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Guidance, Navigation, and Control Study for a Solar Electric Propulsion Spacecraft *

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I. Introduction

Low-thrust propulsion systems have been identified as an efficient means for performing space missions. Spacecraft propelled by low-thrust engines are capable of delivering a greater payload fraction compared to spacecraft using conventional chemical propulsion systems. Recently, several research efforts have investigated numerous applications of low-thrust propulsion including a manned Mars mission [1], scientific missions to Jupiter, Uranus, Neptune and Pluto [2], and lunar missions leading to a permanent lunar colony [3]-[5]. Aston [6] has also demonstrated the merits and feasibility of using low-thrust propulsion to ferry cargo between low-Earth orbit and low-lunar orbit. The study of optimal trajectories and guidance, control, and navigation (GN&C) for low-thrust spacecraft is an integral part of these research efforts.

In response to the release of NASA's 1994 Announcement of Opportunity (AO) for Discovery class interplanetary exploration missions, a preliminary investigation of a lunar-comet rendezvous mission using a solar electric propulsion (SEP) spacecraft was performed. The Discovery mission (eventually named Diana) was envisioned to be a two-phase scientific exploration mission: the first phase involved exploration of the moon and second phase involved rendezvous with a comet. The initial phase began with a chemical propulsion

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translunar injection and chemical insertion into a lunar orbit, followed by a low-thrust SEP transfer to a circular, polar, low-lunar orbit (LLO). After scientific data was collected at the moon, the SEP spacecraft performed a spiral lunar escape maneuver to begin the interplanetary leg of the mission. After escape from the Earth-moon system, the SEP spacecraft maneuvered in interplanetary space and performed a rendezvous with a short-period comet. The immediate goal of this study was to demonstrate the feasibility of using a low-thrust, SEP spacecraft for orbit transfer to both the moon and to a comet. Another primary goal was to develop a computer optimization code which would be robust enough to obtain minimum-fuel rendezvous trajectories for a wide range of comets.

This final report is a summary of the initial research efforts that were undertaken in support of the Discovery mission proposal that was submitted to NASA Headquarters in October 1994. Section II discusses the initial interplanetary phase of the study which involves developing a robust, efficient trajectory optimization program for computing minimum-fuel rendezvous trajectories with various comets. Sections III and IV discuss the computation of the optimal lunar capture and escape trajectories using the SEP spacecraft. Finally, section V presents the conclusions of this research effort.

II. Comet Rendezvous Study

Although the comet rendezvous phase of the Diana mission is after the lunar capture/escape phase, it was deemed to be of primary importance in the preliminary investigation since selecting the "best" comet for rendezvous would drive the mission's performance. The initial problem was to develop a trajectory optimization code that was capable of obtaining the minimum-fuel rendezvous trajectory for a wide variety of target comets. Since a catalog of about 30 comets with orbital periods under 6 years and inclinations under 10 deg existed, a quick and efficient method for computing optimal SEP trajectories was required. For the sake of completeness, the optimal control problem for the comet rendezvous is presented in detail in the next sub-section.

Trajectory Optimization

The objective is to compute the minimum-fuel, continuous-thrust trajectory for a comet rendezvous in heliocentric space. Since the low-thrust SEP engine is assumed to be continuously operating at a constant mass flow rate, the minimum-time trajectory will correspond to the minimum-fuel trajectory. The complete optimal control problem is given below:

For the free end-time problem, find the orientation of the hyperbolic excess velocity vector $\vec{v}_\infty$, the pitch and yaw thrust steering angles $u(t)$ and $v(t)$, and the Julian date for Earth sphere of influence (SOI) departure t_0 which minimize

$$J = t_f \tag{1}$$

subject to the two-body equations of motion

$$\dot{x} = f(t, x, u, v) \tag{2}$$

with the initial conditions

$$x(0) = g(t_0, \vec{v}_\infty) \tag{3}$$

and the terminal state constraints

$$\psi[x(t_f), t_f] = \begin{pmatrix} a(t_f) - a_c \\ e(t_f) - e_c \\ i(t_f) - i_c \\ \Omega(t_f) - \Omega_c \\ \omega(t_f) - \omega_c \\ \nu(t_f) - \nu_c(t_f) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{4}$$

The differential equations of motion are represented by Eq. (2) and are the two-body equations of motion for the thrusting SEP spacecraft in a sun-centered equinoctial coordinate system. The state vector x is comprised of the six equinoctial elements $x = [a, h, k, p, q, F]^T$ which are functions of the classical orbital elements a (semi-major axis), e (eccentricity), i (inclination), Ω (right ascension of the ascending node), ω (argument of periapsis), and E (eccentric anomaly):

$$a = a \tag{5}$$

$$h = e\sin(\omega + \Omega) \tag{6}$$

$$k = e\cos(\omega + \Omega) \tag{7}$$

$$p = \tan(i/2)\sin\Omega \tag{8}$$

$$q = \tan(i/2)\cos\Omega \tag{9}$$

$$F = \Omega + \omega + E \tag{10}$$

The right-hand sides of the equations of motion are denoted by the vector f and the detailed equations can be found in Ref. [7]. The pitch (in-plane) and yaw (out-of-plane) thrust steering angles of the SEP spacecraft are $u(t)$ and $v(t)$, respectively. The initial conditions as denoted by Eq. (3) are a function of the Julian departure date t_0 and the hyperbolic excess velocity $\vec{v}_\infty$. The initial velocity $\vec{v}_0$ of the SEP spacecraft with respect to the sun is calculated by the below vector equation

$$\vec{v}_0 = \vec{v}_\infty + \vec{v}_E \tag{11}$$

where $\vec{v}_E$ is the velocity of the Earth with respect to the sun at the Julian date t_0. The hyperbolic velocity $\vec{v}_\infty$ is assumed to have a magnitude of 1.24 km/s which represents an estimate of the excess energy after the SEP spacecraft has escaped the Earth-moon system and reached the Earth's SOI. Since Eq. (11) is a vector equation, the orientation of $\vec{v}_\infty$ needs to be defined.

