

TURBULENCE PROGRAM FOR PROPULSION SYSTEMS

N95- 27883

Tsan-Hsing Shih
Institute for Computational Mechanics in Propulsion and
Center for Modeling of Turbulence and Transition
NASA Lewis Research Center
Cleveland, Ohio

BACKGROUND

- CMOTT group at LeRC has been in existence for about 4 years. In the first 3 years, its main activities were in developing and validating turbulence and combustion models for propulsion systems, in an effort to remove the deficiencies of the existing models. Two workshops on computational turbulence modeling were held at LeRC (1991, 1993).
- A peer review of turbulence modeling activities at LeRC was held in September, 1993. Seven peers (GE, P&W, RocketDyne, Cornell, Berkeley and NASA Ames) conducted the peer review. The objective of the peer review was to assess the turbulence program at LeRC/CMOTT and to suggest the future direction of turbulence modeling activities for propulsion systems.
- Important messages from the peer review:
 - ◇ “LeRC should spend substantial effort being responsive to industry’s current pressing perceived needs; this involves extensive discussion with industry during every phase of model development, analysis of industry’s problems, goal oriented model development, evaluation of models relative to industry’s intended application ...”
 - ◇ “LeRC has an obligation not only to respond to industry’s requests for help, but to play an autonomous, independent leadership role in providing models of the highest quality, ..., which can be employed not only by the aircraft gas turbine and rocket industries but also by other industries ...”
 - ◇ “In the present financial climate, industry does not have the resources to undertake model development and evaluation. LeRC’s help in this regard via the creation of its turbulence modeling effort, is, therefore, welcome from the industry’s standpoint.”
 - ◇ “It is important to work with the industry to evaluate the models and rank-order them by performance and cost in order to identify the most appropriate models for particular situations.”
 - ◇ Many other useful suggestions and comments including collaboration with industry, joint programs, industry-wide workshop ...

PROGRAM GOALS AT CMOTT

- **Develop reliable turbulence (including bypass transition) and combustion models for complex flows in propulsion systems**
- **Integrate developed models into deliverable CFD tools for propulsion systems in collaboration with industry.**

PROGRAM APPROACH

- **Develop turbulence and combustion modules for industry customers**
- **Industry collaboration and technology transfer**
- **Model development for propulsion systems**
 - ◇ **One-point moment closures for non-reacting flows**
 - ◇ **Scalar PDF method for turbulent reacting flows**
 - ◇ **Validation of existing and newly developed models**

Development of Turbulence and Combustion Modules

- Objective
 - ◇ Build a quick and efficient vehicle for technology transfer to industry
- The features of the turbulence module:
 - ◇ It contains various turbulence models from which users can choose the appropriate model for flows of interest
 - ◇ It is self-contained, i.e., it contains its own solver for turbulence model equations
 - ◇ It can be easily linked to industry's CFD codes
- Turbulence module for NPARC code has been developed, tested, and is ready to be released
 - ◇ The models built-in at the present time:
Mixing length, Chien $k - \epsilon$, CMOTT $k - \epsilon$ models
 - ◇ The model to be built-in:
CMOTT algebraic Reynolds stress, Reynolds stress transport equation models and other models based on the request from industries.
 - ◇ Built-in robust, realizable numerical solver for model equations.
- General turbulence modules
 - ◇ Can be used for both compressible and incompressible flows.
 - ◇ Interface programs for different industry CFD codes
 - ◇ Built-in models will be periodically updated.

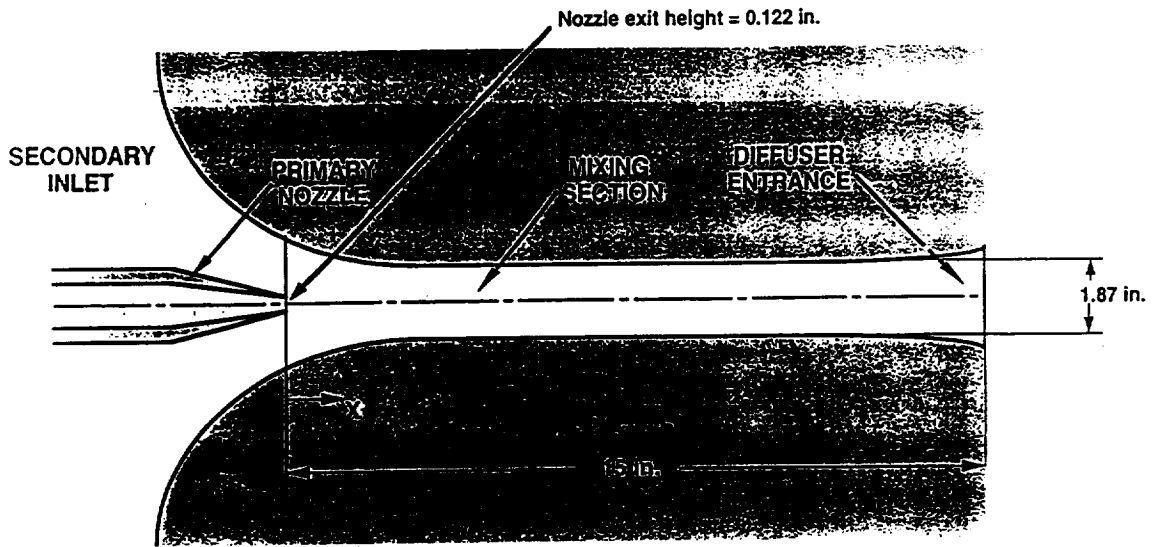
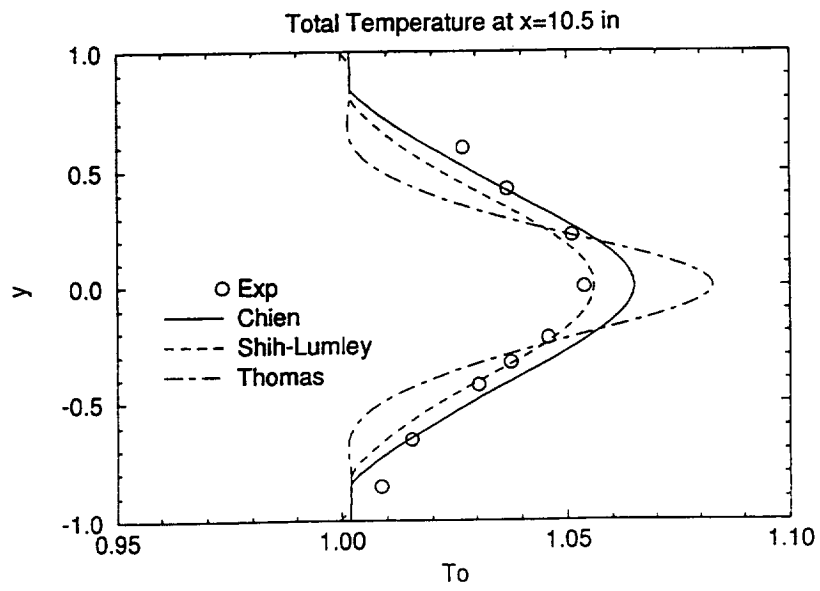
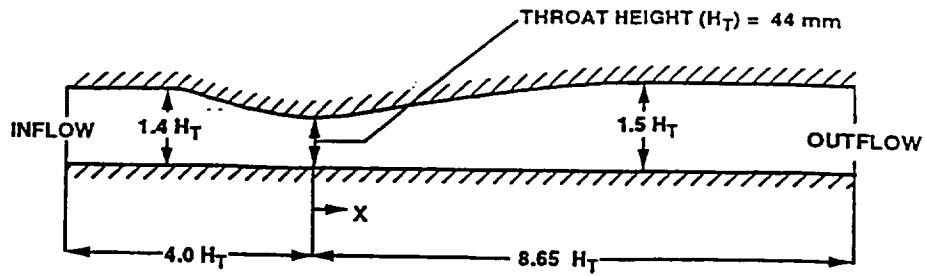
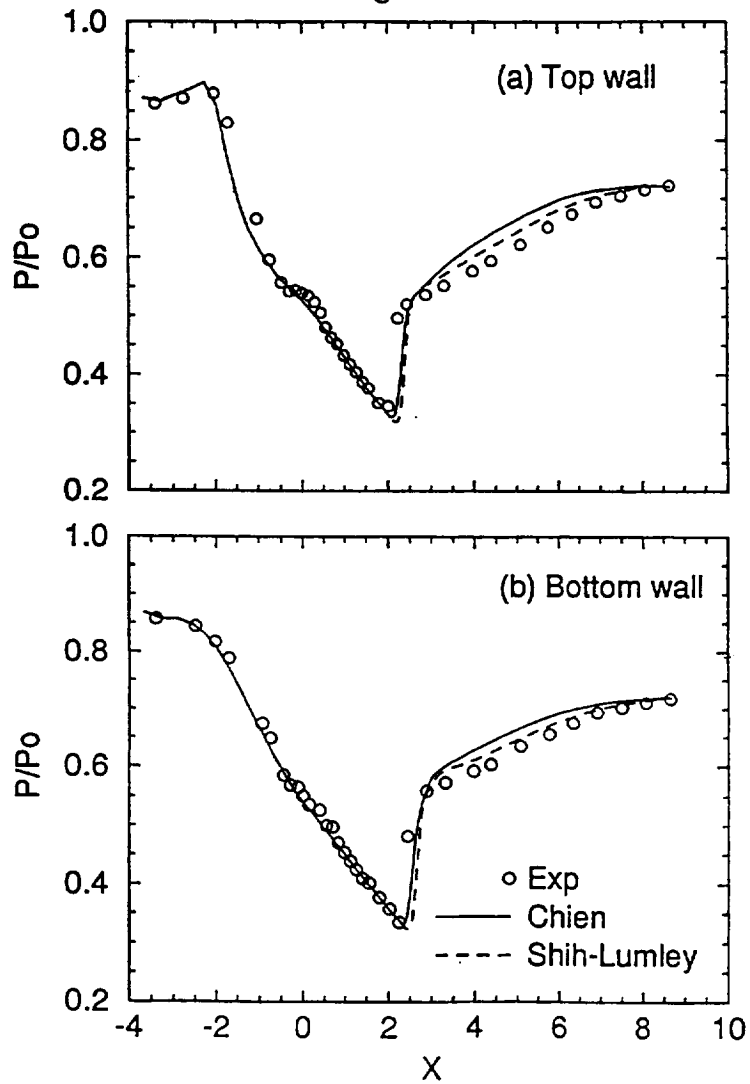


