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FUNDAMENTAL CONCEPTS IN THE SUPPRESSION OF DELAMINATION BUCKLING BY STITCHING

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ABSTRACT

Elementary results are presented for the buckling of stitched, laminated composites containing delamination cracks. The stitching fibers are assumed to provide continuous, linear restoring tractions opposing the deflection of the delaminated layer adjacent to the crack. It is shown that there exists a characteristic length a_o for buckling: if the length, 2a, of the delamination crack exceeds $2a_o$, then, when buckling occurs, it will consist of waves of period $2a_o$ and will usually not span the whole delamination. Simple expressions are derived for the critical buckling load and the minimum stitching density required to suppress buckling of the delaminated layer.

INTRODUCTION

One of the principal obstacles to using relatively cheap graphite epoxy laminates in the commercial aircraft industry is their susceptibility to delamination, especially during impact, and the subsequent catastrophic growth of the delamination crack when the delaminated layer buckles under in-plane compression [see, for example, ref. 1]. One promising solution to this problem is the incorporation of fiber tows normal to the laminate plane by stitching. While stitching tows do not eliminate delamination during impact, they do minimize loss of strength under subsequent compression [refs. 2,3]. It appears that the stitching tows bridge the delamination crack and prevent or reduce buckling of the adjacent delaminated layer. The in-plane stiffness then survives relatively unimpaired and, if the delaminated layer remains flat, the delamination crack does not grow, since it experiences no driving force. Failure under compressive loading occurs by some other mechanism and the compressive strength after impact is restored to an acceptably high value.

This paper presents the simplest possible description of the mechanics of this buckling problem, which can be modeled as that of a buckling plate on a Winkler foundation of damping springs [e.g., refs. 4,5]. However, in contrast to the usual assumption, buckling deflections in the present problem can occur in one direction only. This constraint gives rise to a characteristic length for buckling and an enhanced value of the critical force for buckling. Simple arguments are presented to show how these fundamental quantities vary with stitching fiber density and the thicknesses of the delaminated layer and the substrate beneath it.

BUCKLING IN THE PRESENCE OF LINEAR DAMPING

Consider a delamination crack of length 2a lying along the x-axis, as shown in Figure 1. Let the crack be in a state of plane strain. Suppose the body containing the crack is subject to a compressive load that results in the compressive force F per unit length being imposed on the thin layer of delaminated material above the delamination crack. Buckling of the thin layer will result in the deflection w(x), which can be determined if the layer is thin enough by consideration of the balance of forces according to the elementary theory of bending plates [refs. 5,6]. If the delamination crack is bridged by stitching fibers, as shown schematically in Figure 1, the problem is modified by the lateral tractions those fibers impose.

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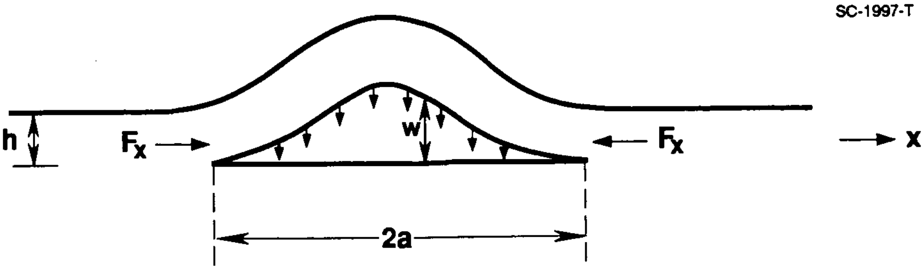

Fig. 1. Schematic of a delaminated and buckled layer, showing lateral tractions opposing the buckling deflection.

If the stitching fiber spacing is appreciably smaller than 2a, the lateral tractions can be considered continuous over x. Furthermore, if the stitching fibers are debonded from the matrix or are much stiffer than the matrix, then the tractions, denoted q, will be a linear function of the deflection:

$$q(x) = \beta w(x) \ , \qquad (\beta > 0) \quad . \tag{1}$$

The spring constant β will be related below to the properties of the stitching tows. The deflection profile is given by the linear differential equation [refs. 4-6]

$$\frac{d^4 w}{dx^4} + \frac{F}{D}\frac{d^2 w}{dx^2} + \frac{\beta}{D} w = 0 \tag{2}$$