The terminal state constraints as denoted by Eq. (4) require that the final classical orbital elements of the SEP spacecraft match the orbital elements of the comet for proper rendezvous conditions. The subscript c indicates the constant orbital elements of the target comet. The equinoctial elements of the spacecraft ($x(t_f)$) must be transformed into the classical orbital elements at the final time. Both true anomaly ν for the spacecraft and the comet have a dependence on final time t_f.

Solution Approach

In general, an optimal control problem may be solved using either direct or indirect methods. An indirect method involves applying calculus of variations principles and solving the corresponding two-point boundary value problem (2PBVP). This is usually an extremely difficult problem except in the case of a simple dynamic system. A direct method utilizes a parameterization of the control and attempts to directly reduce the performance index value at each iteration. Typically, direct methods are more robust than indirect methods since indirect methods rely on fairly accurate knowledge of the initial costate or adjoint variables. Furthermore, our problem involves sensitive boundary conditions and a mix of control functions and control parameters and therefore a direct optimization method is used here. The optimal control problem is replaced with an approximate nonlinear programming problem (NLP) with the continuous control histories ($u(t)$ and $v(t)$) replaced with a finite number of parameters. The control functions are parameterized by cubic spline interpolation through a fixed number of control points. The nonlinear programming problem is numerically solved using sequential quadratic programming (SQP) which is a constrained parameter optimization method [8]. The SQP algorithm used here utilizes first-order finite differences to approximate the gradients and is due to Pouliot [9].

The SQP problem formulation involves 26 optimization parameters and six equality constraints. Four SQP design variables are required for the orientation of $\vec{v}_\infty$ (two angles), the Julian date at Earth SOI departure t_0, and the total time of flight t_f. Eleven evenly-spaced control nodes are used to parameterize the thrust steering angles $u(t)$ and $v(t)$.

The six equality constraints enforce the required matching between the classical orbital elements of the spacecraft and the orbital elements of the comet at $t = t_f$ as indicated by Eq. (4). The equations of motion are numerically integrated by using a standard fourth-order, fixed-step, Runge-Kutta integration scheme with 500 steps.

The robustness of the direct optimization approach is enhanced by utilizing a penalty function method. That is, the complete minimization problem is not solved in one step since convergence to a complete rendezvous would be very difficult to obtain without a good initial guess. Therefore, a penalty function is formed by augmenting the performance index $J = t_f$ with a penalty term:

$$\tilde{J} = t_f + \sum_{i=1}^{m} \psi_i^2 \tag{12}$$

and the augmented performance index $\tilde{J}$ is minimized by the SQP optimization code. The penalty function term is the sum of the squares of the elements of the terminal state constraint vector defined by Eq. (4). A sequence of problems is solved for an increasing value of the integer m. Initially, $m = 2$ and the first sub-problem involves matching the size and shape (a and e) of the comet's orbit at $t = t_f$ with only two equality constraints. Once a solution is obtained, m is set to 3 and a second sub-problem is solved to match a, e, and i. The procedure is repeated until $m = 6$ and all six terminal state constraints are met. Finally, the true minimum-fuel trajectory is obtained without a penalty function (i.e., $J = t_f$) and all six equality constraints are enforced. The penalty function approach enhances the convergence properties of the optimization process since the SQP code can simultaneously work on reducing transfer time and errors in the terminal state constraints. Therefore, convergence is greatly improved for poor initial guesses.

Results

The optimal minimum-fuel rendezvous trajectories were computed for an SEP spacecraft derived from the Transfer Orbit Plasma Investigation Experiment (TROPIX) Project [10]. The fixed spacecraft characteristics are summarized in Table 1. Estimates of the low-thrust spiral time for the lunar capture and escape trajectories are used to compute the initial mass of the SEP spacecraft at the start of the heliocentric phase. For this preliminary research, the engine power P is assumed to be constant during the entire heliocentric trajectory. Therefore, thrust T and propellant mass flow rate $\dot{m}$ are both constant during the orbit transfer.

Table 1: SEP vehicle parameters - start of heliocentric phase

Initial mass (kg)	Power (kW)	I_{sp} (s)	Thrust (N)	$\dot{m}$ (kg/day)
280	1.8	3800	0.077	0.18

The first rendezvous target attempted is the comet Wilson-Harrington. The sequence of sub-problems approach with the penalty function method guided the SQP optimization code to the minimum-fuel rendezvous trajectory. Optimal departure date from the Earth's SOI was found to be May 19, 2000 and the resulting arrival date at Wilson-Harrington was found to be November 18, 2001. Therefore, the heliocentric flight time is 1.502 years and the resulting fuel mass is 98.2 kg. The final SEP spacecraft mass at rendezvous is 181.8 kg and the final mass ratio m_f/m_0 is 0.65. The rendezvous occurs at a distance of about 3.0 AU from the sun at a true anomaly of $\nu = 137.8$ deg. Therefore, the SEP spacecraft "catches up" and matches the comet's orbit as Wilson-Harrington is moving away from the sun and approaching apohelion. The minimum-fuel heliocentric trajectory is presented in Fig. 1. It is observed that the SEP spacecraft completes only about 1/2 revolution about the sun before rendezvous with Wilson-Harrington. Since input solar power typically decreases at a rate approximately proportional to the inverse square of the distance to the sun, the power at 3 AU will be approximately 10% of the initial power at 1 AU. In this preliminary analysis, power is assumed to be constant; subsequent work utilized more realistic solar

