# DESIGN PREDICTION FOR LONG TERM STRESS RUPTURE SERVICE <br> OF COMPOSITE PRESSURE VESSELS ${ }^{\dagger}$ 

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## SUMMARY

Extensive stress rupture studies on glass composites and Kevlar composites were conducted by the Lawrence Radiation Laboratory beginning in the late 60s and extending to about 8 years in some cases. Some of the data from these studies published over the years were incomplete or were tainted by spurious failures, such as grip slippage. These data have been carefully resurveyed by cognizant staff. Updated verified data sets have been defined for both fiberglass and Kevlar composite strand test specimens. These updated data are analyzed in this report by a convenient form of the bivariate Weibull distribution, to establish a consistent set of design prediction charts that may be used as a conservative basis for predicting the stress rupture life of composite pressure vessels.

The updated glass composite data exhibit an invariant Weibull modulus with lifetime. The data are analyzed in terms of homologous service load (referenced to the observed median strength). The equations relating life, homologous load, and probability are given, and corresponding design prediction charts are presented. A similar approach is taken for Kevlar composites, where the updated strand data do show a turndown tendency at long life accompanied by a corresponding change (increase) of the Weibull modulus. This turndown characteristic is not present in stress rupture test data of Kevlar pressure vessels. A modification of the stress rupture equations is presented to incorporate a latent, but limited, strength drop, and design prediction charts are presented that incorporate such behavior.

The methods presented utilize Cartesian plots of the probability distributions (which are a more natural display for the design engineer), based on median normalized data that are independent of statistical parameters and are readily defined for any set of test data. A technique is shown for estimating the Weibull modulus from each observed value. The design prediction equations and the corresponding design prediction charts can be set up to provide selected levels of conservatism in those regions where data are sparse or unavailable. Design values based on these single-end data should be conservative for multiple-end roving and massive composite structures like those on pressure vessels.
$\dagger$ This work was presented at the Ninth DoD/NASA/FAA Conference on Fibrous Composites in Structural Design, 4-7 November, Lake Tahoe, Nevada. Funding for this effort was processed through Contract F04701-88-C-0089 under an interagency agreement from the U.S. Department of the Navy.


## INTRODUCTION

The purpose of this report is to provide a useful engineering tool for estimating stress rupture life of S-glass and Kevlar composites subject to long-term tensile stress. The data base is drawn from the extensive 10 year period test program conducted on S-glass and Kevlar singleend composite strands at the Lawrence Livermore National Laboratory (LLNL).

Previously published results of S-glass exhibited a turndown characteristic at long times. This characteristic, when analyzed by a quadratic maximum likelihood method (Ref. 1), produced unrealistic life prediction extrapolation and did not yield realistic and useful long life predictions. The LLNL S-glass data base has recently been subject to a searching re-examination by the cognizant staff (Refs. 2,3) to eliminate spurious failures such as grip pullouts. These updated data are analyzed here by the Weibull model and tabulated in Appendix A. These data do not exhibit a turndown and are well approximated by invariant Weibull moduli for the life and strength distributions. It should be noted that the stress rupture analysis will produce straight line probability contours on log stress vs log life coordinates if the Weibull moduli for life and strength are invariant with time. The converse is also true.

Convenient analytical methods based on the Weibull model are presented. These methods allow an estimation of the Weibull modulus based on each ranked observation and allow estimation of the population median life based on partial (censored or incomplete) data. The methods of analysis are simple and straightforward, and may be applied in the same fashion to other stress rupture data. The Weibull modulus for S-glass life is $b=0.9$, based on the updated stress rupture data presented. LLNL measurements of S-glass composite strand strength show scatter with a representative coefficient of variation $\mathrm{CV}=0.48+$, corresponding with $\mathrm{m}=24.9$, which is used in the tabulated data and design prediction. These parameters for S-glass are used to produce the design chart of Figure 1.