Fig. 3. Schematic of ejector nozzle test case.





Strong-shock case



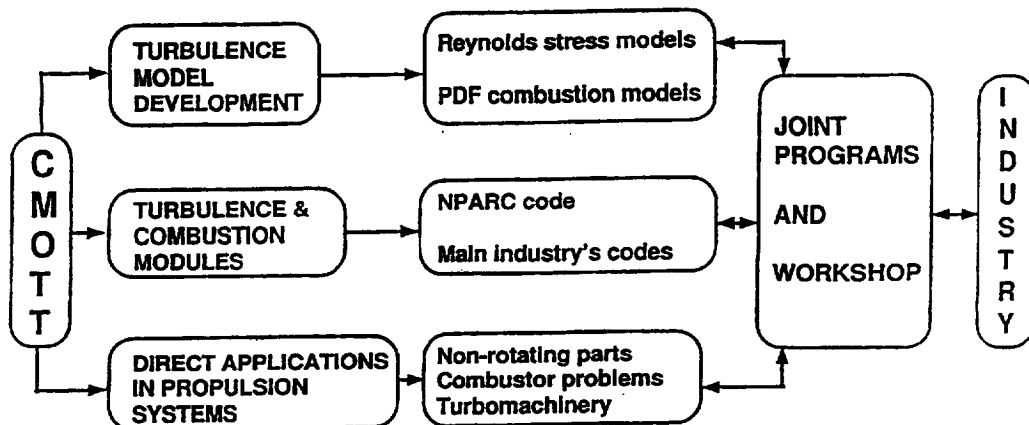
Collaboration with Industry and Technology Transfer

- Joint research programs with industry
 - ◇ Preliminary programs with engine companies and others have been initiated (GE, P&W, RocketDyne, Naval Research Laboratories)
 - ◇ Develop further joint research programs related to the industry's projects
- Industry-wide workshops will be a regular program (once every two years)
 - ◇ Release Lewis turbulence and combustion modules to industries
 - ◇ Discuss the needs of industry

Models developed at CMOTT

1. Isotropic eddy viscosity models
2. Reynolds stress & scalar flux algebraic equation models
3. Second moment transport equation models
4. Multiple-scale models for compressible turbulent flows
5. Bypass transition models
6. PDF models for turbulent reacting flows

PROGRAM SUMMARY



Isotropic eddy viscosity models

- Objective

- ◇ To examine the deficiencies of existing models
- ◇ To develop better eddy viscosity models

- Current status of existing $k - \varepsilon$ eddy viscosity models

$$-\overline{u_i u_j} = \nu_T (U_{i,j} + U_{j,i}) - \frac{2}{3} k \delta_{ij}, \quad \nu_T = C_\mu f_\mu \frac{k^2}{\varepsilon}$$

$$\frac{Dk}{Dt} = T^{(k)} + P^{(k)} - \varepsilon + \text{W.C.}, \quad \frac{D\varepsilon}{Dt} = T^{(\varepsilon)} + P^{(\varepsilon)} - D^{(\varepsilon)} + \text{W.C.}$$

- ◇ They are not tensorially invariant due to $f_\mu(y^+)$, $\text{W.C.}(y^+)$
- ◇ Model constants are not consistent for flows with and without wall
- ◇ Normal stresses may violate realizability
- ◇ Do not work very well for flows with pressure gradients

- Development of a Galilean-, tensorially invariant, realizable, $k - \varepsilon$ model

- ◇ New damping function $f_\mu(k/S\nu)$ is proposed to remove the dependence on y
- ◇ New dissipation ε equation is introduced to give better response to pressure gradients
- ◇ Consistent model coefficients for all flows
- ◇ Realizability of the normal stresses is guaranteed
- ◇ Modified wall function for cases with pressure gradients

- CMOTT $k - \varepsilon$ eddy viscosity model

$$-\overline{u_i u_j} = \nu_T (U_{i,j} - U_{j,i}) - \frac{2}{3} k \delta_{ij}, \quad \nu_T = C_\mu f_\mu \frac{k^2}{\varepsilon}$$

$$\frac{Dk}{Dt} = T_k + P_k - \varepsilon$$

$$\frac{D\varepsilon}{Dt} = T_\varepsilon + C_1 f_1 S \varepsilon - C_2 \frac{\varepsilon^2}{k + \sqrt{\nu \varepsilon}} + f_\phi \Phi$$

- ◇ f_μ, f_1, f_ϕ are functions of $R = k/S\nu$, which is tensorially invariant
- ◇ $C_\mu = \frac{1}{A_0 + A_* U_* k/\varepsilon}$, which ensures realizability for normal stresses
- ◇ Φ represents the effect of inhomogeneity

$$\Phi = b_1 \nabla k \cdot \nabla k + b_2 \frac{k^2}{\varepsilon} \nabla S \cdot \nabla k + b_3 \frac{k^4}{\varepsilon^2} \nabla S \cdot \nabla S$$

- Validation

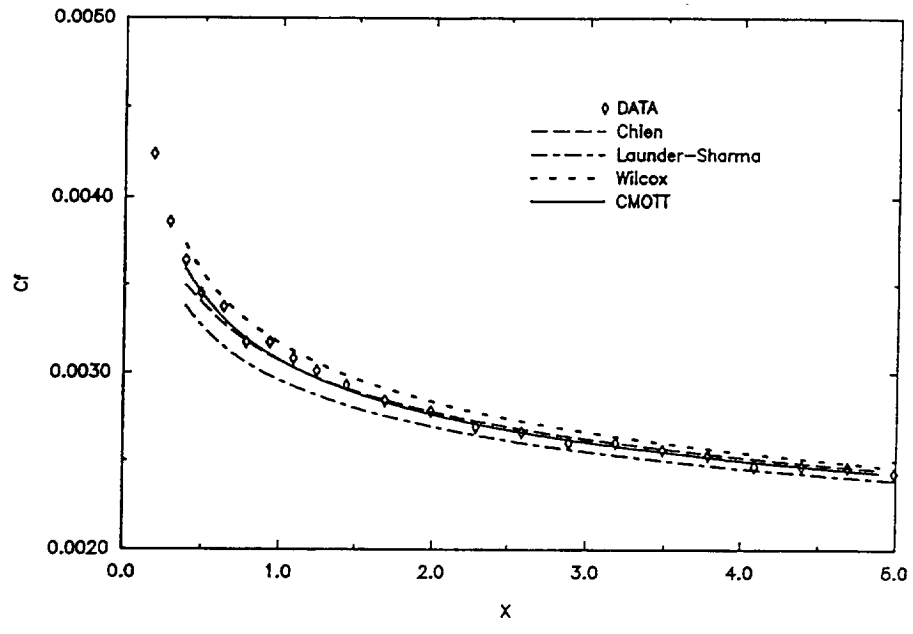
Flows:

- ◇ Channel flows
- ◇ Boundary layer flows with and without pressure gradients
- ◇ Planar jet, round jet and mixing layer
- ◇ Backward-facing step flows
- ◇ Complex flows related to industrial applications

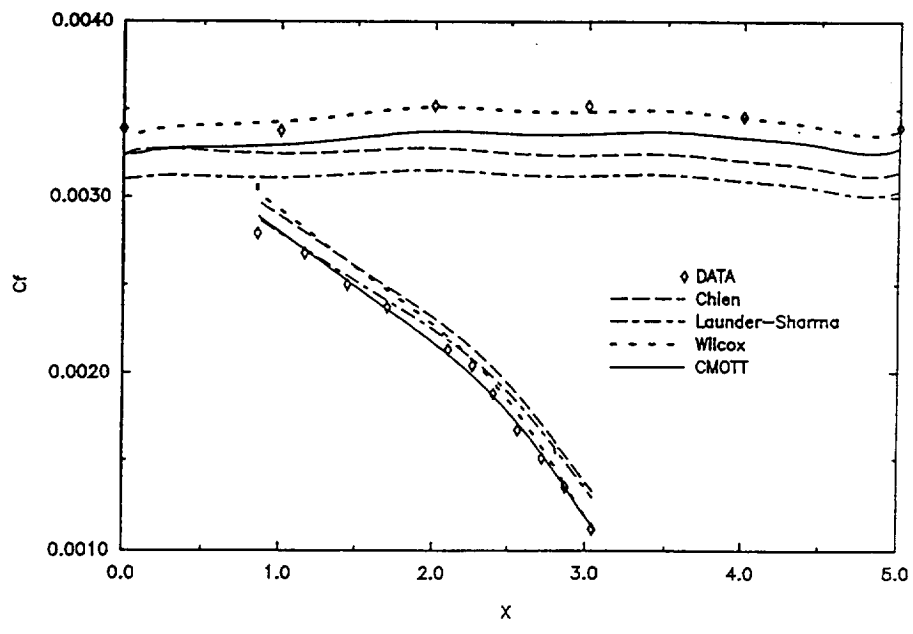
Models:

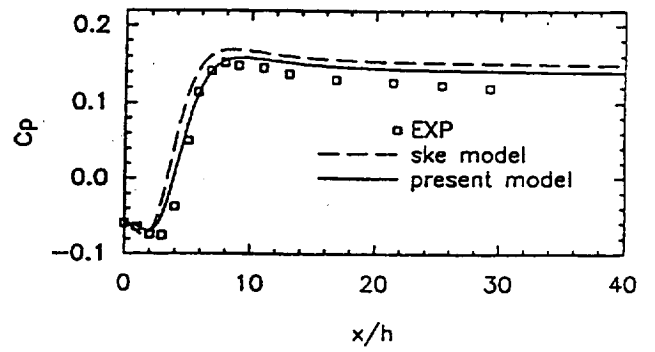
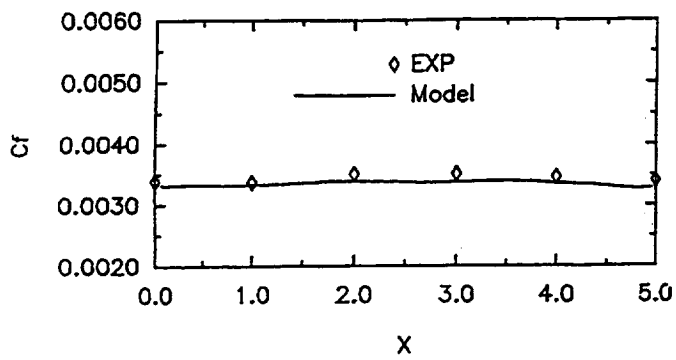
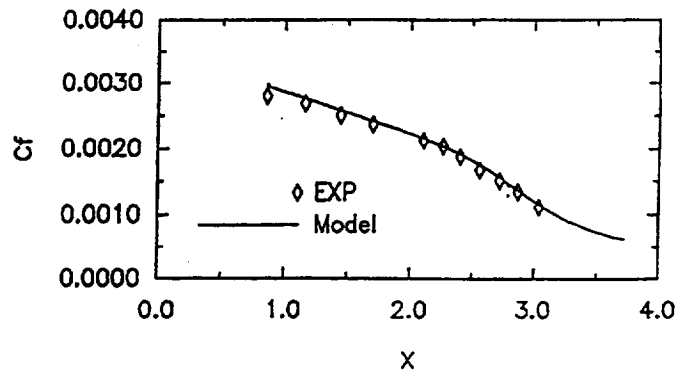
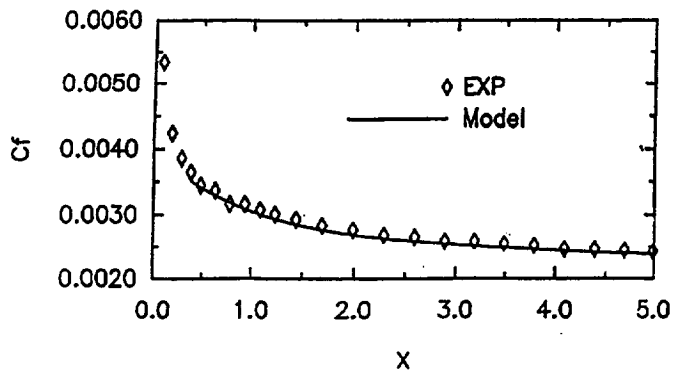
- ◇ Launder-Sharma, Lam-Bremhorst, Chien, Nagano-Hishida, ...
- ◇ $k - \omega$ model (Wilcox)
- ◇ CMOTT $k - \varepsilon$ model

Turbulent Boundary Layer



Turbulent Boundary Layer

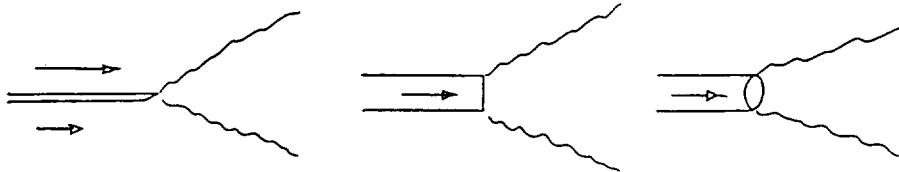




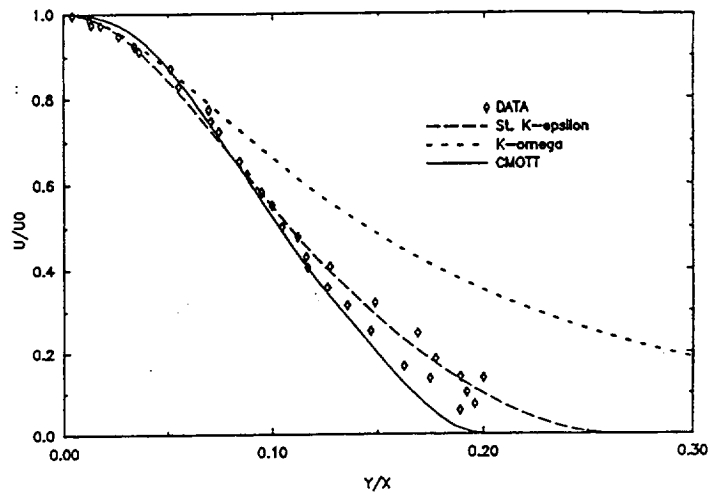
Present model with the modified wall function

Spreading Rate of Free Shear Flows

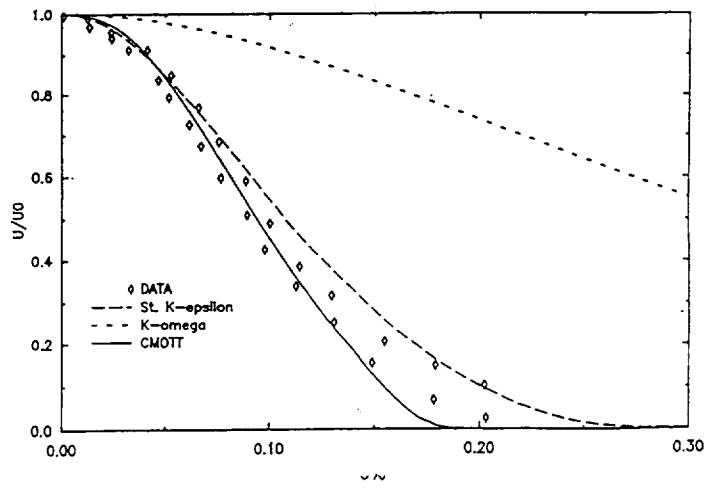
| | exp. | st. $k - \epsilon$ | Chien | $k - \omega$ | CMOTT |
|--------------|-------------|--------------------|-------|--------------|-------|
| Planar Jet | 0.10-0.11 | 0.108 | 0.098 | 0.14* | 0.102 |
| Round Jet | 0.085-0.095 | 0.116 | 0.104 | 0.32* | 0.095 |
| Mixing Layer | 0.13-0.17 | 0.152 | 0.152 | 0.16* | 0.154 |