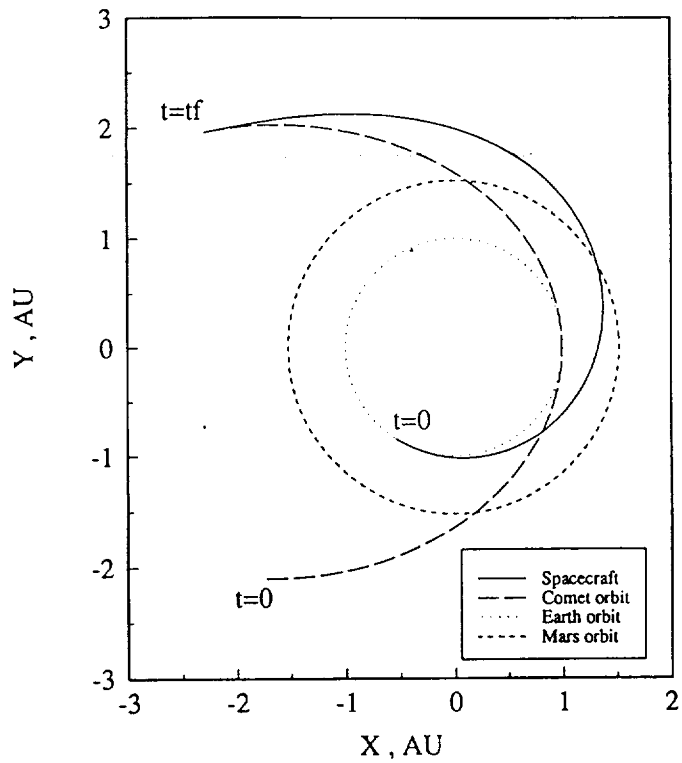

Figure 1: Minimum-fuel heliocentric trajectory to Wilson-Harrington

power ratios as a function of distance to the sun.

The second rendezvous target is the comet Clark. The optimal departure date from the Earth's SOI is September 12, 1999, the heliocentric time of flight is 1.388 years, and the resulting arrival date at Clark is January 31, 2001. Therefore, the resulting fuel mass is 90.8 kg, the final SEP spacecraft mass at rendezvous is 189.2 kg and the final mass ratio m_f/m_0 is 0.68. The rendezvous occurs at a distance of about 1.7 AU from the sun at a true anomaly of ν = 40.5 deg. Therefore, the SEP spacecraft completes the rendezvous with Clark closer to perihelion than the rendezvous with Wilson-Harrington. The minimum-fuel heliocentric trajectory is presented in Fig. 2. It is observed that the SEP spacecraft nearly completes a full revolution about the sun before rendezvous with Clark and remains closer to the sun during the entire trajectory compared to the Wilson-Harrington rendezvous trajectory.

The respective minimum-fuel, continuous-thrust, comet rendezvous trajectories were

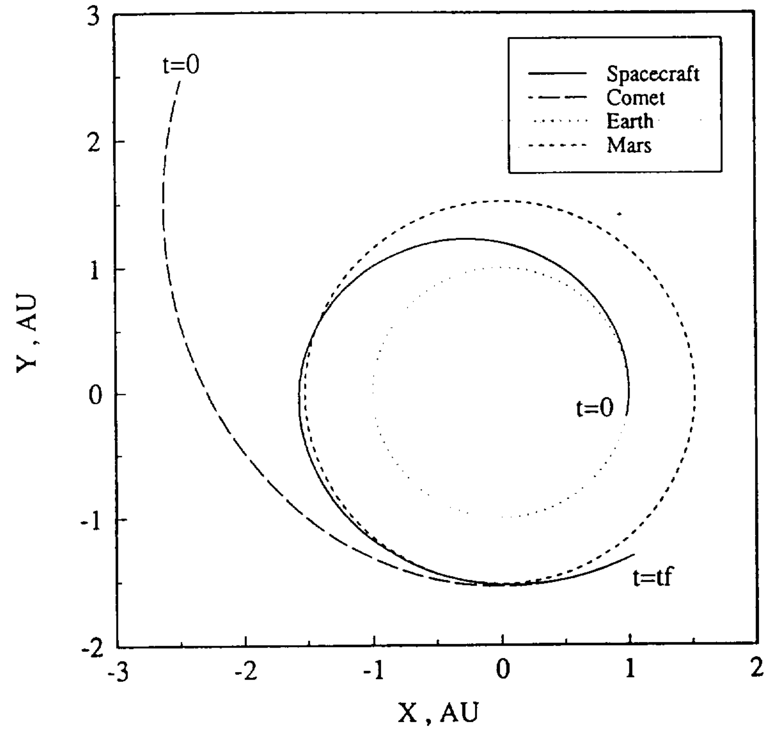

Figure 2: Minimum-fuel heliocentric trajectory to Clark

utilized as initial guesses for a calculus of variations-based trajectory optimization code. Subsequently, many optimal minimum-fuel comet rendezvous trajectories with multiple burn and coast arcs were obtained. It was through this process that Wilson-Harrington was identified as the "best" comet for minimum-fuel rendezvous for the Diana mission.

