View metadata, citation and similar papers at core.ac.uk
brought to you by CORE
provided by NASA Technical Reports Server

1995121380

N95-27801

A MAGNETIC HYSTERESIS MODEL

Thomas W. Flatley
NASA/GSFC Code 712
Greenbelt, MD 20771

Debra A. Henretty
NASA/GSFC Code 712
Greenbelt, MD 20771

Abstract

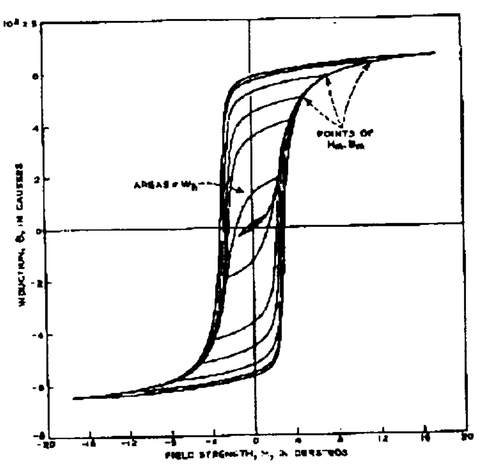
The Passive Aerodynamically Stabilized Magnetically Damped Satellite (PAMS) will be deployed from the Space Shuttle and used as a target for a Shuttle-mounted laser. It will be a cylindrical satellite with several corner cube reflectors on the ends. The center of mass of the cylinder will be near one end, and aerodynamic torques will tend to align the axis of the cylinder with the spacecraft velocity vector. Magnetic hysteresis rods will be used to provide passive despin and oscillation-damping torques on the cylinder.

The behavior of the hysteresis rods depends critically on the "B/H" curves for the combination of materials and rod length-to-diameter ratio ("l-over-d"). These curves are qualitatively described in most Physics textbooks in terms of major and minor "hysteresis loops".

Mathematical modeling of the functional relationship between B and H is very difficult. In this paper, the physics involved is not addressed, but an algorithm is developed which provides a close approximation to empirically-determined data with a few simple equations suitable for use in computer simulations.

I. List of Symbols

$B(H)$	Magnetic Flux Density (primary dependent variable)
$B'(B)$	Slope (dB/dH) on the "major loop" boundaries
B_r	Remanence
B_s	Saturation value of B
H	Magnetizing Field (primary independent variable)
H_c	Coercive Force
$H_L(B)$	Value of H on the left side boundary curve
f	$(H-H_L)/(2H_c)$
k	A TBD constant
q	Boundary slope multiplier
q_o	Value for q for f = 0



PRECEDING PAGE BLANK NOT FILMED

II. Motivation For Analysis

A few years ago, a proposed Gravity and Magnetic Earth Surveyor (GAMES) spacecraft design included a small, totally passive subsatellite which would be released to fly at controlled distances behind the main spacecraft. It was to carry a single laser-reflecting corner cube which ideally would be pointed directly at GAMES and provide a target for a laser-ranging experiment.

A novel subsatellite passive attitude stabilization method, using a combination of aerodynamic and magnetic torques, was proposed by Dave Skillman and Jim Abshire of the Goddard Space Flight Center (GSFC). The work reported here concerns the behavior of magnetic hysteresis rods which were to be used for subsatellite despin and oscillation control.

Although a GAMES new start did not materialize, the Project recommended, and received approval for, a Shuttle test flight of the aero/mag stabilization concept. Preparations are currently underway for the flight of the Passive Aerodynamically Stabilized Magnetically Damped Satellite (PAMS) in 1996.

III. Development of the Boundary Curves

Magnetic hysteresis rods (long, slender, cylindrical samples of a permeable material) become magnetized when exposed to an ambient magnetic field. The "magnetizing field" (H) is the component of the external field parallel to the axis of the cylinder, and the "magnetic flux density" (B) developed in the material produces a magnetic moment parallel to the axis. The interaction of the magnetic moment and the external field produces a torque which is sometimes used to despin satellites.

For the type of materials used, there is a significant phase lag between B and H and most physics books show plots of "B/H curves" or "hysteresis loops". The function B(H) is extremely nonlinear and multi-valued.

There is no physics addressed here. The mathematical model suggested merely attempts to reproduce empirical B vs. H behavior in a form suitable for use in computer simulations of dynamic systems which employ magnetic hysteresis rods.