where F > 0 denotes a compressive force and D is the flexural rigidity of the delaminated layer. (The body has been assumed to be isotropic and homogeneous, a crude representation of a stitched laminate but a useful simplification when exploring fundamental aspects of the problem.) The deflection must also satisfy clamped end boundary conditions and be positive, since negative deflections would imply interpenetration. Equation (2) has both symmetric and antisymmetric solutions, but only the symmetric solutions can satisfy w > 0. The symmetric solutions have the form

$$w(x) = A_+ \cos\xi_+ x + A_- \cos\xi_- x \tag{3a}$$

$$\text{where} \quad \xi_\pm \equiv \sqrt{f(1 \pm \sqrt{1 - b/f^2})} \tag{3b}$$

$$\text{with} \quad f \equiv F/2D \quad \text{and} \quad b \equiv \beta/D \quad . \tag{3c}$$

The boundary conditions determine both the ratio A_+/A_- and the critical buckling load $f_c^{(m)}(a)$ for the m^{th} buckling mode for delamination crack length 2a. Numerically determined values of $f_c^{(m)}(a)$ are shown for the first few modes in Fig. 2 (see also [Ref. 4]). Each $f_c^{(m)}$ is a monotonically decreasing function of a, in contrast to the case of a plate with simply supported ends, for which the buckling load decreases for small a and then increases as a^2 for large a [e.g., ref. 5].

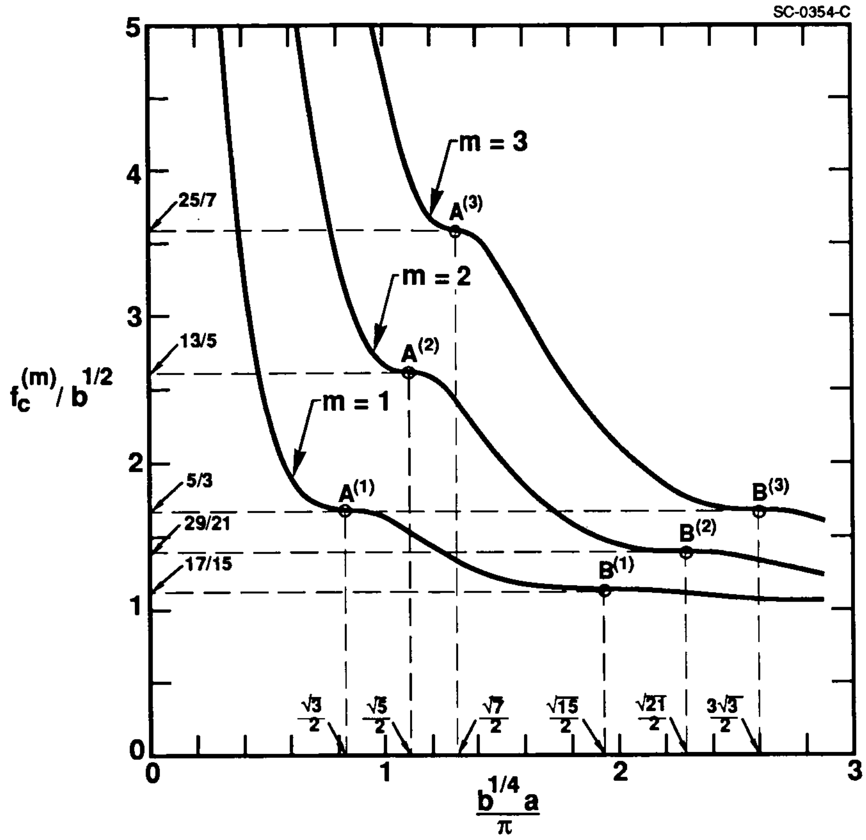

Fig. 2 The critical force for buckling for the first few symmetrical buckling modes as a function of delamination crack length.

With $f_c^{(m)}$ determined, the corresponding deflection $w^{(m)}(x)$ is also determined to within a multiplicative constant by the boundary conditions. To the left of the points $A^{(m)}$ in Fig. 2, $w^{(m)}(x)$ is of one sign, i.e. it contains no zeroes in (-a,a); and, if it is taken to be positive, it contains m maxima. At the points $A^{(m)}$, $B^{(m)}$, etc., $d^2w^{(m)}/dx^2$ evaluated at $x = \pm a$ passes through zero and $w^{(m)}(x)$ acquires two zeroes, which begin at $x = \pm a$ and move into the interval (-a,a) as a increases. At all values of a to the right of $A^{(m)}$, $w^{(m)}(x)$ possesses at least two zeroes, i.e. it is no longer of one sign.