Planar Jet



Round Jet



Algebraic Reynolds stress models

- Objective

- ◇ To examine the deficiencies of existing ARS models
- ◇ To develop better ARS models

- Current status of ARS models

- ◇ Second-order closure based ARS models (Rodi, 1980)

$$\frac{\overline{u_i u_j}}{k} (P - \varepsilon) = -\overline{u_i u_k} U_{j,k} - \overline{u_j u_k} U_{i,k} - \frac{1}{\rho} (\overline{p_{,i} u_j} + \overline{p_{,j} u_i}) - 2\nu \overline{u_{i,k} u_{j,k}}$$

Comments:

- * Assumption: $\overline{u_i u_j} / k = \text{Const.}$, $(\overline{u_i u_j u_k})_{,k} = (\overline{k u_i})_{,i} = 0$
- * Numerical difficulties
- ◇ Pope's explicit ARS model (2-D flows), Taulbee's ARS model (3-D), Gatski and Speziale's ARS model
- ◇ Other methods: RNG, DIA and invariant theory

- General constitutive relations from invariant theory

$$\begin{aligned}
\overline{u_i u_j} &= \frac{2}{3} k \delta_{ij} + 2a_2 \frac{K^2}{\varepsilon} (U_{i,j} + U_{j,i} - \frac{2}{3} U_{i,i} \delta_{ij}) + 2a_4 \frac{K^3}{\varepsilon^2} (U_{i,j}^2 + U_{j,i}^2 - \frac{2}{3} \Pi_1 \delta_{ij}) \\
&+ 2a_6 \frac{K^3}{\varepsilon^2} (U_{i,k} U_{j,k} - \frac{1}{3} \Pi_2 \delta_{ij}) + 2a_7 \frac{K^3}{\varepsilon^2} (U_{k,i} U_{k,j} - \frac{1}{3} \Pi_2 \delta_{ij}) \\
&+ 2a_8 \frac{K^4}{\varepsilon^3} (U_{i,k} U_{j,k}^2 + U_{i,k}^2 U_{j,k} - \frac{2}{3} \Pi_3 \delta_{ij}) + 2a_{10} \frac{K^4}{\varepsilon^3} (U_{k,i} U_{k,j}^2 + U_{k,j} U_{k,i}^2 - \frac{2}{3} \Pi_3 \delta_{ij}) \\
&+ 2a_{12} \frac{K^5}{\varepsilon^4} (U_{i,k}^2 U_{j,k}^2 - \frac{1}{3} \Pi_4 \delta_{ij}) + 2a_{13} \frac{K^5}{\varepsilon^4} (U_{k,i}^2 U_{k,j}^2 - \frac{1}{3} \Pi_4 \delta_{ij}) \\
&+ 2a_{14} \frac{K^5}{\varepsilon^4} (U_{i,k} U_{l,k} U_{l,j}^2 + U_{j,k} U_{l,k} U_{l,i}^2 - \frac{2}{3} \Pi_5 \delta_{ij}) \\
&+ 2a_{16} \frac{K^6}{\varepsilon^5} (U_{i,k} U_{l,k}^2 U_{l,j}^2 + U_{j,k} U_{l,k}^2 U_{l,i}^2 - \frac{2}{3} \Pi_6 \delta_{ij}) \\
&+ 2a_{18} \frac{K^7}{\varepsilon^6} (U_{i,k} U_{l,k} U_{l,m}^2 U_{j,m}^2 + U_{j,k} U_{l,k} U_{l,m}^2 U_{i,m}^2 - \frac{2}{3} \Pi_7 \delta_{ij})
\end{aligned}$$

- RDT and realizability constraints (Reynolds, Lumley)
- CMOTT algebraic Reynolds stress model

$$\overline{u_i u_j} = \frac{2}{3} k \delta_{ij} - C_\mu \frac{k^2}{\varepsilon} 2S_{ij}^* + 2C_2 \frac{k^3}{\varepsilon^2} (-S_{ik}^* \Omega_{kj}^* + \Omega_{ik}^* S_{kj}^*)$$

$$k_{,t} + U_j k_{,j} = [(\nu + \frac{\nu_t}{\sigma_k}) k_{,j}]_{,j} - \overline{u_i u_j} U_{i,j} - \varepsilon$$

$$\varepsilon_{,t} + U_j \varepsilon_{,j} = [(\nu + \frac{\nu_t}{\sigma_\varepsilon}) \varepsilon_{,j}]_{,j} - C_{e1} \frac{\varepsilon}{k} \overline{u_i u_j} U_{i,j} - C_{e2} \frac{\varepsilon^2}{k}$$

where

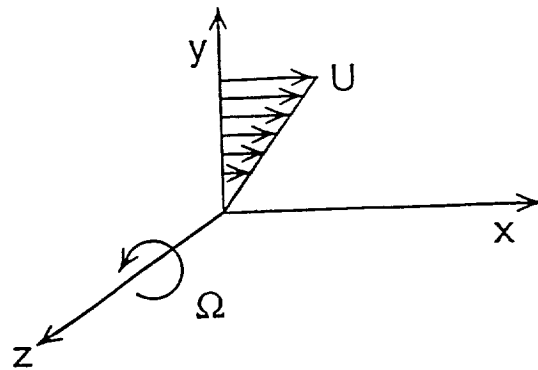
$$C_\mu = \frac{1}{A_0 + A_s^* \frac{U^* k}{\varepsilon}}, \quad C_2 = \frac{\sqrt{1 - 9C_\mu^2 (\frac{S^* k}{\varepsilon})^2}}{C_0 + 6 \frac{S^* k \Omega^* k}{\varepsilon}}$$

$$\nu_t = C_\mu \frac{k^2}{\varepsilon}, \quad A_0 = 6.5, \quad C_0 = 1.0$$

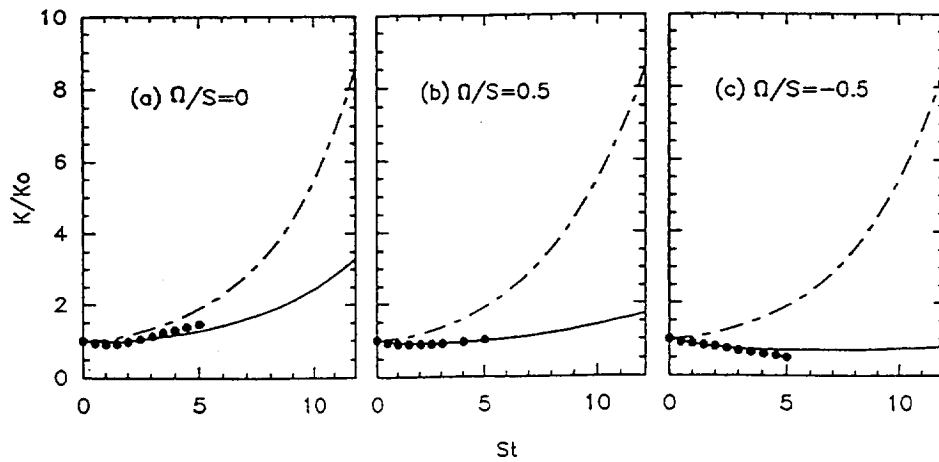
$$C_{e1} = 1.44, \quad C_{e2} = 1.92, \quad \sigma_k = 1, \quad \sigma_\varepsilon = 1.3$$

- Validation

- ◇ Rotating homogeneous shear flows
- ◇ Backward-facing step flows
- ◇ Confined jets
- ◇ Complex flows related to industrial applications



Configuration of rotating homogeneous shear flow



Evolution of turbulent kinetic energy with time.
 — : present model; - - - : SKE; • : LES