We begin with the familiar function y = tan(x) shown on the left below in Figure 1a. Interchanging the axes produces the not-as-familiar function y = arctan(x) as shown below in Figure 1b.

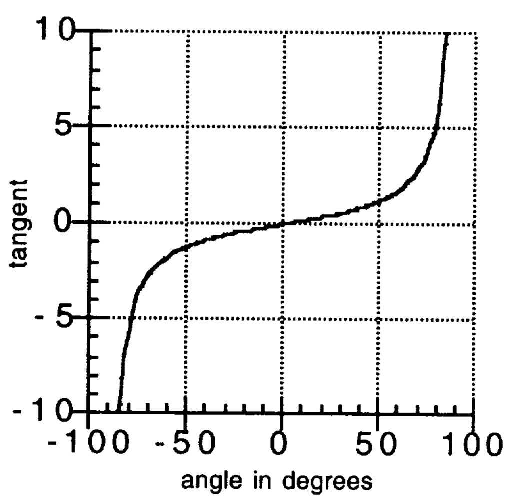


Figure 1a y=tan(x)

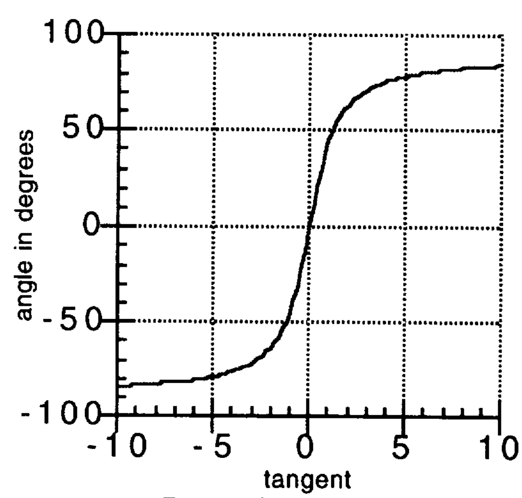


Figure 1b y=arctan(x)

We now let x = kH where k is a selectable constant and H is an independent variable. We also let y be proportional to a dependent variable (B) and wish to have the values of B limited between "saturation levels" $-B_s$ and $+B_s$. The appropriate relationship is given by

$$y = \frac{\pi}{2}\frac{B}{B_s}. \tag{1}$$

In terms of the new variables, we now have the function

$$B = \frac{2}{\pi} B_s \tan^{-1}(kH). \tag{2}$$

Figure 1c shows this function, where k has been arbitrarily set to 5.92 and B_s to 9872.

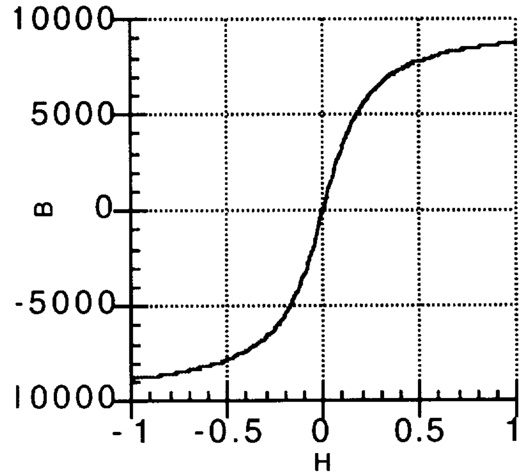


Figure 1c $B = \frac{2}{\pi} B_s \tan^{-1}(kH)$

Figure 1d shows the result of shifting the previous curve to the left by an amount H_c arbitrarily set to 0.135 units. Note that, where this curve crosses the ordinate axis, we let $B = B_r$. The equation for this curve is