For $w > 0$, the minimum buckling load, f_o, is thus found at the point $A^{(1)}$, i.e.

$$f_o = \frac{5}{3} b^{\frac{1}{2}} \quad . \tag{4}$$

The crack length corresponding to point $A^{(1)}$ is given by

$$a_o = \frac{\sqrt{3}}{2} \frac{\pi}{b^{\frac{1}{4}}} \quad . \tag{5}$$

It denotes a <u>characteristic buckling length</u>: whenever $a > a_o$, buckling will occur not over the whole delamination (-a,a), but over some subinterval or series of subintervals of length $2a_o$. The buckling profile over each subinterval is that for $\xi_+ a = 3\pi/2$ and $\xi_- a = \pi/2$, i.e.

$$w_o(x) = A\left[\cos\frac{3\pi x}{2a} + 3\cos\frac{\pi x}{2a}\right] \quad , \tag{6}$$

with A an undetermined constant. The appearance of buckling when $a > a_0$ is therefore qualitatively different from the case of an unbridged delamination crack. If $a < a_0$, buckling occurs over the whole interval $(-a,a)$ and the critical load rises with decreasing a as in Fig. 2.

STITCHED LAMINATES

For stitched laminates, the stiffness parameter β depends on how the delaminated layer is coupled to the rest of the material. Figure 3(a) shows the case of a delamination crack lying in the mid-plane of a thin panel. Buckling upon compressive loading occurs in a symmetric manner. Figure 3(b) shows the case of a thin delaminated layer lying over a thick substrate, which remains straight while the delaminated layer buckles. If the stitching fiber tows pass from top to bottom of the panel and they are initially unstrained, then the stiffness parameter is approximately

$$\beta = \begin{cases} \dfrac{\upsilon_s E_f}{h} & \text{(case I)} \quad (7a) \\[2ex] \dfrac{\upsilon_s E_f}{t} & \text{(case II)} \quad , \quad (7b) \end{cases}$$

where case I refers to the case of Fig. 3(a) and case II to that of Fig. 3(b), υ_s is the volume fraction of the stitching fibers, E_f is the fiber modulus, and the dimensions h and t are defined in Figure 3.

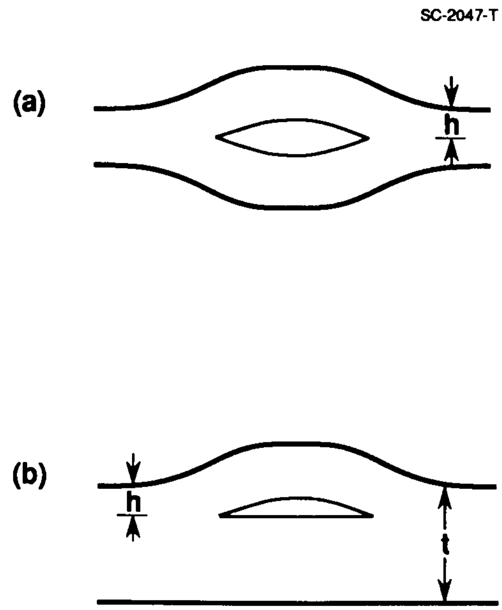

Fig. 3 (a) Symmetric buckling. (b) Buckling of a thin layer over a thick substrate.

Now the results of Section 2 were obtained for an isotropic material, whereas a stitched laminate is more closely orthotropic. Nevertheless, useful results in terms of orders of magnitude can be obtained by making some crude approximations concerning elastic properties. Since buckling of thin plates is determined mainly by in-plane elastic properties, the flexural rigidity of Eq. (3) can be approximated by

$$D = \frac{\upsilon_f E_f h^3}{12(1 - \nu^2)} \quad . \tag{8}$$

where ν is Poisson's ratio, υ_f is the volume fraction of fibers lying in the axial direction (parallel to the x-axis and the applied compressive load), and it has been assumed that these fibers have the same modulus as the stitching fibers. The stitching and axial fibers are indeed often of the same kind. The characteristic buckling length can now be approximated by

$$a_o = \begin{cases} c_1 \left(\frac{\upsilon_f}{\upsilon_s}\right)^{\frac{1}{4}} h & \text{(case I)} \qquad (9a) \\ c_1 \left(\frac{\upsilon_f}{\upsilon_s}\right)^{\frac{1}{4}} \left(\frac{t}{h}\right)^{\frac{1}{4}} h & \text{(case II)} \qquad (9b) \end{cases}$$