The LLNL Kevlar composite strand data (Ref. 2) are presented in Appendix B. These data show a turndown trend. Kevlar spherical vessel data (Ref. 4) do not show this trend. The Weibull model was modified, as discussed later, to incorporate a strength degradation that develops later in time according to a first-order reaction rate (Ref. 5) with a specific time constant. This model predicts that within the time period where degradation is expected, the Weibull modulus for life should show an apparent increase. That is exactly what the Kevlar strand data show, and this modified form is used to construct a rational design chart that includes a partial degradation ( $f=0.3$ ) with a time constant $t c \approx 120,000$ hours. The Weibull modulus for Kevlar composite strand life is also $\mathrm{b}=0.9$, while the Weibull modulus for strength is $\mathrm{m} \approx 30$, reflecting a typically slightly lower scatter $(C V=0.04)$ for Kevlar composite strands than for the S-glass strands. The resulting design chart is shown in Figure 2. Over-plotted on this chart are the pressure vessel data, which do not exhibit the turndown and show greater life at the same homologous load.

Published data for carbon fiber composite strands are very sparse (Ref. 6) and extend to shorter times than the S-glass and Kevlar data. Figure 3 is a preliminary design chart, constructed using the indicated parameters, and should be useful for first-order life estimates of carbon composite pressure vessel stress rupture life.

## DISCUSSION

## The Bivariate Weibull Distribution

An expression for the stress rupture (Ref. 7) of a multifilament strand is given by

$$
S=\operatorname{Exp}-H\{R, t\}
$$

where H is a function of the applied load and time. Assuming the function to be separable and of
exponential (power-law) form, it can be expressed as a bivariate form of the well known Weibull distribution function (Ref. 7).

The two-parameter form of the Weibull distribution is given by

$$
S=\operatorname{Exp}-\left\{X / X_{o}\right\}^{m}
$$

As a matter of convenience, the function is normalized here to the median value Xm , or to any other percentile $\mathrm{Sr}, \mathrm{Xr}$, which may be appropriate, as in the case of a partial sample where the median has not yet been reached in the experimental program and is unknown:

$$
S=\operatorname{Exp}-\left\{\ln \left(1 / S_{r}\right)\left(X / X_{r}\right)^{\mathrm{m}}\right\}
$$

If the median is known, then $\mathrm{S}_{\mathrm{r}}=0.5$ and the distribution scale parameter becomes the median. From a design engineer's point of view, this is intuitively more meaningful than the Weibull scale parameter Xo, which corresponds with the $63 \%$ quantile. Forms of the Weibull distribution that use mean normalized data are more awkward because the quantile of the mean value involves a gamma function of the Weibull modulus (shape parameter):

$$
\mathrm{S}=\operatorname{Exp}-\left\{\Gamma(1+1 / \mathrm{m})\left(\mathrm{X} / \mathrm{X}_{\mathrm{avg}}\right)\right\}^{\mathrm{m}}
$$

The bivariate Weibull form, normalized to median strength and life, is given by

$$
S=\operatorname{Exp}-\left\{\ln (2)(R m)^{\mathrm{m}}(\mathrm{t} / \mathrm{tm})^{\mathrm{b}}\right\}
$$

The symbol R is the homologous load referenced to the median value. This form is used in subsequent discussions for the analysis of the composite stress rupture data.

## From Filaments to Composite Structures

The susceptibility of single filaments to stress rupture is invariably more severe than the susceptibility of multifilament strands for several reasons. The filament surface is totally exposed to the surrounding environment; the access of environmental factors such as moisture, air, or radiation is not impeded; and access remains unimpeded throughout the exposure time. The filament failure under these influences is total, i.e., no load can be transmitted after filament failure. The situation is less severe for the multifilament strand, especially if twist is present. Within a multifilament strand, there is some inhibition of diffusion by geometric effects and by the gradual development of internal concentration gradients. Furthermore, individual filament breaks might not lead to load-carrying reduction because frictional coupling and twist act like the matrix of a composite strand. The composite strand is even less susceptible to individual filament rupture failure because the matrix encapsulates the filaments and can offer considerable protection from diffusion and effects of the environments. Consequently, stress rupture experiments show relatively early failure times and turndown characteristics for single filaments and bare strands as compared with composite strands.

When comparing lifetimes at equal homologous loads, still another factor enters that causes the multifilament strands to exhibit longer life. The prevailing stress within the constituent filaments is reduced because the median strength of a multifilament strand is expected to be less than the median strength of single filaments, and the discrepancy increases as the variability of filament strength increases. At the same homologous load, the average filament stress in massive composites is reduced from the single filament or single-end strand.