$$B = \frac{2}{\pi} B_s \tan^{-1}\left[k\left(H + H_c\right)\right]. \tag{3}$$

For H = 0 then, we have

$$B_r = \frac{2}{\pi} B_s \tan^{-1}\left[k\left(H_c\right)\right]. \tag{4}$$

or equivalently

$$k = \frac{1}{H_c}\tan\left(\frac{\pi}{2}\frac{B_r}{B_s}\right). \tag{5}$$

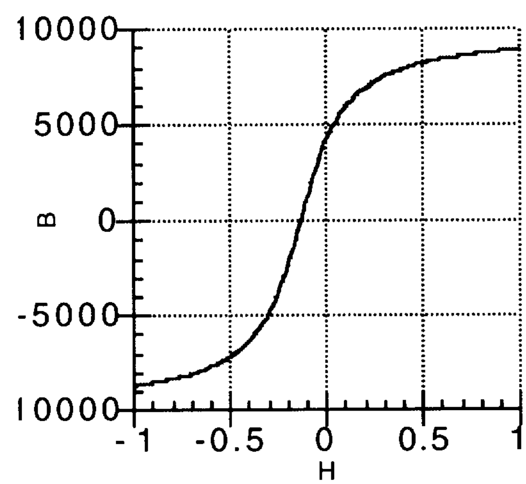


Figure 1d $B = \frac{2}{\pi} B_s \tan^{-1}\left[k\left(H + H_c\right)\right]$

Figure 1e shows the function

$$B = \frac{2}{\pi} B_s \tan^{-1}\left[k\left(H - H_c\right)\right], \qquad (6)$$

obtained by shifting the original curve to the right by the same amount H_c

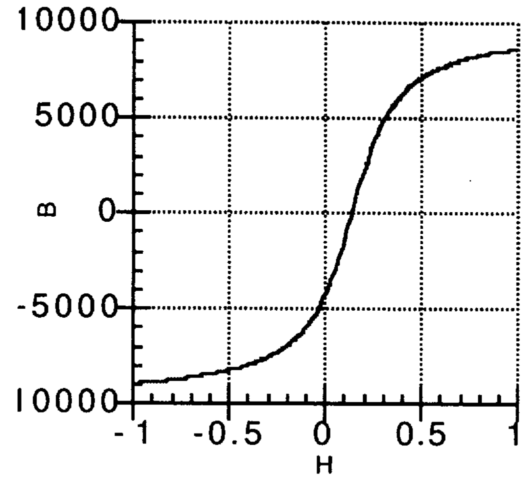


Figure 1e $B = \frac{2}{\pi} B_s \tan^{-1}\left[k\left(H - H_c\right)\right]$

Finally, in this construction sequence, we superimpose the last two figures to obtain Figure 1f which bears a close resemblance to the "magnetic hysteresis loops" shown in the Electromagnetics chapter in most elementary Physics books. Usually, H is called the Magnetic Field Strength and B is called the Magnetic Induction. The units involved are discussed in an Appendix to this report. Note that the left boundary curve crosses the ordinate axis at $B=B_r$. In textbook B/H curves, this value is called the Remanence. The right boundary curve crosses the abscissa at $H=H_c$. This value is called the Coercive Force.

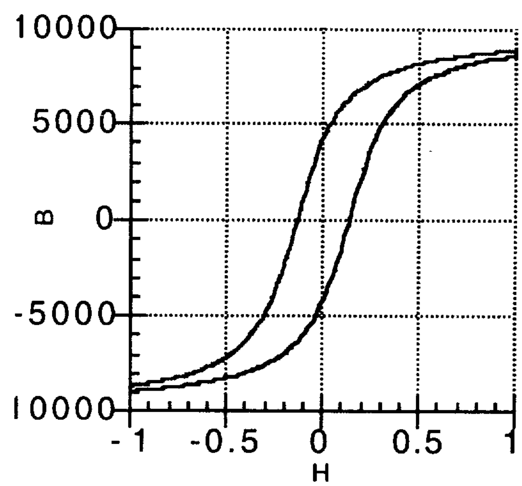


Figure 1f "Hysteresis Loop"

IV. Initial Magnetization Phase and Behavior Inside the Boundaries

Most Physics texts mention an initial magnetization phase in which $B(0)=H(0)=0$ and H is gradually increased in a positive direction. Figure 2 shows a typical "S"-shaped curve showing B first slowly increasing, then rising more sharply, then asymptotically approaching some upper limit.

