42272
P. 17

A Linguistic Geometry for 3D Strategic Planning

Boris Stilman
Department of Computer Science & Engineering, University of Colorado at Denver
Campus Box 109, Denver, CO 80217-3364, USA.
Email: bstilman@cse.cudenver.edu

Abstract. This paper is a new step in the development and application of the Linguistic Geometry. This formal theory is intended to discover the inner properties of human expert heuristics, which have been successful in a certain class of complex control systems, and apply them to different systems. In this paper we investigate heuristics extracted in the form of hierarchical networks of planning paths of autonomous agents. Employing Linguistic Geometry tools the dynamic hierarchy of networks is represented as a hierarchy of formal attribute languages. The main ideas of this methodology are shown in this paper on the new pilot example of the solution of the extremely complex 3D optimization problem of strategic planning for the space combat of autonomous vehicles. This example demonstrates deep and highly selective search in comparison with conventional search algorithms.

1. INTRODUCTION

Aerospace problems such as long and short-range mission planning, especially for autonomous navigation, scheduling, aerospace robot control, long-range satellite service, aerospace combat operations control, etc. can be formally represented as reasoning about complex large-scale control systems. The field of efficient aerospace control systems needs new technology from the science of artificial intelligence (Rodin, 1988; Lirov, Rodin et al., 1988).

The classic approach based on the theory of Differential Games alone is insufficient, especially in case of dynamic, 3D models (Garcia-Ortiz et al., 1993). Following (Rodin, 1988; Shinar, 1990) discrete-event modeling of complex control systems can be implemented as a purely interrogative simulation. These techniques can be based on generating geometrically meaningful states rather than time increments with due respect to the timeliness of actions. By discretizing time, a finite game tree can be obtained. The nodes of the tree represent the states of the game, where the players can select their controls for a given period of time. It is also possible that players do not make their decisions simultaneously and in this case, the respective moves of the two sides can be easily distinguished. Thus, the branches of the tree are the moves in the game space. The pruning of such tree is the basic task of heuristic search techniques. Interrogative approach to control problems offers much faster execution and clearer simulator definition (Lirov et al., 1988). For this kind of approach a series of hierarchical dynamic goal-oriented systems should be developed and investigated.

There are many such problems where human expert skills in reasoning about complex goal-oriented systems are incomparably higher than the level of modern computing systems. Unfortunately, problems of tactics planning and automatic control of autonomous agents such as aerospace vehicles, space stations and robots with cooperative and opposing interests are of the type where human problem-solving skills can not be directly applied. Moreover, there are no highly-skilled human experts in these fields ready to substitute for robots (on a virtual model) or transfer their knowledge to them. There is no grand-master in robot control, although, of course, the knowledge of existing experts in this field should not be neglected – it is even more valuable. Due to the special significance of these problems and the fabulous costs of mistakes, the quality of solutions must be very high and usually subject to continuous improvement.

In this respect it is very important to study human expert reasoning about similar complex systems in the areas where the results are successful, in order to discover the keys to success, and then apply and adopt these keys to the new, as yet, unsolved problems, and first and foremost to the aerospace critical complex systems. It should be considered as investigation, development, and consequent expansion of advanced human expert skills into new areas.

The question that remains then, is this: what language tools do we have for the adequate representation of human expert skills? An application of such language tools to an area of successful results achieved by a human expert should yield a *formal, domain independent knowledge* ready to be transferred to different areas. Neither natural nor programming languages satisfy our goal. The first are informal and ambiguous, while the second are usually detailed,

lower-level tools. Actually, we have to learn how we can formally represent, generate, and investigate a *mathematical model* based on the *abstract images* extracted from the expert vision of the problem.

2. BACKGROUND

The difficulties we encounter trying to find the optimal operation for real-world complex control systems are well known. While the formalization of the problem, as a rule, is not difficult, an algorithm that finds its solution usually results in the search of many variations. For small-dimensional "toy" problems a solution can be obtained; however, for most real-world problems the dimension increases and the number of variations increases significantly, usually exponentially, as a function of dimension (Garey and Johnson, 1991). Thus, most real-world search problems are not solvable with the help of exact algorithms in a reasonable amount of time. This becomes increasingly critical for the real-time aerospace autonomous and semiautonomous vehicles and robots (Lirov et al., 1988; Strosnider and Paul, 1994).

There have been many attempts to find the optimal (suboptimal) operation for real-world complex systems, in particular, for aerospace applications (Leitmann, 1990; Drabble, 1991; Pigeon et al., 1992). Basically, all the approaches for the limited time search can be broken into four categories: the imprecise computation (Chung et al., 1990), real-time search (e.g., Korf, 1990), approximate processing (Lesser et al., 1988), and anytime algorithms (Dean and Boddy, 1988). According to Strosnider and Paul (1994) the correct pruning in its many manifestations is still the only technique that reduces the worst-case execution time without compromising the goal state. But for real-world applications this reduction is usually insufficient: it does not overcome the combinatorial explosion. Another techniques, such as approximate processing, scoping, and use of domain knowledge, can reduce execution time significantly but they might compromise the goal state.

One of the basic ideas is to decrease the dimension of the real-world system following the approach of a *human expert in the field*, by breaking the system into smaller subsystems. This process of decomposition can be applied recursively until we end up with a collection of basic subproblems that can be treated (in some sense) independently. These ideas have been implemented for many problems with varying degrees of success (see, e.g., Albus, 1991; Knoblock, 1990; Mesarovich et al, 1989; Botvinnik, 1984). Implementations based on the formal theories of linear and nonlinear planning meet hard efficiency problems (McAllester and Rosenblitt, 1991; Chapman, 1987; Nilsson, 1980; Stefik, 1981; Sacerdoti, 1975). An efficient planner requires an intensive use of heuristic knowledge. Moreover, it is possible to use both dynamic and static heuristic knowledge in reducing the search variations. The dynamic knowledge can be acquired during the run time and immediately applied for search reduction (Strosnider and Paul, 1994). On the other hand, a pure heuristic implementation is unique. There is no general constructive approach to such implementations. Each new problem should be carefully studied, and previous experience usually can not be applied. Basically, we can not answer the question: what are the formal properties of the human expert heuristics that drove us to a successful hierarchy of subsystems for a given problem, and how can we apply the same ideas in a an altered or even different problem domain? Moreover, every attempt to evaluate the computational complexity and quality of a pilot solution necessitates implementing its program, which in itself is a unique task for each problem.

In the 1960's, a formal syntactic approach to the investigation of properties of natural language resulted in the fast development of a theory of formal languages by Chomsky (1963), Ginsburg (1966), and others. This development provided an interesting opportunity for dissemination of this approach to different areas. In particular, there came an idea of analogous linguistic representation of images. This idea was successfully developed into syntactic methods of pattern recognition by Fu (1982), Narasimhan (1966), and Pavlidis (1977), and picture description languages by Shaw (1969), Feder (1971), and Rosenfeld (1979).

Searching for adequate mathematical tools formalizing human heuristics of dynamic hierarchies, we have transformed the idea of linguistic representation of complex real-world and artificial images into the idea of similar representation of complex hierarchical systems (Stilman, 1985). However, the appropriate languages should possess more sophisticated attributes than languages usually used for pattern description. The origin of such languages can be traced back to the research on programmed

attribute grammars by Knuth (1968), Rozenkrantz (1969).

A mathematical environment (a "glue") for the formal implementation of this approach was developed following the theories of formal problem solving and planning by Nilsson (1980), Fikes and Nilsson (1971), Sacerdoti (1975), McCarthy (1980), McCarthy and Hayes (1969), and others based on first order predicate calculus.

In the beginning of 80's Botvinnik, Stilman, and others developed one of the most interesting and powerful heuristic hierarchical models. It was successfully applied to scheduling, planning, control, and computer chess. The hierarchical networks were introduced in (Botvinnik, 1984; Stilman, 1977) in the form of ideas, plausible discussions, and program implementations (see below). We consider this model as an ideal case for transferring the developed search heuristics to the aerospace domain employing formal linguistic tools.

An application of the developed model to a chess domain was implemented in full as program PIONEER (Botvinnik, 1984). Similar heuristic model was implemented for power equipment maintenance in a number of computer programs being used for maintenance scheduling all over the USSR (Botvinnik et al., 1983; Reznitskiy and Stilman, 1983; Stilman, 1985, 1993a). All these earlier developed programs were the direct implementations of the specific dynamic hierarchies of subsystems. The first pilot implementation of the elements of the generic hierarchy of formal languages for the 2D case was done at the University of Colorado at Denver by King (1993) and Mathews (1993) employing CLIPS tools (Giarratano, 1991) and C language, respectively.

The results shown by these programs in solving complex chess and scheduling problems indicate that implementations of the dynamic hierarchy resulted in the extremely goal-driven algorithms generating search trees with a branching factor close to 1.

In order to discover the inner properties of human expert heuristics, which have been successful in a certain class of complex control systems, we develop a formal theory, the so-called *Linguistic Geometry* (Stilman, 1993-94). This research includes the development of syntactic tools for *knowledge representation* and *reasoning* about large-scale hierarchical complex systems. It relies on the formalization of *search heuristics*, which allow one to decompose complex system into a hierarchy of subsystems, and thus solve intractable problems by reducing the search. These *hierarchical images* in the form of networks of paths were extracted from the expert vision of the problem. The hierarchy of subsystems is represented as a *hierarchy of formal attribute languages.*

3. SHORT SURVEY OF LINGUISTIC GEOMETRY

In order to pursue our objectives formally we have to define a class of problems to be studied and introduce a hierarchy of languages for decomposition of these problems.

