

# A PC-Based Magnetometer-Only Attitude and Rate Determination System for Gyroless Spacecraft\*

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## Abstract

This paper describes a prototype PC-based system that uses measurements from a three-axis magnetometer (TAM) to estimate the state (three-axis attitude and rates) of a spacecraft given no a priori information other than the mass properties. The system uses two algorithms that estimate the spacecraft's state—a deterministic magnetic-field only algorithm and a Kalman filter for gyroless spacecraft. The algorithms are combined by invoking the deterministic algorithm to generate the spacecraft state at epoch using a small batch of data and then using this deterministic epoch solution as the initial condition for the Kalman filter during the production run. System input comprises processed data that includes TAM and reference magnetic field data. Additional information, such as control system data and measurements from line-of-sight sensors, can be input to the system if available. Test results are presented using in-flight data from two three-axis stabilized spacecraft: Solar, Anomalous, and Magnetospheric Particle Explorer (SAMPEX) (gyroless, Sun-pointing) and Earth Radiation Budget Satellite (ERBS) (gyro-based, Earth-pointing). The results show that, using as little as 700 sec of data, the system is capable of accuracies of 1.5 deg in attitude and 0.01 deg/sec in rates; i.e., within SAMPEX mission requirements.

## 1 Introduction

The coarseness of the attitude information derived from the Earth's magnetic field,  $\vec{B}$ , limits the usefulness of magnetometers in attitude determination systems. However, magnetic field measurements offer two advantages: (1) measurements can be made at any time, regardless of the spacecraft's orientation, and (2)  $\vec{B}$  usually changes direction rapidly enough to make computation of its time derivative possible. These changes made during the orbit are large enough to enable determination of all three Euler angles using only a three-axis magnetometer (TAM). The second feature suggests that the spacecraft's rates can be computed, in principle, by examining the time derivatives of  $\vec{B}$ . These advantages have prompted us to study contingency attitude algorithms which can use only TAM measurements.

It should be emphasized that the problem is nontrivial: we want to reliably estimate both attitude *and* rates of the spacecraft using *only* TAM measurements and *no* a priori information. A successful algorithm can then accommodate a Sun-sensor failure on a gyroless spacecraft such as the Solar, Anomalous, and Magnetospheric Particles Explorer (SAMPEX), as well as for a gyro-based spacecraft such as the Earth Radiation Budget Satellite (ERBS) when the gyros are not functional. In fact, our work is partly motivated by a control anomaly on ERBS (Kronenwetter et al., 1988) during a thruster-induced yaw maneuver that resulted in the spacecraft tumbling with rates of more than 2 deg/sec. As a result, both Sun and Earth sensor readings became unreliable and the gyro output was saturated. Similarly, control of the Relay Mirror Experiment (RME) satellite was lost after the failure of the Earth sensors (Natanson, 1992). In both the ERBS and RME cases, a TAM became the only functional attitude instrument. In addition to such contingencies, a TAM-only algorithm can also be of use during Sun/Earth acquisition wherein the attitude and rate estimates will enable a more efficient maneuver.

We present here a combined computational scheme invoking two different algorithms, deterministic attitude determination from magnetometer-only data (DADMOD) and the real-time sequential filter (RTSF), both of which have been successful in TAM-only situations. The DADMOD (Natanson et al., 1990 and 1991; Natanson, 1992) is an algorithm that relates the time derivatives of  $\vec{B}$  in inertial and spacecraft body coordinates to determine the attitude and the body rates. The DADMOD has been successfully tested for ERBS under normal conditions, as well as for RME after the aforementioned horizon sensor

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failure (Natanson, 1992). The RTSF (Challa, 1993; Challa et al., 1994) is a novel extended-Kalman filter, originally developed for SAMPEX, which estimates, in addition to the attitude, errors in rates propagated via Euler's equation. The RTSF can estimate rate errors as small as 0.0003 deg/sec (Natanson et al., 1993), and this feature makes it a very robust and accurate real-time algorithm. In particular, it has been shown (Challa, 1993; Challa et al., 1994) that the RTSF converges successfully in TAM-only situations using "inertial" initial conditions (IIC); i.e., the spacecraft is assumed at rest in the geocentric inertial coordinates (GCI) with its axes coinciding with the GCI axes.

Both DADMOM and RTSF, although successful, have their own drawbacks when used independently. The DADMOM provides at least two solutions and there is no a priori way (using residuals, etc.) to determine which is correct. The RTSF's IIC, on the other hand, do not guarantee convergence, and even then the convergence times are long (nearly 2000 sec). The solution we have adopted to overcome these difficulties involves using the DADMOM solutions at epoch to initialize the RTSF, the RTSF's residuals and rate errors being used to identify the correct solution. These deterministic initial conditions (DIC) for the RTSF are determined by the DADMOM, using a small (100 sec) batch of TAM measurements. The DICs then ensure as well as speed up convergence. Results using SAMPEX in-flight data on the combined scheme (Natanson et al., 1994) have demonstrated that accuracies of 1.5 deg in attitude and 0.01 deg/sec in the rates are possible even with an uncalibrated TAM. Remarkably, these accuracies are even within SAMPEX requirements under Sun-sensor-supported conditions. Encouraged by these preliminary results, we have developed a prototype PC-based system that automates this combined scheme; thus providing a tool for use in situations such as those mentioned earlier. The objective of this paper is to give an overview of the above system and to present additional results using SAMPEX and ERBS data.

The rest of the paper is organized as follows. Section 2 briefly describes the algorithms, Section 3 describes the system concepts and capabilities, Section 4 presents TAM-only results, and Section 5 provides conclusions and future directions.

## 2 Overview of Algorithms

### 2.1 Deterministic Attitude Determination From Magnetometer-Only Data

The attitude can be determined via the tri-axis attitude determination (TRIAD) algorithm (Wertz, 1985) if the components of two independent vectors can be obtained in the reference and body frames. The DADMOM approaches the problem by specifying these vectors to be  $\vec{B}$  and its first time derivative,  $\dot{\vec{B}}$ . The components of these vectors are related via

$$A\vec{B}^R = \vec{B}^A \quad (1)$$

$$A\dot{\vec{B}}^R = \dot{\vec{B}}^A + \vec{\omega}^A \times \vec{B}^A$$

where  $A$  is the attitude matrix,  $\vec{\omega}$  is the angular velocity vector, and superscripts  $R$  and  $A$  imply that the corresponding vectors are resolved in the reference and body frames, respectively. The crucial difficulty in implementing the TRIAD algorithm is that  $\vec{\omega}$  is unknown.

