FINAL TECHNICAL REPORT

The Evolution of Condensates in Shock Tube Flow

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I. Summary

When a simple model for the relationship between the density-temperature fluctuation correlation and mean values is used, we determine that the rate of change of turbulent intensity can influence directly the accretion rate of droplets. We find, experimentally, that the droplet particle size is both temperature and Reynolds number dependent. We also find that the rate of droplet growth has an important dependence on Reynolds number, in a manner stronger than its simple temperature dependence, thereby suggesting a previously unsuspected role for turbulence in the control of condensate accretion.

II. Background

Heterogeneous nucleation and the subsequent droplet and crystal growth play a critical role in the contribution from PCS and vapor trails to atmospheric environmental concerns and in the development of ice crystals on environmentally exposed aerofoils. Yet, the best growth rate theories[1,2,3,4] are essentially ad hoc and/or empirical with very weak experimental confirmation[5,6] of underlying physical principles. In a recent work,[7] U. De Silva , A. Gardner and J. A. Johnson III have shown experimentally that the droplet size increases at fixed Reynolds number when the temperature decreases. They have also shown, independent of temperature, a pronounced dependence of droplet size on the Reynolds num-

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[1] Lothe,J.,and Pound,G.M., "Reconsideration of Nucleation Theory", Journal of Chemical Physics, Vol.36, No.8, 1962, pp.2080-2085.

[2] Lothe,J.,and Pound,G.M., "Statistical Mechanics of Nucleation", in Nucleation edited by A. C. Zettlemoyer, Marcel Dekker, New York, 1969, Chap. 3.

[3] Dunning, W. J., "General and Theoretical Introduction," in Nucleation edited by A. C. Zettlemoyer, Marcel Dekker, New York, 1969, Chap.1.

[4] Wegner,P.P. "Gas Dynamics of Expansion Flows with Condensation and Homogenous Nucleation of Water Vapor" Nonequilibrium Flows, Pt.1, Vol.1, edited by P. P. Wegner, Marcel Dekker , New York, 1970, Chap. 4.

[5] Brian, A. B.,Zuev, A. P. , and Khmelevskhi , A. N. ,"The effect of Condensation on the Parameters of the Flow behind Shock Wave in Water Vapor" Zhurnal Tekhnicheskoi Fiziki, Vol.62, July 1992, pp.765-770.

[6] Schnerr, G. H. and Bohing, R. "Compressible Turbulent Boundary Layers with Heat Addition by Homogenous Condensation", AIAA Journal Vol.30, No.30, 1992, pp. 1284-1289.

[7] U. De Silva, A. Gardner, J. A. Johnson III, AIAA Journal, 33, 1995, pp. 368-370.

ber, specifically suggesting an explicit sensitivity of droplet size to turbulence. In that work, the measurements relied on Mie scattering at a single port at ten different Reynolds numbers and at three different temperatures. Here we will provide a theoretical introduction to the problem of droplet size growth rate sensitivity and then report on our preliminary determinations of a sensitivity in droplet size growth rate (by using two ports) to both temperature and turbulence explicitly.

III. A Theoretical Model for Turbulent Condensate Accretion

Considerable interest exists in the accretion rate for condensates in nonequilibrium flow with icing.[8] Turbulence is thought to play an important role in such flows. It has already been experimentally determined, as cited above, that turbulence influences the sizes of droplets in the heterogeneous nucleation of supersaturated vapors. This section addresses the issue of the possible influence of turbulence on the accretion rate of droplets.

According to a nonequilibrium model developed earlier[9] the droplet growth rate for flow with changing local pressure is given by:

$$\frac{dr(t)}{dt} = -3BN_n \frac{d}{dt} \log \frac{P(t)}{P_\infty} \tag{1}$$

with $B = (2\sigma)/[\rho_c T(R/\mu_v)]$ where σ = surface tension coefficient; ρ_c = condensate density; and μ_v = molecular weight of vapor. N_n is a correction coefficient such that $0 \le N_n \le 4$ and $N_n = [(n-3)^{n-3}]/[(n-4)^{n-2}]$ for $n>4$. P_∞ is obtained through the Clausius-Clapeyron equation. For adiabatic flow of a compressible ideal gas the mean value of the square of the local speed of sound is defined as

$$\overline{b^2} = \left(\frac{\partial \overline{P}}{\partial \overline{\rho}}\right)_{\overline{s},g} = \gamma \frac{\overline{P}}{\overline{\rho}} \tag{2}$$

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[8]T. Cebeci, AIAA Journal, 33, 1995, pp. 1351-2

[9]R. Hazoumé and J. Chabi Orou, "Pressure Sensitivity in the Droplet Growth Rate in Nonequilibrium Condensation," in Mathematical and Experimental Developments for Nonequilibrium Statistical Systems, Institut de Mathematiques et de Sciences Physiques, Université Nationale du Benin, May 25-27, 1994, Contonu, Benin (to be published)

where: $\gamma = C_P/C_V$ is the ratio of the specific heats; $\overline{g}$ is the mean value of the condensate mass fraction of water; $\overline{P}$ is the mean pressure function; $\overline{s}$ is the mean entropy; and $\overline{\rho}$ is the mean density function.