3.1 Class of Problems

A **Complex System** is the following eight-tuple:

$< X, P, R_p, \{ON\}, v, S_i, S_t, TR>$,where

$X=\{x_i\}$ is a finite set of *points*;

$P=\{p_i\}$ is a finite set of *elements*; P is a union of two non-intersecting subsets P_1 and P_2;

$R_p(x, y)$ is a set of binary relations of *reachability* in X (x and y are from X, p from P);

ON(p)=x, where ON is a partial function of *placement* from P into X;

v is a function on P with positive integer values describing the *values* of elements.

The Complex System searches the state space, which should have initial and target states;

S_i and S_t are the descriptions of the *initial* and *target* states in the language of the first order predicate calculus, which matches with each relation a certain Well-Formed Formula (WFF). Thus, each state from S_i or S_t is described by a certain set of WFF of the form $\{ON(p_j) = x_k\}$;

TR is a set of operators, TRANSITION(p, x, y), of transitions of the System from one state to another one. These operators describe the transition in terms of two lists of WFF (to be removed from and added to the description of the state), and of WFF of applicability of the transition. Here,

Remove list: ON(p)=x, ON(q)=y;

Add list: ON(p)=y;

Applicability list: $(ON(p)=x)\wedge R_p(x,y)$, where p belongs to P_1 and q belongs to P_2 or vice versa. The transitions are carried out with participation of one or many elements p

from P_1 and P_2.

According to the definition of the set P, the elements of the System are divided into two subsets P_1 and P_2. They might be considered as units moving along the reachable points. Element p can move from point x to point y if these points are reachable, i.e., $R_p(x, y)$ holds. The current location of each element is described by the equation ON(p)=x. Thus, the description of each state of the System $\{ON(p_j) = x_k\}$ is the set of descriptions of the locations of the elements. The operator TRANSITION(p, x, y) describes the change of the state of the System caused by the move of the element p from point x to point y. The element q from point y must be withdrawn (eliminated) if p and q do not belong to the same one of the two subsets P_1 and P_2.

The problem of the optimal operation of the System is considered as a search for the optimal sequence of transitions leading from one of the initial states of S_i to a target state S of S_t.

It is easy to show formally that a robotic system can be considered as a Complex System (see below). Many different technical and human society systems (including military battlefield systems, systems of economic competition, positional games) that can be represented as twin sets of movable units (representing two or more opposing sides) and their locations can be considered as Complex Systems.

With such a problem statement for the search of the optimal sequence of transitions leading to the target state, we could use formal methods like those in the problem-solving system STRIPS (Fikes and Nilsson, 1971), nonlinear planner NOAH (Sacerdoti, 1975), or in subsequent planning systems. However, the search would have to be made in a space of a huge dimension (for nontrivial examples). Thus, in practice no solution would be obtained.

We devote ourselves to finding an approximate solution of a reformulated problem.

3.2 Geometry of Complex Systems: Measurement of Distances

To create and study a hierarchy of dynamic subsystems, we have to investigate geometrical properties of the Complex System.

A ***map of the set*** **X** relative to the point x and element p for the Complex System is the mapping: $\mathbf{MAP_{x,p}}$: X —> $\mathbf{Z_+}$ (where x is from X, p is from P), which is constructed as follows.

We consider a *family of reachability areas* from the point x, i.e., a finite set of the following nonempty subsets $\{M^k_{x,p}\}$ of X (Fig.1):

k=1: $M^k_{x,p}$ is a set of points *m reachable in one step* from x: $R_p(x,m)$=T;

k>1: $M^k_{x,p}$ is a set of points *reachable in k steps and not reachable in k-1 steps,* i.e., points *m* reachable from points of $M^{k-1}_{x,p}$ and not included in any $M^i_{x,p}$ with *i* less than *k*.

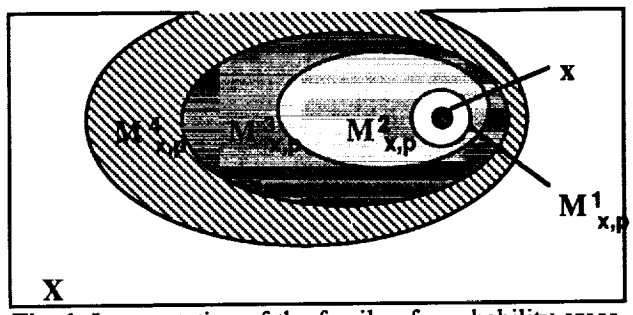


Fig. 1. Interpretation of the family of reachability areas

Let $MAP_{x,p}(y)=k$, for y from $M^k_{x,p}$ (*the number of steps from x to y*). For the remaining points, let $MAP_{x,p}(y)=2n$, if y≠x (n is the number of points in X); $MAP_{x,p}(y)=0$, if y = x.

It is easy to verify that the map of the set X for the specified element p from P defines an *asymmetric distance function* on X:

1. $MAP_{x,p}(y) > 0$ for x≠y; $MAP_{x,p}(x)=0$;
2. $MAP_{x,p}(y)+MAP_{y,p}(z) \geq MAP_{x,p}(z)$.

If R_p is a symmetric relation,

3. $MAP_{x,p}(y)=MAP_{y,p}(x)$.

In this case each of the elements p from P specifies on X its *own* ***metric.***

Various examples of measurement of distances for robotic vehicles are considered in (Stilman, 1993a, 1993c).

3.3 Set of Paths: Language of Trajectories

This language is a formal description of the set of lowest-level subsystems, the set of all paths between points of the Complex System. An element might follow a path to achieve the goal "connected with the ending point" of this path.

A ***trajectory*** for an element p of P with the beginning at x of X and the end at the y of X (x ≠ y) with a length *l* is following formal string of symbols $\boldsymbol{a}(x)$ with points of X as parameters:

$$t_0=\boldsymbol{a}(x)\boldsymbol{a}(x_1)\ldots\boldsymbol{a}(x_l),$$

where x_l = y, each successive point x_{i+1} is

reachable from the previous point x_i, i.e., $R_p(x_i, x_{i+1})$ holds for i = 0, 1,..., l–1; element p stands at the point x: ON(p)=x. We denote by $t_p(x, y, l)$ the set of all trajectories for element p, beginning at x, end at y, and with length l. $\mathcal{P}(t_0)=\{x, x_1, ..., x_l\}$ is the set of parameter values of the trajectory t_0. (To avoid confusion we should emphasize that $a(x)a(x_1)...a(x_l)$ is a formal record and does not mean anything else except what is given above.)

A ***shortest trajectory*** t of $t_p(x, y, l)$ is the trajectory of minimum length for the given beginning x, end y, and element p.

Properties of the Complex System permit us to define (in general form) and study formal grammars for generating the shortest trajectories. A general grammar and its application to generating the shortest trajectory for a aerospace robotic vehicle will be presented later.

Reasoning informally, an analogy can be set up: the shortest trajectory is analogous with a straight line segment connecting two points in a plane. An analogy to a k-element segmented line connecting these points is called an ***admissible trajectory of degree*** **k**, i.e., the trajectory that can be divided into k shortest trajectories. The admissible trajectories of degree 2 play a special role in many problems. As a rule, elements of the System should move along the shortest paths. In case of an obstacle, the element should move around this obstacle by tracing an intermediate point aside and going to and from this point to the end along the shortest trajectories. Thus, in this case, an element should move along an admissible trajectory of degree 2.

A ***Language of Trajectories*** $L_t^H(S)$ for the Complex System in a state S is the set of all the shortest and admissible (degree 2) trajectories of length less than H. Different properties of this language and generating grammars were investigated in (Stilman, 1993a).

3.4 Networks of Paths: Languages of Trajectory Networks

After defining the Language of Trajectories, we have new tools for the breakdown of our System into subsystems. According to the ideas presented in (Botvinnik, 1984), these subsystems should be various types of trajectory networks, i.e., the sets of interconnected trajectories with one singled out and called the *main trajectory*. An example of such network is shown in Fig. 2. The basic idea behind these networks is as follows. Element p_0 should move along the main trajectory $a(1)a(2)a(3)a(4)a(5)$ to reach the ending point 5 and remove the target q_4 (an opposing element). Naturally, the opposing elements should try to disturb those motions by controlling the intermediate points of the main trajectory. They should come closer to these points (to the point 4 in Fig. 2) and remove element p_0 after its arrival (at point 4). For this purpose, elements q_3 or q_2 should move along the trajectories $a(6)a(7)a(4)$ and $a(8)a(9)a(4)$, respectively, and wait (if necessary) on the next to last point (7 or 9) for the arrival of element p_0 at point 4. Similarly, element p_1 of the same side as p_0 might try to disturb the motion of q_2 by controlling point 9 along the trajectory $a(13)a(9)$. It makes sense for the opposing side to include the trajectory $a(11)a(12)a(9)$ of element q_1 to prevent this control.

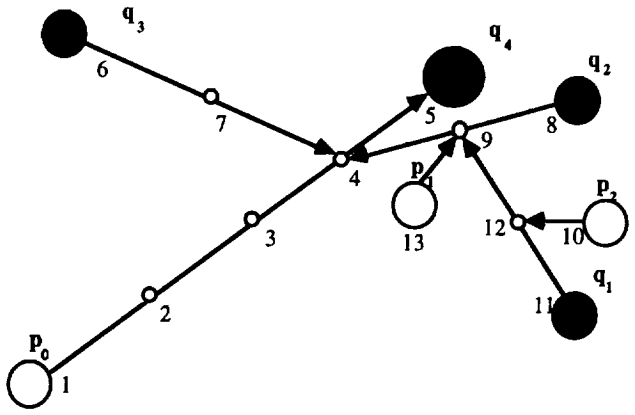


Fig. 2. Network language interpretation.