Here, we assume that the slope dB/dH of this curve and that of any point within the boundary constraints depends on the horizontal distance between the current point (H,B) and the boundary curve which the point is moving away from, in this case, the left side boundary. As an example, suppose that the slope is zero coming off the left boundary and approaches the boundary slope as it approaches the right boundary. Note from the previous section that the two boundary curves are always $2H_c$ apart in the horizontal direction and define the fraction

$$f = \frac{H - H_L}{2H_c} \qquad (7)$$

where H_L is the value on the left boundary curve corresponding to the current value of B. Since the left side boundary is given by

$$B = \frac{2}{\pi} B_s \tan^{-1}\left[k\left(H_L + H_c\right)\right], \qquad (8)$$

then,

$$H_L = \frac{\tan\left(\frac{\pi}{2}\frac{B}{B_s}\right)}{k} - H_c. \qquad (9)$$

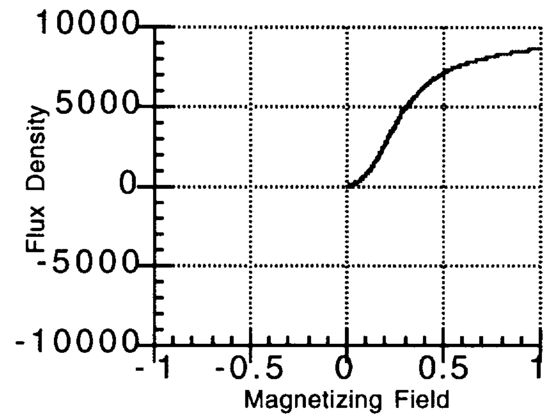


Figure 2 Initial Magnetization Phase

Now, Let B' be the boundary curve slope (either boundary) corresponding to the current value of B. Differentiating the previous equation with respect to H, we find

$$0 = \frac{\pi}{2kB_s} \sec^2\left(\frac{\pi}{2}\frac{B}{B_s}\right) B' \qquad (10)$$

or

$$B' = \frac{2}{\pi} kB_s \cos^2\left(\frac{\pi}{2}\frac{B}{B_s}\right). \qquad (11)$$

Now, a simple function providing the desired behavior could be

$$\frac{dB}{dH} = f\,B'. \qquad (12)$$

Another possibility would be

$$\frac{dB}{dH} = f^p\,B' \qquad (13)$$

where p is an exponent of the fractional distance f.

Actually, in the specific example addressed below, the slopes at the left boundary seen in the test data were small but non-zero. To account for this possibilty, a more general expression (with one more selectable constant) was used in fitting the data. We let

$$\frac{dB}{dH} = \left[q_o + \left(1 - q_o\right) f^p\right] B' \qquad (14)$$

where $q_o B'$ is the slope dB/dH near the boundary.

V. Specific Application

Some empirical B/H curve data was generously provided to the author by Frederick Mobley of the Johns Hopkins University Applied Physics Laboratory (APL). Tests were run on a long, slender rod made from AEM 4750 material, a nickel-iron alloy. The magnetizing field was oscillated between nine different peak values ranging from +/- 0.04 to +/- 0.50 units. At the higher levels, boundary curve limits were clearly achieved.

Examination of the boundary curves showed immediately that H_c=0.135 units and B_r=4240 units were reasonable approximations. Placing these values and H=0 in the left-hand boundary curve

$$B = \frac{2}{\pi} B_s \tan^{-1}\left[k\left(H + H_c\right)\right] \qquad (15)$$

produces an equation showing a required relationship between k and B_s, i.e.

$$4240 = \frac{2}{\pi} B_s \tan^{-1}[0.135k]. \tag{16}$$

Another point on the left hand boundary was located at (-0.5,-7150). Using these values for H and B, respectively, produces a second requirement for k and B_s, i.e.

$$-7150 = \frac{2}{\pi} B_s \tan^{-1}[-0.365k]. \tag{17}$$

These equations were solved numerically to find B_s=9872. With this known, we can compute k = 5.925. The resulting boundary curve then becomes

$$B = \frac{2}{\pi}(9872)\tan^{-1}[5.925(H \pm 0.135)]. \tag{18}$$

Figure 3a shows a reproduction of some of the hysteresis rod B/H test data received from APL. The outermost contour was obtained by cycling the magnetizing field (H) between the limits of ±0.5 units and plotting the resultant steady-state B/H contour curve. The peak values of B observed were ±7150 units. Points on the left-hand boundary curve were used to determine appropriate values for k and B_s for the math model described above.

