consistent calculation is required to obtain specific configurations. However, much that happens in the magnetopause does not depend on its exact location, so results for specific models with analytically prescribed magnetopause locations can be instructive. A particular model is developed in this paper.

The model studied here approximates the magnetopause as a paraboloid of revolution. It has the advantage that it is analytically tractable, and is of approximately the correct shape. The analytic treatment of this configuration has been explored previously, but we made certain significant advances that are incorporated in our paper. We feel that this model will become important and that the progress made in our paper will help establish its usefulness.

Our particular interest in this paper was to understand some of the global effects of the currents flowing in the magnetopause. Since they are determined by the condition that the normal field vanish at the magnetopause, we call them screening currents. In particular, we wished to understand the importance of screening in the tail region.

We found that the magnetopause currents almost completely cancel the dipole field in the tail. The result is that the predicted field normal to the plasma sheet decays exponentially with distance down the tail. Thus it is extremely weak, and can be easily reversed beyond twenty to twenty-five earth radii. It opens the possibility that under extreme conditions the screening currents at the magnetopause might facilitate the reversal of the normal field at the plasma sheet rather close to the earth.

Another conclusion of this paper is that, since the screened dipole field so nearly vanishes in the tail region, the screened field can be more accurately calculated than either the dipole or the screening field separately. That is, models in which an approximate screening field is added to the dipole field tend to overestimate the normal field at the plasma sheet. One consequence is that the flare of the tailward magnetosphere cavity can be overestimated.

4. Classical Tearing Modes and Reconnection

In this section the second paper of our bibliography ("Reconnection of vorticity lines and magnetic lines") is discussed. Due to deadlines, there was a divergence between the motivations and the results of this investigation. Since the motivation has not been set out before, we take advantage of this opportunity to do so here.

The original tearing mode model was introduced to understand MHD behavior in the vicinity of closed magnetic field lines. Since the flows perpendicular to the closed field line tend to be hyperbolic, this central line will be called an X-line. An approximation was introduced in which the time for an Alfvén wave to traverse the closed field line was much less than the time required for the evolution of the considered modes. That is, the MHD spectrum consisted of a slow mode and a sequence of fast modes separated in frequency by the inverse of the time for an Alfvén wave to traverse the loop. An approximation was introduced in which the mode treated was aligned along the X-line so that the high-frequency evolution was decoupled. Since the role of Alfvén waves in enforcing coordination of the mode along the X-line was obscured, a folklore arose that the field strength along the X-line was unimportant.

The situation is quite different when the Alfvén time for propagation along the X-line is long, either because the field is weak or the line is very long. Then the spectrum is nearly degenerate, with separations between modes again of the order of the inverse of the Alfvén time. This has a very significant effect on the behavior of an initial value problem. The evolution of an initial perturbation in which a number of eigenfunctions is excited is unconstrained by the presence or absence of closed field lines for early times. This evolution is dominated by the fastest growing eigenmode only after a time long compared to the inverse of the difference of the growth rates of the several modes initially excited. That is, a global tearing mode gets organized only after Alfvén waves have propagated several times around the X-line. In the interval before that time the evolution is dominated by the dispersal of Alfvén waves. The dispersal of energy by the radiation of Alfvén waves can significantly affect the growth rate of the modes.

It would seem from this that, in configurations where every magnetic line wanders to infinity, it is impossible to store or focus energy; every perturbation of the magnetic field would immediately relax by emission of Alfvén waves, and magnetospheric dynamics would mimic the dynamics of a soft pillow. Yet this is not so. The solution of this paradox is that there is nontrivial topology in the magnetosphere, and that this topology does affect the dynamics. The additional topology of lines of force in three-dimensional space, beyond that of closed lines, depends on null points of the field. They will be discussed in the next section.

5. A Simple Magnetospheric Model Exhibiting Tearing

In this section we discuss the last paper in our Bibliography ("An Approach to a Spherical Tearing Mode"). Here we explore an analog of the tearing mode that is genuinely three-dimensional. We show how the difficulties, described in the previous section, of adapting classical tearing modes to three-dimensional configurations can be overcome.

The first step is to pick an equilibrium configuration. Here we make the very unfashionable choice of a completely closed configuration, containing a closed surface on which the normal component of the magnetic field vanishes. The sixth paragraph of this section details how the standard objections to such a configuration can be overcome.