Similar networks are used for the breakdown of complex systems in different areas. Let us consider a linguistic formalization of such networks. The Language of Trajectories describes "one-dimensional" objects by joining symbols into a string employing a reachability relation $R_p(x, y)$. To describe networks, i.e., "multi-dimensional" objects made up of trajectories, we use the relation of *trajectory connection.*

A ***trajectory connection*** of the trajectories t_1 and t_2 is the relation $C(t_1,t_2)$. It holds if the ending link of the trajectory t_1 coincides with an intermediate link of the trajectory t_2; more precisely, t_1 is connected with t_2 if among the parameter values $\mathcal{P}(t_2)=\{y,y_1,...,y_l\}$ of trajectory t_2 there is a value $y_i = x_k$, where

t_1=***a***(x_0)***a***(x_1)...***a***(x_k). If t_1 belongs to a set of trajectories with the common end-point, then the entire set is said to be connected with the trajectory t_2.

For example, in Fig. 2 the trajectories ***a***(6)***a***(7)***a***(4) and ***a***(8)***a***(9)***a***(4) are connected with the main trajectory ***a***(1)***a***(2)***a***(3)***a***(4)***a***(5) through point 4. Trajectories ***a***(13)***a***(9) and ***a***(11)***a***(12)***a***(9) are connected with ***a***(8)***a***(9)***a***(4).

To formalize the trajectory networks, we define and use routine operations on the set of trajectories: $C_A{}^k(t_1,t_2)$, a ***k-th degree of connection***, and $C_A{}^+(t_1,t_2)$, a ***transitive closure***.

Trajectory ***a***(11)***a***(12)***a***(9) in Fig. 2 is connected degree 2 with trajectory ***a***(1)***a***(2)***a***(3)***a***(4)***a***(5), i.e., C^2(***a***(11)***a***(12)***a***(9), ***a***(1)***a***(2)***a***(3)***a***(4)***a***(5)) holds. Trajectory ***a***(10)***a***(12) in Fig. 2 is in transitive closure to the trajectory ***a***(1)***a***(2)***a***(3)***a***(4)***a***(5) because C^3(***a***(10)***a***(12), ***a***(1)***a***(2)***a***(3)***a***(4)***a***(5)) holds by means of the chain of trajectories ***a***(11)***a***(12)***a***(9) and ***a***(8)***a***(9)***a***(4).

A ***trajectory network*** **W** relative to trajectory t_0 is a finite set of trajectories $t_0,t_1,\ldots,t_k$ from the language $L_t{}^H(S)$ that possesses the following property: for every trajectory t_i from W (i = 1, 2,...,k) the relation $C_W{}^+(t_i,t_0)$ holds, i.e., each trajectory of the network W is connected with the trajectory t_0 that was singled out by a subset of interconnected trajectories of this network. If the relation $C_W{}^{\mathbf{m}}(t_i, t_0)$ holds, i.e., this is the **m**-th degree of connection, trajectory t_i is called the **m** ***negation trajectory***.

Obviously, the trajectories in Fig. 2 form a trajectory network relative to the main trajectory ***a***(1)***a***(2)***a***(3)***a***(4)***a***(5). We are now ready to define network languages.

A ***family of trajectory network languages*** $\mathbf{L_C(S)}$ in a state S of the Complex System is the family of languages that contains strings of the form

$$t(t_1, param)t(t_2, param)\ldots t(t_m, param),$$

where *param* in parentheses substitute for the other parameters of a particular language. All the symbols of the string $t_1, t_2,\ldots, t_m$ correspond to trajectories that form a trajectory network W relative to t_1.

Different members of this family correspond to different types of trajectory network languages, which describe particular subsystems for solving search problems. One such language is the language that describes specific networks called Zones. They play the main role in the model considered here (Botvinnik, 1984; Stilman, 1977, 1993b, 1993c, 1994a). A formal definition of this language is essentially constructive and requires showing explicitly a method for generating this language, i.e., a certain formal grammar, which is presented in (Stilman, 1993b, 1993c,1994a). In order to make our points transparent here, we define the Language of Zones informally.

A ***Language of Zones*** is a trajectory network language with strings of the form

$$Z=t(p_0,t_0,\tau_0)\ t(p_1,t_1,\tau_1)\ldots t(p_k,t_k,\tau_k),$$

where $t_0,t_1,\ldots,t_k$ are the trajectories of elements $p_0,p_1,\ldots,p_k$ respectively; $\tau_0,\tau_1,\ldots,\tau_k$ are nonnegative integers that "denote the time allotted for the motion along the trajectories" in a correspondence to the mutual goal of this Zone: to remove the target element – for one side, and to protect it – for the opposing side. Trajectory $t(p_0,t_0,\tau_0)$ is called the *main trajectory* of the Zone. The element q standing on the ending point of the main trajectory is called the *target*. The elements p_0 and q belong to the opposing sides.

To make it clearer, let us show the Zone corresponding to the trajectory network in Fig. 2.

$$Z=t(p_0,a(1)a(2)a(3)a(4)a(5),4)$$
$$t(q_3,a(6)a(7)a(4),3)$$
$$t(q_2,a(8)a(9)a(4),3)t(p_1,a(13)a(9),1)$$
$$t(q_1,a(11)a(12)a(9),2)\ t(p_2,a(10)a(12),1)$$

Assume that the goal of the white side is to remove target q_4, while the goal of the black side is to protect it. According to these goals, element p_0 starts the motion to the target, while black starts in its turn to move elements q_2 or q_3 to intercept element p_0. Actually, only those black trajectories are to be included into the Zone where the motion of the element makes sense, i. e., the *length of the trajectory is less than the amount of time (third parameter* τ*) allocated to it.* For example, the motion along the trajectories ***a***(6)***a***(7)***a***(4) and ***a***(8)***a***(9)***a***(4) makes sense, because they are of length 2 and time allocated equals 3: each of the elements has 3 time intervals to reach point 4 to intercept element p_0 assuming one would go along the main trajectory without move omission. According to definition of Zone, the trajectories of white elements (except p_0) could only be of the

length 1, e.g., $a(13)a(9)$ or $a(10)a(12)$. As element p_1 can intercept the motion of the element q_2 at the point 9, black includes into the Zone the trajectory $a(11)a(12)a(9)$ of the element q_1, which has enough time for motion to prevent this interception. The total amount of time allocated to the whole group of black trajectories connected (directly or indirectly) with the given point of the main trajectory is determined by the number of that point. For example, for the point 4, it equals 3 time intervals.

Besides Zones considered above we introduce *retreat* and *unblock* Zones. They include a target (or blocking element) with all possible trajectories of the length 1 with the beginning at the location of this element.

A language $L_Z^H(S)$ generated by the certain grammar G_Z (Stilman, 1993b, 1993c,1994a) in a state S of a Complex System is called the ***Language of Zones***.

Network languages allow us to describe the "statics", i.e., the states of the System. In order to describe the "dynamics" of the System, i.e., the motions from one state to another, we have to regenerate the entire hierarchy of languages. Of course, it is an inefficient procedure. To improve the efficiency of applications in the search process it is important to describe the change of the hierarchy of languages (Stilman, 1994a). A study of this change helped us in modifying the hierarchy instead of regenerating it in each state. This change is represented as a mapping (translation) to some other hierarchy (actually, to the new state of the same hierarchy). Thus, the functioning of the system, in a search process, generates a tree of translations of the hierarchy of languages. This tree is represented as a string of the highest level formal language, the Language of Translations (Stilman, 1994b, 1994c).

A practicality of the formal constructions considered in Section 3 as well as the entire hierarchy of languages are demonstrated on the following 3D example of the problem of strategic planning for the system of simplified space autonomous vehicles.

4. STRATEGIC PLANNING FOR SPACE COMBAT

The combat robotic model can be represented as a Complex System naturally (Section 3.1). The set X represents the operational district, which could be the area of combat operation, broken into smaller square or cubic areas, "points", e.g., in the form of the big square or cubic grid. It could be a space operation, where X represents the set of different orbits, or an air force battlefield, etc. P is the set of robots or autonomous vehicles. It is broken into two subsets P_1 and P_2 with opposing interests; $R_p(x,y)$ represent moving capabilities of different robots for different problem domains: robot p can move from point x to point y if $R_p(x, y)$ holds. Some of the robots can crawl, others can jump or ride, sail and fly, or even move from one orbit to another. Some of them move fast and can reach point y (from x) in "one step", i.e., $R_p(x, y)$ holds, others can do that in *k* steps only, and many of them can not reach this point at all. ON(p)=x, if robot p is at the point x; v(p) is the value of robot p. This value might be determined by the technical parameters of the robot. It can include the immediate value of this robot for the given combat operation; S_i is an arbitrary initial state of operation for analysis, or the starting state; S_t is the set of target states. These might be the states where robots of each side reached specified points. On the other hand, S_t can specify states where opposing robots of the highest value are destroyed. The set of WFF $\{ON(p_j) = x_k\}$ corresponds to the list of robots with their coordinates in each state. TRANSITION(p, x, y) represents the move of the robot p from the location x to location y; if a robot of the opposing side stands on y, a removal occurs, i.e., robot on y is destroyed and removed.

