Designing for Time-Dependent Material Response in Spacecraft Structures

M.W. Hyer [1] and Lynda L.S. Oleksuk [2]
Department of Engineering Science and Mechanics
Virginia Polytechnic Institute and State University
Blacksburg, VA

D.E. Bowles [3]
Applied Materials Branch
NASA-Langley Research Center
Hampton, VA

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ABSTRACT

To study the influence on overall deformations of the time-dependent constitutive properties of fiber-reinforced polymeric matrix composite materials being considered for use in orbiting precision segmented reflectors, simple sandwich beam models are developed. The beam models include layers representing the face sheets, the core, and the adhesive bonding of the face sheets to the core. A three-layer model lumps the adhesive layers with the face sheets or core, while a five-layer model considers the adhesive layers explicitly. The deformation response of the three-layer and five-layer sandwich beam models to a midspan point load is studied. This elementary loading leads to a simple analysis, and it is easy to create this loading in the laboratory. Using the correspondence principle of viscoelasticity, the models representing the elastic behavior of the two beams are transformed into time-dependent models. Representative cases of time-dependent material behavior for the face-sheet material, the core material, and the adhesive are used to evaluate the influence of these constituents being time-dependent on the deformations of the beam. As an example of the results presented, if it assumed that, as a worst case, the polymer-dominated shear properties of the core behave as a Maxwell fluid such that under constant shear stress the shear strain increases by a factor of 10 in 20 years, then it is shown that the beam deflection increases by a factor of 1.4 during that time. In addition to quantitative conclusions, several assumptions are discussed which simplify the analyses for use with more complicated material models. Finally, it is shown that the simpler three-layer model suffices in many situations.

INTRODUCTION

As part of the development phase of the use of composite materials for long-duration space applications, it would be useful to have a simple analytical tool which models the important features of sandwich construction and allows for the evaluation of the influence of the time-dependent behavior of the various constituents in the construction; namely the face sheets, the core, and the adhesive. In addition, it would be beneficial to have a simple laboratory specimen which could be used to gather empirical data regarding time-dependent material behavior and screen candidate materials. Both of these requirements can be satisfied to a

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[1] Professor

[2] Former Graduate Student

[3] Assistant Branch Head

large degree by considering sandwich beams, and by studying their time-dependent behavior as a function of the time-dependent behavior of the constituents. Beams are one-dimensional in nature, leading to somewhat simpler analyses than plate or shell specimens and laboratory loading of beams is generally direct and free of unwanted secondary effects Three- or four-point loadings are examples of simple yet effective loadings. This paper discusses the time-dependent response of both three- and five-layer symmetric sandwich beams. The five-layer model includes the face sheets and the core, and the adhesive bonding these constituents together. The three-layer model lumps the adhesive layers into either the face sheets or the core. The response of these two models to a three-point loading is considered. Numerical predictions regarding the deflections over a 20 year time span are made in the context of the various constituents of the sandwich construction exhibiting time-dependent behavior. Since 20 year data are not available, the behavior of the constituents is hypothesized. For more complete details of the work discussed, the reader should consult ref. 1.

DEVELOPMENT OF THE THREE-LAYER MODEL

Nomenclature and Problem Definition

In fig. 1 the three-layer model is described, as is the loading. The beams considered are of length L. The three-point bending load consists of a simply supported beam with a point load P at midspan. Because of symmetry about the midspan, this loading is studied here as a cantilever beam with a tip load P/2, the center of the simply-supported beam being clamped in the analogous cantilever problem. The extensional moduli in the x direction of the face sheet and core are denoted as E_1 and E_2, respectively. The shear moduli in the x − z plane are denoted as G_1 and G_2. The thickness of the face sheets is t_1 and that of the core 2h. The overall thickness is 2H.

Equations Governing Elastic Response

For this problem it is assumed that the stress components σ_y, σ_z, τ_{xy}, and τ_{yz} are zero Hence the elastic stress-strain relation is given by

$$\sigma_x = E\varepsilon_x \qquad \tau_{xz} = G\gamma_{xz} \ . \tag{1}$$

In the above, for a particular layer, E is the extensional modulus in the x direction and G the shear modulus in the x − z plane.

The assumed displacement field for the three-layer beam is also illustrated in fig. 1. In particular, the sandwich cross section is assumed to displace uniformly as-a-whole in the x direction an amount $u^o(x)$ and downward as-a-whole an amount $w^o(x)$. In addition, the cross sections of the face sheet layers are assumed to rotate independently of the cross section of the core; the angles of rotation, ψ and ϕ, respectively, being defined in the figure. With these kinematics, the displacement field is written as

$$u(x,z) = \begin{cases} u^o(x) + h\phi(x) - (z + h)\psi(x) & (-H \le z \le -h) \\ u^o(x) - z\phi(x) & (-h \le z \le +h) \\ u^o(x) - h\phi(x) - (z - h)\psi(x) & (+h \le z \le +H) \end{cases} \tag{2}$$