III. Lunar Capture Trajectories

The initial phase of the Diana mission involves the combined chemical-electric propulsion transfer to a polar, circular, 100 km altitude LLO. The chemical translunar insertion (TLI) burn is performed by the upper stage of a Delta II booster and a subsequent lunar orbit insertion (LOI) chemical burn places the SEP spacecraft into a prescribed elliptical orbit about the moon. The SEP system is then used to perform the remaining orbit transfer to polar LLO. In this section, the optimal minimum-fuel, lunar capture and circularization trajectory is computed for the SEP spacecraft.

Trajectory Optimization

The objective is to compute the minimum-fuel, continuous-thrust trajectory for the circularization maneuver from a given polar elliptical orbit to a circular, polar, 4000 km altitude high lunar orbit (HLO). The initial polar elliptical lunar orbit is the result of the LOI chemical burn and is fixed at a 1000 $\times$ 50,000 km ellipse. For this preliminary analysis, it was determined that the apolune should be less than the lunar sphere of influence (SOI) and that the perilune should be safely above the moon's surface. The chemical fuel required for the LOI burn could be reduced by allowing a higher apolune distance, but three-body effects could adversely alter the elliptical lunar orbit before the SEP circularization maneuver is initiated if the apolune distance is well outside the SOI. The final HLO represents a proposed relay satellite orbit and the main SEP spacecraft will continue on down to polar LLO after releasing the relay spacecraft.

The complete optimal control problem is given below:

For the free end-time problem, find the pitch and yaw thrust steering angles $u(t)$ and $v(t)$, and the coast time to the start of the SEP initiation t_{coast} which minimize

$$J = -m(t_f) \tag{13}$$

subject to the three-body equations of motion

$$\frac{dr}{dt} = v_r \tag{14}$$

$$\frac{dv_r}{dt} = \frac{v_\theta^2}{r} + a_T \sin u \cos v + \nabla U_r \tag{15}$$

$$\frac{dv_\theta}{dt} = -\frac{v_r v_\theta}{r} + a_T \cos u \cos v + \nabla U_\theta \tag{16}$$

$$\frac{d\Omega}{dt} = \frac{\sin\theta}{v_\theta \sin i}\left(a_T \sin v + \nabla U_h\right) \tag{17}$$

$$\frac{di}{dt} = \frac{\cos\theta}{v_\theta}\left(a_T \sin v + \nabla U_h\right) \tag{18}$$

$$\frac{d\theta}{dt} = \frac{v_\theta}{r} - \frac{\sin\theta\cos i}{v_\theta \sin i}(a_T \sin v + \nabla U_h) \tag{19}$$

where $$a_T = \frac{T}{m_{LOI} - \dot{m}t}, \quad 0 \le t \le t_f$$

with the initial conditions

$$r(0) = 2,738 \quad \text{km} \tag{20}$$

$$v_r(0) = 0 \tag{21}$$

$$v_\theta(0) = 1.84427 \quad \text{km/s} \tag{22}$$

$$\Omega(0) = 261.55 \quad \text{deg} \tag{23}$$

$$i(0) = 90 \quad \text{deg} \tag{24}$$

$$\theta(0) = 137.38 \quad \text{deg} \tag{25}$$

and the terminal state constraints

$$\psi[x(t_f), t_f] = \begin{pmatrix} v_r(t_f) \\ v_\theta(t_f) - \sqrt{\mu_m/r} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{26}$$

The states are radial position r, radial velocity v_r, circumferential velocity v_θ, longitude of the ascending node angle Ω, inclination i, and in-plane longitude angle θ. The radius r is the distance from the center of the moon to the spacecraft and v_r and v_θ are the inertial velocity components measured in the instantaneous orbit plane. The ascending node angle Ω is measured counter-clockwise from the fixed $+x$ axis to the ascending node direction. The inertial $+x$ axis is initially pointing from the moon's center to the Earth at $t = 0$. The inclination i is with respect to the $x - y$ or Earth-moon orbit plane. Longitude angle θ is

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the in-plane angle measured from the ascending node to the spacecraft in the direction of motion. Therefore, θ is the sum of argument of perilune ω and true anomaly ν.

The gravity potential gradient ∇U for the combined Earth and moon gravity field is

$$\nabla U = -\frac{\mu_m \vec{r}}{r^3} - \mu_e \left[\frac{\vec{r}_e}{r_e^3} - \frac{\vec{r}_{e-m}}{D^2} \right] \tag{27}$$

where the gravitational parameters of the Earth and Moon are represented by μ_e and μ_m, respectively, $\vec{r}_e$ is the radius vector from the Earth to the spacecraft, $\vec{r}_{e-m}$ is the radius vector from the Earth to the moon, and D is the constant separation distance between the Earth and moon. The components of ∇U are

$$\nabla U_r = -\frac{\mu_m}{r^2} - \frac{\mu_e r_{e_r}}{r_e^3} + \frac{\mu_e r_{e-m_r}}{D^3} \tag{28}$$

$$\nabla U_\theta = -\frac{\mu_e r_{e_\theta}}{r_e^3} + \frac{\mu_e r_{e-m_\theta}}{D^3} \tag{29}$$

$$\nabla U_h = -\frac{\mu_e r_{e_h}}{r_e^3} + \frac{\mu_e r_{e-m_h}}{D^3} \tag{30}$$

where the subscripts r and θ correspond to components along the radial and circumferential in-plane directions and the subscript h corresponds to the direction normal to the instantaneous orbit plane.