4.1. Problem Statement

Space robotic vehicles with different moving capabilities are shown in Fig. 3. The operational district X is the space grid of $8 \times 8 \times 8$. The total number of cubic areas n = 512. Robot W-CENTER (White Command & Control Space Center) located at 818 (x = 8, y = 1, z = 8), can move to any next location within the current orbital plane x1z, e.g., from its' current location — to 817, 717, 718. Robot B-CENTER (Black Command & Control Space Center) located at 615, can move to any next area within the same plane x1z similarly to the robot W-CENTER. Two other vehicles W-CARRIERs (White Space Buster Carriers) from 715 and 815, respectively, can move only straight ahead towards the strategic goal areas 718 and 818, one square at a time, e.g., from 715 to 716, from 716 to 717, etc. Basically, any of the cubes with the coordinates y = 1, z = 8 is desirable for these CARRIERs. Each of the

CARRIERs carries on the top an advanced W-AS-FIGHTER (White Space Fighter) which can take off from the CARRIER only in the strategic districts considered above. After take-off W-AS-FIGHTER can move in any direction, diagonally or straight forward or backward moving through several cubic areas during one time unit. The B-CARRIER at 351 is analogous to W-CARRIERs. It can move only straight ahead toward the strategic goal area 311 where the B-AS-FIGHTER, the cargo, can take off.

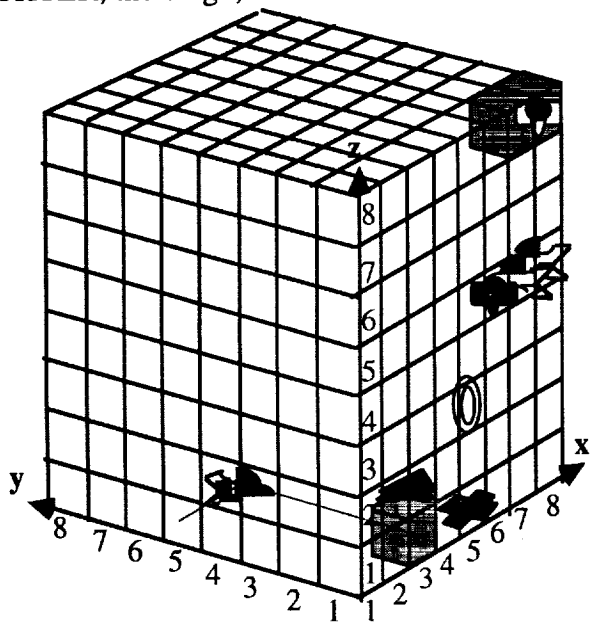


Fig. 3. A problem for space robotic vehicles.

The vehicle W-STATION (White Space Station) located at 513 can move only straight ahead one cubic area at a time, i.e., from 513 to 514, from 514 to 515; its mobility is limited by two areas, 514 and 515 only. The rest of Black vehicles are B-INTERCEPTOR (Black Interceptor) and B-SCOUT (Black Scout-Fighter). B-INTERCEPTOR located at 312 can move diagonally with several cubic areas at a time, e.g., from 312 to 114 or to 514. Finally, B-SCOUT looking for a strategic information can leap forward, backward or right or left two squares at a time, e.g., from 511 it can move to 712, 613, 413. All these vehicles can move only within the current orbital plane x1z.

Theoretically, B-SCOUT at 511 can reach any of the points $z \in \{712, 613, 413, 312, 721\}$ in one step, i.e., $R_{B\text{-SCOUT}}(511,z)$ holds, while B-INTERCEPTOR can reach $z \in \{211, 413, 514, 615, 716, 817, 411, 213, 114\}$ in one step, i.e. $R_{B\text{-INTERCEPTOR}}(312, z)$ holds. Assume that the grid is so fine that none of the vehicles can move through the cubic area where another vehicle is currently located (or stop in this area). This means that in the current state B-INTERCEPTOR actually can not move to 615, 716, 817, while B-SCOUT can not leap to 312.

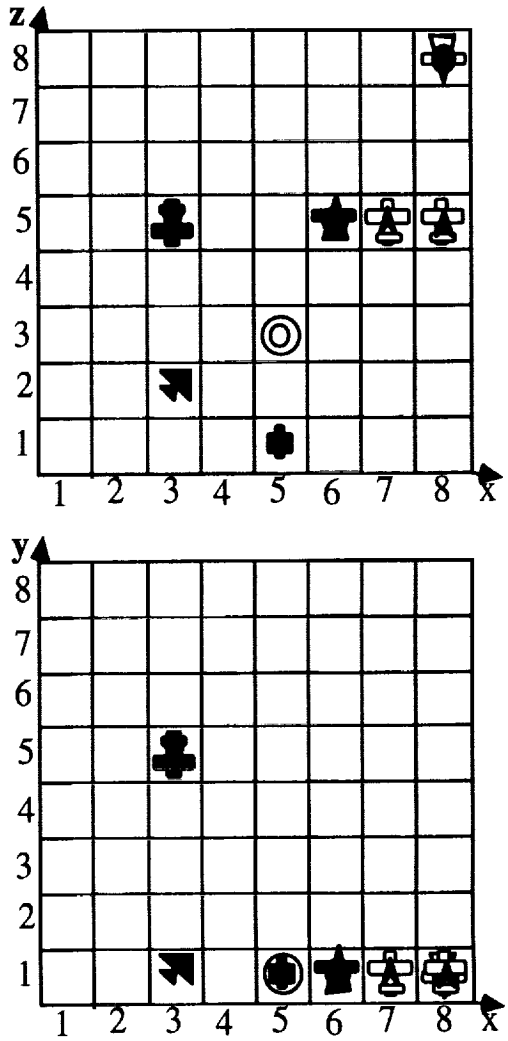


Fig. 3. A problem for space vehicles (2 projections).

Assume also that vehicles W-CENTER, W-STATION, and W-CARRIERs including, of course, their cargo, W-AS-FIGHTERs, belong to one side, while B-CENTER, B-INTERCEPTOR, B-SCOUT, and B-CARRIER with its cargo belong to the opposing side. This means that these agents belong to the sets P_1 and P_2, respectively. Each of the vehicles has powerful weapons able to destroy opposing vehicles ahead of the course, and this way move through the area where this vehicle is currently located. For example, B-CENTER from 615 can move to 815 through 715 in two steps destroying both W-CARRIERs along

the way. The only difference is with the White and Black CARRIERs and W-STATION. While routinely they can move only straight ahead (and be blocked by any of the friendly or opposing vehicles), they can destroy opposing vehicles at the next diagonal locations ahead of the course and then move to their respective areas. For example, W-CARRIER from 715 can destroy opposing B-CENTER at 616 and 816 and move to its' location. In particular, this diagonal attack ability extends the mobility of W-STATION to the areas 616 and 416 where it can hit the opposing side vehicle. Obviously, each of the opposing sides must avoid loosing a respective W(B)-CENTER which means a complete destruction of the command and control battlefield communications and immediately ends the combat in a loss to this side. On the other hand, launching a totally powerful Aerospace Fighter (AS-FIGHTER) and preventing lunch of the opposing AS-FIGHTER (or destroying it) is considered as a win. The conditions considered above give us S_t, the description of target states of the Complex System. The description of the initial state S_i is obvious and follows from Fig. 3 (xz- and yz-projections).

Assume that our time scale discretization is such that motions of the opposing sides alternate, and due to the shortage of resources (which is typical in a real combat operation) or some other reasons, each side can not participate in two missions simultaneously. It means that during the current time unit, in case of the White turn, only one of the White vehicles can move. Analogous condition holds for Black. Of course, it does not mean that if one side began participating in one of the missions, it must complete it. Any time on its turn each side can switch from one mission to another, e.g., transferring resources (fuel, weapons, human resources, etc.), and later switch back.

Similar to the real world operation it is hard to predict the result of this simplified combat. However, it seems that the locations of the W-CARRIERs are advantageous in comparison with the Black agents, B-CENTER, B-INTERCEPTOR, and B-SCOUT, while B-CARRIER is too far from the strategic area at 311. It is likely that Black can not prevent lunches of W-AS-FIGHTERs (or destroy both of them). Is there a strategy for the Black side to win or, at least, end this combat in a draw lunching its B-AS-FIGHTER on time?

Of course, this question can be answered by a direct search employing, for example, the minimax algorithm with alpha-beta cut-offs. Theoretical evaluations and experiments with computer showed that finding a solution of this problem requires generation of the search tree that includes about 30^{25} moves (transitions). Of course, this is beyond reasonable time constraints of the most advanced modern computers. It is very interesting to observe the dramatic reduction of search employing the Linguistic Geometry tools.

In order to demonstrate generation of the Hierarchy of Languages for this problem, we have to generate the Language of Trajectories and the Language of Zones in each state of the search. The details of generation of trajectories and Zones for 2D and 3D problems are considered in (Stilman, 1993b, 1993c, 1993d, 1994b, 1994c).

4.2. Search Generation for Space Combat

Consider how the hierarchy of languages works for the optimal control of the Space Combat System introduced above (Fig. 3). We generate the string of the Language of Translations (Stilman, 1994a) representing it as a conventional search tree (Fig.1) and comment on its generation.

In our comments on this generation we will emphasize the major steps and skip some technical details considered, e.g., in (Stilman, 1994c).