For these reasons, it is concluded that using single-end composite strands to establish design charts for massive composites, such as filament-wound pressure vessels, will provide an inherently conservative basis for estimation.

## ANALYTICAL FORMS

## Invariant Weibull Moduli

The median normalized bivariate Weibull equation (Ref. 7) relates load, life, and probability, as shown below:

$$
S=\operatorname{Exp}-\left\{\operatorname{Ln}(2)\left(\frac{\sigma}{\sigma_{\mathrm{m}}}\right)^{\mathrm{m}}\left(\frac{\mathrm{t}}{\mathrm{t}_{\mathrm{m}}}\right)^{\mathrm{b}}\right\}
$$

The equation is the power law form with the underlying linear log-log relation :

$$
\left(\frac{\sigma_{0}}{\sigma_{m}}\right)^{\mathrm{m}}\left(\frac{\mathrm{t}}{\mathrm{t}_{\mathrm{m}}}\right)^{\mathrm{b}}=\left\{\begin{array}{l}
\operatorname{Ln}\left(\frac{1}{S}\right) \\
\operatorname{Ln}(2)
\end{array}\right\}
$$

This relation defines a family of straight lines on $\log$-log coordinates in which the following proportionality holds (where R represents the load fraction of median strength):

$$
\mathrm{R}^{\mathrm{m}} \mathrm{t}^{\mathrm{b}} \quad \alpha\left\{\frac{\operatorname{Ln}\left(\frac{1}{\mathrm{~S}}\right)}{\operatorname{Ln}(2)}\right\}
$$

The preceding relations may be used as shown in the following steps to compute desired combinations of load fraction, $R$, life, $t$, and survival probability, $S$. The shape parameters (Weibull moduli) for strength, $m$, and for life, $b$, are determined by analysis of experimental strength and life distributions. For example, the LLNL glass data give $m=24.9$ and $b=0.9$. The procedure to construct the probability, load, life design chart follows.

1. Select a reference set of values from the data or design: Rr , tr , and Sr (usually $\mathrm{Sr}_{r}=0.5$ ). 2. Select the load and life Weibull moduli $m$ and $b$ to give desired conservatism for the design chart.
2. Compute the constant K

$$
K=\frac{R_{r}^{m} t_{\mathrm{r}}{ }^{\mathrm{b}} \operatorname{Ln}(2)}{\operatorname{Ln}\left(\frac{1}{S_{\mathrm{r}}}\right)}
$$

4. Define the general $R, t, S$ relation by

$$
\mathrm{R}^{\mathrm{m}} \mathrm{t}^{\mathrm{b}}=\left[\frac{\mathrm{KLn}\left(\frac{1}{S}\right)}{\operatorname{Ln}(2)}\right]
$$

5. Construct the design chart using the $\log -\log$ form $\left(\mathrm{S}_{\mathrm{r}}=0.5\right)$

$$
\log (R)=\log \left(R_{r}\right)-\frac{b}{m} \log \left(\frac{t}{t_{r}}\right)+\left(\frac{1}{m}\right) \log \left[\frac{\operatorname{Ln}\left(\frac{1}{S}\right)}{\operatorname{Ln}(2)}\right]
$$

This equation, representing time invariant Weibull moduli for both strength and life, is typified by the glass design chart of Figure 1.
6. Compute life for particular load fraction, R, and survival probability, S, by

$$
t=\left\{\frac{\mathrm{K}}{\mathrm{R}^{\mathrm{m}}}\left[\frac{\operatorname{Ln}(1 / \mathrm{S})}{\operatorname{Ln}(2)}\right]\right\}^{(1 / \mathrm{b})}
$$

7. Compute load fraction for particular life, $t$, and survival probability, $S$, by

$$
R=\left\{\frac{K}{t^{b}}\left[\frac{\operatorname{Ln}(1 / S)}{\operatorname{Ln}(2)}\right]\right\}^{(1 / m)}
$$

8. Compute the survival probability for particular load fraction, $R$, and life, $t$, by

$$
S=\operatorname{Exp}-\left\{\frac{R^{m_{t} b}}{K} \operatorname{Ln}(2)\right\}
$$