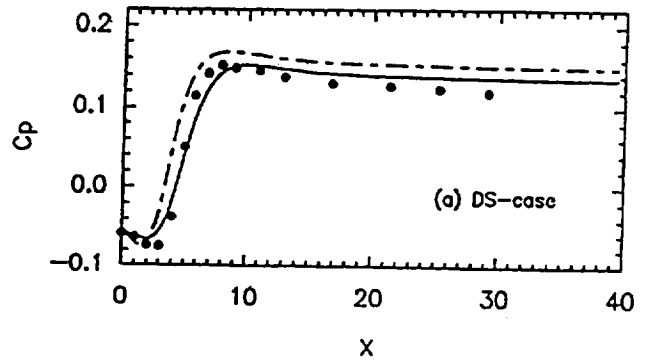
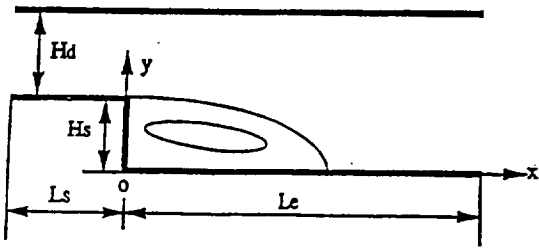
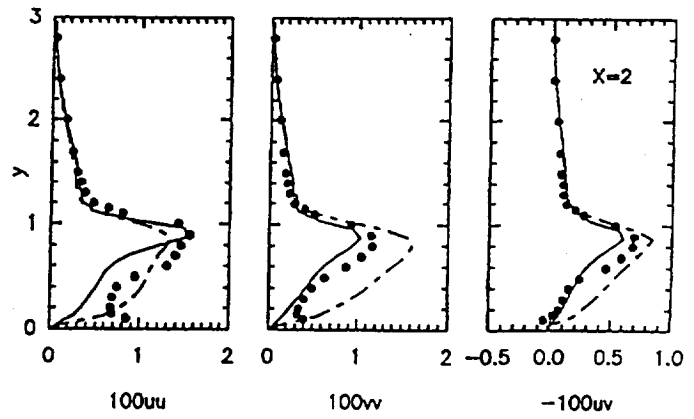
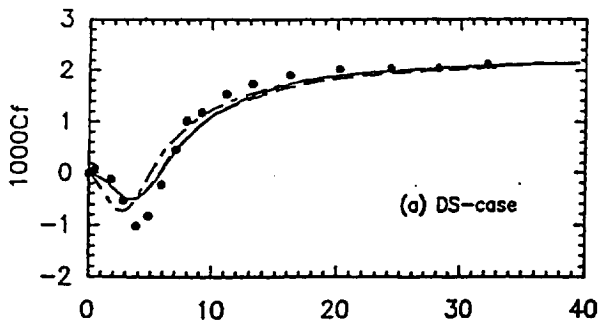
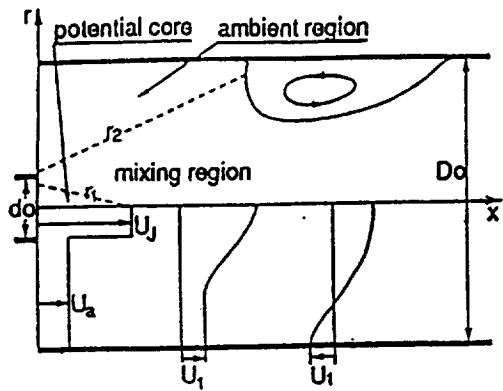


Table Comparison of the reattachment points

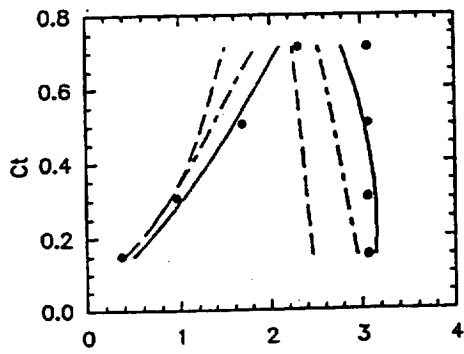
| Case | measurement | SKE | PRESENT |
|------|-------------|------|---------|
| DS | 6.1 | 4.99 | 5.82 |
| KKJ | 7 ± 0.5 | 6.35 | 7.35 |



— : present model; - - - : SKE; • : EXP

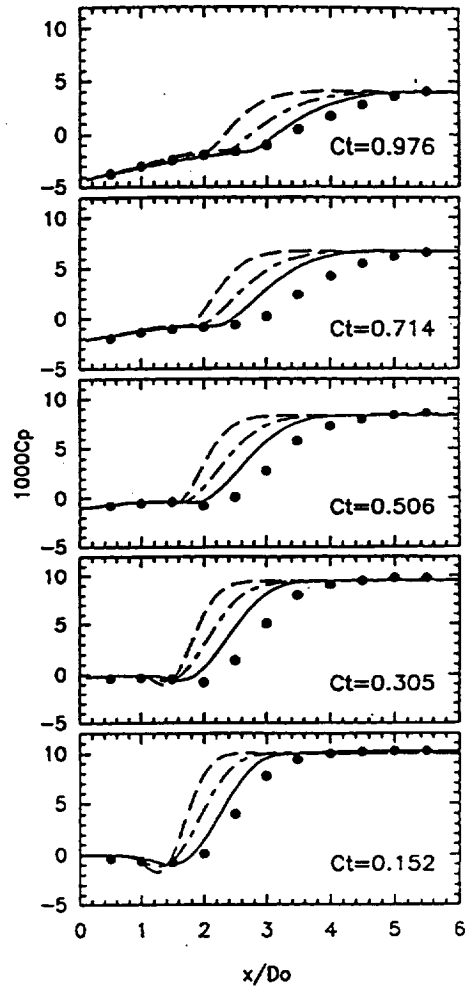


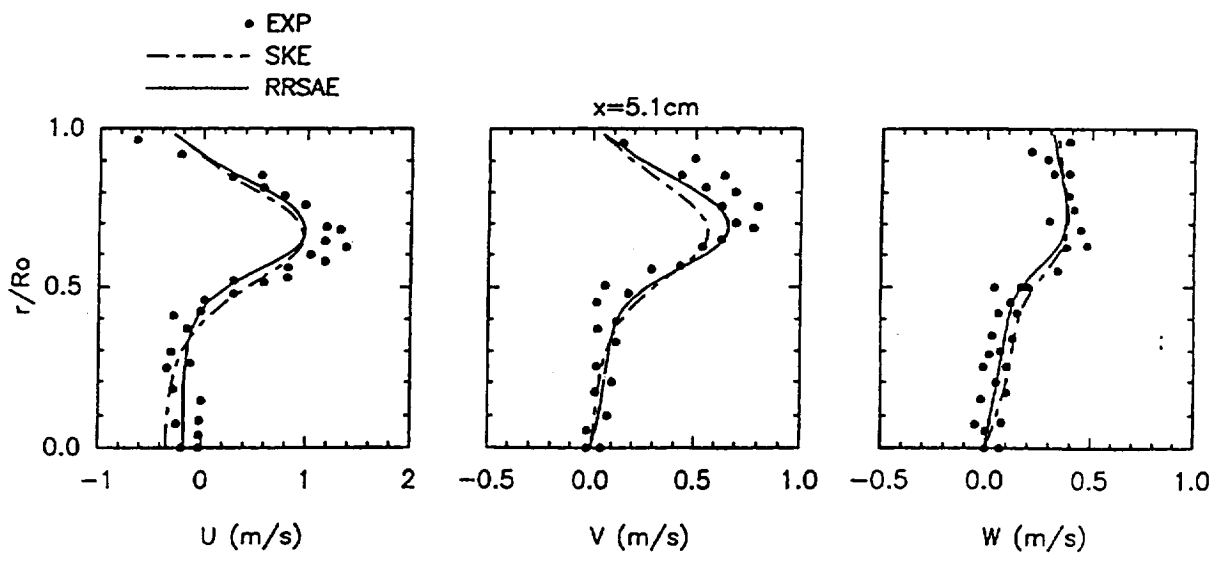
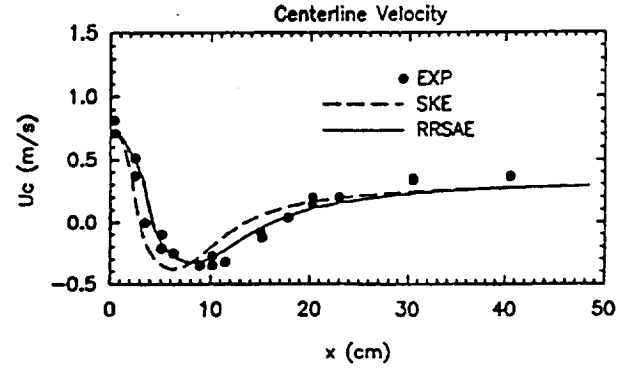
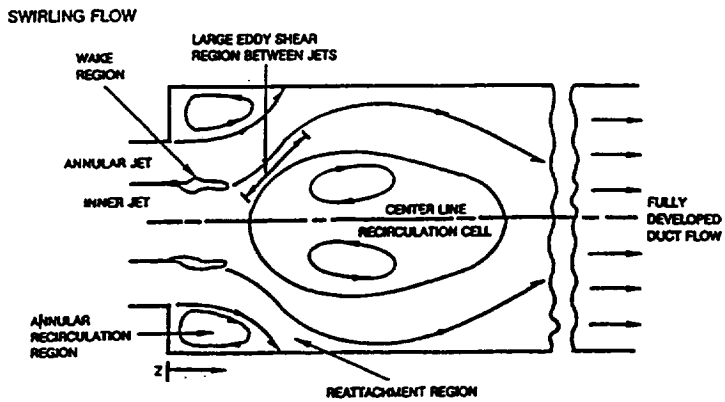
Flow configuration and notations