The in-plane pitch thrust steering angle u is measured positive above the local horizon to the projection of the thrust vector onto the orbit plane. The out-of-plane yaw thrust steering angle v is measured positive above the orbit plane to the thrust vector and is between ± 90 degrees. The thrust acceleration of the spacecraft, a_T, is computed by dividing the constant thrust magnitude, T, by the current spacecraft mass. The mass of the spacecraft is denoted by m, and propellant mass flow rate $\dot{m}$ is considered positive out of the vehicle.

The initial conditions (20-25) represent the 1000 $\times$ 50,000 km lunar elliptical polar orbit. Since the optimal LOI insertion burn occurs near perilune, the spacecraft is assumed to be at perilune at $t = 0$. The initial eccentricity of the 1000 $\times$ 50,000 km orbit is 0.8995.

The two terminal state constraints (26) define a circular, lunar orbit with unspecified radius or inclination. The remainder of the 3-D low-thrust transfer to the polar HLO is approximated by Edelbaum's analytic expression for 3-D quasi-circular orbit transfers [11] so that the circularization maneuver can be removed from the long-duration, many-revolution, near-circular transfer to HLO.

The goal is to find the thrust steering angles $u(t)$ and $v(t)$, the duration of the coast arc before the SEP engine is initiated, and the final time t_f for the end of the circularization maneuver such that the final spacecraft mass $m(t_f)$ is maximized (or, equivalently, such that the total fuel is minimized) and the spacecraft terminates in a circular, polar HLO. The initial powered circularization maneuver is numerically simulated and the remaining 3-D circle-to-circle transfer to polar LLO is approximated by Edelbaum's analytical expression for quasi-circular transfers. Therefore, fuel mass is accounted for in both the numerically integrated trajectory from elliptical orbit to intermediate circular orbit and the subsequent quasi-circular transfer to polar HLO.

Minimum Eccentricity-Rate Steering

Initial attempts to solve the minimum-fuel circularization transfer problem via SQP resulted in convergence problems due to discontinuities in the pitch thrust steering angle time history. The pitch steering $u(t)$ becomes discontinuous if the control nodes for the cubic spline interpolation are limited to -180 to 180 deg or from 0 to 360 deg since the thrust vector is continually rotating with respect to the local horizon as the SEP circularization maneuver occurs. Therefore, the discontinuities can be removed by guessing a steering profile between 0 and 360 deg for the first revolution, 360 and 720 deg for the second revolution, etc so that the resulting optimal profile will be smooth. However, this approach requires a fairly accurate estimate of the total revolutions completed before the circularization is complete. A simpler approach would be to utilize the minimum eccentricity-rate steering law as a reference and parameterizing a steering angle bias with a cubic spline interpolation through a set of control nodes.

The minimum eccentricity rate steering law is derived from the governing differential equation for eccentricity for two-body motion

$$\dot{e} = \frac{a_T}{v}\left[2(e + \cos\nu)\cos\alpha + \frac{r}{a}\sin\nu\sin\alpha\right] \tag{31}$$

where v is the velocity magnitude and r the radius. The in-plane thrust steering angle α is measured from the velocity vector to the projection of the thrust vector onto the orbit plane and is considered positive above the velocity vector in the direction of motion. The partial derivative of Eq. (31) with respect to steering angle α is

$$\frac{\partial\dot{e}}{\partial\alpha} = \frac{-2a_T}{v}\cos\nu\sin\alpha + \frac{ra_T}{av}\sin\nu\cos\alpha \tag{32}$$

By equating $\partial\dot{e}/\partial\alpha$ to zero, the steering angle which results in an extremal rate (maximum or minimum) is determined. Therefore, equating Eq. (32) to zero and solving for α results in the following extremal steering law:

$$\tan\alpha^* = \frac{r}{2a}\tan\nu \tag{33}$$

In order to determine if this law provides a maximum or minimum eccentricity rate, the second partial derivative is computed:

$$\frac{\partial^2\dot{e}}{\partial\alpha^2} = \frac{-2a_T}{v}\cos\nu\cos\alpha - \frac{ra_T}{av}\sin\nu\sin\alpha \tag{34}$$

The common terms v and a_T are canceled and the following expressions for $\sin\alpha$ and $\cos\alpha$ from the extremal steering law are substituted:

$$\sin \alpha^* = \frac{r \sin \nu}{\sqrt{r^2 \sin^2 \nu + 4a^2 \cos^2 \nu}} \tag{35}$$

$$\cos \alpha^* = \frac{2a \cos \nu}{\sqrt{r^2 \sin^2 \nu + 4a^2 \cos^2 \nu}} \tag{36}$$

Therefore, Eq. (34) becomes

$$\frac{\partial^2 \dot{e}}{\partial \alpha^2} = \frac{-4a \cos^2 \nu - \frac{r^2}{a} \sin^2 \nu}{\sqrt{r^2 \sin^2 \nu + 4a^2 \cos^2 \nu}} < 0 \quad \text{for all} \quad \nu \tag{37}$$

Since the second partial derivative is always negative, the steering law presented by Eqs. (35-36) provides the *maximum* rate for increasing eccentricity. To derive the minimum eccentricity-rate steering laws (maximum negative eccentricity rate), the signs are reversed on Eqs. (35-36):

$$\sin \alpha^* = \frac{-r \sin \nu}{\sqrt{r^2 \sin^2 \nu + 4a^2 \cos^2 \nu}} \tag{38}$$

$$\cos \alpha^* = \frac{-2a \cos \nu}{\sqrt{r^2 \sin^2 \nu + 4a^2 \cos^2 \nu}} \tag{39}$$

These expressions represent the minimum eccentricity rate steering law since substitution into Eq. (34) results in a positive second partial derivative.

