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A CONTROLLED VARIATION SCHEME FOR CONVECTION TREATMENT IN PRESSURE–BASED ALGORITHM

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Convection effect and source terms are two primary sources of difficulties in computing turbulent reacting flows typically encountered in propulsion devices. The present work intends to elucidate the individual as well as the collective roles of convection and source terms in the fluid flow equations, and to devise appropriate treatments and implementations to improve our current capability of predicting such flows. A controlled variation scheme (CVS) has been under development in the context of a pressure–based algorithm, which has the characteristics of adaptively regulating the amount of numerical diffusivity, relative to central difference scheme, according to the variation in *local* flow field. Both the basic concepts and a pragmatic assessment will be presented to highlight the status of this work.

A CONTROLLED VARIATION SCHEME FOR CONVECTION TERM TREATMENT IN PRESSURE-BASED ALGORITHM

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Overview of Algorithm

- Handle Different Physics & Varying Number of Unknowns: u, v, w, p, ϱ, T, k, ε, etc. Without Reformulating the Algorithm
- Suitable for Incompressible & Compressible Flows
- Suitable for Steady & Unsteady *Flows at All Speeds*
- Modern Concepts, e.g., Modern Discretization Schemes, Multigrid, Composite Grids, etc. Should be Implementable, in Principle

▶ **Pressure–Based Sequential Solver**

Treatment of Convection

- **Convection terms:** Strong Nonlinearity

- **Critical Situations**
  - High Local Cell Peclet Numbers (Convection Dominates Diffusion)
  - Sharp Gradients in Flowfield
  - Recirculation
  - Interaction of Convection with Turbulence, Chemical Reactions, etc.
  - Presence of Source Terms

- **Flows with Sharp Gradients, e.g., Shocks**
  - Any First–Order Scheme → too diffusive
  - Any <u>Linear</u> Second–Order Scheme → spurious oscillations near sharp gradients
  - **Remedy** → <u>Nonlinear</u> Second–Order TVD (Total Variation Diminishing) Schemes

- **Source Terms:** Cause Numerical Difficulties Due to Different Length and Time Scales

Present Approach

- **Sequential Solver**
- **Explicit Control of Numerical Viscosity**
  - Based on Total Variation Diminshing (TVD) Concept
  - Controlled Variation Scheme (CVS)
- **Cases Studied**
  - Compressible Shock Tube Flows
  - Longitudinal Combustion Instability
  - Incompressible Recirculating Flows (Laminar and Turbulent)

Harten's Implicit TVD Scheme

- **Scalar Conservation Law**

$$\frac{\partial w}{\partial t} + \frac{\partial f(w)}{\partial x} = 0$$

- **Implicit TVD Scheme**

$$w_i^{n+1} + \left(\tilde{f}^{n+1}_{i+1/2} - \tilde{f}^{n+1}_{i-1/2}\right) = w_i^n$$

- **Numerical Flux**

$$\tilde{f}^{n+1}_{i+1/2} = \frac{1}{2}\Big[\underbrace{f_i + f_{i+1}}_{\bar{F}} + \underbrace{g_i + g_{i+1}}_{\bar{G}} - \underbrace{Q\left(a_{i+1/2} + \gamma_{i+1/2}\right)\Delta_{i+1/2} w}_{\bar{Q}} \Big]$$

Central Diff. Flux	Anti–diffusion Flux	Numerical Dissipation Flux
$\overline{F}$	$\overline{G}$	$\overline{Q}$

Model Problem I

- **Linear Steady Burgers' Equation**

$$\alpha\phi_x = \beta\phi_{xx} \qquad \alpha,\ \beta = constants > 0$$

$$\phi(0) = 0\ , \quad \phi(1) = 1$$

- **Cell Peclet Number (P)**

$$P = \frac{\alpha}{\left[\frac{\beta}{h}\right]} = local\left[\frac{convection}{diffusion}\right]$$

- **Central Difference Scheme**

$$(2 - P)\,\phi_{i+1} - 4\phi_i + (2 + P)\,\phi_{i-1} = 0$$

Critical Value : $|P| > 2$

Effective Cell Peclet Number of CVS

- **Ratio of Local Convection to Diffusion Strength**

Diffusion (CVS) = Physical + Numerical

$$P_{cvs} = \frac{\alpha}{\left[\frac{\beta}{h} + \alpha \left\{ \frac{\overline{Q}_{i+1/2} - \overline{G}_{i+1/2}}{\Delta_{i+1/2}\phi} \right\} \right]}$$

- **Normalized Viscosity**

$$Q^* - G^* = \frac{\alpha \left\{ \frac{\overline{Q}_{i+1/2} - \overline{G}_{i+1/2}}{\Delta_{i+1/2}\phi} \right\}}{\beta / h}$$