As shown by Natanson et al. (1990), the problem can be cast in the form of transcendental equations as follows. Taking into account that the vector lengths must be the same in the different frames, the projection,  $\vec{\omega}_\perp$ , of  $\vec{\omega}$  onto the plane perpendicular to  $\vec{B}$ , can be expressed as a function of an unknown angle,  $\Phi$ , between the vectors  $A[\vec{B}^R \times \dot{\vec{B}}^R]$  and  $[\vec{B}^A \times \dot{\vec{B}}^A]$ . The attitude matrix  $A$  then depends only on the angle  $\Phi$ , and the problem involves determining two unknowns: the angle  $\Phi$  and the component,  $\omega_1$ , of  $\vec{\omega}$  in the direction of  $\vec{B}$ . These unknowns can be related by combining the kinematic relationship between the second derivatives,  $\ddot{\vec{B}}^A$  and  $\ddot{\vec{B}}^R$ , with the dynamics for the  $\dot{\vec{\omega}}$  given by Euler's equation. This results in the following schematic equation (Natanson et al., 1990 and 1992):

$$\vec{\Lambda}_0(\Phi) + \vec{\Lambda}_1(\Phi)\omega_1 + \vec{\Lambda}_2\omega_1^2 = \vec{0} \quad (2)$$

where  $\vec{\Lambda}_0$ ,  $\vec{\Lambda}_1$ , and  $\vec{\Lambda}_2$  parametrically depend on the inertia tensor and control torques. Two coupled equations, quadratic in  $\omega_1$  and transcendental in  $\Phi$ , are then obtained by projecting Equation (2) along two directions perpendicular to  $\vec{B}$ . By expressing  $\omega_1$  as a function of  $\Phi$  from one of the quadratic equations and substituting into the other, one obtains an equation transcendental in  $\Phi$ , which is then solved numerically.

## 2.2 Real-Time Sequential Filter

The RTSF's state vector  $X$  comprises the four components of the attitude quaternion,  $\vec{q}$ , and the three components of the rate correction,  $\vec{b}$ , to  $\vec{\omega}^A$ :

$$\vec{X} = [\vec{q}^T \ \vec{b}^T]^T \quad (3)$$

The RTSF uses sensor data to estimate  $\vec{q}$  as well as  $\vec{b}$ , with  $\vec{b}$  being estimated kinematically in the same manner as gyro biases for a gyro-based spacecraft; i. e., by attributing differences between the measured and propagated attitudes to errors in  $\vec{\omega}^A$ . The  $\vec{b}$  estimates are then used to correct  $\vec{\omega}^A$ , and these corrected rates are used as initial conditions to propagate Euler's equation to the next measurement time. The propagation of  $\vec{b}$  is modelled via a first-order Markov model:

$$\frac{d\vec{b}}{dt} = -\tau^{-1}\vec{b} + \vec{\eta}_b \quad (4)$$

where  $\vec{\eta}_b$  is a white noise term, and  $\tau$  is a finite time constant. The novel feature of the RTSF is that, because  $b$  represents rate errors accumulated between measurements, a suitable value for  $\tau$  is the time between measurements: 5 seconds for the SAMPEX data and 8 seconds for the ERBS data used here. (In contrast, the same model, when used for gyro bias estimation, requires  $\tau$  of several hours.)

## 3 System Concepts and Description

The system presently has only algorithmic capabilities; thus graphics are not currently included. System input and output are performed through ASCII files or the screen. The executable file for the system occupies roughly 260 kilobytes of memory on an IBM PC-compatible workstation. Input to the system consists of a dataset of processed spacecraft data with minimal requirements of timetags, and measured and reference magnetic fields. Control torques from momentum wheels and magnetic torquers are used when available, with the RTSF's rate-error estimates,  $b$ , providing robustness against any missing control data when the rates are low. (See, for example, the ERBS nominal mode results in Section 4 where no control data were available.) The system can also use data from line-of-sight sensors (such as Sun-sensors) to utilize intermittently valid data from such sensors.

Broadly, the system performs the following steps during a run:

- Use a batch of between 20 and 50 data records (1 to 4 min) to generate DADMOM solutions for the epoch.
- Use the RTSF to process a short batch of data records (20 to 50) to identify the correct DADMOM solution based on acceptance criteria for TAM residuals and rate errors. If no DADMOM solution is acceptable, use IIC.
- Make a production run using the available data.
- At the end of the run, optionally generate predicted attitude and rates for a user-specified time span.

## 4 Results

### 4.1 SAMPEX Eclipse Data of 11/15/92

SAMPEX is the first of the Small Explorer satellites and has the following features:

- 550 × 675 km orbit with 82 deg inclination
- Sun pointing of the pitch axis subject to a velocity-avoidance constraint that requires a minimum angular separation between the yaw axis and the spacecraft's velocity
- Nominal pitch rate of one rotation per orbit (RPO)
- Attitude accuracy requirements of 2 deg on each axis
- Attitude-determination hardware: fine Sun sensor (FSS) for roll/yaw and TAM for pitch
- Attitude-control hardware: momentum wheel for pitch and magnetic torquer assembly (MTA) for roll/yaw

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The FSS and TAM measurements are used in a single-frame TRIAD algorithm on board the spacecraft for attitude determination (Frakes et al., 1992). These TRIAD attitude solutions, together with rates obtained by differencing them, are used here as truth models.

Two special situations should be noted: eclipses and Sun-magnetic field coalignments. During an eclipse, SAMPEX uses the last observed Sun vector along with the TAM measurements to generate TRIAD solutions. In addition, the MTA is turned off and attitude control is performed by the wheel only, under the assumption that the pitch axis remains directed along the Sun vector. It has been verified (Natanson et al., 1993) that, in the absence of maneuvers, the assumption of a constant Sun vector during an eclipse does not introduce serious attitude errors. The second situation, coalignment, is of particular interest because the pitch angle is not observable, which introduces large errors in the TRIAD solutions. Thus, onboard attitude determination and control are turned off during coalignment. These features of the SAMPEX control law are seen in the Figures 1 through 3, which span the duration of an eclipse on 11/15/92 and respectively present results using: (1) only the DADMOD; (2) only the RTSF (using IIC), and (3) the combined scheme (the RTSF using DIC).