First, the Language of Zones in the start state is generated. Every element tries to attack every element of the opposing side. The targets for attack are determined within the limit of four steps, the horizon. This is a "view range" of this problem. It means that horizon H of the language $L_Z(S)$ is equal to 4, i.e., the length of the main trajectories of all Zones must not exceed 4 steps. The reasons and the algorithm for choosing the right value of the horizon are considered in (Stilman, 1994c). One of the Zones for W-CARRIER at 715, Z_{WC} is shown in Fig.1. In formal notation this Zone is as follows:

Z_{WC}=

t(W-CARRIER,*a*(715)*a*(716)*a*(717)*a*(718),4)
t(B-CENTER, *a*(615)*a*(716), 2)
t(B-CENTER, *a*(615)*a*(616)*a*(717), 3)
t(B-CENTER, *a*(615)*a*(616)*a*(617)*a*(718), 4)
t(B-INTERCEPTOR,*a*(312)*a*(817)*a*(718), 4)
t(W-CARRIER, *a*(815)*a*(715), 1)
t(W-CENTER, *a*(818)*a*(718), 1)
t(W-CENTER, *a*(818)*a*(717), 1)
t(W-CENTER, *a*(818)*a*(817), 1)

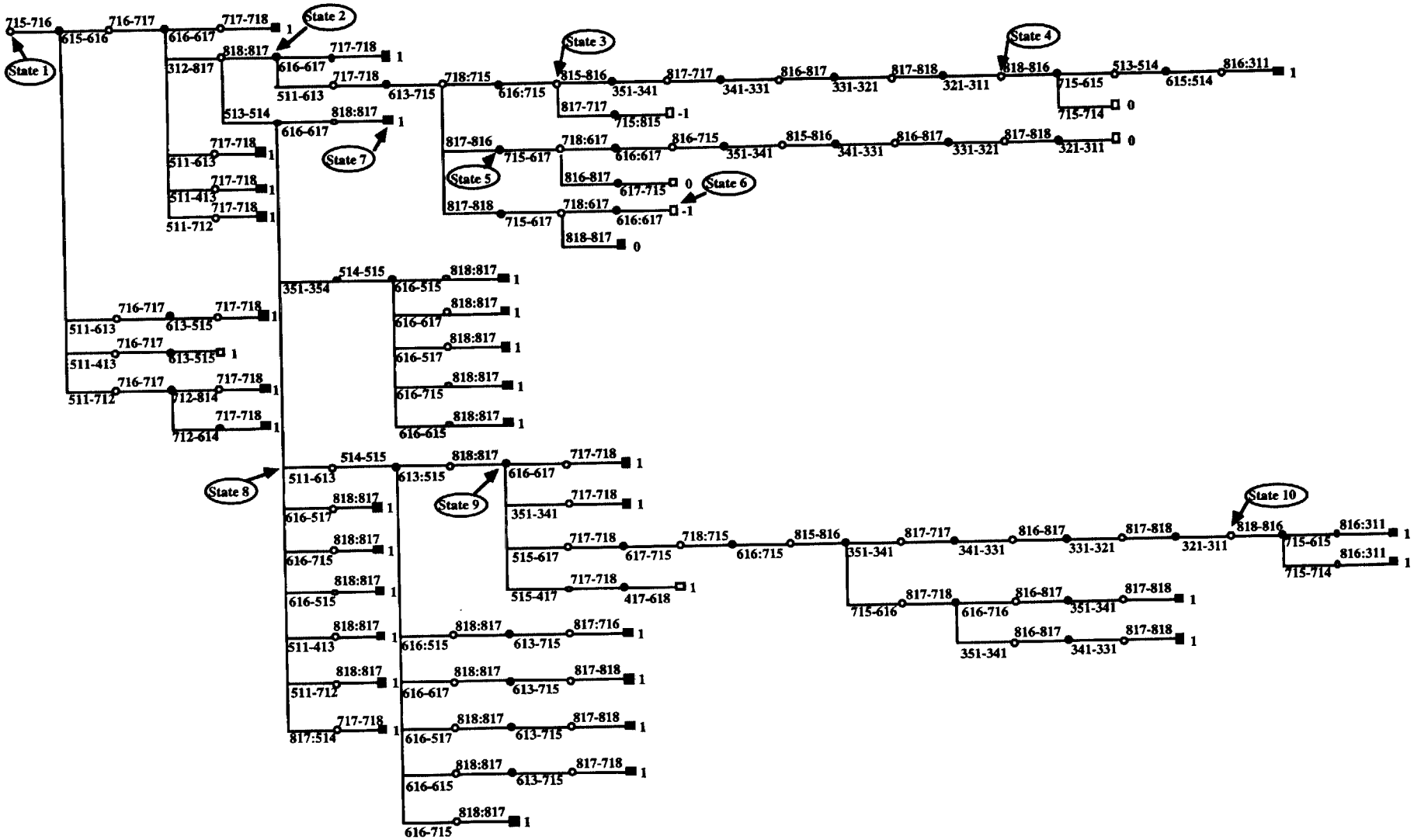


Fig. 4. Search tree for the optimization problem for space vehicles within the horizon 4.

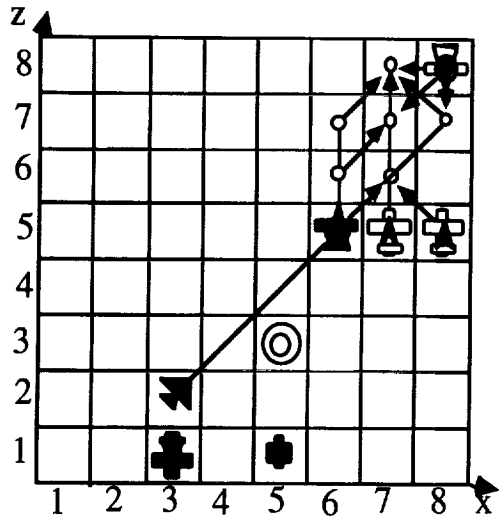


Fig. 5. State 1

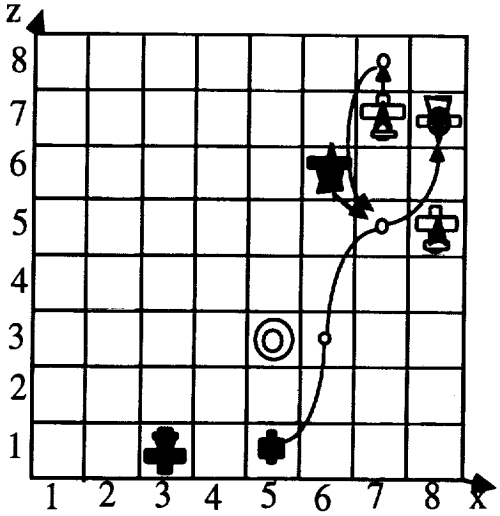
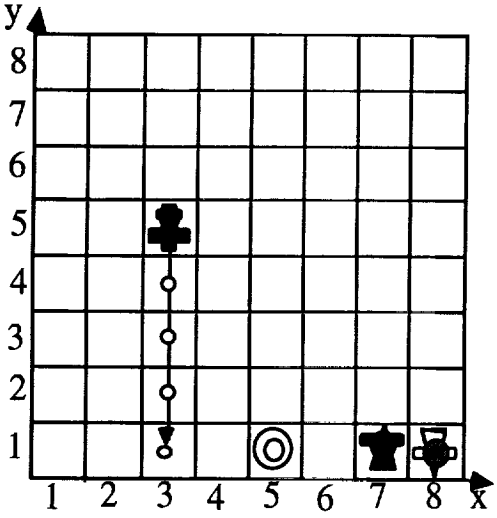


Fig. 6. State 2



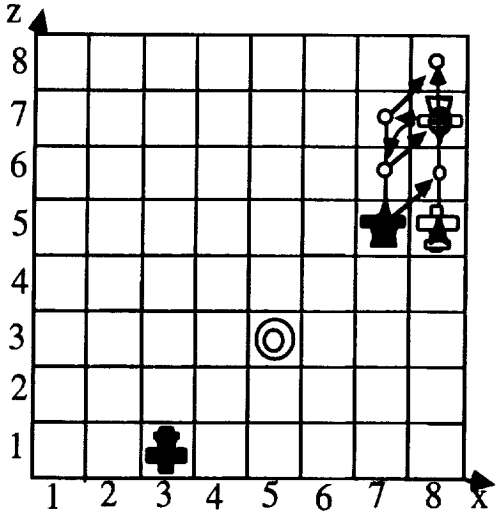


Fig. 7. State 3 (xy- and xz-projections)