Separation and reattachment points

- EXP
- SKE
- RNG
- RRSAE





Scalar turbulence model

- Objective

To improve the predictive capability of current scalar turbulence $(\overline{\theta^2} - \epsilon_\theta)$ models

- ◇ A new scalar flux constitutive relation
- ◇ A new scalar dissipation rate model equation

$$\overline{u_i \theta} = -C_\lambda \frac{k^2}{\epsilon} \left(\frac{2}{r}\right)^{1/2} \Theta_{,i} + \frac{k^3}{\epsilon^2} \left(\frac{2}{r}\right)^{1/2} (a_2 U_{i,j} + a_3 U_{j,i}) \Theta_{,j}$$

$$U_j \frac{\partial \overline{\theta^2}}{\partial x_j} = \left(\frac{\alpha_T}{\sigma_t} \overline{\theta^2}_{,j}\right)_{,j} - 2\overline{u_i \theta} \frac{\partial \Theta}{\partial x_i} - 2\epsilon_\theta$$

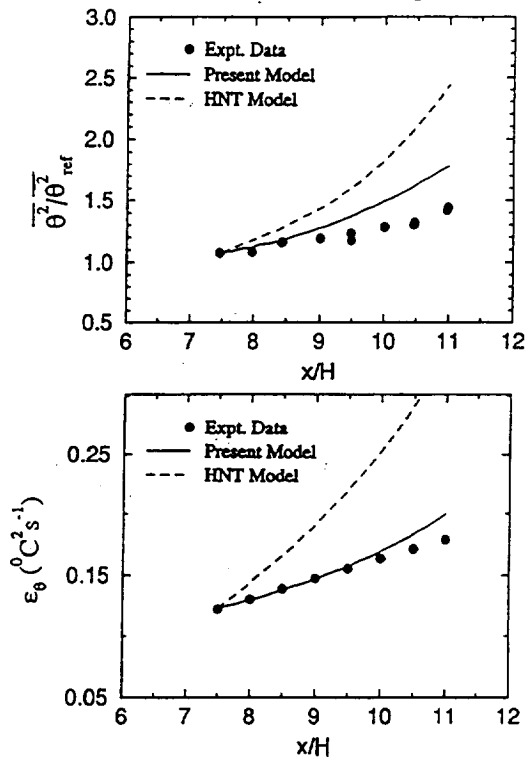
$$U_j \frac{\partial \epsilon_\theta}{\partial x_j} = \left(\frac{\alpha_T}{\sigma_\phi} \epsilon_{\theta,j}\right)_{,j} + C_{\theta 1} \epsilon_\theta S + C_{\theta 2} \sqrt{\frac{\epsilon_\theta \epsilon}{Pr}} S_T - C_{\theta 3} \frac{\epsilon_\theta \epsilon}{k}$$

$$C_\lambda = \frac{(2 + 2r + 0.5r^2)}{26 + 3.2\eta^2 + 2\xi^2}$$

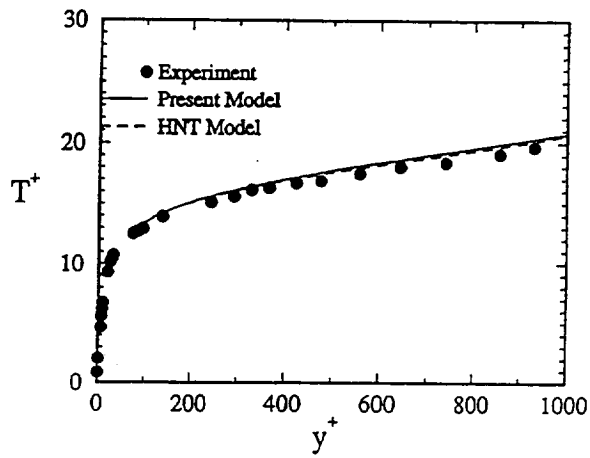
$$S_T = \sqrt{\Theta_{,i} \Theta_{,i}}, \quad S = \sqrt{2S_{ij} S_{ij}}, \quad \eta = Sk/\epsilon, \quad \xi = \frac{k}{\epsilon} \left(\frac{k}{\theta^2}\right)^{1/2} S_T, \quad r = \frac{2k}{\epsilon} \frac{\epsilon_\theta}{\theta^2}$$

$$C_{\theta 1} = C_1 - 0.13, \quad C_{\theta 2} = 0.63, \quad C_{\theta 3} = C_2 - 1, \quad \sigma_t = 1.0, \quad \sigma_\phi = 1.8$$

Tavoularis & Corrsin Expt (1981)



Flat plate boundary layer with constant surface temperature



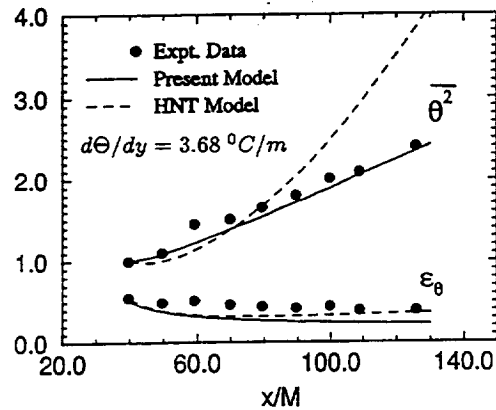
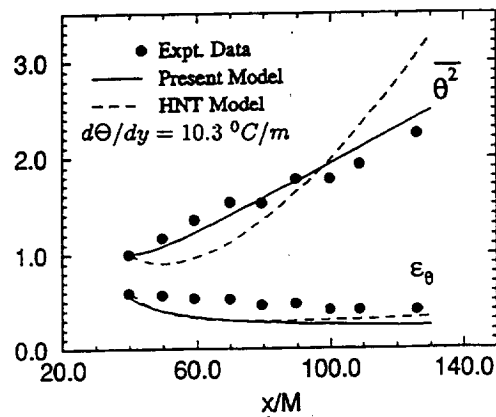
- Validation

- ◇ Homogeneous turbulence subjected to $\partial\Theta/\partial y$
- ◇ Homogeneous turbulence subjected to $\partial U/\partial y, \partial\Theta/\partial y$
- ◇ Flat plate boundary layer with constant surface temperature

- Work in progress

- ◇ Model assessment for different scalar boundary conditions
- ◇ Model extension for integration to the wall

Sirivat & Warhaft Expt (1983)



Second Order Closure Models

$$\frac{D\overline{u_i u_j}}{Dt} = T_{ij} + P_{ij} + \Pi_{ij}^{\text{Rapid}} + \Pi_{ij}^{\text{Return}} - \frac{2}{3}\varepsilon\delta_{ij}$$

- Objective

- ◇ To assess existing models
- ◇ To find the direction of improving closure models