Figure 1a presents the DADMOD solutions for the 3–2–3 Euler angles parametrizing the GCI-to-body frame attitude, with the TRIAD solutions serving as the truth model. The first and second Euler angles here identify the orientation of the wheel axis and should be essentially constant because there are no control torques, and environmental torques acting on the spacecraft are negligible. This situation is clearly seen in the top plot of Figure 1a where, although at least two DADMOD solutions exist for any given time, the correct solution agrees with the truth model and remains essentially constant. The third Euler angle of this 2–3–2 scheme identifies rotations about the wheel axis and nominally varies at  $\pm 1$  RPO. This situation is evident initially in the middle plot; during the period 1000–1800 sec, however, the velocity-avoidance constraint necessitated a wheel-induced maneuver away from Sun-pointing so that the third Euler angle remains constant during this period. Coalignment occurs briefly at about 450 sec into the eclipse and the large errors in the TRIAD solutions are clearly seen in the bottom plot of Figure 1a, which is an enlarged section of the middle plot. We see from Figure 1a that the DADMOD attitude estimates are accurate to within 5 deg.

Figure 1b presents the DADMOD solutions for the spacecraft rates along the body axes, with the truth here being the RTSF solutions. We see from Figure 1b that the DADMOD rate estimates are accurate to within 0.01 deg/sec.

Figure 2 presents RTSF results using IIC (i.e., with very large initial errors), the TRIAD results serving as truth models. In Figure 2a, roll and pitch represent the orientation of the body frame with respect to the nominal Sun frame, and the RTSF errors are defined as the deviations from the TRIAD solutions. We see that, although the initial attitude errors exceed 100 deg, these errors fall below 1.5 deg in about 2000 sec. This is corroborated by the TAM residuals (bottom plot), which are very large initially, but drop to below 5 mG around 2000 sec.

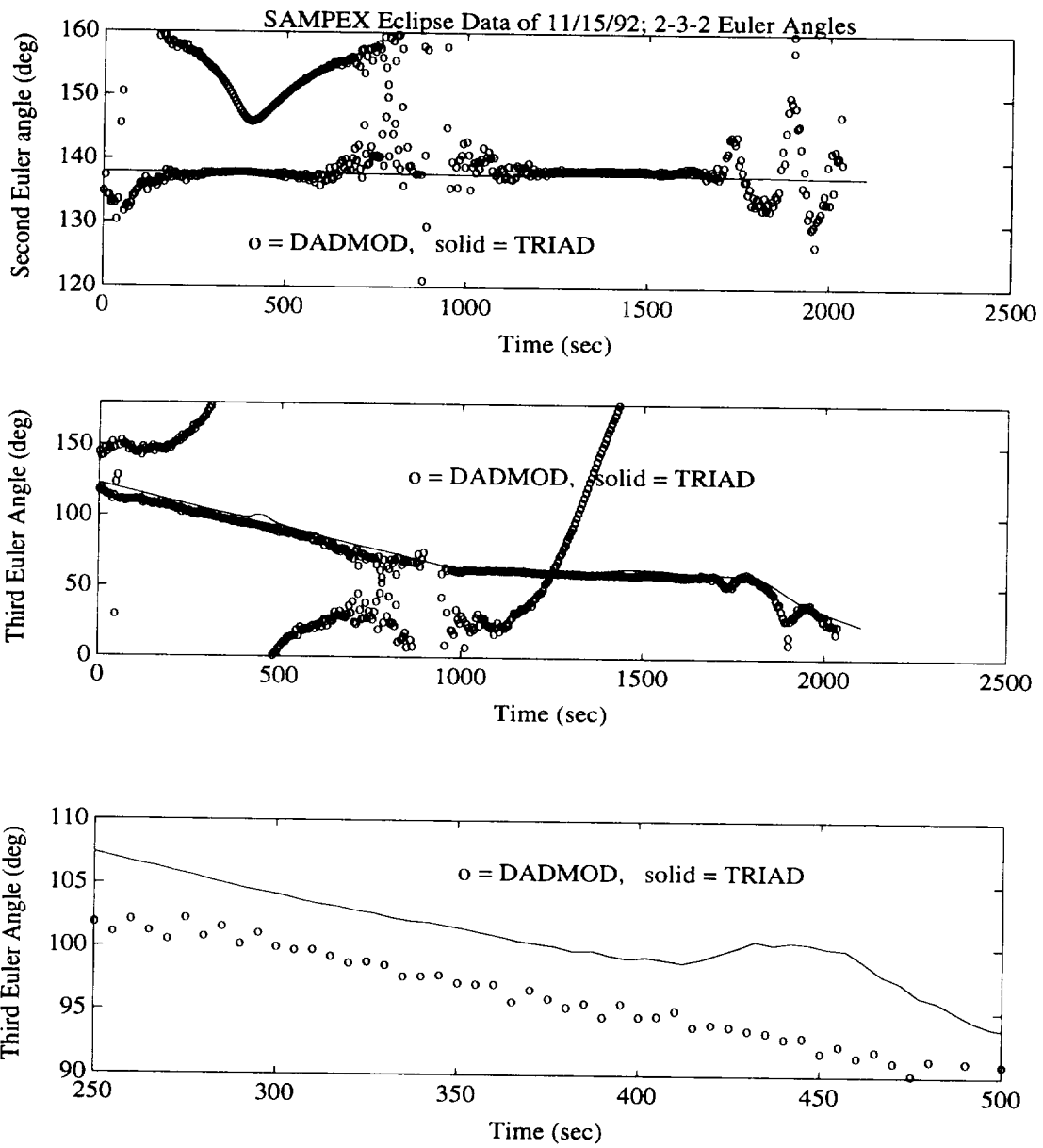
Figure 2b shows similar features with respect to the rates. The rates (top and middle plots) converge to within 0.01 deg/sec of the TRIAD solutions in about 1800 sec. The convergence of the RTSF rate estimates by 1800 sec agrees with the behavior of the rate-error estimates, which by then have fallen below 5 deg/hour (note the units). Another noteworthy feature in this figure is the oscillation in the RTSF's yaw rate. These oscillations occur at the spacecraft's nutational period of 120 sec and arise from integrating Euler's equation with large transverse rates during the initial stages of the run. However, the RTSF's rate-error estimates correctly damp the amplitude to negligible values by about 1800 sec.