with $$c_1 = \frac{3^{\frac{1}{4}}}{2^{3/2}} \frac{\pi}{(1-\nu^2)^{\frac{1}{4}}} \simeq 1.5 \text{ when } \nu = 0.3 \quad ; \tag{9c}$$

and the critical force f_o of Eq. (4) is approximately

$$f_o = \begin{cases} c_2 (\upsilon_f \upsilon_s)^{\frac{1}{2}} E_f h & \text{(case I)} \qquad (10a) \\ c_2 (\upsilon_f \upsilon_s)^{\frac{1}{2}} \left(\frac{h}{t}\right)^{\frac{1}{2}} E_f h & \text{(case II)} \qquad (10b) \end{cases}$$

with $$c_2 = \frac{5}{3^{3/2}(1-\nu^2)^{\frac{1}{2}}} \simeq 1.0 \text{ when } \nu=0.3 \quad . \tag{10c}$$

Equations (9) and (10) illustrate in a simple way how geometry and fiber density control the efficacy of stitching in suppressing buckling. In particular, Eq. (9) shows that the characteristic buckling length in stitched materials will usually be no more than an order of magnitude greater than the thickness, h, of the delaminated layer. Even if $\upsilon_s \sim 10^{-2}\upsilon_f$, i.e., sparse stitching, one still has $a_o \simeq 5h$ for case I, since the ratio υ_f/υ_s appears to the power $\frac{1}{4}$. The additional factor $(t/h)^{\frac{1}{4}}$ in case II is also unlikely to be much greater than 2 in practice.

In the absence of impact damage, stitched laminates generally fail under in-plane compression by the formation of a kink band of buckled and broken axial fibers. The stress at which this occurs corresponds to a particular value, f_o', of the force acting on the ends of a delaminated layer if it has not yet buckled. Substituting f_o' for f_o, Eq. (10) provides a simple estimate of the stitching fiber density required to suppress buckling and eliminate delamination crack growth as a potential failure mechanism. Thus the required value of υ_s is proportional to $(f_o'/h)^2$. Conversely, the critical force, f_o, is more sensitive to fiber volume fractions than is the characteristic buckling length, being proportional to $\upsilon_s^{\frac{1}{2}}$.

Equation (10) also shows that f_o in case II varies as $t^{\frac{1}{2}}$, which simply reflects the fact that stitching fibers of shorter initial through-thickness length experience greater strain for a given buckling deflection. Thus an effective method of raising f_o for a given density of stitching fibers and laminate thickness is to pass stitching tows only part way through the laminate, achieving through-thickness reinforcement by staggering the stitching at different depths. Laminates with such stitching patterns are indeed available, but no results concerning their delamination and buckling behavior have yet been published.

CONCLUSIONS

The presence of stitching tows introduces a minimum compressive load, f_o, required to buckle a delaminated layer in a stitched laminate, regardless of the length of the delamination crack; and a characteristic buckling length, a_o. If the delamination crack length 2a exceeds $2a_o$, buckling has the form of waves of length $2a_o$, which arise when the load exceeds f_o. In typical stitched laminates, the length a_o will not exceed the thickness, h, of the buckling delaminated layer by more than one order of magnitude. The critical force, f_o, can be enhanced by increasing stitching density or passing stitching tows only part way through the laminate.

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REFERENCES

1. Horton, R.E., and McCarty, J.E., 1987, "Damage Tolerance of Composites," in Engineered Materials Handbook, Vol. 1: Composites (ASM International, Metals Park, OH).

2. Dow, M.B., and Smith, D.L., 1989, "Damage Tolerant Composite Materials Produced by Stitching Carbon Fabrics," Int. SAMPE Technical Conf. Series, Vol. 21, pp. 595-605.

3. Smith, P.J., and Wilson, R.D., 1985, "Damage Tolerant Composite Wing Panels for Transport Aircraft," Boeing Commercial Airplane Company, NASA Contractor Report 3951.

4. Hetényi, M., "Beams on Elastic Foundation," University of Michigan Press, Ann Arbor, MI, 1946.

5. Timoshenko, S.P., and Gere, J.M., 1961, "Theory of Elastic Stability," 2nd edition, McGraw-Hill, New York, p. 94 and Chap. 8.

6. Saint Venant, B. de, 1883, discussion in "Théorie de l'Elasticité des Corps Solides," by A. Clebsch, Gauthier-Villars, Paris, p. 704.