- Basic model forms

$$\Pi_{ij}^{\text{Rapid}} = F_{ij}(S_{ij}, \overline{u_i u_j}),$$

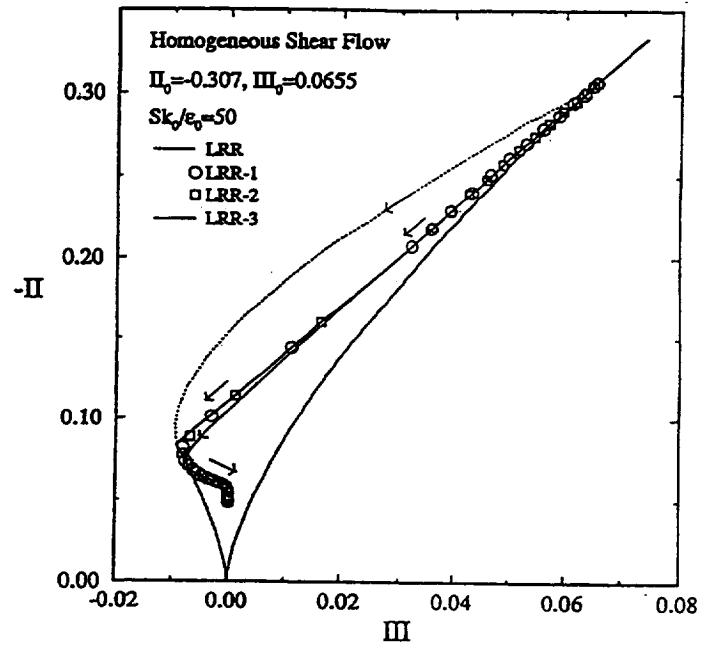
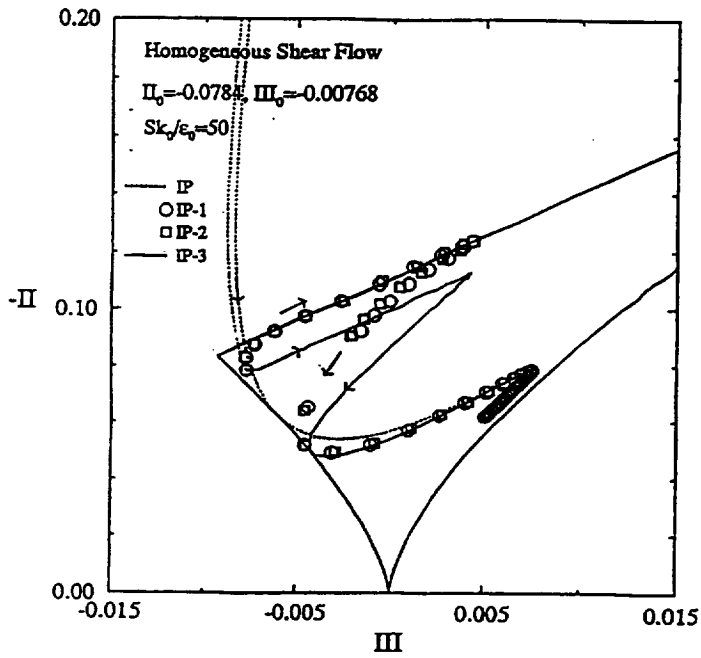
$$\Pi_{ij}^{\text{Return}} = F_{ij}(\overline{u_i u_j}, \nu, k, \varepsilon),$$

$$T_{ij} = F_{ij}((\overline{u_i u_j})_{,k}, k, \varepsilon)$$

- General comments on second order closures:

- ◇ The model, Π_{ij}^{Rapid} , is relatively well developed compared with other terms
- ◇ The model, Π_{ij}^{Return} , is least developed
- ◇ A Galilean and tensorially invariant second order closure model has not been well developed yet
- ◇ All models have large errors near the wall, especially in the buffer layer; therefore, for engineering application, the wall function approach is suggested at the present time

- Application of realizability to IP and LRR models



Multiple scale k - ϵ model

- **Objective:**

- ◇ To consider the effect of a non-equilibrium energy spectrum on eddy viscosity for compressible turbulence

- **Approach:**

- ◇ Use multiple scale concept introduced by

□ Large-Scale

$$\bar{\rho} \frac{D \tilde{k}_p}{Dt} = \frac{\partial}{\partial y} \left[\left(\bar{\mu} + \frac{\mu_T}{\sigma_{\tilde{k}_p}} \right) \frac{\partial \tilde{k}_p}{\partial y} \right] + \mu_T \left(\frac{\partial \tilde{u}}{\partial y} \right)^2 - \bar{\rho} \tilde{\epsilon}_p + \text{fc}_1$$

$$\bar{\rho} \frac{D \tilde{\epsilon}_p}{Dt} = \frac{\partial}{\partial y} \left[\left(\bar{\mu} + \frac{\mu_T}{\sigma_{\tilde{\epsilon}_p}} \right) \frac{\partial \tilde{\epsilon}_p}{\partial y} \right] + C_{p1} \frac{\tilde{\epsilon}_p}{k_p} \mu_T \left(\frac{\partial \tilde{u}}{\partial y} \right)^2 - C_{p2} \bar{\rho} \frac{\tilde{\epsilon}_p^2}{k_p} + \text{fc}_2$$

- fc_1 - exchanges between the turbulent kinetic energy and internal energy
- fc_2 - increased spectral energy transfer due to compressibility effects

□ Small Scale

$$\bar{\rho} \frac{D \tilde{k}_t}{Dt} = \frac{\partial}{\partial y} \left[\left(\bar{\mu} + \frac{\mu_T}{\sigma_{\tilde{k}_t}} \right) \frac{\partial \tilde{k}_t}{\partial y} \right] + \bar{\rho} \tilde{\epsilon}_p - \bar{\rho} \tilde{\epsilon}_t$$

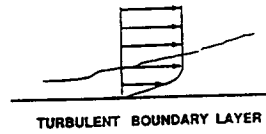
$$\bar{\rho} \frac{D \tilde{\epsilon}_t}{Dt} = \frac{\partial}{\partial y} \left[\left(\bar{\mu} + \frac{\mu_T}{\sigma_{\tilde{\epsilon}_t}} \right) \frac{\partial \tilde{\epsilon}_t}{\partial y} \right] + C_{t1} \bar{\rho} \frac{\tilde{\epsilon}_t \tilde{\epsilon}_p}{\tilde{k}_t} - C_{t2} \bar{\rho} \frac{\tilde{\epsilon}_t^2}{\tilde{k}_t}$$

□ Eddy Viscosity

$$\mu_T \approx \bar{\rho} u l \approx \bar{\rho} (\tilde{k}_p + \tilde{k}_t)^{\frac{1}{2}} \frac{(\tilde{k}_p + \tilde{k}_t)^{\frac{3}{2}}}{\tilde{\epsilon}_p}$$

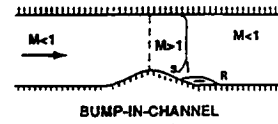
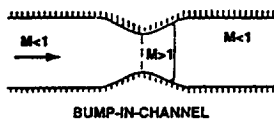
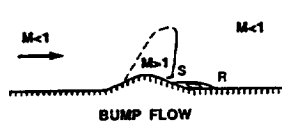
Model Evaluation

- Turbulent Shear Flow

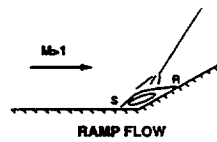
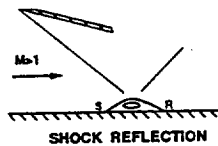


- Shock/Turbulent-Boundary-Layer Interactions

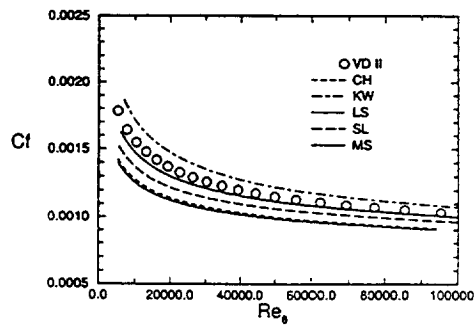
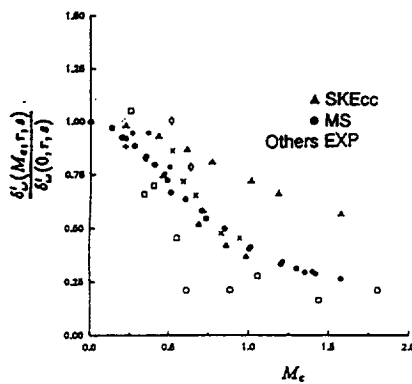
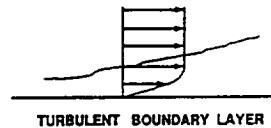
- ◇ transonic flow