Model Problem II

- **Linear Steady Burgers' Equation With Source**

$$\alpha\phi_x = \beta\phi_{xx} + \alpha\psi(x)$$

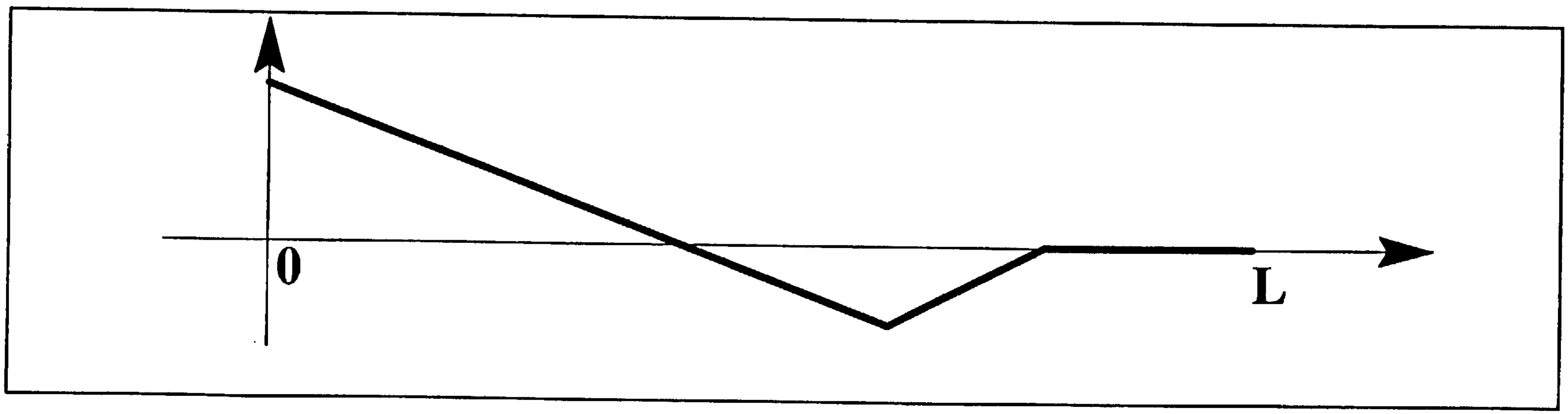
$$\phi(0) = 0$$

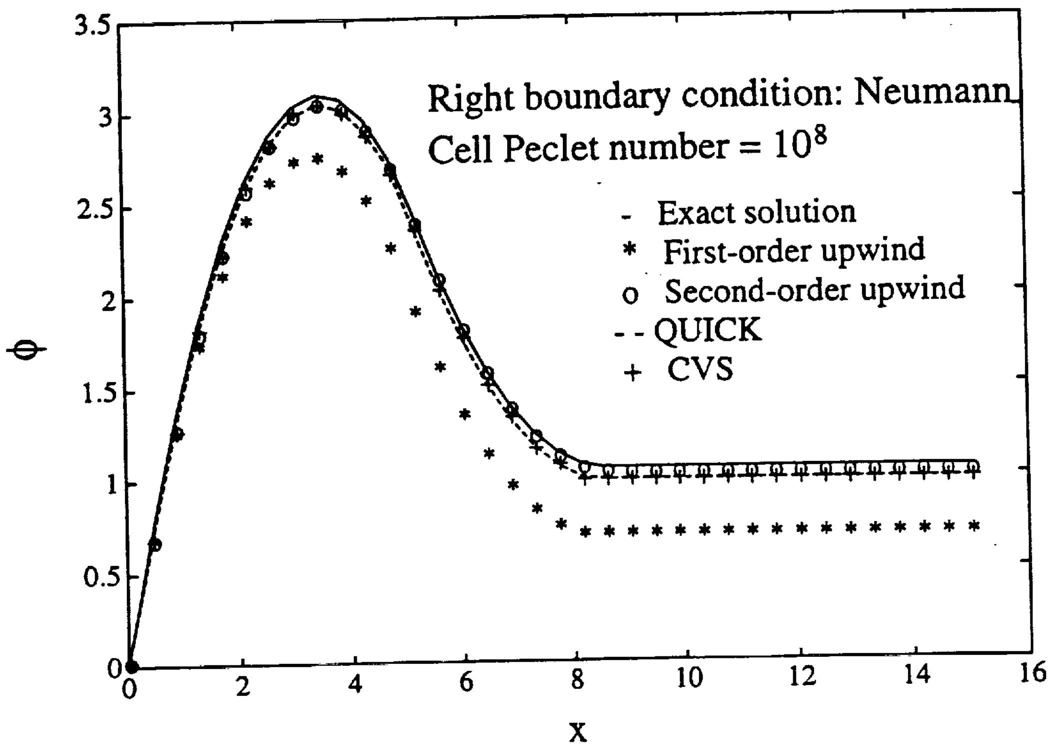
Right Boundary Condition:

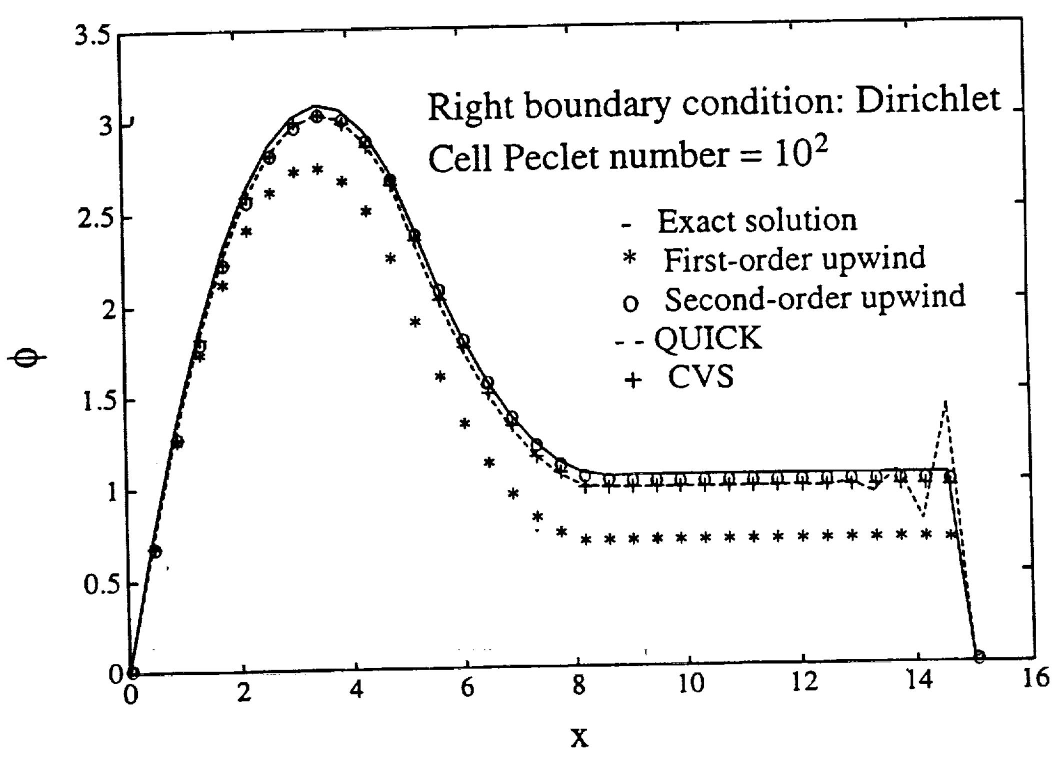
Neumann: $(\phi_x)_{x=L} = 0$

Dirichlet : $\phi(L) = 0$

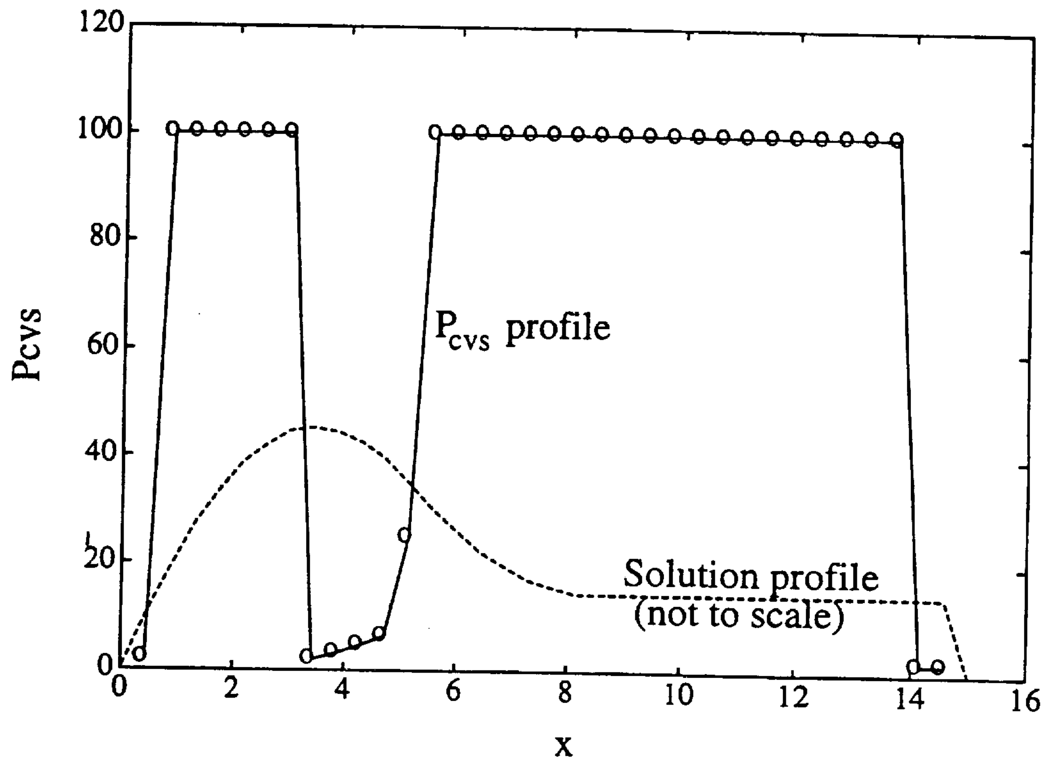
- **Source Term $\psi(x)$**







Model Problem II: Different Schemes



Model Problem II: Dirichlet B.C., P=100
Effective Cell Peclet Number

i	x	P_{cvs}
2	0.42	1.96
9	3.42	1.96
10	3.85	3.15
11	4.28	4.66
12	4.71	6.32
13	5.14	24.85
34	14.14	1.97
35	14.57	1.96
Remaining i	Remaining x	100.00

Treatment of Source Terms

- Using TVD Type Sequential Solver
- Special Techniques :
  - MacCormack's Predictor–Corrector
  - Strang's Operator–Splitting
- 1–D Longitudinal Combustion Instability Problem

TREATMENT OF SOURCE TERMS: A LONGITUDINAL COMBUSTION INSTABILITY PROBLEM

Besides convection terms, source terms (if present) in the Navier–Stokes equations can be strong enough to cause numerical difficulties such as a loss of accuracy in the form of spurious oscillations in the solution profiles or numerical instability. This is so because strong source terms can be sufficiently stiff and the time and length scales imposed by them may not be commensurate with those imposed by convection, for example. Thus, due attention has to be paid to the source terms and not just to the convection terms.

A one–dimensional longitudinal combustion instability problem is chosen which has a strong heat release source term. The high accuracy TVD type of convection treatment in a sequential solver (second fig. clockwise: top right) is seen to provide higher accuracy than the first–order upwind scheme (first figure: top left), as evident from the amplitudes of the ten pressure modeshapes shown in the viewgraph. However, the TVD type of convection treatment without any special source term treatment yields spurious oscillations in modehapes numbered 5, 6 and 7 (second figure clockwise). From the corresponding heat release modeshapes (third fig. clockwise), it is clear that modes 5, 6 and 7 are the modes of maximum heat release, thus demonstrating that when source terms become stiff enough they may lead to spurious oscillations. This can be resolved by increasing the amount of numerical dissipation in the scheme (by varying δ) but this is accompanied by an overall smearing of solution profiles. However, special source term treatment such as MacCormack's predictor–corrector method or Strang's time–splitting method (here, the latter) can resolve the problem by suppressing any spurious oscillations without the need of any extra numerical damping. This is clearly evident from the bottom left plot (fourth fig. clockwise).