$$w(x,z) = w^o(x) \ , \text{all } z \ .$$

Hereafter, for convenience, the superscript o will be dropped and hence the strains required for use in the stress-strain relation are given by

$$\varepsilon_x = \begin{array}{c} \dfrac{du}{dx} + h\,\dfrac{d\phi}{dx} - (z+h)\,\dfrac{d\psi}{dx} \\[2mm] \dfrac{du}{dx} - z\,\dfrac{d\phi}{dx} \\[2mm] \dfrac{du}{dx} - h\,\dfrac{d\phi}{dx} - (z-h)\,\dfrac{d\psi}{dx} \end{array}
\qquad
\gamma_{xz} = \begin{array}{ll} -\psi + \dfrac{dw}{dx} & (\,-H \le z \le -h) \\[2mm] -\phi + \dfrac{dw}{dx} & (\,-h \le z \le +h) \\[2mm] -\psi + \dfrac{dw}{dx} & (\,+h \le z \le +H)\ . \end{array}
\tag{3}$$

To be determined are u(x), ϕ(x), ψ(x), and w(x). For the viscoelastic problem the functions are time-dependent and should be written as u(x,t). ....

The total potential energy used for determining the elastic response simplifies here to

$$\Pi = \frac{1}{2} \int_0^{\frac{L}{2}} \int_{-\frac{H}{2}}^{\frac{H}{2}} \left(E\varepsilon_x^2 + G\,\gamma_{xz}^2 \right) dx\,dx - \frac{P}{2}\,w\!\left(\frac{L}{2}\right)\ ,
\tag{4}$$

where the limits on the integral reflect the fact that the analog cantilever problem is being considered, the tip being loaded with known load P/2. Substituting the moduli and strains into eq. 4 and integrating with respect to z leads to

$$\begin{aligned}
\Pi = \int_0^{\frac{L}{2}} &\left[c_0\!\left(\frac{du}{dx}\right)^2 + c_1\!\left(\frac{d\phi}{dx}\right)^2 + c_3\!\left(\frac{d\phi}{dx}\right)\!\left(\frac{d\psi}{dx}\right) \right. \\[2mm]
&+ c_6\!\left(\frac{d\psi}{dx}\right)^2 + \frac{1}{2}(c_7 + c_9)\!\left(\frac{dw}{dx}\right)^2 \\[2mm]
&\left. + \frac{1}{2}c_7\psi^2 + \frac{1}{2}c_9\phi^2 - c_7\psi\!\left(\frac{dw}{dx}\right) - c_9\phi\!\left(\frac{dw}{dx}\right) \right] dx \\[2mm]
&- \frac{P}{2}\,w\!\left(\frac{L}{2}\right)\ .
\end{aligned}
\tag{5}$$

The constants c_i are as follows:

$$\begin{aligned}
c_0 &= E_1 t_1 + E_2 h \\[2mm]
c_1 &= h^2\!\left(E_1 t_1 + \frac{1}{3}E_2 h\right) \\[2mm]
c_3 &= E_1 h t_1^2 \\[2mm]
c_6 &= \frac{1}{3}E_1 t_1^3 \\[2mm]
c_7 &= 2G_1 t_1 \\[2mm]
c_9 &= 2G_2 h\ .
\end{aligned}
\tag{6}$$

From eq. 5, by taking the first variation and integrating by parts, the governing equations can be shown to be

$$2c_0\left(\frac{d^2u}{dx^2}\right) = 0$$

$$2c_1\left(\frac{d^2\phi}{dx^2}\right) + c_3\left(\frac{d^2\psi}{dx^2}\right) + c_9\left(\frac{dw}{dx} - \phi\right) = 0$$

$$c_3\left(\frac{d^2\phi}{dx^2}\right) + 2c_6\left(\frac{d^2\psi}{dx^2}\right) + c_7\left(\frac{dw}{dx} - \psi\right) = 0 \qquad (7)$$

$$c_7\left(\frac{d\psi}{dx} - \frac{d^2w}{dx^2}\right) + c_9\left(\frac{d\phi}{dx} - \frac{d^2w}{dx^2}\right) = 0 \ .$$

The boundary conditions associated with the variational statement are

at $x = 0$ at $x = \dfrac{L}{2}$

$$u = 0 \qquad\qquad 2c_0\frac{du}{dx} = 0$$

$$\phi = 0 \qquad\qquad 2c_1\frac{d\phi}{dx} + c_3\frac{d\psi}{dx} = 0 \qquad (8)$$

$$\psi = 0 \qquad\qquad c_3\frac{d\phi}{dx} + 2c_6\frac{d\psi}{dx} = 0$$

$$w = 0 \qquad\qquad (c_7 + c_9)\frac{dw}{dx} - c_7\psi - c_9\phi = \frac{P}{2} \ .$$

On the right side, the 1st and 4th terms can be identified with inplane and shear force re-sultants, respectively, while the 2nd and 3rd terms are moment resultants.