The steering law denoted by Eqs. (38-39) ranges from $-180 < \alpha^* < 180$ deg for $0 < \nu < 360$ deg. Therefore, the "discontinuity" of the pitch steering law has been accounted for. Since the minimum eccentricity-rate steering law does not necessarily correspond to the minimum-fuel pitch steering for the circularization maneuver, a bias steering angle u_b is added to reference steering angle u^*

$$u = u^* + u_b \tag{40}$$

where the reference steering u^* is

$$u^* = \alpha^* + \gamma \tag{41}$$

and α^* is the minimum eccentricity-rate steering law from Eqs. (38-39) and γ is the flight path angle. The pitch bias angle u_b is now the control that is parameterized by a cubic spline fit through 11 control nodes. The yaw (out-of-plane) thrust steering angle v is not discontinuous so parameterization is maintained with a cubic spline fit through 11 additional control nodes. Therefore, the total optimization problems has 24 SQP design variables (22 control nodes, t_{coast}, and t_f) and two equality constraints requiring termination in a circular lunar orbit.

Results

The optimal minimum-fuel circularization trajectory was computed for the TROPIX-derived SEP spacecraft. As a result, the spacecraft coasts for approximately one day after the LOI chemical burn at perilune and initiates the SEP circularization maneuver at apolune. The optimal continuous-thrust circularization maneuver lasts 14.7 days and completes about four revolutions about the moon as indicated by Fig. 3. The integrated ΔV for the circularization maneuver is 215.5 m/s and the circular lunar altitude after four revolutions is 19,300 km. The corresponding apolune and perilune altitudes for the optimal circularization maneuver are presented in Fig. 4. It is observed that the optimal circularization maneuver trades between reducing (increasing) apolune (perilune) while holding perilune (apolune) constant until both apolune and perilune meet at 19,300 km. Edelbaum's approximate analytic equation is used to compute the subsequent circle-to-circle transfer to polar HLO and the result is a 29.6 day transfer with an integrated ΔV of 442.7 m/s. The resulting optimal thrust steering histories for the circularization maneuver are presented in Fig. 5 and the "discontinuities" in the pitch steering profile are noted.

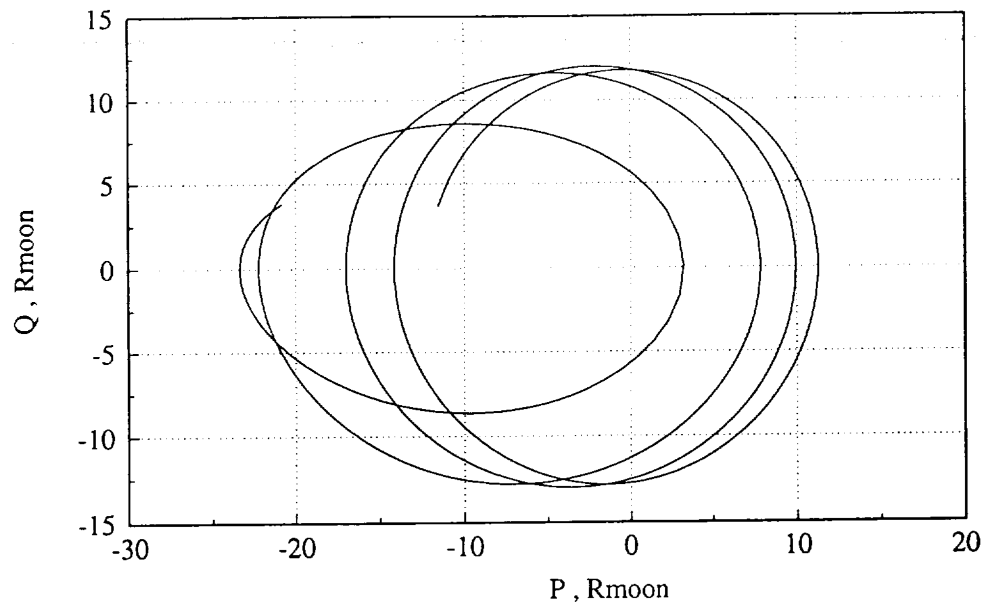

Figure 3: Optimal SEP lunar circularization maneuver

The optimal yaw steering angle oscillates at the orbit period frequency and is fairly small in magnitude.

IV. Lunar Escape Trajectories

After a prolonged stay at polar LLO for scientific data collection, the spacecraft uses the SEP system to escape the Earth-moon system enroute to the comet rendezvous. For preliminary mission planning, the baseline escape trajectory was determined to have zero excess energy at the Earth's heliocentric SOI (i.e., $C_3 = 0$ km^2/s^2). In this section, the optimal minimum-fuel, Earth-moon system powered escape trajectory is computed for the SEP spacecraft.