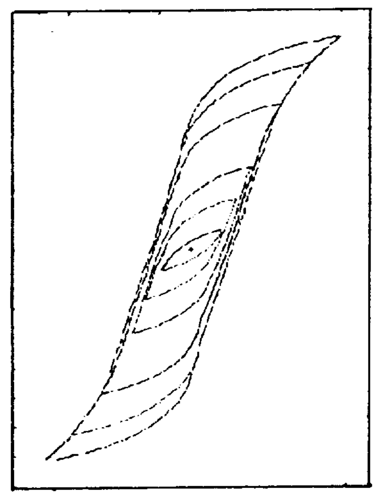


Figure 3a Test Data

The outermost contour of Figure 3b shows the math model output for the same ±0.5 units input. Note that the test data slope appears to have a fairly large linear range compared to that produced by the model. The arctangent model shows more curvature and flattens out a little more at the extreme values of H.

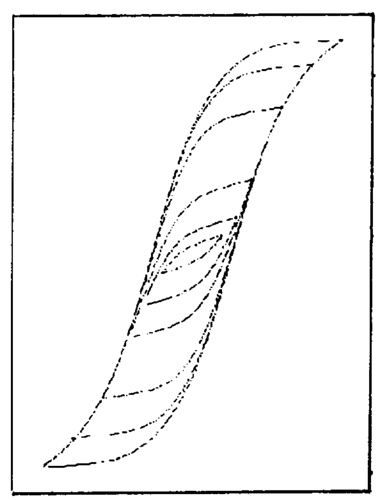


Figure 3b Model Data

Figure 3c shows test data for a smaller range of H values, i.e. ±0.1 units. The outermost curve here is the same as the innermost curve of Figure 3a. This curve was used in the selection of the other two model parameters, p and q_o Values of p = 4.75 and q_o= 0.085 were obtained by a trial-and-error iterative procedure which matched a peak value of B of 637 units and an ordinate crossing value of 275 units as observed in the test data. Figure 3d shows that a very close match was obtained.

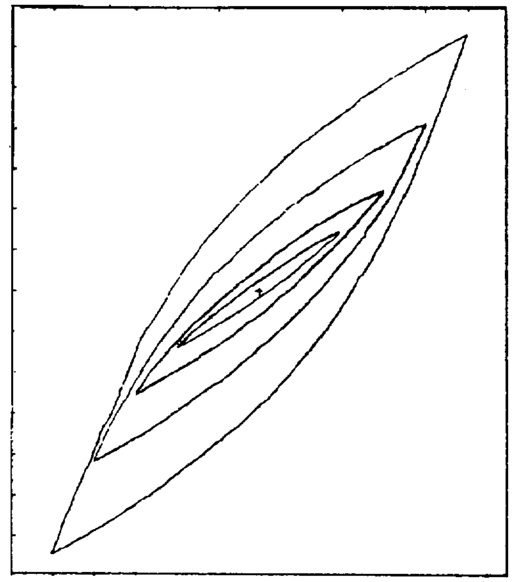


Figure 3c Test Data

The four parameters selected above were then used in the model andB(H) contours were generated for the other ranges of H for which test data was available, i.e. ±0.4, ±0.3, ±0.2, ±0.15, ±0.08, ±0.06, and ±0.04 units. All test data and model outputs are shown in Figures 3a through 3d.

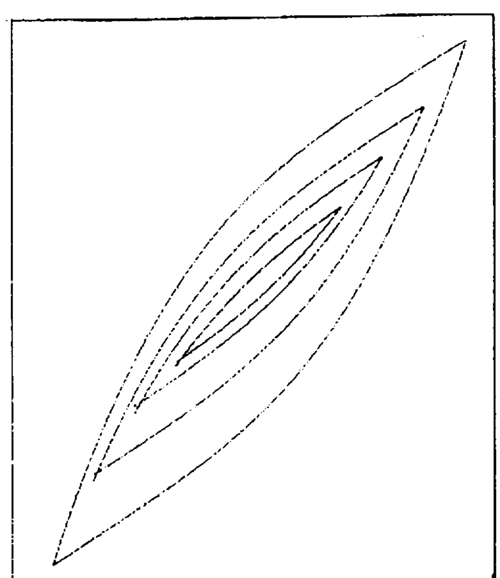


Figure 3d Model Data

VI. An Algorithm for Determining dB/dt

Given B, H and dH/dt, with k, q_o, and p predetermined constants,

1. Compute H_L, the value of H on the left boundary curve corresponding to B. Since this curve is given by $B = \frac{2}{\pi} B_s \tan^{-1}[k(H_L + H_c)]$, we find

```
HL=TAN(PI*B/2/BS)/K-HC
```