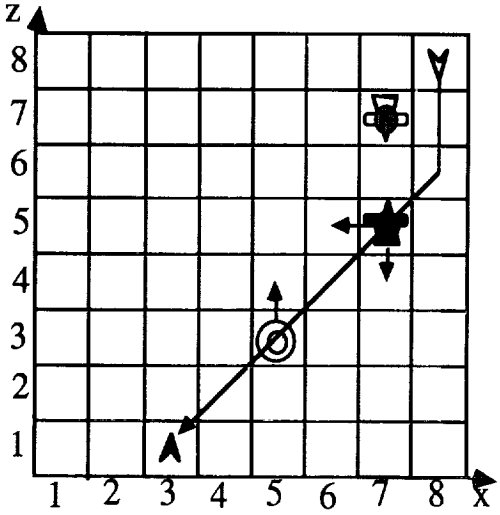


Fig. 8. State 4

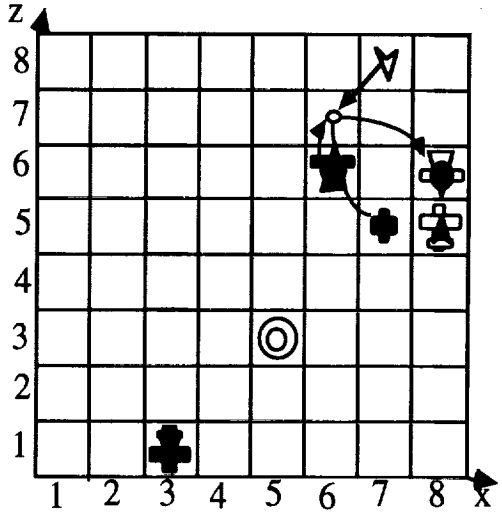


Fig. 9. State 5

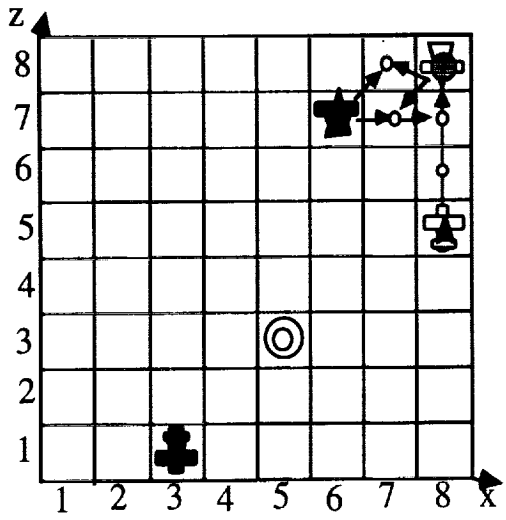


Fig. 10. State 6

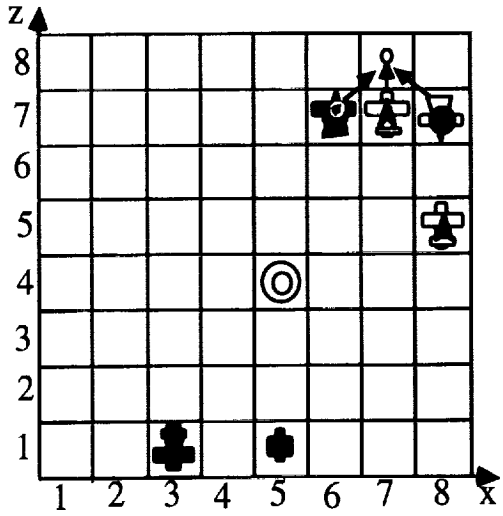


Fig. 11. State 7

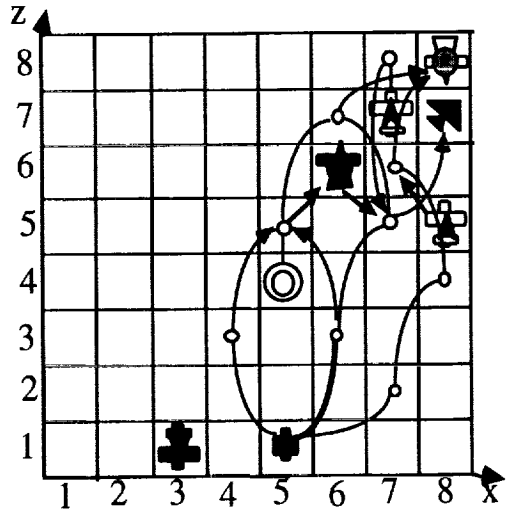


Fig. 12. State 8

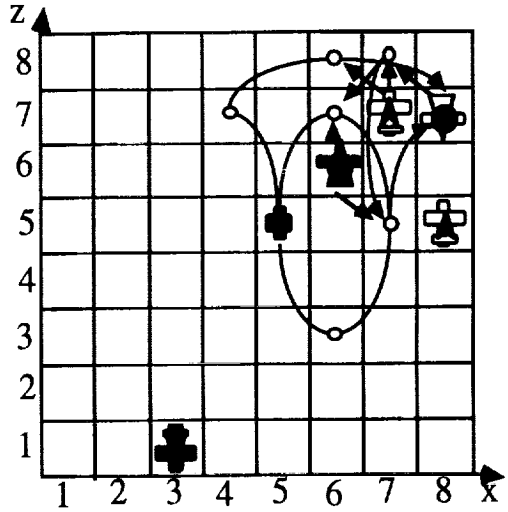


Fig. 13. State 9

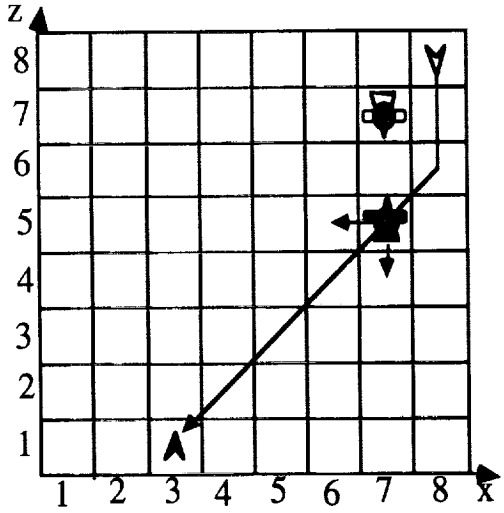


Fig. 14. State 10

Search tree generation (Fig. 4) begins with the move 1. 715-716 in the most traversable White Zone with the vulnerable target of the highest value. This Zone Z_{WC} of W-CARRIER is shown in Fig. 5 (yz-projection). The order of consideration of Zones and particular trajectories is determined by the Grammar of Translations. The computation of move-ordering constraints is the most sophisticated procedure in the Grammar of Translations. It takes into account different parameters of Zones, trajectories, and the so-called chains of trajectories. We should keep in mind that after each move the model moves to the new current state S_c, so the entire Language of Zones, $L_Z(S_c)$, must be regenerated. With respect to efficiency of the model it was very important to solve a technical problem relative to the well known Frame Problem (McCarthy and Hayes, 1969; Fikes and Nilsson, 1971; Nilsson, 1980). This allowed us to avoid recomputation of the entire language recomputing only the changing part. An approach to the formal solution of this problem is considered in (Stilman, 1994a).

The next move, 1. ... 615-616, is in the same Zone along the first negation trajectory. B-CENTER is trying to intercept motion of the W-CARRIER at 717 or 718. The interception continues: 2. 716-717 616-617 3. 717-718. Interception failed and here the grammar terminates this branch with the value of 1 (as a win of the White side). This value is given by the special state evaluation procedure built into the grammar. This procedure evaluated this state as a winning state for the White after analysis of the "traversability" of all the Zones active in this state. In particular, it figured out that the exchange at 718: 3. ... 617:718 4. 818:718 would destroy B-CENTER and, thus, it is unacceptable for Black. (Here and in the search tree symbol ":" means the removal of an element.) Moreover, the safe arrival of W-CARRIER at the strategic area 718 would cause the lunch of W-AS-FIGHTER ending the combat in a win for the White side. Also, the analysis of the Black Zones showed that Black have nothing to oppose.

The grammar initiates the backtracking climb. After the climb up to the move 2. ... 616-617 different intercepting trajectory in the same Zone (Fig. 5) has been activated $a(312)a(817)a(718)$: 2. ... 312-817. After the arrival at 817 B-FIGHTER has been destroyed by W-CENTER, and the following interception failed: 3. 818:817 616-617 4. 717-718.

The backtracking climb up to the move 3. 818:817 is interrupted at the State 2 shown in Fig. 6. This is the state where the new attacking Zone of B-SCOUT from 511 to 817 has been registered when we visited this state earlier during descent. This information has been stored to be brought to the upper levels of the search tree; the grammar stores these newly generated Zones as idle for possible activation in different states. Each backtracking move is followed by the inspection procedure, the analysis of the subtree generated in the process of the earlier search. After the climb up to the State 2 (Fig. 6), the tree to be analyzed consists of the only branch: 3. ... 616-617 4. 717-718. The inspection procedure determined that the current minimax value (+1) can be "improved" (in favor of the Black side) by destroying the new target at 817, the W-CENTER. This target was staying at 817 in the analyzed subtree. The improvement can be achieved by participation of W-SCOUT from 511, i.e., by inclusion of the currently idle attack Zone with the main trajectory from 511 to 817 (Fig. 6).

The motion of B-SCOUT along the main trajectory $a(511)a(613)a(715)a(817)$ is accompanied by the motion of intercepting element, initially as W-CARRIER, then from 718 as W-AS-FIGHTER 3. ... 511-613 4. 717-718 613-715 5. 718:715 616:715. Thus, W-SCOUT is intercepted but the newly lunched W-AS-FIGHTER is destroyed also. The current state, State 3, is shown in Fig. 7. In this state the state evaluation procedure could not generate a definite value in favor of either side because two attack Zones for W-CARRIER at 815 and B-CARRIER at 351 are traversable (Fig. 7). Both Zones are activated: 6. 815-816 351-341.

Now the unblock Zone of W-CENTER should be activated in order to free the motion of W-CARRIER through 817. The exact location for the unblock, 7. 817-717, is chosen in order to keep protected the most of the squares of the main trajectory: 816, 817, and 818. The race of CARRIERs continues: 7. ... 341-331 8. 816-817 331-321 9. 817-818 321-311. Both White and Black AS-FIGHTERs are ready be lunched, and the state evaluation procedure still can not terminate the branch. The current state, State 4, is shown in Fig. 8.