## Single Point Estimates for Median Life and Weibull Modulus

The following equations show how the median life may be estimated, when the shape factor, $m$, is known, from each early observed failure life at tr corresponding with the survival probability, $\mathrm{S}_{\mathrm{r}}$, where
$\mathrm{S}_{\mathrm{r}}=1-(\mathrm{r}-0.5) / \mathrm{N}$ and r is the rank serial number. The use of $(\mathrm{r}-0.5)$ as the effective rank for computing the corresponding probability is a convenience. Alternative formulations are $\mathrm{r} / \mathrm{N}$, $\mathrm{r} /(\mathrm{N}+1),(\mathrm{r}-0.3) /(\mathrm{n}+0.4)$, as well as probabilistic treatments of the "correct" rank assignments. Extensive computer simulations of Weibull distributions (Ref. 8) show that ( $\mathrm{r}-0.5$ )/ N is an effective and convenient rank assignment. The value of $S_{r}$ was taken to be $1-(r-0.5) / \mathrm{N}$ in the tabulated computations. The median life value is estimated by the following equations (with m known):

$$
\begin{aligned}
& S=\operatorname{Exp}-\left\{\operatorname{Ln}(2)\left(\frac{\mathrm{t}}{\mathrm{t}_{\mathrm{m}}}\right)^{\mathrm{m}}\right\} \\
& \mathrm{t}_{0.5}=\mathrm{t}_{\mathrm{m}}=\mathrm{t}_{\mathrm{r}}\left(\frac{\operatorname{Ln}(2)}{\operatorname{Ln}\left(\frac{1}{\mathrm{~S}_{\mathrm{r}}}\right)}\right)^{\frac{1}{\mathrm{~m}}}
\end{aligned}
$$

The shape factor may also be estimated from a few early observations, and in this case we take as an interim normalizing value the longest observed life tr , corresponding to survivability Sr , and use the preceding data at lesser life $t$ having survival probability S (greater than Sr ) to estimate the Weibull Modulus by the relation:


The two estimators are combined to produce individual median life estimates from each observed value, with $t$ as the normalizing life at survival probability, $\mathrm{S}_{\mathrm{r}}$, as shown below

$$
\mathrm{t}_{0.5}=\mathrm{t}_{\mathrm{r}}\left\{\frac{\operatorname{Ln}(2)}{\operatorname{Ln}\left(\frac{1}{S_{\mathrm{r}}}\right)}\right)\left(\frac{\operatorname{Ln}\left(\frac{\mathrm{t}}{\mathrm{t}_{\mathrm{r}}}\right)}{\operatorname{Ln}\left\{\frac{\operatorname{Ln}\left(\frac{1}{\mathrm{~S}}\right)}{\operatorname{Ln}\left(\frac{1}{S_{\mathrm{r}}}\right)}\right)}\right\}
$$

These predictions are tabulated in the data appendices.

## Degradation and Strength Turndown

The possibility of a latent strength reduction, which may appear after a substantial amount of time under sustained load, was addressed by Christensen (Ref. 5). He showed the effect on stress rupture life of degradation that follows first-order chemical reaction rates, with the strength decreasing progressively until failure:

$$
\sigma=\sigma_{\mathrm{r}} \operatorname{Exp}\left(-\mathrm{t} / \mathrm{t}_{\mathrm{c}}\right)
$$

This idea, with some additional constraints, can be used to make a simple and useful modification of the design charts. One can postulate such degradation, which might be related to moisture effects, and the breakdown of locally susceptible regions in the load-carrying filaments. These locally susceptible regions are finite; after all of them are degraded, the breakdown process will stop. This leads to a fraction, f , of strength lost in accordance with the time constant, tc , of this assumed reaction

$$
\sigma=\sigma_{\mathrm{r}}\left[1-\mathrm{f}\left(1-\operatorname{Exp}\left(-t / \mathrm{t}_{\mathrm{c}}\right)\right)\right]
$$

This fractional strength loss is incorporated into the stress rupture equations as shown below:

$$
R=R_{r}\left(\frac{t_{r}}{t}\right)^{\frac{b}{m}}\left[\frac{\operatorname{Ln}\left(\frac{1}{S}\right)}{\operatorname{Ln}(2)}\right]^{\frac{1}{m}}\left\{1-f\left[1-\operatorname{Exp}\left(-\frac{t}{t_{c}}\right)\right]\right]
$$