The equation for u(x) is decoupled from the other three equations. Here attention will be focused on the displacement w(x) and thus the first equation will not be considered further. The solutions for the other three displacement variables are taken to be of the form

$$\phi(x) = fe^{\lambda x}$$

$$\psi(x) = se^{\lambda x} \qquad (9)$$

$$w(x) = we^{\lambda x}$$

Substituting these forms into the last three of eq. 7 results in the following polynomial that must be satisfied by λ:

$$\lambda^6\left(4c_1c_6 - c_3^2\right)(c_7 + c_9) - \lambda^4(2c_7c_9)(c_1 + c_3 + c_6) = 0 \ . \qquad (10)$$

The roots to this equation, and application of the boundary conditions, lead to

$$w(x) = w_3x^3 + w_2x^2 + w_1x + w_0 + w_5\sinh(\lambda x) + w_6\cosh(\lambda x)$$

$$\phi(x) = 3w_3x^2 + 2w_2x + \left[w_1 + \frac{6(2c_1 + c_3)}{c_9}w_3\right] + w_5A_6\cosh(\lambda x) + w_6A_6\sinh(\lambda x)$$

$$\psi(x) = 3w_3x^2 + 2w_2x + \left[w_1 + \frac{6(c_3 + 2c_6)}{c_7}w_3\right] + w_5B_6\cosh(\lambda x) + w_6B_6\sinh(\lambda x)$$

$$(11)$$

where

$$w_0 = w_5$$

$$w_1 = \frac{P}{2(R_1 + R_2)^2}\left(\frac{R_1^2}{c_9} + \frac{R_2^2}{c_7}\right)$$

$$w_2 = \frac{PL}{8(R_1 + R_2)}$$

$$w_3 = \frac{-P}{12(R_1 + R_2)}$$

$$w_5 = \frac{-P(c_7 R_1 - c_9 R_2)^2}{2c_7 c_9 \lambda(c_7 + c_9)(R_1 + R_2)^2}$$

$$w_6 = -w_5 ,$$

(12)

with

$$\lambda = \sqrt{\frac{2c_7 c_9(c_1 + c_3 + c_6)}{(4c_1 c_6 - c_3^2)(c_7 + c_9)}}$$

$$R_1 = 2c_1 + c_3$$

$$R_2 = c_3 + 2c_6 .$$

(13)

Interest here will focus on the response at the tip of the cantilever, this being representative of what the beam is doing. In that regard, using $x = L/2$, eqs. 11 and 12,

$$w_{tip} = w\left(\frac{L}{2}\right) = \frac{PL^3}{48(R_1 + R_2)} + \frac{PL}{4(R_1 + R_2)^2}\left(\frac{R_1^2}{c_9} + \frac{R_2^2}{c_7}\right)$$
$$- \frac{P(c_7 R_1 - c_9 R_2)^2}{2c_7 c_9 \lambda(c_7 + c_9)(R_1 + R_2)^2}\left(1 - e^{\frac{-\lambda L}{2}}\right) .$$

(14)

It has been found that for a very wide range of values of elastic properties, the last term contributes very little. It is thus dropped as it considerably simplifies the algebra.

Time-Dependent Behavior of the Three-Layer Beam

In the present study, for obtaining an understanding of the overall effects of the time-dependence of the various constituents, and at the same time considering a worst-case scenario, the constituents are modelled as Maxwell fluids. With a Maxwell fluid, for a constant level of applied stress the material strains indefinitely. For a Maxwell fluid the constitutive equation takes the form

$$\sigma + p_1 \dot{\sigma} = q_1 \dot{\varepsilon} ,$$

(15)

where it is understood that σ can represent a normal or a shear stress and ε can represent an extensional or a shear strain. Taking the Laplace transform of both sides results in

$$\mathbb{P}(s)\bar{\sigma}(s) = \mathbb{Q}(s)\bar{\varepsilon}(s) .$$

(16)

where $\bar{\sigma}(s)$ and $\bar{\varepsilon}(s)$ are the Laplace transforms of the stress and strain functions of time, respectively. In the above

$$\mathbb{P}(s) = 1 + p_1 s$$
$$\mathbb{Q}(s) = q_1 s . \tag{17}$$

It should be noted that the subscript 1 on p_1 and q_1 in eqs. 15, 17, etc. have nothing to do with the face sheets. The nomenclature p_1 and q_1 are standard for viscoelastic materials. See ref. 2, for example.

If a particular constituent is considered to be time-dependent, then the application of the correspondence principle for the sandwich beam problem at hand calls for replacing, in the formulation for the static elastic response, the elastic modulus of the constituent with the ratio $\mathbb{Q}(s)/\mathbb{P}(s)$, i.e.,

$$E \text{ or } G \rightarrow \mathbb{Q}(s)/\mathbb{P}(s) . \tag{18}$$

The static load P is then assumed to be applied at time zero in a stepwise fashion so the load must be replaced with its Laplace transform, i.e.,

$$P \rightarrow P/s \tag{19}$$

The resulting expression is then the Laplace transform of the time-dependent response of the beam. Taking the inverse transform yields the response of the beam as a function of time.