Among different attack Zones for W-AS-FIGHTER the Zone with the main trajectory $a(818)a(816)a(311)$ is chosen. This is a traversable "time gaining" trajectory attacking two targets simultaneously, B-CENTER at 715 and B-AS-FIGHTER at 311. After 10. 818-816 the

retreat Zone of W-CENTER at 715 is activated. With two possible safe areas for retreat, 714 and 715, the wrong one is chosen first: 10. ... 715-615. New attack Zone of W-STATION $a(513)a(514)a(615)$ is activated immediately because it is the time-gaining unblock trajectory as well: 11. 513-514. This motion of W-STATION actually gained time. W-CENTER has been engaged and it must respond either destroying W-STATION or retreating, and, thus, losing a time interval and passing a move turn to the White side. W-AS-FIGHTER immediately attacks B-AS-FIGHTER along the trajectory just being unblocked: 11. ... 615:514 12. 816:311. The state evaluation procedure terminates the branch and evaluates as +1 in favor of White. The following backtracking climb up to the move 10. 818-616 where the retreat Zone of B-CENTER is activated again. Now the right area for retreat is chosen 10. ... 715-714. In absence of the vulnerable or time-gaining threats from either side the branch is terminated in a draw (0). The "guilty party" for this draw value is W-STATION at 513. The unblock Zone registered in this terminal state as idle is stored to be activated at the upper levels of the search tree.

It seems that our preliminary estimate about easy win of the White side was incorrect. With the precise planning Black forced a draw in the variations analyzed so far. Let us continue the tree generation.

The grammar initiates the backtracking climb up to the State 3 (Fig. 7). Now when we propagate the draw value as an optimum White is changing moves looking for a win. An attempt of the earlier activation of the W-CARRIER unblock Zone fails because White lose the last W-CARRIER with its valuable cargo: 6. 817-717 715:815. The optimum value is still a draw. The climb continues and move 5. 718:715 with B-SCOUT removal (while W-CENTER is under direct threat) is changed for W-CENTER retreat 5. 817-816. The current State 5 is shown in Fig. 9. A new Zone of B-SCOUT with the main trajectory $a(715)a(617)a(816)$ is immediately activated (Fig. 9): 5. ... 715-617 6. 718-617 616:617. B-SCOUT at 617 is intercepted by W-AS-FIGHTER while W-AS-FIGHTER itself is destroyed by B-CENTER. The state evaluation procedure does not generate a definite value in favor of either side and branch generation continues. The following branch is quite similar to the previous long branch which includes the race of W-CARRIER from 815 and B-CARRIER from 351. The difference is that in this variation W-CENTER unblocks the main trajectory from 816 to 715: 7. 816-715, and stays there while B-CENTER is at 617 all the time (compare with Fig. 8). These new locations of White and Black CENTERs result in a draw after the arrival of both CARRIERs at the respective strategic locations, 818 and 311. The state evaluation procedure does not register vulnerable time-gaining threats and terminates this branch.

The grammar initiates the backtracking climb up to the move 5. ... 715-617. In this state the tree inspection procedure activates the W-CENTER retreat Zone from 816 changing B-SCOUT interception 6. 718:617 for the only W-CENTER retreat 6. ... 816-817. The new attack Zone of B-SCOUT with the main trajectory $a(617)a(715)a(817)$ is activated: 7. 617-715. Here the state evaluation procedure registered state repetition in the current branch (compare with the state before State 5 shown in Fig. 9), and terminated the branch with the draw value (0).

The following climb is interrupted in the state after 5. ... 613-715, and W-CENTER retreat move 6. 817-816 is changed for the last possible retreat: 6. 817-818. The new B-SCOUT attack Zone is immediately activated via $a(715)a(617)a(818)$. The intercepting trajectories are similar to the Zone shown in Fig. 9. The following variation 6. ... 715-617 7. 718:617 616:617 is terminated in the state, State 6, shown in Fig. 10. The state evaluation procedure detected that the Zone for W-CARRIER at 815 is non-traversable (because the unblocking of W-CENTER is impossible) while B-CARRIER Zone from 351 is traversable, and evaluated this state as (-1) in favor of Black. The following climb and change of 7. 718:617 for W-CENTER retreat 7. 818-817 results in the state which has already occurred in the search tree and was evaluated as a draw (0).

The backtracking climb continues propagating the value of 0 (a draw) as a minimax value of the currently generated subtree. The climb stops at the move 3. 818:817, which is changed for 3. 513-514. The tree inspection procedure has chosen this move as a move of a very high preference. This is the first time when new Zone of W-STATION at 513 with the main trajectory $a(513)a(514)a(515)a(616)$ is activated. In the backtracking climb B-CENTER returned to 616, and now White could attack this target within the horizon 4. (The actual length of the main trajectory is 3 steps.) Moreover, this is a time-gaining

motion because this is the motion in the unblock Zone of W-STATION. This Zone registered in the bottom of the search tree (Fig. 8), has been idle for a long time, and now is activated as well.

The following motion continues in the Zone of W-CARRIER with participation of the intercepting and protecting elements, B-CENTER and W-CENTER: 3. ... 616-617 4. 818:817. This state is shown in Fig. 11 (State 7). It is evaluated in favor of White (+1), and the branch is terminated. From now on the current minimax value of the subtree generated so far is a win for White. Now Black try to branch. After the climb Black side activates the attack Zone of B-CARRIER at 351, while W-STATION continues attack of B-CENTER: 3. ... 616-617 4. 514-515. In response, Black explore the destruction of the attacker and all possible retreats. In all these cases White continue 5. 818:817 and these branches terminated with the value in favor of White.

After multiple descents and ascents the grammar returns to the State 8 shown in Fig. 12. The tree inspection procedure activates motion of B-SCOUT along the intercepting trajectory $a(511)a(613)a(515)$ (Fig. 12). This trajectory is of high preference because it partly coincides with the main trajectories of two different Zones: $a(511)a(613)a(715)a(617)a(818)$ or $a(511)a(613)a(715)a(617)a(818)$ with W-CENTER as a target. Moreover, this motion is also the motion along the main trajectory in the control Zone $a(511)a(613)a(715)a(817)$ with the square 817 as a location of the future target, W-CENTER, whose arrival is expected by the tree inspection procedure. As usual, this control Zone was registered in the bottom of the search tree and kept idle until now. Thus 3. ... 511-613 should be considered as a highly time-gaining move. The State 9 generated after 3. ... 511-613 4. 514-515 613:515 5. 818:817 is shown in Fig. 13.

After the futile attempts to continue interception of W-CARRIER by W-CENTER or attack by B-CARRIER, the grammar returns to the State 9. At this moment the tree inspection procedure activates new attack Zone of B-SCOUT from 515 to 817. Among the bundle of such Zones (Fig. 13) the Zone with the most traversable main trajectory $a(515)a(617)a(715)a(817)$ is picked up. After 5. ... 515-617 6. 717-718 617-715 7. 718:715 616:715, the state is exactly the same as State 3 (Fig. 7) generated earlier in the search tree. The only difference is that in the current state there is no W-STATION at 513. As we know the minimax value for the State 3 propagated from the bottom of the search subtree was a draw (0). So, it seems that Black which is currently looking for this value have found one. This, probably, means that after 3. 513-514, Black eventually have found the right variation leading to a draw. But, because of the different location of W-STATION mentioned above we can not just consider this state as the state visited before, terminate this branch, and assign the value. Analogously to the State 3 (on descent), the state evaluation procedure can not assign a definite value to this state, so the branch continues. All the following moves, the CARRIERs race, are exactly the same as in the earlier branch generated from the State 3. The race is complete when both CARRIERs have reached their respective strategic areas. The corresponding State 10 is shown in Fig. 14. The only difference of this state with the State 4 (Fig. 8) is the absence of W-STATION at 513. But this tiny change makes big difference. The motion of W-AS-FIGHTER along the time-gaining trajectory $a(818)a(816)a(311)$ is a simultaneous immediate attack of both B-CENTER and B-AS-FIGHTER. This means that at least one of the targets will be destroyed. The continuation is as follows: 12. 818-816 715-615 (or 12. ... 715-714) 13. 816:311. In both variations W-AS-FIGHTER is destroyed and they are terminated with the value (+1) in favor of White. Thus, despite of this long 25-move(!) resistance, Black achieved nothing. The current minimax value is still in favor of White.

The following climb and branching when Black tries, e.g., most efficiently activate the retreat Zone of B-CENTER from 715 at the upper levels of the search tree or explore different B-SCOUT attack trajectories from 511, does not change the minimax value. The following tree generation does not even yield a "better" (longer) resistance variation than the best variation generated so far. Basically, this longest variation is the optimal variation which is likely to be followed by both sides in the actual combat. In order to generate this branch the grammar used the information, the key networks (W-STATION retreat Zone) learned at the bottom of the search tree in the previously generated non-optimal branches.

The search tree generated by the grammar consists of 152 moves. Obviously, this is a dramatic reduction in comparison with billion-move trees generated by conventional search procedures and still insufficient for solving this problem.

5. DISCUSSION

The example considered in this paper demonstrates the power of the Linguistic Geometry tools that allowed to transfer heuristics discovered in the 2D problem domain of positional games, to another domain of simplified aerospace robotic vehicles. The conventional approaches employing search algorithms with alpha-beta pruning require approximately 30^{25} move search tree to solve this problem, while the tree presented in this paper consists of 152 moves. Moreover, the branching factor of this search, i.e., the average number of moves in each node, is about 1.12(!) while the depth of the search required to solve this problem must be at least 25 moves. This means that the algorithm is actually goal-oriented, i.e., it approaches the goal almost without branching to different directions. Looking at the complexity of the hierarchy of languages which represents each state in the search process, we can suppose that the growth from the problems with the lesser number of agents with limited moving capabilities and smaller 2D operational district (Stilman, 1994b, 1994c) to the current essentially more complex problem is linear with the factor close to one. This means that the complexity of the entire algorithm may be about linear with respect to the length of the input.