We take the closed surface to be spherical, and we call this the tearing surface. This is done in the same spirit that produced, in the original papers discussing tearing modes, a slab model that was topologically equivalent to a general toroidal configuration. Then the magnetic field lies tangentially on the tearing surface. Inside this surface we place a dipole, and outside an external magnetic field that is constant at infinity and points in an arbitrary direction. The currents producing this field configuration are limited to the dipole, a region close to the tearing surface, and at infinity. A general field meeting these conditions has a discontinuity at the tearing surface. Such a discontinuity obscures essential features of the topology, and is unphysical. Thus we make the current carrying layer at the tearing surface to be of finite thickness, and require the fields to be continuous.

It is well-known that the field at the tearing surface must have singularities, and that in typical situations these are null points. The topological statement describes only the tangential component of the field, but clearly, if the field is continuous at the tearing surface and the normal component vanishes, these points must be three-dimensional null points. We will call them cusps. In the configurations studied here, one is in the northern hemisphere, and one in the southern. It is important to note that the existence of cusps is topologically stable. If they exist in one configuration they exist in any plausibly similar configuration.

The field line structure in the vicinity of a cusp is as follows. The field lines tangential to the magnetopause that connect to a cusp form a sheet, that is the tearing surface, and the isolated lines that join a cusp normal to the tearing surface, here called cusp lines, connect to the dipole or to infinity. The tangential field lines that join to the other cusp also form a sheet. In the equilibrium configuration we are considering here, these two sheets are coincident. Under general perturbations, cusps persist, their sheets persist, but the two sheets are not coincident. There is a gap between them that contains magnetic lines that join to the dipole inside the magnetopause, and to infinity outside.

Here we come to a crucial point. When the dipole and external fields are not aligned, the cusp occurs at some compromise location between these two directions. This has the consequence that the inner cusp line is not aligned with the dipole. Another way of saying this is that a component of the external field appears inside the tearing surface, and this component bends the cusp line. *The appearance of a component of the external field inside the magnetosphere can be perfectly consistent with the magnetopause being completely closed.* In the work described here, only one simple case has been worked out. However, it is clear to us that if there are bulges or furrows in the magnetopause, they can lead to significant modifications of the field inside the magnetosphere, even if the configuration is rigorously closed. Apparent penetration of the external field can not be used to show that the configuration is open or closed. As a better test of the open or closed character of the field configuration, we propose that if the magnetospheric and magnetosheath plasma are well separated, the configuration is closed regardless of the interior field configuration. Since this is in fact observed, we believe a closed configuration is a good model for magnetospheric ground state.

There is an analogy between the equilibrium described here and the equilibrium used in tearing theory. An essential element on the latter case is the existence of a surface composed of closed magnetic field lines, which is to be compared with the requirement here of coincidence between the sheets arising at two different cusps. In either case the desired configuration is quite unlikely for arbitrarily prescribed fields, but there is observational evidence that it is a reasonable starting point.

The question of interest is the evolution away from the given equilibrium. In classical tearing theory a good place to start is to consider nearby equilibria. That is, we assume that the evolution is slow compared to that associated with magnetosonic or Alfvén waves, and thus progresses through states that are in balance against the emission of such waves. The neighboring states that we studied were characterized by a weakening of the currents in the magnetopause. It is well-known that if there are no magnetopausal currents, there is in general a significant magnetic flux, here called open flux, composed of lines that connect the dipole to the external field. The two cusp sheets separate, leaving a gap that is filled by the open flux. The open flux lines proceed from the dipole to the magnetopause along a cusp line, through the magnetopause in the gap between the cusp sheets, and at the other end connects to the external field along another cusp line. Attaining such a configuration from the closed configuration requires reconnection of the magnetic field lines. Thus evolution of the closed state is likely to include reconnection.

We now turn from a consideration of the static nature of the perturbations to a more speculative discussion of their dynamic aspects. It is clear that everything interesting happens near the cusps. Field lines deep in the magnetosphere or in the magnetosheath suffer no impediment to the radiation of Alfvén waves and can immediately adjust their equilibrium configuration. On the other hand, Alfvén waves slow down and thus their amplitude increases near cusps. Further, Alfvén waves can propagate into a cusp from several directions, so that wave energy is focused. This can variously lead to reflection or absorption of these waves. Since the energetic portion of the evolution is concentrated near a point cusp, there is less fluid to be moved than in classical tearing modes. This should lead to faster evolution.

6. Conclusion

Our efforts confirm that a closed, spherical configuration is a good model for studying tearing modes in a magnetosphere. Our initial calculations are in preparation for publication but there is still much to be learned about these modes.

7. Bibliography

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