Special Source Term Treatment

- Conservation law with a source term

$$w_t + f(w)_x = \psi(w)$$

- **Treatment**

a) MacCormack's Predictor-Corrector Method

b) **Operator Splitting (Strang's Time-Splitting)**

$$W^{n+1} = S_\psi(\Delta t/2)\ S_f(\Delta t)\ S_\psi(\Delta t/2)\ W^n$$

where S_f represents the numerical solution operator for

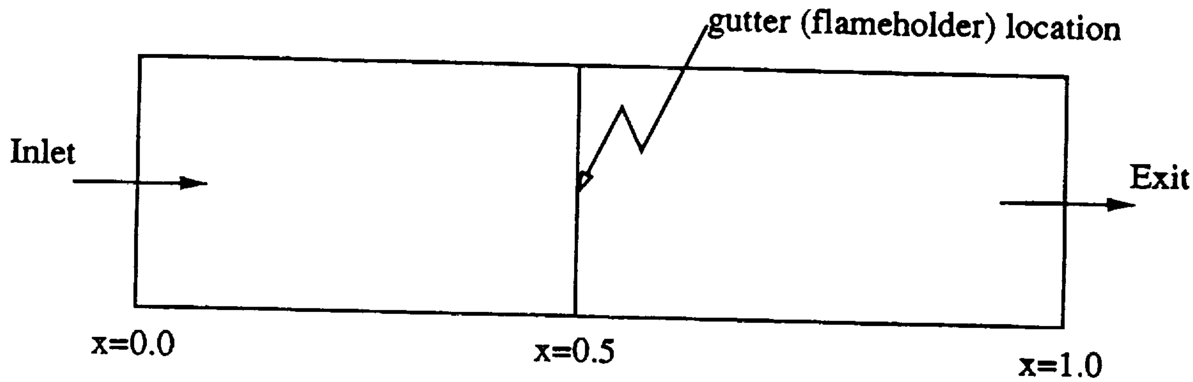
$$w_t + f(w)_x = 0$$

and S_ψ is the numerical solution operator for the ODE

$$w_t = \psi(w)$$

1–D Combusting Flow in a Duct

- To illustrate the effect of a strong source term (heat release) on numerical accurcay
- To demonstrate the efficacy of special source term treatment for a strong source term



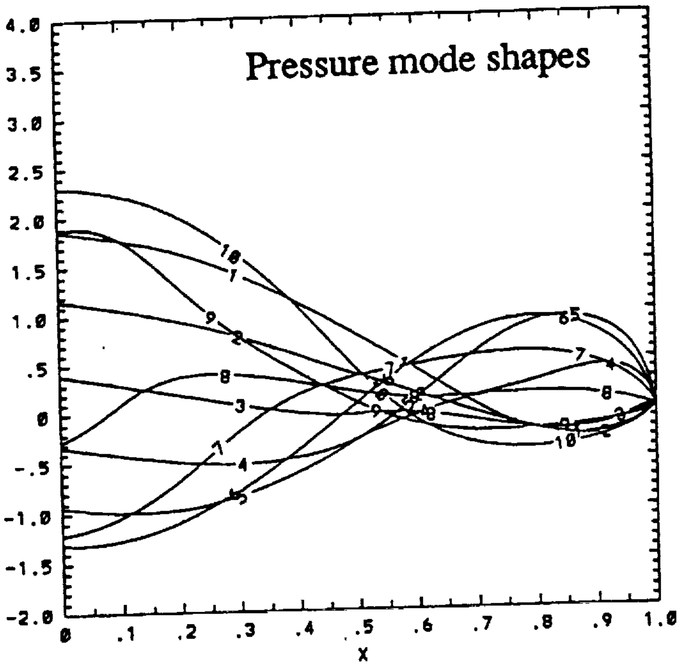
Basic Equations

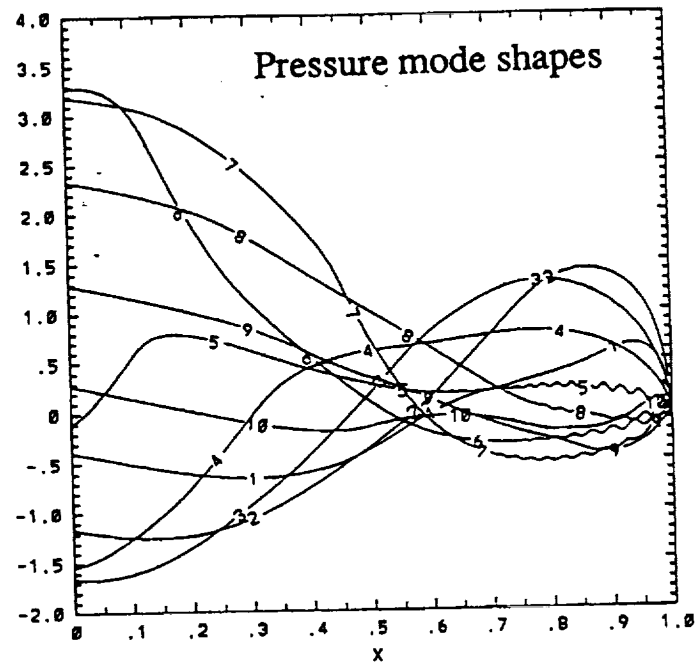
$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} = 0$$

$$\frac{\partial m}{\partial t} + \frac{\partial (mu)}{\partial x} = -\frac{\partial p}{\partial x} + \frac{1}{Re_\infty}\frac{\partial \tau_{11}}{\partial x}$$