2. Compute B', the boundary curve slope corresponding to the value of B. It may be shown that

```
BP=2*K*BS/PI*COS(PI*B/2/BS)**2
```

3. Find f,

```
F=(H-HL)/2/HC
```

If dH/dt is negative, measure f from the right hand boundary,

```
IF (DHDT.LT.0) F=1-F
```

4. Find q,

```
Q=Q0+(1-Q0)*F**P
```

5. Find dB/dH,

```
DBDH=Q*BP
```

6. Find dB/dt,

```
DBDT=DBDH*DHDT
```

VII. Appendix on Magnetic Units

The use of units for the variables B and H has been avoided in the body of this report. The history of magnetic units is long and sometimes contentious and it was feared that including a discussion of this subject would introduce an unnecessary distraction in the presentation of the mathematical modelling of magnetic hysteresis.

As mentioned in the body of the report, empirical B/H curves were received from Mr. Fred Mobley of The Johns Hopkins University Applied Physics Laboratory (APL) of Silver Spring, MD. The data referred to here was produced during testing of a rod made from a material known as AEM 4750 (a nickel-iron alloy) with a length of of 58 inches (1.47 meters) and a diameter of 0.1084 inches (0.00275 meters). The independent variable, the **magnetizing field H**, was expressed in **oersteds** and the dependent variable, the **flux density B** was given in **gauss.**

In modern times, people have become comfortable with **magnetic fields in tesla**, **magnetic moments in ampere-meters-squared** and **torques in newton-meters**. A magnetic hysteresis rod in space will have a certain magnetic moment **M** and will interact with the Earth's magnetic field **B** to generate a torque **T**, with the pertinent equation being the familiar cross product law **T** = **M** x **B**.

In this Appendix, we attempt to bridge the gap between raw data in the form of B/H curves, with B in gauss and H in oersteds, and computer simulations of the performance of magnetic hysteresis rods in space.

An immediate possible source of confusion is the fact that the letter B is now commonly used for the external magnetic field, where historically, the letter H was used. In early-space-age literature[1] and textbooks[2], when magnetic materials were exposed to a **magnetic field strength** or magnetizing field H, a **magnetic flux density** or

magnetic induction B was developed in the material. The unit for H was oersted and the unit for B was gauss. These unit names honored two of the early researchers in the field, Hans Christian Oersted (1777-1851) and Carl Friedrich Gauss (1777-1855).

With a magnetic hysteresis rod, the magnetic moment or **dipole moment** depended on the flux density and the volume of material with the governing relationship being $m = BV/4\pi$. Here the magnetic moment m is measured in **pole-centimeters** and the volume V is in **cubic centimeters. Pole-cm** is a unit probably familiar to most readers and many have probably used the conversion factor of 1000 pole-cm = 1 ampere-meter-squared. Why this is so is probably not very well known.

The ampere-meter-squared unit is probably the simplest and most obvious magnetic unit. With a current in **amperes** flowing in a loop enclosing an area in **square-meters**, the magnetic moment is simply the current times the area. Now, the magnetic field in the vicinity of such a loop is of interest. The physics involved is expressed by the Biot and Savart equation

$$dH = I \, ds \times r \, / \, r^3. \tag{19}$$

In this differential equation, **H** is the magnetic field, I is the current, **ds** is an incremental length of coil and **r** is a vector from **ds** to the point of interest. Unitwise, with **r** and ds in centimeters, a current in **abamperes** produces a magnetic field in oersted. There are 10 amperes in one abampere.

Figure A-1 shows a circular coil of radius "a" carrying a current "i" (in amperes) and a point **R** in the plane of the coil at which the magnetic field is to be determined. For points far away from the coil, the following analysis shows that the magnetic field is given approximately by

$$H = \pi a^2 \, i/10 \, R^{-3} \tag{20}$$

oersted "into the paper".

