# Accelerated Panel Methods using the Fast Multipole Method

by

N95- 23307

James F. Leathrum, Jr.
Dept. of Electrical and Computer Engineering
Old Dominion University
Norfolk, Virginia 23529

Panel methods are commonly used in computational fluid dynamics for the solution of potential flow problems. The methods are a numerical technique based on the surface distribution of singularity elements. The solution is the process of finding the strength of the singularity elements distributed over the body's surface. This process involves the solution of the matrix problem $Pq = p'$ for a set of unknowns $q$. The Fast Multipole Method is used to directly compute $q$ without using matrix solvers. The algorithm works in $O(N)$ time for $N$ points, a great improvement over standard matrix solvers.

In panel methods, the surface of a body is divided into a series of quadrilateral panels. The methods involve the computation of the influence of all other panels on each individual panel. The influence is based on the surface distribution, though this can be approximated by the area for distant panels. An alternative approximation, though with arbitrary accuracy, is to develop a multipole expansion about the center of the panel to describe the effect of a given panel on distant points in space. The expansion is based on the moments of the panel, thus allow the use of various surface distributions without changing the basic algorithm, just the computation of the various moments. The expansions are then manipulated in a tree walk to develop Taylor series expansions about a point in space which describe the effect of all distant panels on any point within a volume of convergence. The effect of near panels then needs to be computed directly, but the effect of all distant panels can be computed by simply evaluating the resulting expansion.

The Fast Multipole Method has been applied to panel methods for the solution of source and doublet distributions. A major feature of the algorithm is that the algorithm does not change to derive the potential and velocity for sources and doublets. The same expansions can be used for both sources and doublets. Since the velocity is related to the potential, and the doublet potential is related to the $z$-component of the source velocity, all values can be derived from the same expansion by taking a series of partial derivatives. This requires more expansion terms to be kept since terms are lost in the process of taking partial derivatives. Thus to maintain accuracy for the doublet computation, more terms are required than if just evaluating for sources. The resulting Fast Multipole code should then parallelize better than classical panel methods due to the locality of data dependencies found in the Fast Multipole Method. Theoretically the parallelized code should execute in $O(\log N)$ time with $O(N)$ processors, though this is not practical. Ongoing work includes implementing the parallel accelerated panel method, including methods to improve the load balancing of the problem by taking advantage of the known geometry of panels, and to encorporate sensitivity analysis into the algorithm.