To estimate the long-term effect of time-dependent behavior in the sandwich beams here, it will be assumed that if a constituent exhibits time-dependent behavior, the strain of the constituent will increase by a certain factor in 20 years if subjected to a constant stress. Two levels of time dependency will be studied, one considered worst-case time dependency and the other considered minimal time dependency. The displacement response of the tip for the 20 year period for these two levels of time dependency will then be computed.

As an example, consider the following: With the face sheet material properties being controlled to a large extent by fiber properties, a large degree of time-dependent behavior is unexpected. Hence, it will be assumed that the face sheet material, in the extreme case, exhibits a 10% increase in extensional strain when subjected to a constant stress for 20 years. As a result, for the face sheet material

$$q_1 = 200E_1 \text{ and } p_1 = 200 \text{ years } . \tag{20}$$

Using eq. 17, the substitutions indicated by eqs. 18 and 19 can now take place in eq. 14 (with the last term dropped). Performing the inverse transformation results in an expression for the time-dependent tip deflection, namely,

$$w_{tip}(t) = w_{tip} + \frac{PL^3}{96} \left(\frac{Aq_1}{BG} \right) \left(1 - e^{-\frac{B}{G}t} \right)$$
$$+ \frac{PL}{16B^2} \left[\frac{1}{c_9} \left(D^2 - \frac{B^2H^2}{G^2} \right) - \frac{1}{c_7} \left(\frac{B^2F^2q_1^2}{G^2} \right) \right] \left(1 - e^{-\frac{B}{G}t} \right) \tag{21}$$
$$- \frac{PL}{16BG} \left[\frac{1}{c_9} \left(D - \frac{BH}{G} \right)^2 + \frac{1}{c_7} \left(\frac{B^2F^2q_1^2}{G^2} \right) \right] te^{-\frac{B}{G}t}$$

where

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C-6 .

$$A = t_1(h^2 + ht_1 + \frac{1}{3}t_1^2)$$

$$B = E_2h(\frac{1}{3}h^2)$$

$$C = t_1(2h^2 + ht_1)$$

$$D = E_2h(\frac{2}{3}h^2)$$

$$F = t_1(ht_1 + \frac{2}{3}t_1^2)$$

$$G = Aq_1 + Bp_1$$

$$H = Cq_1 + Dp_1$$

(22)

and w_{tip} represents the elastic response as given in eq. 14. Consider a sandwich beam with quarts epoxy face sheets in an 8 layer quasi-isotropic lay-up and a honeycomb core. Table 1 illustrates nominal constituent elastic properties. Note that table 1 includes information for the five-layer model to be discussed shortly. For the three-layer model, since the adhesive layer is so thin, the elastic properties of the core are not adjusted to account for lumping the adhesive into the core. Only the thickness of the core is adjusted.

Table 1. Nominal Material Properties and Geometries for Beam Model

Face Sheets	Honeycomb Core	Adhesive
$E_1 = 2.5E6$ psi	$E_3 = 1.0E3$ psi	$E_2 = 0.5E6$ psi
$G_1 = 0.96E6$ psi	$G_3 = 29E3$ psi	$G_2 = 0.179E6$ psi
$t_1 = 0.040$ in.	$h = 0.255$ in., 3-layer model	$t_2 = 0$, 3-layer model
	$h = 0.250$ in., 5-layer model	$t_2 = 0.005$ in., 5-layer model

From eq. 21 it can be seen that the time dependency is exponential in form but, as shown in fig. 2, over the 20 year period it appears linear. For the quartz-epoxy/honeycomb sandwich, the percent increase in tip deflection, relative to the static elastic deflection at t=0, is illustrated in fig. 2 for both the worst-case, or maximum, time dependency and the minimal time-dependency of the face-sheet material. Minimal time dependency is defined to be the case when the face-sheet material exhibits only a 1% increase in extensional strain when subjected to a constant tensile stress for 20 years. For the worst case it is seen that the cantilever tip deflection increases by about 8% in 20 years. For the case of minimal time dependency, the tip deflection increases just under 1% in 20 years. These numerical values reflect an almost one-to-one relationship between face-sheet material extensional properties and tip deflection.

As another example, if the core exhibits the behavior of a Maxwell fluid, the appropriate substitutions of eqs. 17, 18, and 19 into eq. 14 leads to an expression for the time-dependent tip deflection as

$$w_{tip}(t) = w_{tip} + \frac{PL}{4(R_1 + R_2)^2} \left(\frac{R_1^2}{2h} \right) \frac{t}{q_1} \ . \tag{23}$$

As can be seen, the tip deflection is linear with time.