The log-log form to be used for plotting the probability contours on the design chart is given by

$$
\left.\log (R)=\log \left(R_{r}\right)-\frac{b}{m} \log \left(\frac{t}{t_{r}}\right)+\left(\frac{1}{m}\right) \log \left[\frac{\operatorname{Ln}\left(\frac{1}{S}\right)}{\operatorname{Ln}(2)}\right]+\left.\log \right|_{\mid} ^{\prime} 1-f\left[1-\operatorname{Exp}\left(-\frac{t}{t_{c}}\right)\right]\right\}
$$

This type of design chart is illustrated by the Kevlar composite strand design chart of Figure 2, although the Kevlar vessel data do not exhibit the turndown.


Figure 1. Glass Composite Strand Stress Rupture Design Chart


Figure 2. Kevlar Composite Strand Stress Rupture Design Chart
with $\mathrm{m}=30 \mathrm{~b}=0.9 \mathrm{f}=0.3 \mathrm{Tc}=12000 \mathrm{hrs}$.

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## CONCLUSIONS

Design charts based on the bivariate form of the Weibull distribution are presented for S-glass composites and Kevlar composites, and for carbon composites in preliminary form. The equations are readily modified to accommodate selected values for the life and strength Weibull moduli in order to determine the most consistent and useful fits to the data.

The methods presented may be applied to any type of stress rupture data that exhibit a consistent power law relation between the Weibull moduli for strength and life. A modification of the linear model is presented and used to accommodate the Kevlar strand data turndown tendency at long life.

The underlying analytical models show that if life and strength are Weibull distributions with invariant moduli (shape factors), then log load vs log life will be linear. Conversely, if the $\log$ load vs $\log$ life is linear, and either the life distribution or the strength distribution is a Weibull, then the other distribution must also be a Weibull.

The data processing by these methods allows each observation to be used for estimating the Weibull modulus, giving a valuable perspective for engineering approximations that seek a single conservative design reference value. Methods for estimating median life from incomplete data are also shown.

A notable point is the dramatic difference seen in stress rupture life between S-glass and Kevlar composite rupture data on the one hand, and the carbon data on the other hand. The carbon data seem to exhibit very little stress rupture degradation, and therefore offer very high homologous stresses in operation. Such high stress potential (and high performance) may not be a practically usable characteristic. The lower design stresses required for the glass and Kevlar also provide a certain amount of damage tolerance during the service life. In addition, both S-glass and Kevlar are inherently resistant to moderate impact and casual damage. Carbon composites, on the other hand, are notorious for susceptibility to physical damage and abuse. Such susceptibility, coupled with very high operating stresses, could lead to premature or catastrophic failures in cases of casual damage to a carbon composite vessel operating so close to its expected strength.

The design of S-glass and Kevlar pressure vessels is controlled by stress rupture characteristics for long-term service. The design of carbon composite pressure vessels for long term service is controlled by the amount and type of damage in the operating environment. The carbon composite pressure vessels must be protected from environmental damage or designed to resist and tolerate the service environment.

Acknowledgement
The author acknowledges the assistance of the Commercial Products Division of Structural Composites Industries Inc. for opportunities to examine the current stress rupture models and the performance records for large commercial composite vessel production.

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## SYMBOLS

S,F Cumulative survival and failure probabilities.
m Weibull modul (shape factor) for strength; subscript designates median.
b Weibull modulus for life.
R Homologous load (referenced to the median strength).
$H, X \quad$ Function of load and time, and generic variable.
$t$ Time, life.
r Subscript for reference value of load and time; also rank number for sorted data.
f , tc $\quad$ Strength fraction lost in first-order degradation reaction, with characteristic time, tc .
$\sigma \quad$ Stress or strength.

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## APPENDIX A

## S-Glass Composite Strand Data Tables and Plots Based on LLNL Data Update


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| Stress = | 332 ksi |
| $\mathrm{R}=$ | 58\% |
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| Normal $=$ | 76032 |
| Snorm $=$ | 0.5929 |
| Remaining: |  |
| $\mathrm{N}_{1}=$ | ${ }^{50} 5$ |
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## APPENDLX B

## Kevlar Composite Strand Data Tables and Plots Based on Reference 2





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