Finally, Figure 3 presents results of the combined scheme (i.e., the RTSF using DIC). As can be seen in Figure 3a, the results are vastly superior to those in Figure 2a (note the differences in scale), which is also indicated by the residuals. Note that, although the pitch errors appear to be substantial around coalignment, these errors are primarily due to the TRIAD estimates. The rate estimates also show a marked improvement, as also evidenced by the rate errors. Note the kink in the TRIAD pitch rate results during coalignment. This kink is entirely spurious and originates from differentiating the erroneous TRIAD attitude estimates (bottom plot of Figure 1a). Coalignment has no noticeable effect on the RTSF's estimates because the filter then relies on its propagated value of the unobservable pitch angle. See also Challa et al. (1994) for additional analysis of the RTSF's performance during a SAMPEX coalignment.

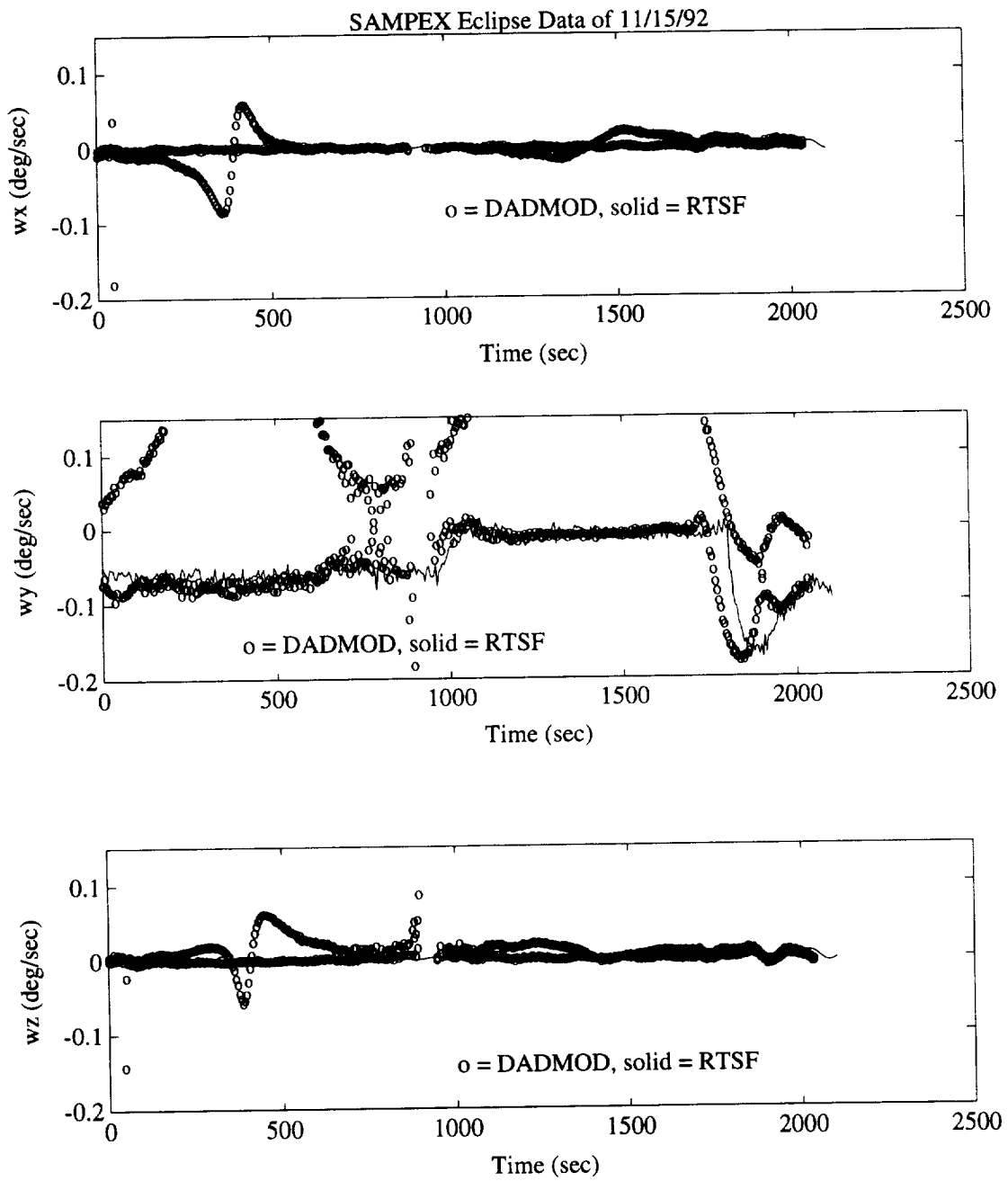
#### 4.2 ERBS Nominal Mode Data of 1/15/89

The ERBS is a conventional scientific satellite and features (Nair et al., 1982) the following:

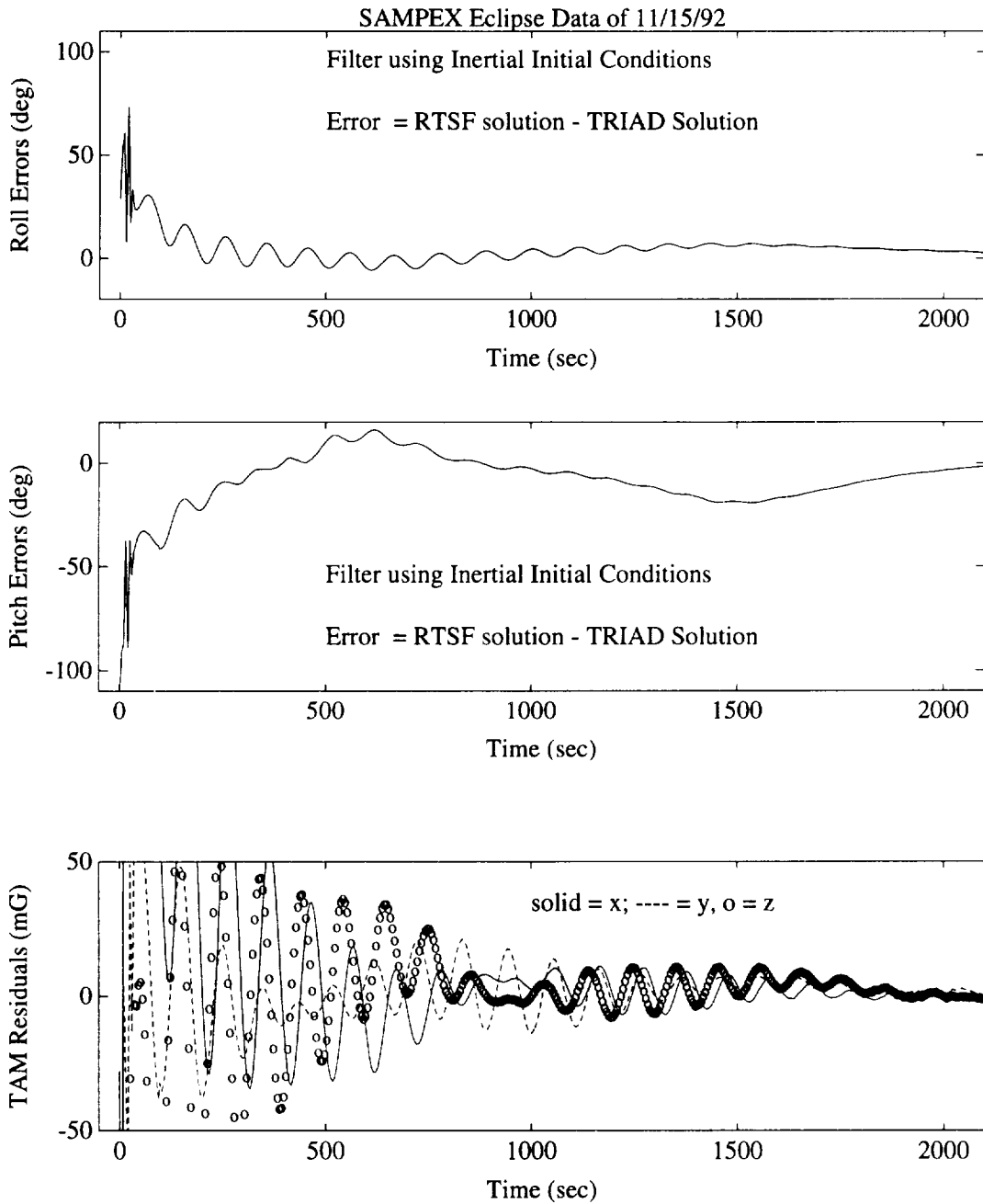
- 600 x 640 km orbit with inclination around 50 deg



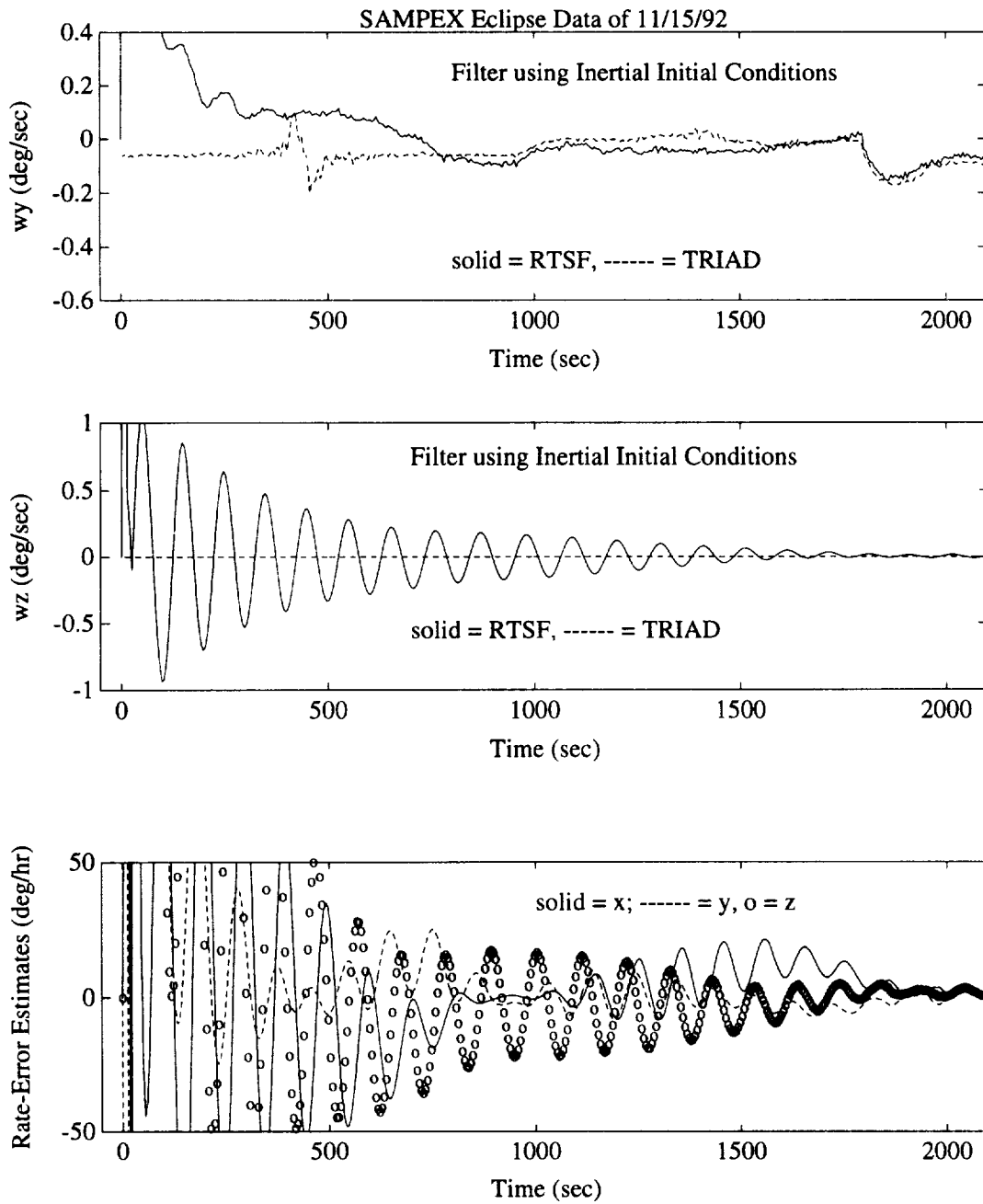
**Figure 1a. DADMOD Attitude Solutions Using SAMPEX Eclipse Data**