Assuming for the case of maximum time dependency that the core strain increases by a factor of 10 in 20 years, and for the case of minimum time dependency that the core strain increases only by a factor of 2 in 20 years, eq. 23 leads to the results shown in fig. 3. It is seen that these cases lead to considerable tip deflection over a 20 year period.

DEVELOPMENT OF THE FIVE-LAYER MODEL

Nomenclature and Problem Definition

To explicitly include the adhesive layer, a five-layer model is necessary. The nomenclature and kinematics of the five-layer model are shown in fig. 4. The properties of the face sheets are subscripted with a 1, the properties of the adhesive subscripted with a 2, and the properties of the core subscripted with a 3. Total sandwich thickness is again 2H.

Equations Governing Elastic Response

The sandwich cross section is again assumed to displace uniformly as-a-whole in the x direction an amount $u^o(x)$ and downward an amount $w^o(x)$. The cross sections of the face sheet layers are assumed to rotate independently of the cross sections of the adhesive layers, and the core has its own cross-section rotation. The displacement field is thus given by

$$u(x,z) = \begin{cases} u^o(x) + h\alpha(x) + t_2\beta(x) - (z + h + t_2)\gamma(x) & -H \leq z \leq -H + t_1 \\ u^o(x) + h\alpha(x) - (z + h)\beta(x) & -H + t_1 \leq z \leq -h \\ u^o(x) - z\alpha(x) & -h \leq z \leq h \\ u^o(x) - h\alpha(x) - (z - h)\beta(x) & h \leq z \leq H - t_1 \\ u^o(x) - h\alpha(x) - t_2\beta(x) - (z - h - t_2)\gamma(x) & H - t_1 \leq z \leq H \end{cases} \tag{24}$$

$$w(x,z) = w^o(x) \ .$$

There are five kinematic quantities to be determined with the five-layer model. Dropping the superscript o, the strains are given by

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$$\varepsilon_x = \begin{cases} \dfrac{du}{dx} + h\dfrac{d\alpha}{dx} + t_2\dfrac{d\beta}{dx} - (z + h + t_2)\dfrac{d\gamma}{dx} & -H \le z \le -H + t_1 \\[2mm] \dfrac{du}{dx} + h\dfrac{d\alpha}{dx} - (z + h)\dfrac{d\beta}{dx} & -H + t_1 \le z \le -h \\[2mm] \dfrac{du}{dx} - z\dfrac{d\alpha}{dx} & -h \le z \le h \\[2mm] \dfrac{du}{dx} - h\dfrac{d\alpha}{dx} - (z - h)\dfrac{d\beta}{dx} & h \le z \le H - t_1 \\[2mm] \dfrac{du}{dx} - h\dfrac{d\alpha}{dx} - t_2\dfrac{d\beta}{dx} - (z - h - t_2)\dfrac{d\gamma}{dx} & H - t_1 \le z \le H \end{cases}$$

$$\gamma_{xz} = \begin{cases} -\gamma + \dfrac{dw}{dx} & -H \le z \le -H + t_1 \\[2mm] -\beta + \dfrac{dw}{dx} & -H + t_1 \le z \le -h \\[2mm] -\alpha + \dfrac{dw}{dx} & -h \le z \le h \\[2mm] -\beta + \dfrac{dw}{dx} & h \le z \le H - t_1 \\[2mm] -\gamma + \dfrac{dw}{dx} & H - t_1 \le z \le H \end{cases} \tag{25}$$

Substituting these strains into the total potential energy, eq. 4, leads to the following differential equations for the kinematic variables:

$$2c_0\left(\frac{d^2u}{dx^2}\right) = 0$$

$$2c_1\left(\frac{d^2\alpha}{dx^2}\right) + c_2\left(\frac{d^2\beta}{dx^2}\right) + c_3\left(\frac{d^2\gamma}{dx^2}\right) + c_9\left(\frac{dw}{dx} - \alpha\right) = 0$$

$$c_2\left(\frac{d^2\alpha}{dx^2}\right) + 2c_4\left(\frac{d^2\beta}{dx^2}\right) + c_5\left(\frac{d^2\gamma}{dx^2}\right) + c_8\left(\frac{dw}{dx} - \beta\right) = 0 \tag{26}$$

$$c_3\left(\frac{d^2\alpha}{dx^2}\right) + c_5\left(\frac{d^2\beta}{dx^2}\right) + 2c_6\left(\frac{d^2\gamma}{dx^2}\right) + c_7\left(\frac{dw}{dx} - \gamma\right) = 0$$

$$c_7\left(\frac{d\gamma}{dx} - \frac{d^2w}{dx^2}\right) + c_8\left(\frac{d\beta}{dx} - \frac{d^2w}{dx^2}\right) + c_9\left(\frac{d\alpha}{dx} - \frac{d^2w}{dx^2}\right) = 0$$

where the constants c_i are given by

$$c_0 = E_1t_1 + E_2t_2 + E_3h$$

$$c_1 = h^2(E_1t_1 + E_2t_2 + \frac{1}{3}E_3h)$$

$$c_2 = ht_2(2E_1t_1 + E_2t_2)$$

$$c_3 = ht_1(E_1t_1)$$

$$c_4 = t_2^2(E_1t_1 + \frac{1}{3}E_2t_2)$$

$$c_5 = t_1t_2(E_1t_1)$$

$$c_6 = t_1^2(\frac{1}{3}E_1t_1)$$

$$c_7 = 2G_1t_1$$

$$c_8 = 2G_2t_2$$

$$c_9 = 2G_3h .$$

(27)