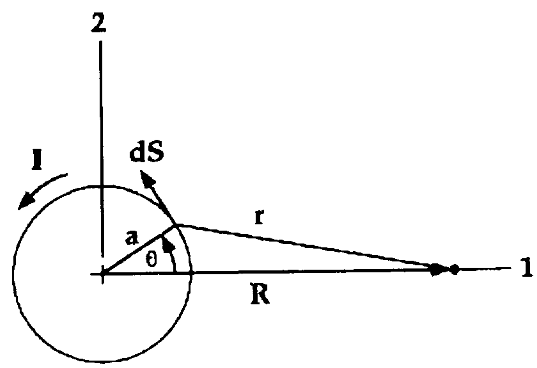


Figure A-1 Current Loop

The elemental length ds is in the 1-2 plane, with the components

$$ds_1 = - a \sin(\theta) \, d\theta \tag{21}$$

and

$$ds_2 = a \cos(\theta) \, d\theta. \tag{22}$$

The vector **r** (also in the 1-2 plane) has the components

$$r_1 = R - a \cos(\theta) \tag{23}$$

and

$$r_2 = - a \sin(\theta). \tag{24}$$

The vector **ds x r** is obviously in the 3 direction with a magnitude of $a[a-R\cos(\theta)]d\theta$. The quantity r^2 is given by $R^2+a^2-2aR\cos(\theta)$. The strength of the field is then obtained by integrating (from $\theta = 0$ to 2π) the differential equation

$$dH = i/10 \, a \, [\, a - R \cos(\theta) \,] \, d\theta \, / \, [\, R^2 + a^2 - 2 \, a \, R \cos(\theta) \,]^{3/2}. \tag{25}$$

Far from the coil. i.e., for R>>a, the denominator in this equation may be approximated by $R^3[1-3(a/R)\cos(\theta)]$ in the denominator or by $R^{-3}[1+3(a/R)\cos(\theta)]$ in the numerator. This makes the approximate differential equation

$$dH = a \, i/10 \, R^{-3} \, [\, a - R \cos(\theta) \,] \, [\, 1 + 3 \, a/R \cos(\theta) \,] \, d\theta. \tag{26}$$

Of the four terms here, only two will contribute to the integral, so we can let

$$dH = a\, i/10\, R^{-3} \,[\, a - 3\, a \cos^2(\theta) \,]\, d\theta. \tag{27}$$

Integrating this equation produces

$$H = a\, i/10\, R^{-3} \,(\, 2\pi a - 3\pi a \,) = -\, \pi a^2 \; i/10\, R^{-3}\,. \tag{28}$$

Now, fictitious "magnetic poles" were commonly employed in the teaching of magnetics years ago. They came in two varities, positive poles and negative poles. The force between two poles was inversely proportional to the square of the distance between them and was measured in **dynes** when the distance was in centimeters. A dyne was the force required to accelerate a mass of one gram to a level of one cm/sec^2. Forces were either repulsive or attractive, depending on whether the signs were the same or different. If a positive pole and a negative pole were placed one centimeter apart, they formed a unit dipole, with a magnetic moment of one pole-cm. Collections of "p" positive poles and "p" negative poles a distance "c" apart formed a dipole with a magnetic moment of m=pc. Now, a unit positive pole in the vicinity of this dipole would experience an attractive force toward the negative end and a repulsive force away from the positive end. The magnetic field (in oersteds) due to the dipole at this point was defined as equal in magnitude and direction to the net force vector (in dynes) acting on the unit pole.

Figure A-2 shows the dipole described above and a point **R** at which we would like to determine the magnetic field strength. It will be shown below that, for R>>c, and with **R** perpendicular to the dipole axis, the field is approximately $H = pc/R^3$. Assume now that the fields due to the coil described above and the dipole described here are equal. We then would have $\pi a^2 i/10 = pc$. Now, let A be the area of the coil in square meters. Then $a^2 = 10^4 A$, and we have $1000Ai = pc$. Therefore, one ampere-meter-squared = 1000 pole-cm as stated above.

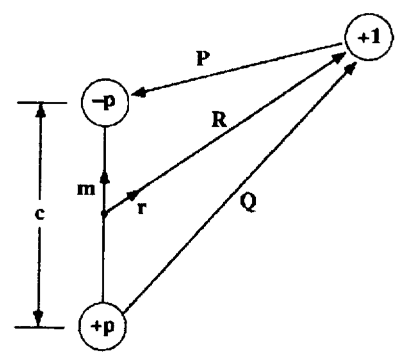


Figure A-2 Magnetic Dipole

Initially, let **R**=R**r** be a general point at a distance R from the center of the dipole, with **r** a unit vector. Let **m** be a unit vector from the positive pole to the negative pole of the dipole. Let **P** be a vector from the test point to the negative pole of the dipole and let **Q** be a vector from the positive pole to the test point as shown. We can write **R+P=mc/2** and **Q=mc/2+R.** The force (in dynes) acting on the test pole will be $p\mathbf{Q}/Q^3 + p\mathbf{P}/P^3$, where P and Q are the magnitudes of the vectors **P** and **Q**.