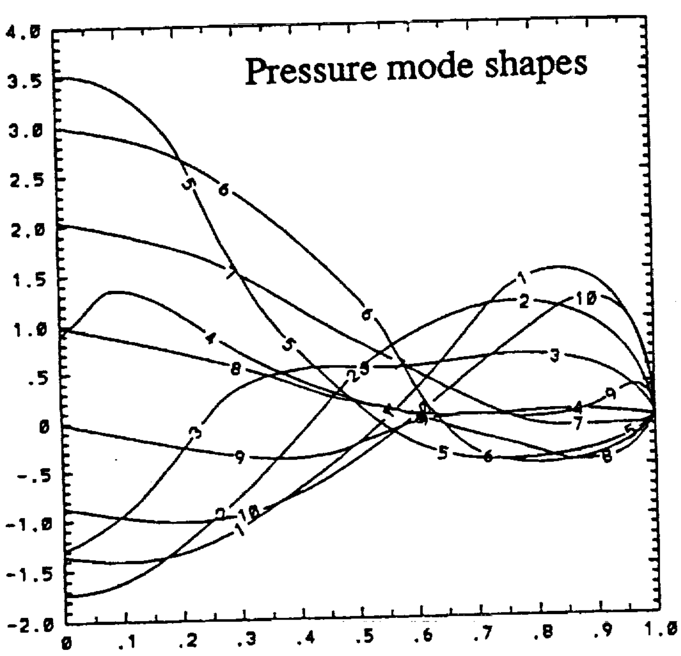
$$\frac{\partial E}{\partial t} + \frac{\partial (Eu)}{\partial x} = -\frac{\partial (pu)}{\partial x} + \frac{1}{Re_\infty}\left[\frac{1}{Pr_\infty}\frac{\partial}{\partial x}\left(\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial x}(u\tau_{11})\right] + Q(x,t)$$

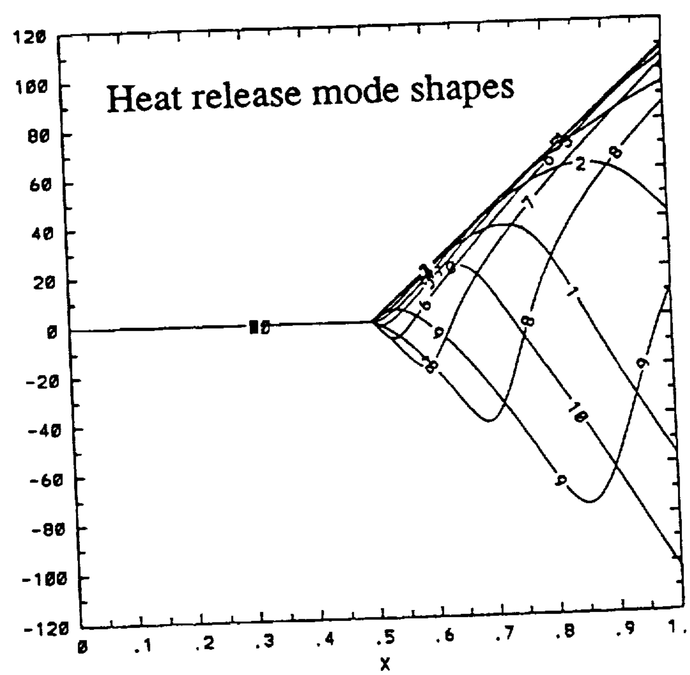
Combustion Instability Problem : Mode Shapes