Trajectory Optimization

The objective is to compute the minimum-fuel, continuous-thrust trajectory for the escape maneuver from circular polar lunar orbit to zero-energy conditions with respect to

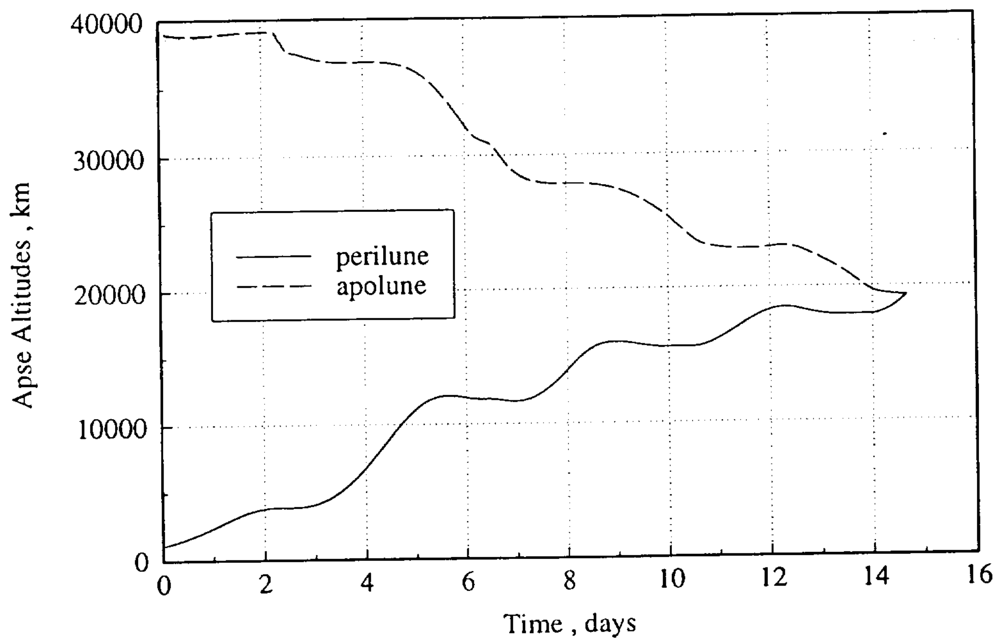

Figure 4: Apolune and perilune during SEP circularization maneuver

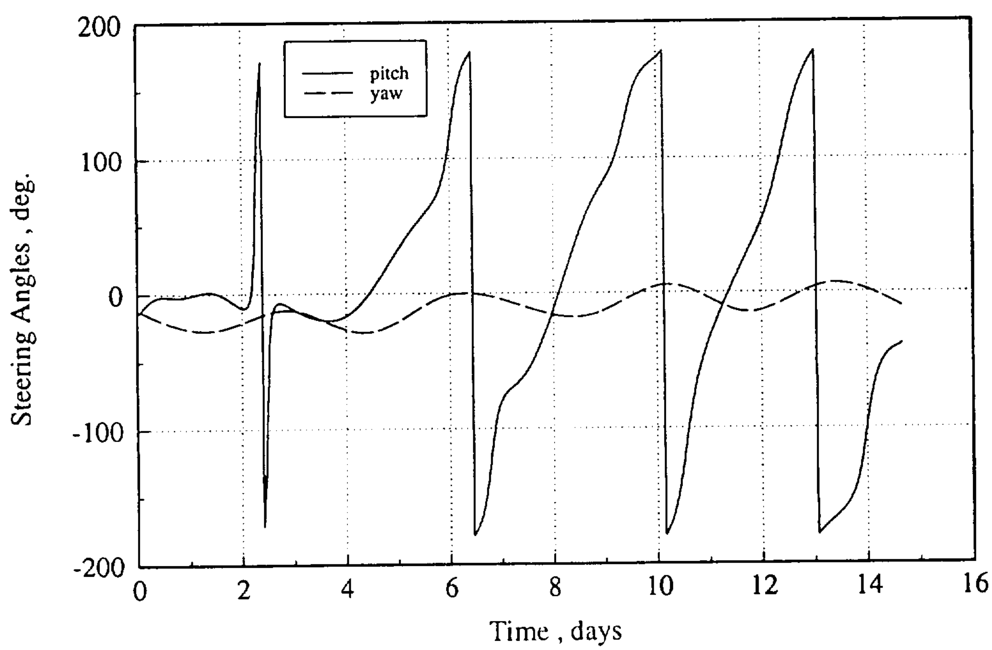

Figure 5: Optimal thrust steering for SEP lunar circularization maneuver

Earth ($C_3 = 0$ km^2/s^2). Therefore, the SEP spacecraft will have zero hyperbolic excess speed ($v_\infty = 0$) at the Earth's SOI for the baseline mission.

The complete optimal control problem is given below:

For the free end-time problem, find the pitch and yaw thrust steering angles $u(t)$ and $v(t)$, and the Julian date for the beginning of the escape which minimize

$$J = -m(t_f) \tag{42}$$

subject to the three-body equations of motion (14-19)

with the initial conditions

$$x(0) = x_0 \tag{43}$$

and the terminal state constraints

$$\psi[x(t_f), t_f] = 0 \tag{44}$$

The states x and governing differential equations of motion are referenced to the moon-centered, inertial coordinate frame as defined in section III. The initial conditions (43) represent a circular, polar lunar orbit at an altitude of 8,500 km. This was determined to be the initial condition for the numerical trajectory optimization problem since the SEP transfer from 450-km polar LLO to 8,500 km altitude is nearly circular and can be approximated by Edelbaum's analytic equation. The corresponding quasi-circular transfer as computed by Edelbaum's equation requires 50 days. The terminal state constraints (44) represent the requirement that the final energy with respect to the Earth at $t = t_f$ be zero. The moon-centered states x must be transformed to an Earth-centered inertial reference frame and the spacecraft's energy with respect to the Earth is calculated

$$\mathcal{E} = \frac{v^2}{2} - \frac{\mu_e}{r} \tag{45}$$

where $\mathcal{E}$ is the energy of the spacecraft with respect to the Earth, and v and r are the velocity and radius of the spacecraft with respect to an Earth-centered inertial frame.