**Figure 1b. DADMOD Rate Solutions Using SAMPEX Eclipse Data**

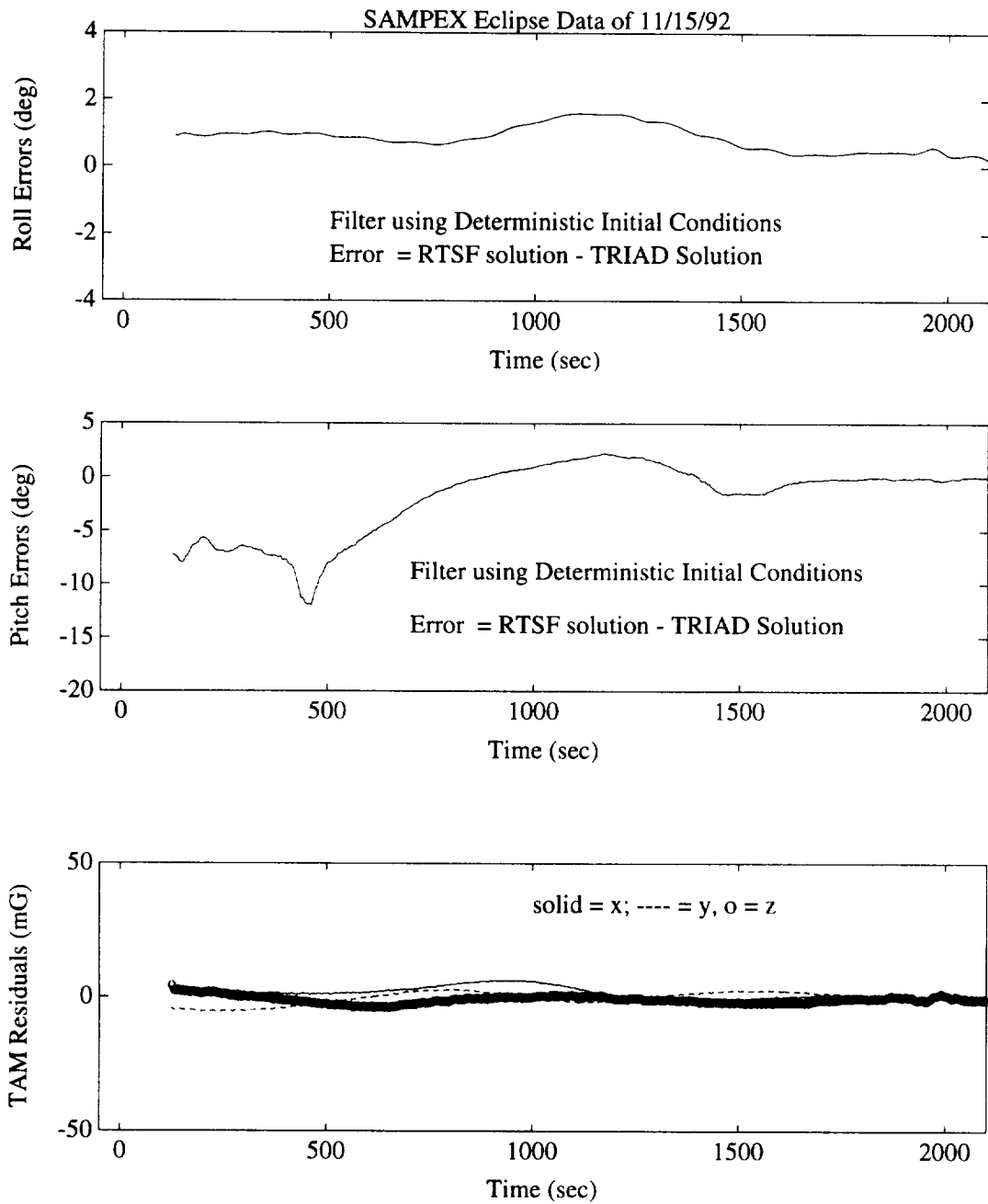


**Figure 2a. RTSF Attitude Estimates and TAM Residuals Using SAMPEX Eclipse Data**

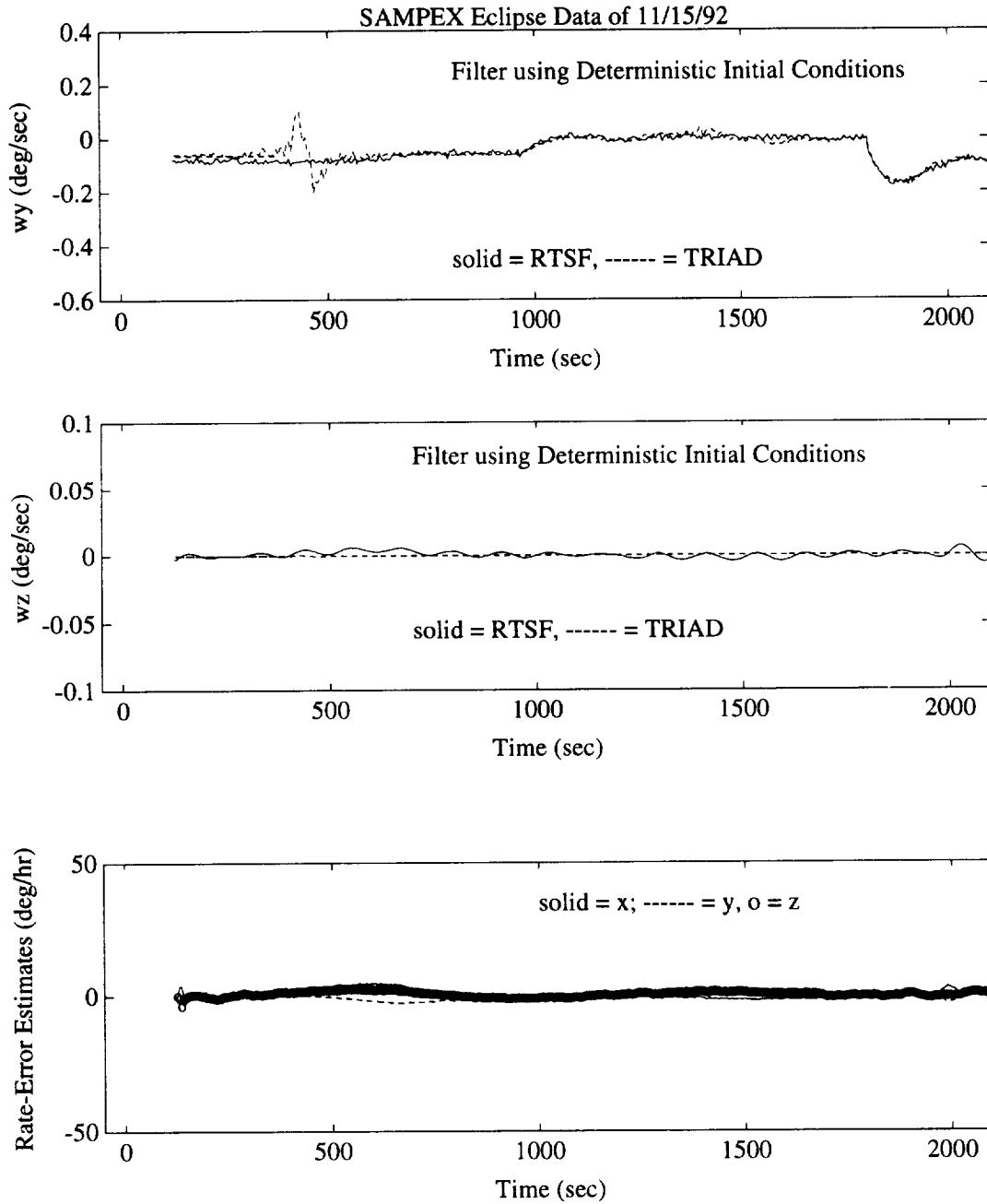


**Figure 2b. RTSF Rate and Rate-Error Estimates Using SAMPEX Eclipse Data**





**Figure 3a. Attitude Estimates and TAM Residuals From the Combined Scheme Using SAMPEX Eclipse Data**



**Figure 3b. Rate and Rate-Error Estimates From the Combined Scheme Using SAMPEX Eclipse Data**

- Earth-pointing yaw axis
- Nominal pitch rate of 1 RPO
- Attitude accuracy requirements per axis of 0.25 deg; rate accuracy requirements per axis of 0.005 deg/sec
- Attitude-determination hardware: FSS, Earth sensor assembly (ESA), and TAM
- Attitude-control hardware: pitch momentum wheel, MTA, gyros, and thrusters

Although the spacecraft was functioning nominally during this data span, only TAM results were input to our system. In particular, no control data (wheel speed and magnetic torques) were made available; therefore, the system was run with the following limitations:

- The attitude truth model was generated by a batch estimator using data from the more accurate Sun sensor, Earth sensor, and gyro data. Gyro data serve as the truth model for the rates.