The relevant boundary conditions are

at $x = 0$:   at $x = \frac{L}{2}$:

$$u = 0 \qquad 2c_0\left(\frac{du}{dx}\right) = 0$$

$$\alpha = 0 \qquad 2c_1\left(\frac{d\alpha}{dx}\right) + c_2\left(\frac{d\beta}{dx}\right) + c_3\left(\frac{d\gamma}{dx}\right) = 0$$

$$\beta = 0 \qquad c_2\left(\frac{d\alpha}{dx}\right) + 2c_4\left(\frac{d\beta}{dx}\right) + c_5\left(\frac{d\gamma}{dx}\right) = 0$$

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$$\gamma = 0 \qquad c_3\left(\frac{d\alpha}{dx}\right) + c_5\left(\frac{d\beta}{dx}\right) + 2c_6\left(\frac{d\gamma}{dx}\right) = 0$$

$$w = 0 \qquad (c_7 + c_8 + c_9)\left(\frac{dw}{dx}\right) - c_7\gamma - c_8\beta - c_9\alpha = \frac{P}{2} .$$

As before, the equation governing $u^o(x)$ decouples from the other four and it can be disregarded at this time. Solution of the equations for $\alpha(x)$, $\beta(x)$, $\gamma(x)$, and $w(x)$ follows the procedure for the three-layer beam. The algebra, however, is considerably more involved. As with the three-layer beam, approximations are used to simplify the algebra, particularly for application of the correspondence principle. Through these approximations, the expression for the tip deflection of the five-layer beam takes the form

$$W_{tip} = W\left(\frac{L}{2}\right) = \frac{PL^3}{48(R_1 + R_2 + R_3)}$$

$$+ \frac{PL}{4(R_1 + R_2 + R_3)^2}\left(\frac{R_1^2}{c_9} + \frac{R_2^2}{c_8} + \frac{R_3^2}{c_7}\right) .$$

(29)

where

$$R_1 = 2c_1 + c_2 + c_3$$

$$R_2 = c_2 + 2c_4 + c_5$$

$$R_3 = c_3 + c_5 + 2c_6 .$$

(30)

This equation is the analog of eq. 14 for the three-layer beam.

Time-Dependent Response of the Five-Layer Beam

Numerical studies with the five-layer beam indicate that little new information, relative to what can be learned with the three-layer model, is obtained by using the five-layer model to study the influence of time-dependent face sheet and core properties. However, since the five-layer model includes the additional feature of the adhesive layer, it is of value to discuss the influence of time-dependent adhesive properties. Assuming the adhesive shear properties behave as a Maxwell fluid, using the expression for the tip deflection, eq. 29, performing the steps given by eqs. 18 and 19, and taking the inverse transformation, the tip deflection as a function of time for a viscoelastic adhesive layer is

$$w_{tip}(t) = w_{tip} + \frac{PL}{4(R_1 + R_2 + R_3)^2} \left(\frac{R_2^2}{2t_2} \right) \left(\frac{t}{q_1} \right) . \tag{31}$$

This expression also depends on time in a linear manner. Using the material properties as given in table 1, the time-histories of the percent increase in tip deflection for the cases of the adhesive shear strain doubling and increasing ten-fold in 20 years are shown in fig. 5. For the worst-case condition, the tip deflection changes by less than 0.5% in 20 years.

CONCLUSIONS

Presented has been the development of models to be used in evaluating the influence of time-dependent face sheet, core, and adhesive constitutive properties on the overall deformations of sandwich beams. The study has its origins in the need to understand the time-dependent deformations of orbiting precision segmented reflectors. Beams may be considered an oversimplification of the structural characteristics of segmented reflectors. However, the basic characteristics of sandwich construction are retained in the beam models, and beams could serve as a screening tools as effectively, and certainly as economically, as plate or shell-like models. Here efforts have been made to involve the important material properties explicitly so parametric studies can easily be made. Some approximations were necessary, but these have been justified and in no way do they compromise the results obtained.

Several recommendations are in order. First, extending the analysis to include thermal effects, such as would occur in the presence of a slight through-the-thickness temperature gradient, would be worthwhile. Such a gradient would cause unwanted curvature in a beam, and over time, the curvature may change. Second, it would be useful to extend the analysis to include the two-dimensional aspects of the reflector, namely its plate-like geometry. If the change of focal length with time, for example, of a reflector is to be determined, such an analysis is necessary.