The goal is to find the thrust steering angles $u(t)$ and $v(t)$, the optimal date for beginning the escape maneuver, and the escape time t_f such that the final spacecraft mass $m(t_f)$ is maximized and the spacecraft terminates with zero excess energy. Since the mass-flow rate is constant, this problem is again equivalent to a minimum-time problem. As before, the optimal control problem is solved with SQP. The thrust steering angles u and v are both parameterized with 31 control nodes fit with a cubic spline. Therefore, the SQP problem has 64 design variables and only one equality constraint that enforces the zero Earth-energy at $t = t_f$.

Results

The minimum-fuel escape trajectory was obtained by SQP and the resulting spiral trajectory is shown in Fig. 6. The total trip time from polar LLO to Earth escape conditions is 83.5 days and the total equivalent ΔV is 1360 m/s. The Earth's orbit about the moon is shown by the dotted curve and the Earth's position is indicated at different time points. The first 50 days of the low-altitude escape spiral is not shown in Fig. 6. Therefore, the Earth-position markings for $t = 0$, $t = 6.6$ days, etc are referenced to time after the 50-day quasi-circular transfer computed via Edelbaum. It is observed that at escape conditions ($t = 33$ days on the figure), the trajectory is directly opposite the Earth's current location. Therefore, the spacecraft maximizes the final energy by timing the escape maneuver such that during the last revolution the distance to the Earth is maximized. Although the

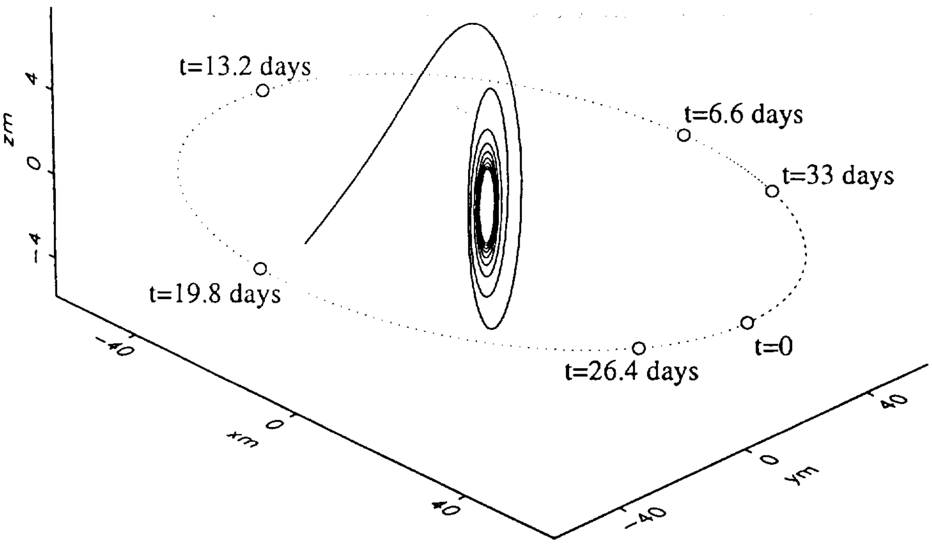

Figure 6: Optimal escape trajectory

Earth's SOI is not shown in Fig. 6, the optimal powered escape trajectory terminates well within the SOI. Therefore, excess Earth-relative energy ($C_3 > 0$) is achievable by continuing the powered SEP escape maneuver out to the SOI.

V. Summary and Conclusions

A preliminary study of the individual phases for a lunar-comet rendezvous mission has been performed. The study was in support of the Diana mission proposal for NASA's 1994 Announcement of Opportunity (AO) for Discovery class exploration missions. The approach taken here was to analyze and optimize each trajectory segment individually. Since the trajectory optimization problems involved a mix of continuous control functions and discrete control parameters, a direct optimization method, namely sequential quadratic programming (SQP), was used.

The comet rendezvous problem in heliocentric space was initially solved using SQP. A penalty function approach was used and proved to enhance the convergence properties of the problem. The minimum-fuel, continuous-thrust rendezvous trajectories were obtained for the comets Wilson-Harrington and Clark. The preliminary analysis helped provide good initial guesses for subsequent calculus of variations-based trajectory optimization programs that allowed multiple thrust and coast arcs.

The optimal minimum-fuel lunar capture and escape problems were solved using SQP in the context of the restricted three-body problem dynamics. A minimum eccentricity-rate steering law was developed for the lunar orbit circularization maneuver. The minimum-fuel lunar escape problem to zero Earth-relative energy was also readily solved using SQP.

The trajectory optimization codes developed in this study were used to individually optimize the respective trajectory segments of the overall mission as spacecraft parameters such as mass, power, specific impulse, and orbit boundary conditions changed during the Diana mission design. The direct optimization approach allowed fairly quick mission iterations and overall mission integration. The trajectory optimization codes are available to NASA Lewis Research Center and reside on the VAX computer system.

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