First-Order Upwind Scheme



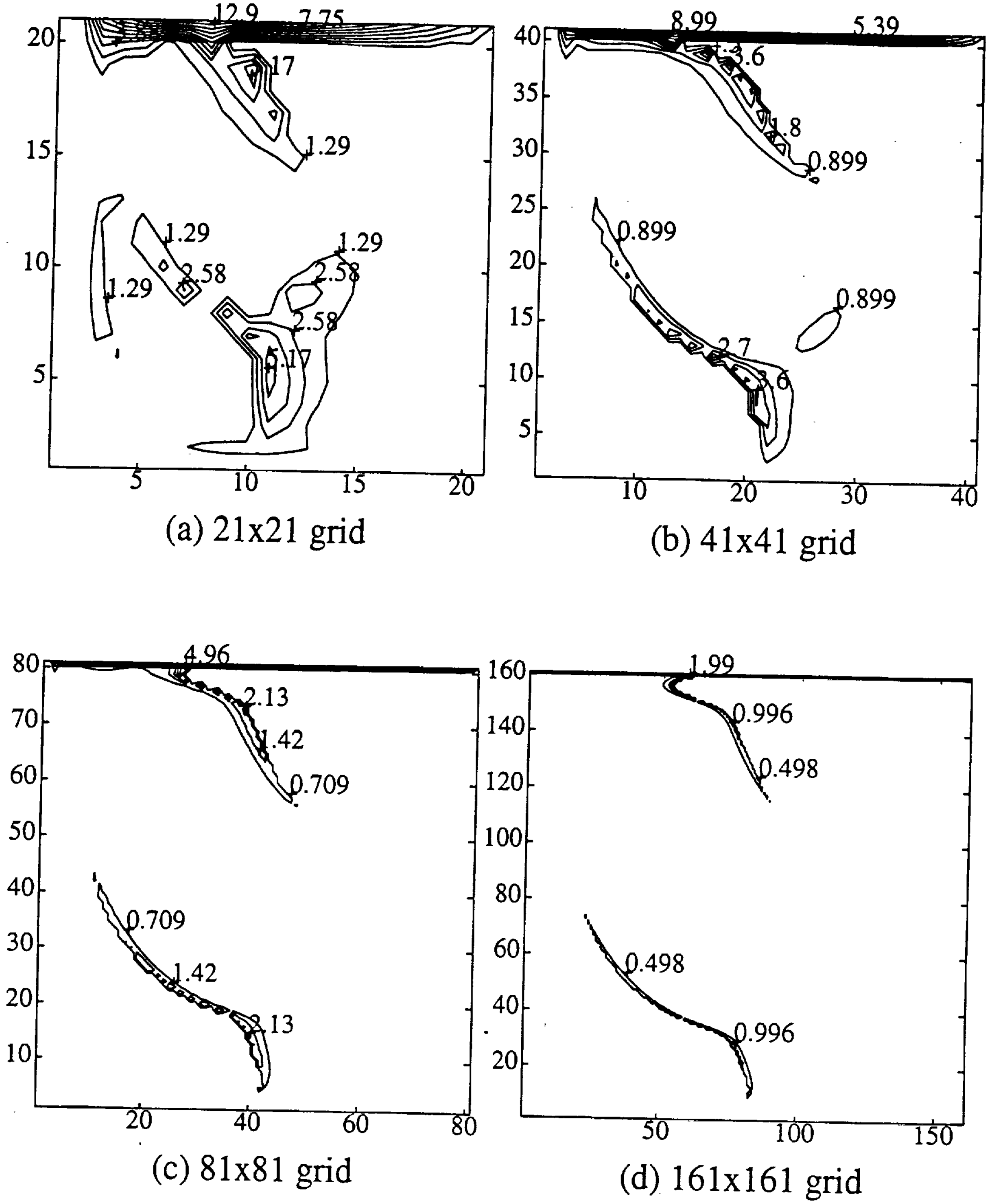


TVD Scheme; Source Treatment

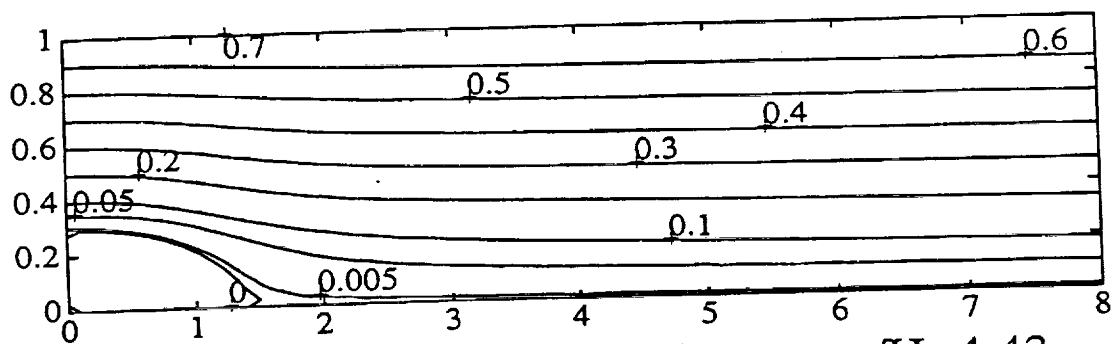
$\delta = 0$

TVD Scheme; No Source Treatment

$\delta = 0$

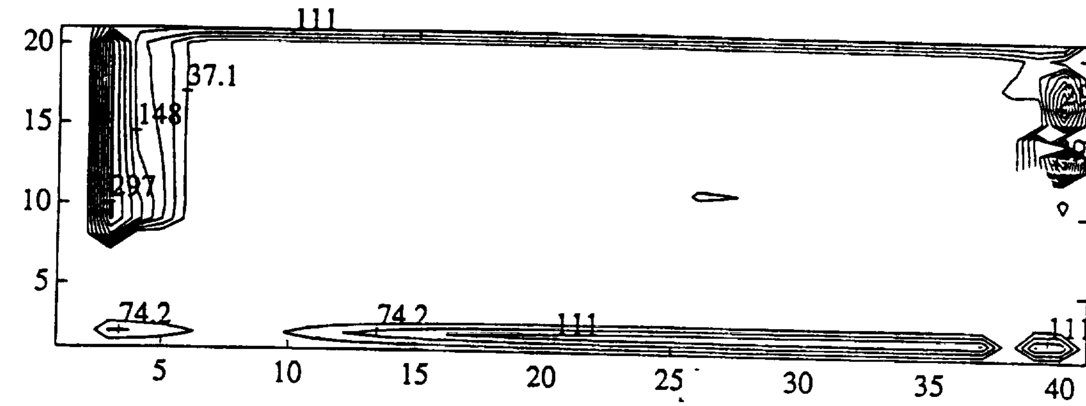


Normalized Viscosity (Q* - G*)

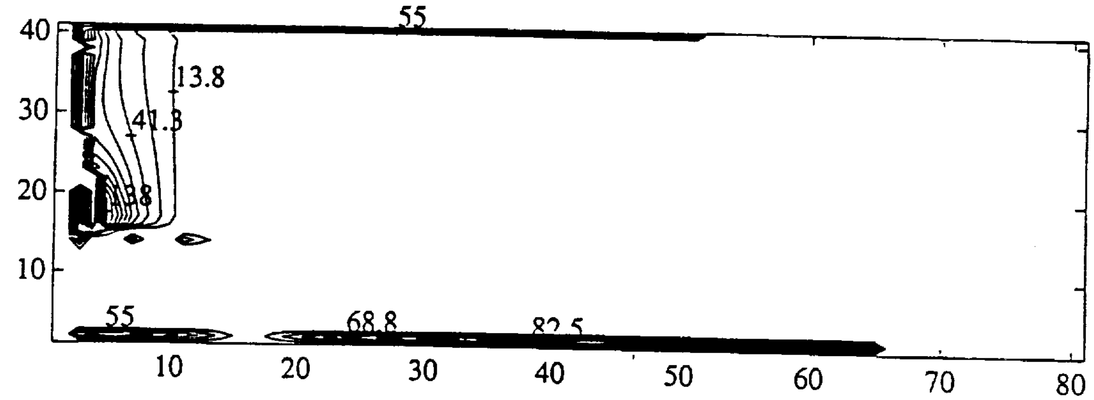


(d) Controlled variation scheme; x_R/H=4.43

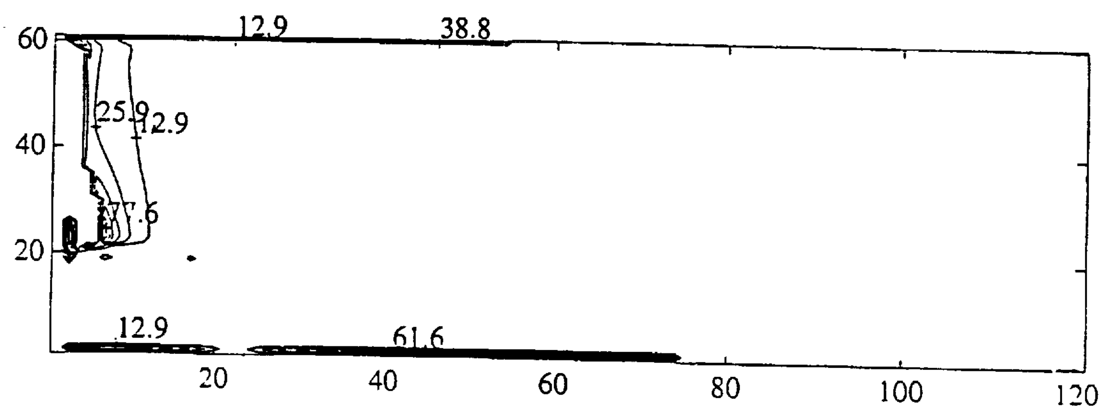
Streamfunction: Backward-Step, Turbulent Flow
Re = 132,000



(a) 41x21 Grid



(b) 81x41 Grid



(c) 121x61 Grid

Q*-G* on Various Grids

CVS: Conclusions

- TVD Schemes can be Effectively Extended to Sequential Solvers
- CVS Effective for High Cell Peclet Number (Convection–Dominated) Recirculating Flows
- Mechanism → Reduces Effective Local Cell Peclet Number Via Controlled Numerical Dissipation
- Numerical Dissipation Decreases Monotonically With Grid Refinement
- CVS Truly Useful When Central Difference Fails, e.g., Turbulent Flows → Enforces Physical Realizability

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