At the same time the simplified aerospace navigation problem considered here is still very close to the original chess domain. It is possible to predict that the power of Linguistic Geometry goes far beyond these limits. The definition of the Complex System (Section 3.1) is generic enough to cover a variety of different problem domains. The core component of this definition is the triple X, P, and R_p. Thus, looking at the new problem domain we have to define X, the finite set of points – locations of elements. We do not impose any constraints on this set while the operational district X considered in this paper as well as the original chess board have different extra features, e.g., 2D or 3D connectivity, which is totally unimportant for these problems. Thus, for example, we can consider X as a set of orbits where the elements are in permanent motion with respect to each other. The moving capabilities of elements P in our example, i.e., the binary relations R_p, are non-sophisticated. This is exactly the place for introduction of the variable speed, the gravity impact, the engine impulse duration, etc.

Also, it should be noted that in example considered in this paper we introduced some additional constraints for the Complex System. These are requirements of the motion alternation for the opposing sides and participation of the only element in each motion. This introduction was done only for a transparent display of ideas and advantages of Linguistic Geometry. The generic definition of the Complex System (Section 3.1) does not include these constraints. The examples where the constraints of single element motion have been relaxed are considered in (Stilman, 1995).

References

Albus, J. (1991). Outline for a Theory of Intelligence. *IEEE Trans. on Systems, Man and Cybernetics* (pp. 473-509), 3.

Boddy, M. and Dean, T. (1989). Solving Time-Dependent Planning Problems, *Proc. of the 11th Int. Joint Conf. on AI*, (pp. 979-984).

Botvinnik, M.M. (1984). *Computers in Chess: Solving Inexact Search Problems*. Springer Series in Symbolic Computation, New York: Springer-Verlag.

Botvinnik, M., Petriyev, E., Reznitskiy, A., et al. (1983). Application of New Method for Solving Search Problems For Power Equipment Maintenance Scheduling. *Economics and Mathematical Methods* (pp. 1030-1041), 6, (in Russian).

Chapman, D. (1987). Planning for conjunctive goals. *Artificial Intelligence* 32(3).

Chomsky, N. (1963). Formal Properties of Grammars. in *Handbook of Mathematical Psychology*, eds. R.Luce, R.Bush, E. Galanter., vol. 2 (pp. 323-418). New York: John Wiley & Sons.

Chung, J., Liu, J. and Lin, K., (1990). Scheduling Periodic Jobs That Allow Imprecise Results. *IEEE Transactions on Computers*, (pp. 1156-1174), 39(9).

Drabble, B., (1991) Spacecraft Command and Control Using Artificial Intelligence Techniques, *J. of the British Interplanetary Society*, Vol. 44, (251-254).

Durfee, E.H. and Lesser, V.R. (1987). Incremental Planning to Control a Blackboard-Based Problem Solver. *Proceedings of the Workshop on Space Telerobotics*, Vol. 3, NASA Jet Propulsion Lab, July, (pp. 91-99).

Feder, J. (1971). Plex languages. *Information Sciences*, 3: 225–241.

Fikes, R.E. and Nilsson, N.J. (1971). STRIPS: A New Approach to the Application of Theorem Proving in Problem Solving. *Artificial Intelligence* 2: 189–208.

Fu, K.S. (1982). *Syntactic Pattern Recognition and Applications*, Prentice Hall, Englewood Cliffs.

Garcia-Ortiz, A. et al. (1993). Application of Semantic Control to a Class of Pursue-Evader Problems, *Computers and Mathematics with Applications*, 26(5), (pp. 97-124).

Garey, M.R. and D.S.Johnson D.S. (1991). *Computers and Intractability: A Guide to the Theory of NP-Completeness*. W.H. Freeman and Co., San Francisco.

Giarratano, J.C., (1991). CLIPS User's Guide, NASA Johnson Space Center, Information Systems Directorate (JSC-25013).

Ginsburg, S. (1966). *The Mathematical Theory of Context-Free Languages*, McGraw Hill, New York.

King, R. D. (1993). Rule Based Approach to Hierarchical Grammars for Geometrical Reasoning, Master Thesis, Dept. of Computer Sci., Univ. of Colorado at Denver.

Knoblock, C.A. (1990). Learning Abstraction Hierarchies for Problem Solving, *Proc. of the 8th AAAI Conf.*, (pp.923-928), Menlo Park, CA.

Knuth, D.E. (1968). Semantics of Context-Free Languages. *Mathematical Systems Theory*, (pp. 127–146), 2.

Korf, R.E. (1990). Real-Time Heuristic Search, *Artificial Intelligence*, (pp. 189-211), 42(2-3).

Leitmann, G., (1990). *Optimization Techniques with Applications to Aerospace Systems*, Academic Press.

Lesser, V.R., Pavlin, J., and Durfee, E. (1988). Approximate Processing in Real-Time Problem Solving, *AI Magazine*, (pp. 49-62), 9(1).

Lirov Y., Rodin, E.Y., McElhaney, B.G., and Wilbur, L.W. (1988). Artificial Intelligence Modeling of Control Systems, *Simulation*, (pp. 12-24), 50(1).

Mathews, E. R. (1993). An Implementation of the Grammars of Trajectories and Zones in C Language, Master Thesis, Dept. of Mathematics, Univ. of Colorado at Denver.

McAllester, D. and Rosenblitt, D. (1991). Systematic Non-Linear Planning, *Proc. of AAAI-91*, (pp. 634-639).

McCarthy, J. (1980). Circumscription–A Form of Non-Monotonic Reasoning. *Artificial Intelligence*,(27-39), 13.

McCarthy, J. and Hayes, P.J. (1969). Some Philosophical Problems from the Standpoint of Artificial Intelligence. *Machine Intelligence* (pp. 463–502), 4.

Mesarovich, M.D. and Takahara Y. (1989). *Abstract Systems Theory*, Berlin: Springer-Verlag.

Narasimhan, R.N. (1966). Syntax–Directed Interpretation of Classes of Pictures. *Comm. of ACM* (pp. 166–173), 9.

Nilsson, N.J. (1980). *Principles of Artificial Intelligence*, Palo Alto, CA: Tioga Publ.

Pavlidis, T. (1977). *Structural Pattern Recognition*, New York: Springer-Verlag.

Pigeon, A., Howard, G., and Seaton B. (1992) Operational Aspects of Space craft Autonomy, *J. of the British Interplanetary Society*, Vol. 45, (pp. 87-92).

Reznitskiy, A.I. and Stilman, B. (1983). Use of Method PIONEER in Automating the Planning of Maintenance of Power-Generating Equipment. *Automatics and Remote Control* (pp. 147-153), 11, (in Russian).

Rodin E. (1988). Semantic Control Theory, *Applied Mathematical Letters*, (pp. 73-78), 1(1).

Rosenfeld, A. (1979). *Picture Languages, Formal Models for Picture Recognition*, Academic Press.

Rozenkrantz, D.J. (1969). Programmed Grammars and Classes of Formal Languages, *J. of the ACM* (pp. 107–131), 1.

Sacerdoti, E.D. (1975). The Nonlinear Nature of Plans, *Proc. Int. Joint Conference on Artificial Intelligence.*

Shaw, A.C. (1969). A Formal Picture Description Scheme as a Basis for Picture Processing System, *Information and Control* (pp.9-52), 19.

Shinar, J., (1990). Analysis of Dynamic Conflicts by Techniques of Artificial Intelligence, INRIA Report, Antipolis.

Simon, H.A. (1980). *The Sciences of the Artificial*, 2-nd ed., The MIT Press, Cambridge, MA.

Stefik, M. (1981). Planning and meta-planning (MOLGEN: Part 2),"*Artificial Intelligence* (pp. 141-169), 2.

Stilman, B. (1977). The Computer Learns. in Levy, D., *1976 US Computer Chess Championship* (pp. 83-90). Computer Science Press, Woodland Hills, CA.

Stilman, B. (1985). Hierarchy of Formal Grammars for Solving Search Problems. In *Artificial Intelligence. Results and Prospects, Proceedings of the International Workshop* (pp. 63-72), Moscow, (in Russian).

Stilman, B. (1993a). A Linguistic Approach to Geometric Reasoning," *Int. J. Computers and Mathematics with Applications* (pp. 29-57), 26(7).

Stilman, B. (1993b). Network Languages for Complex Systems, *Int. J. Computers and Mathematics with Applications* (pp. 51-79), 26(8).

Stilman, B. (1993c). Syntactic Hierarchy for Robotic Systems, *Integrated Computer-Aided Engineering* (pp. 57-81), 1(1).

Stilman, B. (1993d). A Formal Language for Hierarchical Systems Control, *Languages of Design* (333-356), 1(4).

Stilman, B. (1994a). Translations of Network Languages. *Int. J. Computers and Mathematics with Applications* (pp. 65-98), 27(2).

Stilman, B. (1994b). A Formal Model for Heuristic Search. *Proc. of the 22nd Annual ACM Computer Science Conf.*, (pp. 380-389), March 8-10, Phoenix, AZ.

Stilman, B., (1994c). A Linguistic Geometry for Space Applications, *Proc. of the 1994 Goddard Conference on Space Applications of Artificial Intelligence*, (pp. 87-101), NASA Goddard Space Flight Center, Greenbelt, MD, USA, May 1994.

Stilman, B., (1995). Multiagent Air Combat with Synchronous Motions, Symposium on Linguistic Geometry and Semantic Control, *Proc. of the First World Congress on Intelligent Manufacturing: Processes and Systems*, Mayaguez, Puerto Rico, Feb. 1995, (to appear).

Strosnider, J.K., Paul, C.J. (1994). A Structured View of Real-Time Problem Solving, *AI Magazine*,(45-66),15(2).