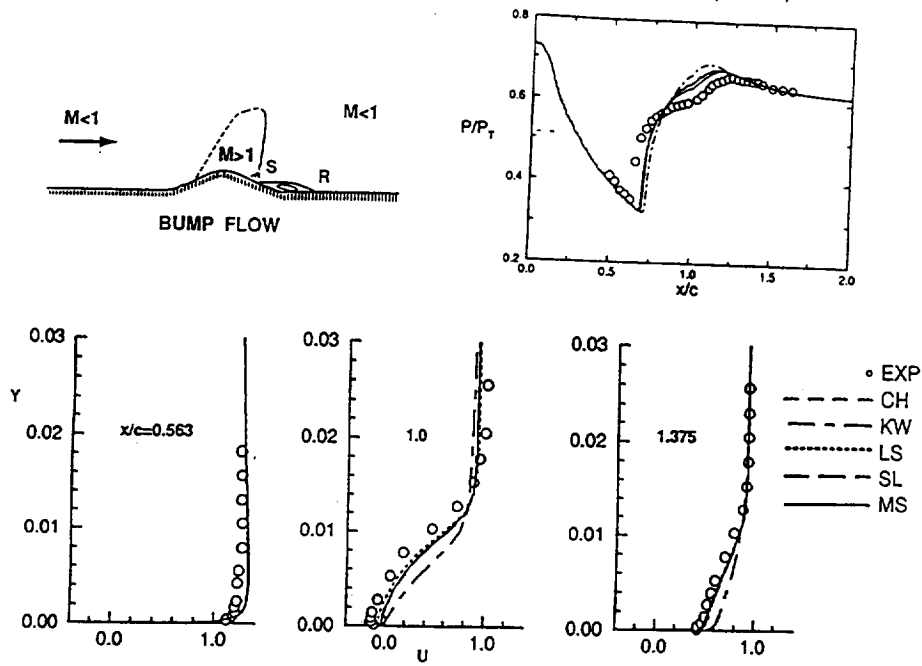
- ◇ supersonic flow



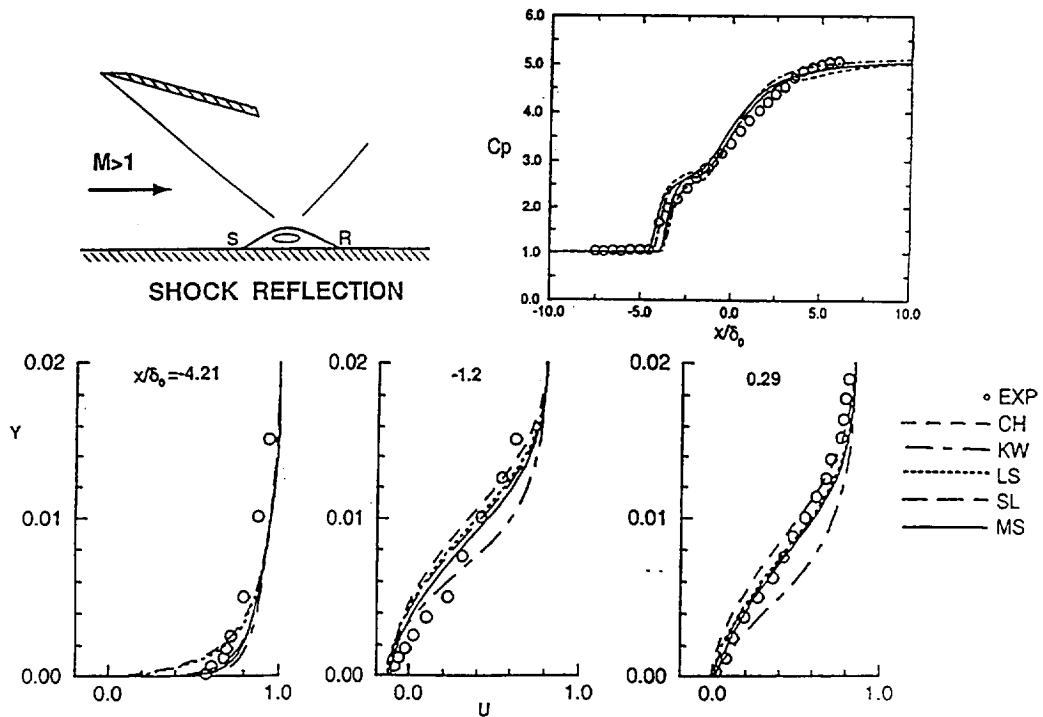
Compressible Turbulent Shear Flow



Flow over a Bump—Bachalo and Johnson (1979)



Shock Reflection—Reda *et. al* (1973-1977)



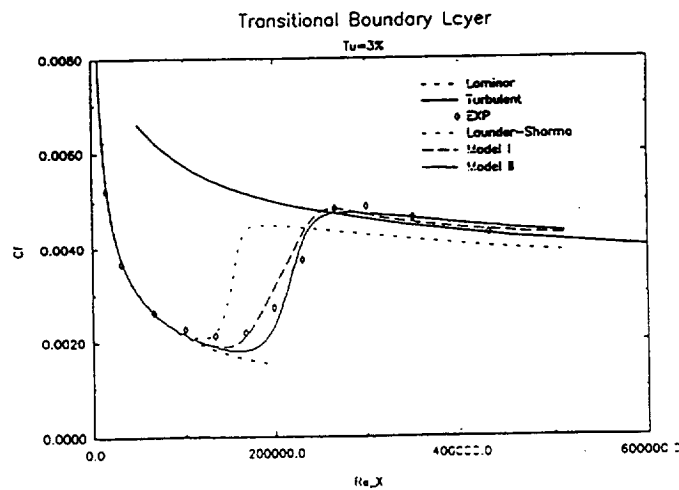
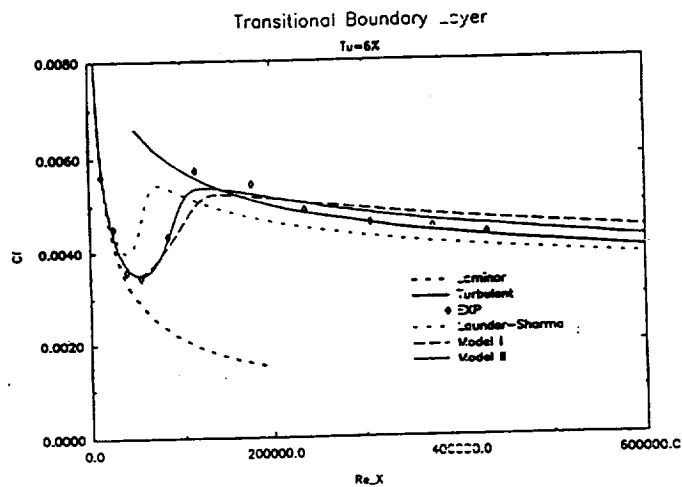
Bypass transition models

- Objective:

- ◊ Develop transition models for flows with free stream turbulence

- Approach:

- ◊ Using $K-\epsilon$ model as the base model
- ◊ Introduce effective intermittency to either the eddy viscosity or the $k-\epsilon$ model equations



PDF modeling of turbulent reacting flows

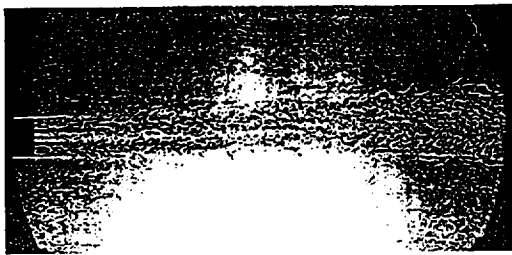
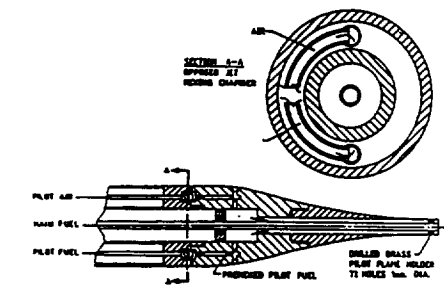
- Objective:

- ◇ Develop models that can accurately simulate finite chemical reactions in turbulent flows.
- ◇ Develop and validate independent PDF models.
- ◇ Technology transfer.

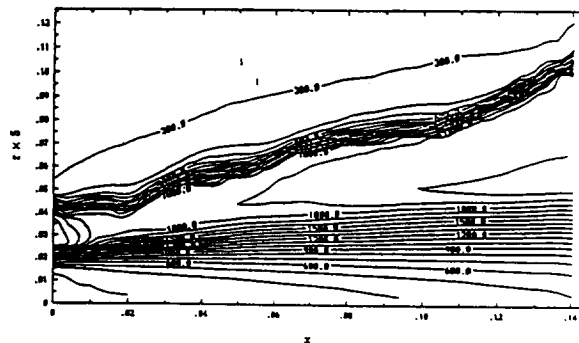
- Approach:

- ◇ Joint pdf for scalar compositions.
- ◇ Moment closure schemes for velocity field.
- ◇ Develop hybrid solver consisting of Monte Carlo method and finite-difference/finite-volume method.

BURNER JET



Without PDF Method



With PDF Method. (CMOTT)