ACKNOWLEDGMENT

REFERENCES

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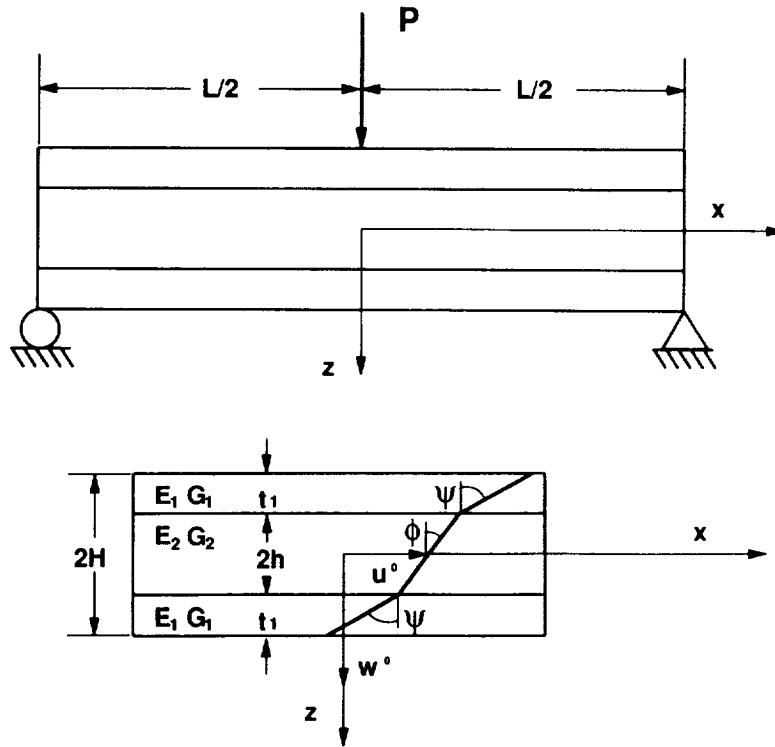2. Flugge, Wilhelm, Viscoelasticity, Springer-Verlag, Berlin, 1975.

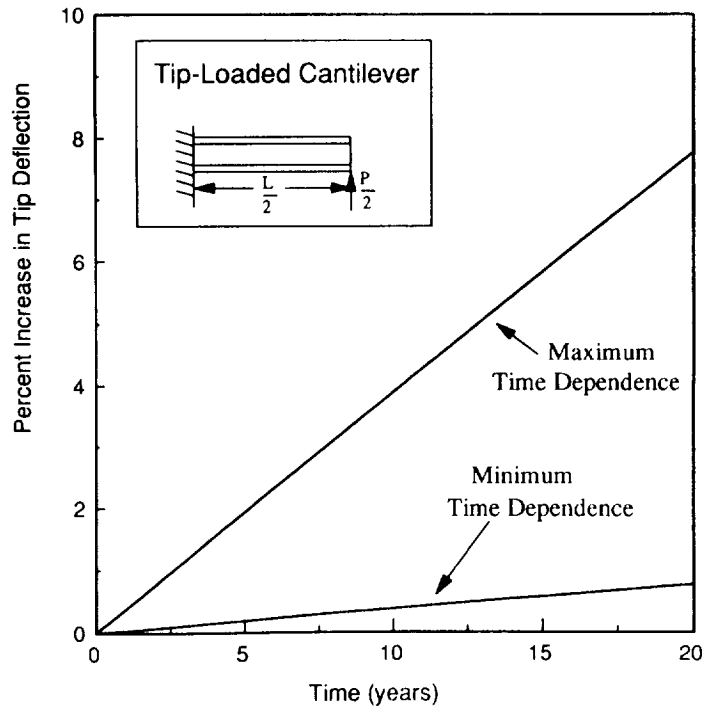**Fig. 1** - **Geometry, nomenclature, loading, and assumed displacement field for the three-layer model.**

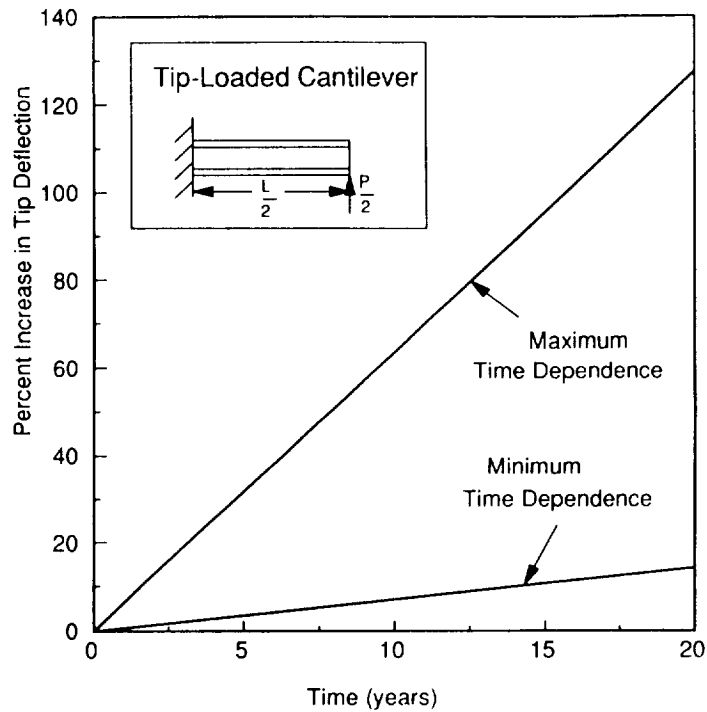**Fig. 2 - Percent increase in tip deflection of three-layer beam due to time-dependent face-sheet extensional properties.**