- No wheel data were input to the system. A constant, nominal wheel speed based on early prelaunch specifications was used. This is an incorrect assumption.
- No magnetic torquer data were input to the system and the magnetic control torques were set to zero. This is an incorrect assumption.
- Spacecraft inertia tensor and wheel parameters were based on early prelaunch specifications. The accuracies of these assumptions are unknown.
- Environmental torques were not computed.
- The RTSF's noise parameters were not tuned, other than setting the Markov time constant to the telemetry period of 8 sec.

The above limitations, together with the availability of accurate truth models, thus enable us to evaluate the robustness of the system as well as quantifying its performance.

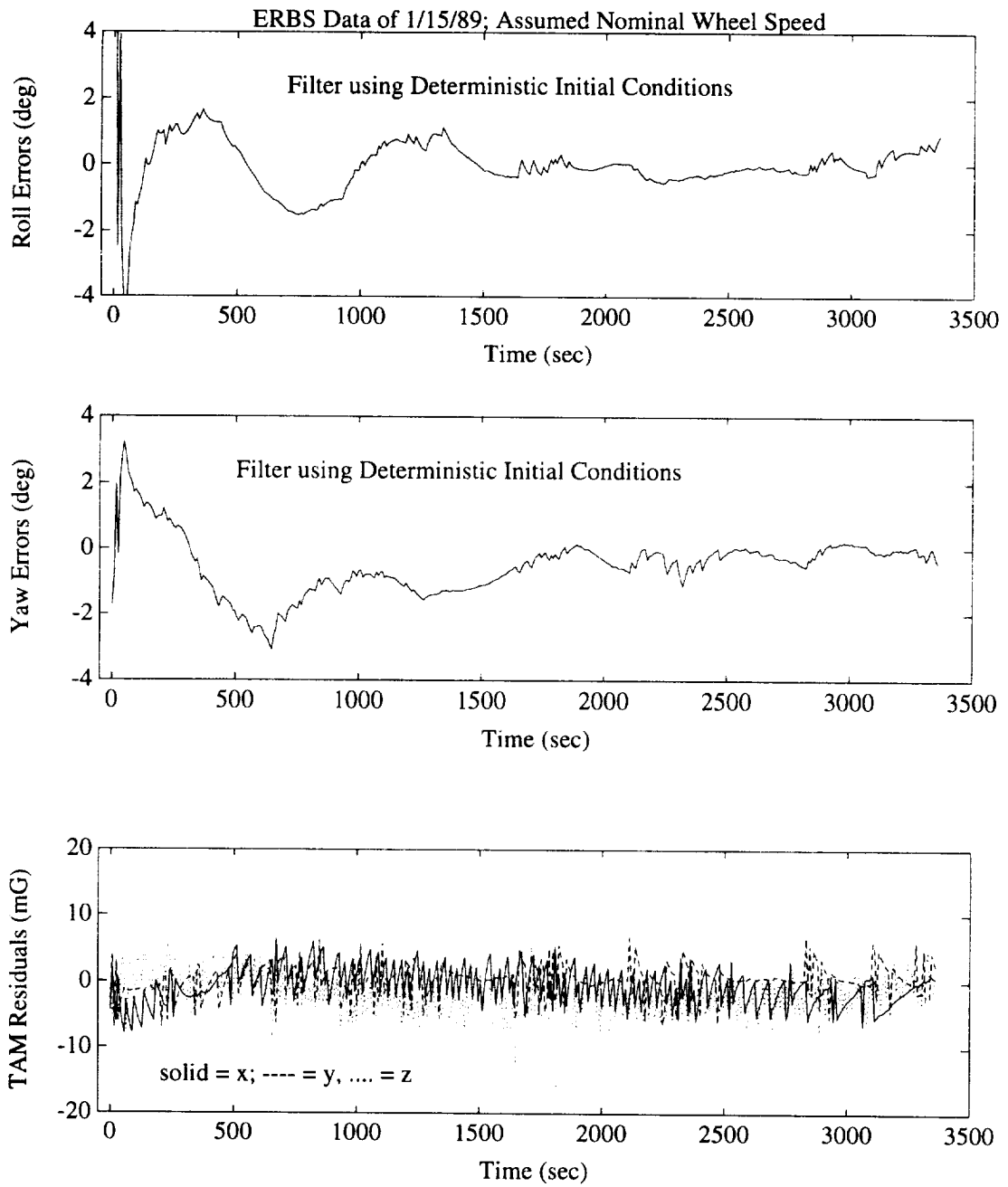
The results using the combined scheme are presented in Figures 4a and 4b, and we see accuracies of 2 deg in attitude and 0.003 deg/sec in the rates. The quick convergence of the RTSF's estimates is also in agreement with the TAM residuals (Figure 4a) as well as the rate-error estimates (Figure 4b), which are very small throughout the run. The power of the combined algorithm is strikingly evident when we note that, despite of the above data deficiencies, the RTSF's rate estimates are better than those from the gyros!