Now, $P^2 = \mathbf{P}\,\mathrm{dot}\,\mathbf{P} = (\mathbf{m}c/2 - \mathbf{R})\,\mathrm{dot}\,(\mathbf{m}c/2 - \mathbf{R})$ or approximately, with R>>c, $P^2 = R^2 - c\, \mathbf{m}\,\mathrm{dot}\,\mathbf{R}$. Obviously, we have introduced the operator "dot" to indicate the scalar product between two vectors. Similarly Q^2 is approximately equal to $R^2 + c\, \mathbf{m}\,\mathrm{dot}\,\mathbf{R}$. The force then becomes, approximately,

$$\mathbf{f} = p\mathbf{Q}\,[\, R^2 + c\, \mathbf{m}\,\mathrm{dot}\,\mathbf{R} \,]^{-3/2} + p\mathbf{P}\,[\, R^2 - c\, \mathbf{m}\,\mathrm{dot}\,\mathbf{R} \,]^{-3/2} \tag{29}$$

$$\approx (p/R^3)\,[\, \mathbf{Q}\,(\, 1 + 3/2\; c/R\; \mathbf{m}\,\mathrm{dot}\,\mathbf{r}) + \mathbf{P}\,(\, 1 - 3/2\;\; c/R\; \mathbf{m}\,\mathrm{dot}\,\mathbf{r})\,]$$

$$\approx (p/R^3)\,[\, \mathbf{P+Q} - 3/2\; c/R\; (\mathbf{P-Q})\; \mathbf{m}\,\mathrm{dot}\,\mathbf{r}\,]$$

$$\approx (p/R^3)\,[\, c\,\mathbf{m} - 3\, c\, \mathbf{r}\; \mathbf{m}\,\mathrm{dot}\,\mathbf{r}\,]$$

$$\approx (pc/R^3)\,[\, \mathbf{m} - 3\; \mathbf{m}\,\mathrm{dot}\,\mathbf{r}\; \mathbf{r}\,]$$

Recalling that the force in dynes is numerically equal to the magnetic field in oersteds, this equation represents a concise expression for a dipole magnetic field. It is similar to that shown in Wertz[3].

At a point on the dipole equator (which is what we were initially interested in) the field reduces to simply pc/R^3 as was stated above.

Getting back to the computer simulation of magnetic hysteresis rod behavior, we have the following rule:
<u>Given a magnetic flux density (B) in gauss , and a volume of material (V) in cubic centimeters, the magnetic moment in modern (SI) units will be $M = BV/4000\pi$ ampere-meters-squared.</u>

Another perhaps familiar expression from the magnetics section of physics books is $B = \mu H$, where μ is the **permeability** of the medium. This equation converts H in oersteds to B in gauss. With the now obsolete (but still widely used) system of units, the permeability of free space was unity. Magnetic fields in space were then expressed in gauss and were numerically equal to magnetic fields in oersteds. This latter unit soon became rarely used and the Earth's magnetic field strength was stated in gauss (or milligauss) for some time. The unit **gamma** was also sometimes used in the early days of orbiting spacecraft with one gauss being equivalent to 10^5 gamma. When the emphasis on SI units arrived on the scene, "gauss" gave way to "webers per square meter" with the conversion factor of 10^4 gauss per Wb/m^2. This unit name was selected to honor Wilhelm Eduard Weber (1804-1891). Some time later, this latter unit was renamed the tesla in honor of Nikola Tesla (1856-1943). This made one gamma equal to one nanotesla.

A second rule to be followed then in dealing with B/H data in gauss and oersteds is:

<u>Working with magnetic hysteresis rods in space, the "H in oersteds" required for the math model would be obtained by multiplying the magnetic field in tesla by 10000.</u>

References:

1. Fischell, Robert E., Magnetic Damping of the Angular Motions of Earth Satellites, ARS Journal, September 1961, pages 1210-1217.

2. Lemon, Harvey Brace and Ference, Michael, Jr., Analytical Experimental Physics, The University of Chicago Press, 1947, pages 356-357.

3. James R Wertz, Spacecraft Attitude Determination and Control, D. Reidel Publishing Company, 1978, page 783.