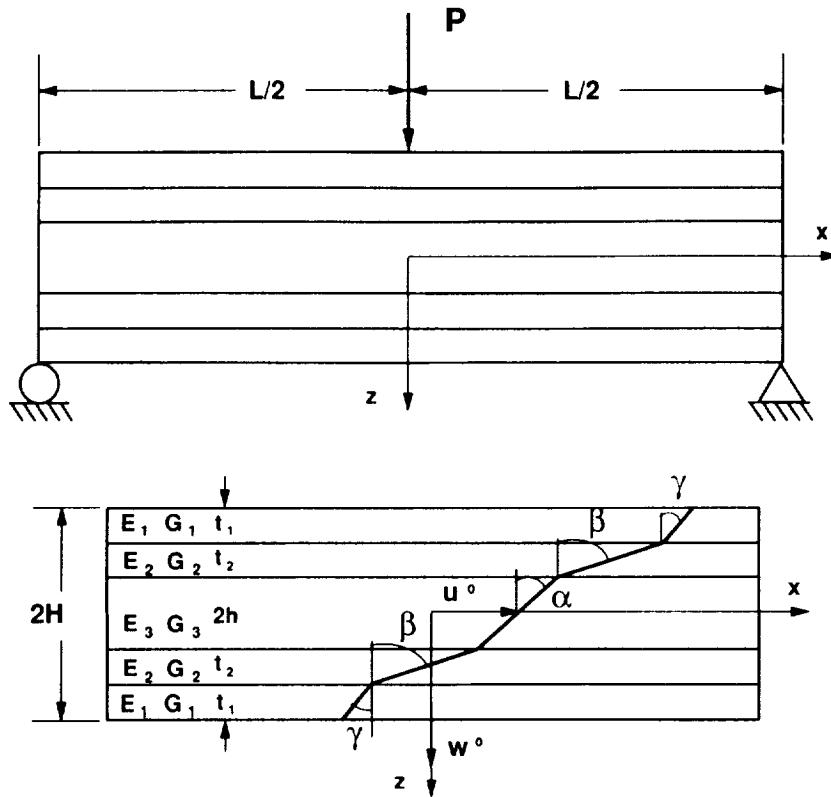**Fig. 3 - Percent increase in tip deflection of three-layer beam due to time-dependent core shear properties.**

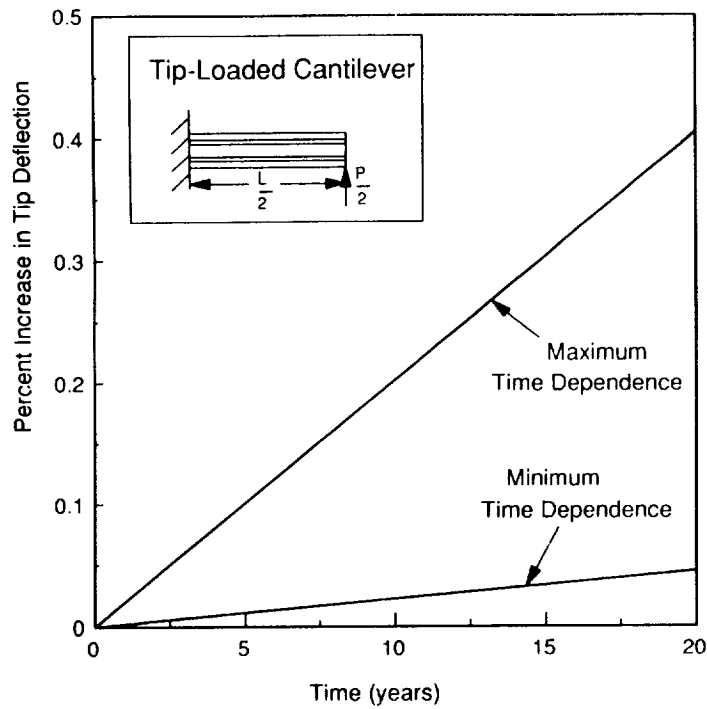**Fig. 4** - Geometry, nomenclature, loading, and assumed displacement field for the five-layer model.

Fig. 5 - Percent increase in tip deflection of five-layer beam due to time-dependent adhesive layer shear properties.