## 5 Summary and Future Work

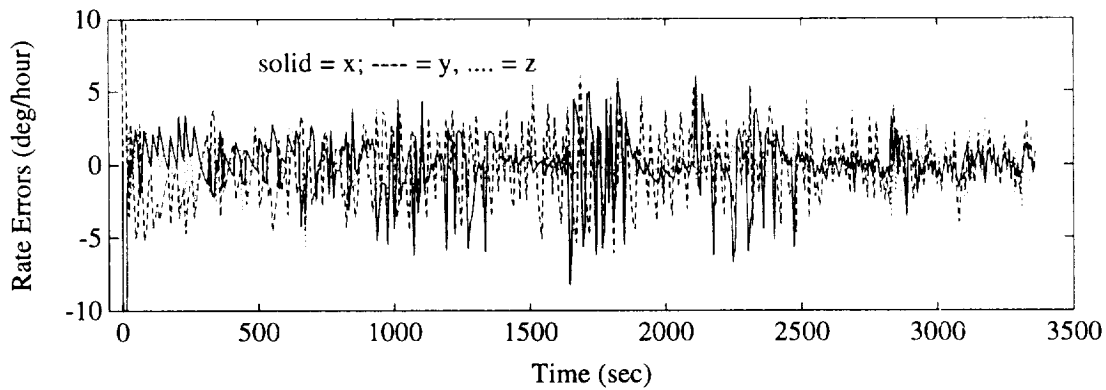
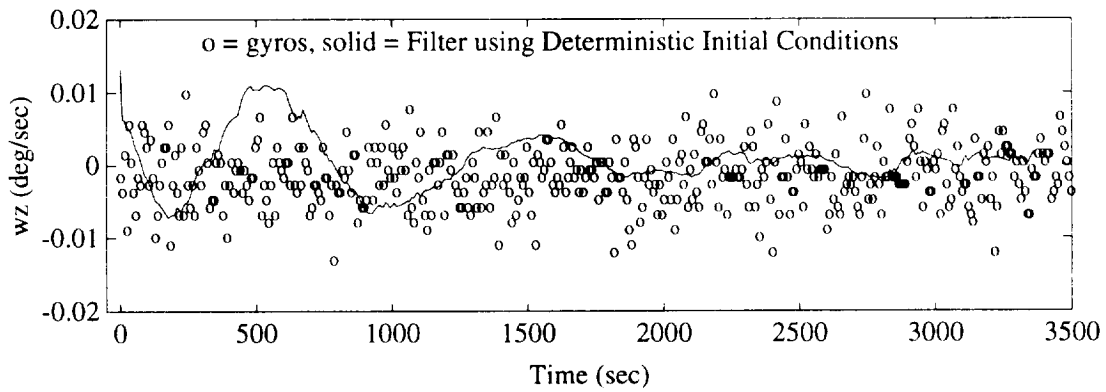
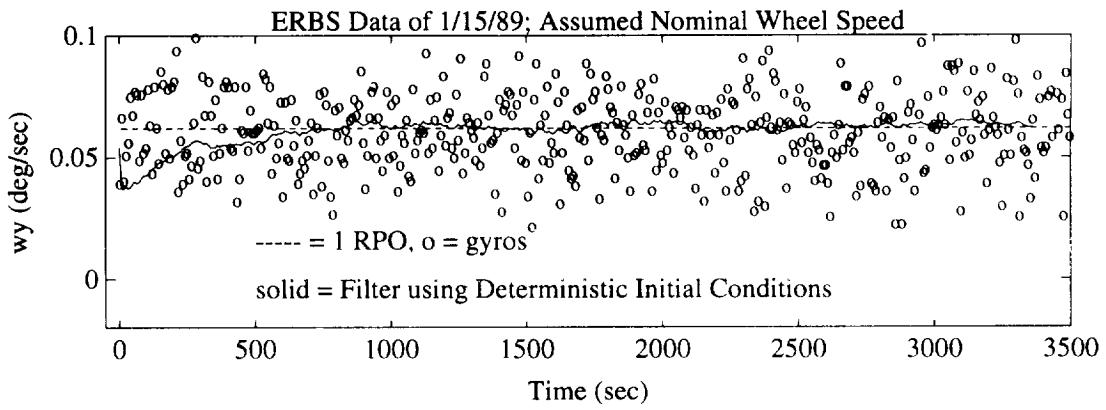
We find that the system provides an effective and easy-to-use tool for use in TAM-only situations, yielding 3- $\sigma$  accuracies around 1.5 deg in attitude and 0.01 deg/sec in the rates. The system is able to provide accurate estimates under varied conditions. Thus, the results using SAMPEX data involved a gyroless spacecraft's complex motion during an eclipse, maneuvers, and coalignment. The results with ERBS data, on the other hand, involved nominal motion of a gyro-based spacecraft and were generated despite missing control data and inaccurate spacecraft mass properties. Remarkably, the SAMPEX results were within nominal mission requirements and the ERBS rate estimates were superior to the gyro measurements.

More studies are currently under way to further probe the effectiveness of the combined scheme and involve data from: (1) the ERBS tumble mentioned in Section 1, and (2) spinning spacecraft such as the Fast Auroral Snapshot Explorer (FAST).

Although the system currently runs on IBM PC-compatible workstations, the source code is fairly generic FORTRAN and can be modified easily for use on other platforms such as mainframe computers. Also, although the system requires a small initial batch of data to generate and test the DADMOD solutions, this initialization is very fast so the system can be adapted for real-time use. A requirement, however, is that processed telemetry and reference data must be supplied in a specific format. However, it is not difficult to generate this processed dataset by developing small computer programs that use the outputs from existing ground software.



**Figure 4a. Attitude Estimates and TAM Residuals From the Combined Scheme Using Nominal ERBS Data**



**Figure 4b. Rate and Rate-Error Estimates From the Combined Scheme Using Nominal ERBS Data**

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