We can readily write from (2) that:

$$\frac{\overline{P}}{\overline{\rho}^{\gamma}} = cnst \text{ and } \frac{\overline{P}}{(\overline{b^2})^{\frac{\gamma}{(\gamma-1)}}} = cnst \tag{3}$$

Equations (2) and (3) were written assuming there are fluctuations of the principle physical properties around their mean values as, e.g., $g = \overline{g} + g'$ with $\overline{g'} = 0$. However $\overline{g'^2} \neq 0$. This means that we can consider $P = \overline{P} + P'$, $\rho = \overline{\rho} + \rho'$, g = $\overline{g} + g'$, and $T = \overline{T} + T'$. The equation of state

$$P = \rho(\frac{1-\omega_o}{\mu_i} + \frac{\omega_o - g}{\mu_v})RT \tag{4}$$

becomes

$$\overline{P} + P' = (\overline{\rho} + \rho')(\frac{1-\omega_o}{\mu_i} + \frac{\omega_o - \overline{g} - g'}{\mu_v})R(\overline{T} + T') \tag{5}$$

with ω_o=the initial humidity, μ_i =the molecular mass of the carrier gas, and R=the ideal gas constant. Averaging equation (4) yields:

$$\overline{P} = (\frac{1-\omega_o}{\mu_i} + \frac{\omega_o - \overline{g}}{\mu_v})R(\overline{T}\overline{\rho} + \overline{\rho' T'}) \tag{6}$$

where a new function, $\overline{\rho' T'}$, has appeared; this is the density-temperature correlation function involving the fluctuations of density and temperature.

Now we make the important simple assumption

$$\overline{\rho' T'} = -\lambda \overline{\rho}\overline{T}. \tag{7}$$

The equation of state then becomes

$$\frac{\overline{P}}{\overline{\rho}} = l(\overline{g})(1-\lambda)R\overline{T} = \frac{\overline{b^2}}{\gamma} \tag{8}$$

where

$$l(\overline{g}) = \frac{1-\omega_o}{\mu_i} + \frac{\omega_o - \overline{g}}{\mu_v}. \tag{9}$$

We must also make it clear that the physical properties of interest are functions of space and time. That is,

$$\overline{T} = \overline{T}(x_i,t), \overline{P} = \overline{P}(x_i,t), \overline{\rho} = \overline{\rho}(x_i,t), \overline{g} = \overline{g}(x_i,t), \lambda = \lambda(x_i,t) \qquad (10)$$

where i = 1,2,3, x_i is the i component of the space variable, and t is the time. To use equation (10), the simplest adequate model for $\lambda(x_i,t)$ would be $-\lambda(x_i,t) = \exp - ft$. With this model, the density temperature correlation function depends only on time and obeys a decay law which is exponential. Finally, it is then usual (with U and Λ as velocity and length parameters respectively) to express all these relations in a nondimensional form: $v_i = u_i/U$; $y_i = x_i/\Lambda$; and $\tau = tU/\Lambda$.

We can now include the influence of fluctuations in temperature (as well as the implicit fluctuations in P and ρ) in the mean square of the local speed of sound $\overline{a^2(y_2,\tau)}$ by explicitly requiring $\frac{\overline{P}(y_2,\tau)}{\overline{\rho}(y_2,\tau)} = \frac{\overline{a^2}}{\gamma}$. This form is reconciled with equation (8) above by

$$\frac{\overline{b^2}}{\gamma} = \frac{\overline{a^2}}{\gamma}(1-\lambda(y_2,\tau)) \qquad (11)$$

We have shown in equation (3) that $\overline{P}\alpha(b^2)^{\frac{\gamma}{(\gamma-1)}}$. Thus

$$\frac{\partial}{\partial\tau}Log\overline{P}(y_i,\tau) = \frac{\gamma}{(\gamma-1)}\left[\frac{\partial\overline{a^2(y_2,\tau)}}{\partial\tau} + \frac{\partial}{\partial\tau}Log(1-\lambda)\right] \qquad (12)$$

$$= \frac{\gamma}{\gamma-1}\{\frac{\partial}{\partial\tau}\overline{a^2(y_2,\tau)} - \frac{f}{\exp f\tau + 1}\}$$

in which f is the real positive constant introduced in the definition of λ. We finally obtain from equation (1)

$$\frac{dr(\tau)}{d\tau} = \frac{6\sigma}{\rho_c(\frac{R}{\mu_v})\overline{T}}\frac{(n-3)^{n-3}}{(n-4)^{n-2}}\frac{\gamma}{(\gamma-1)}\left\{\frac{f}{\exp f\tau+1} - \frac{\partial\overline{a^2(y_2,\tau)}}{\partial\tau}\right\} \qquad (13)$$

The interpretation of equation (13) is straightforward. (i)At small values of τ and constant turbulent intensity, $(\partial\overline{a^2(y_2,\tau)}/\partial\tau=0)$, the rate of change in droplet size is determined entirely by the density correlation exponential factor f. (ii)At large values of τ and constant turbulent intensity, $(\partial\overline{a^2(y_2,\tau)}/\partial\tau=0)$, the rate of change in droplet size has no dependence on turbulence at all. (iii)If the turbulent intensity is not constant, i.e., $(\partial\overline{a^2(y_2,\tau)}/\partial\tau\neq0)$, then the rate of change in droplet size is sensitive to the turbulence at all times. The case $\partial\overline{a^2(y_2,\tau)}/\partial\tau\neq0$ requires that a measure of the actual evolution in the strength of turbulent fluctuations be determined and that a model for determining the connection between the mean square local speed of sound and the evolution of turbulence be determined. It has already been shown[2] that, at fixed thermodynamic conditions, the size of a droplet in condensing flow is influenced by the value of the Reynolds number and, by inference, the strength of local turbulence. Equation (13) above sets conditions for the sensitivity of the condensate accretion to turbulence, based on basic physics principles, which can now be tested by experiments.

IV. Experimental Results

To realize the purpose of droplet rate of growth measurement, we looked at the scatter signals from two ports 30 centimeters apart along the shock tubes axis each other for ten different Reynolds numbers and five different temperatures. A pressure driven shock tube was used consisting of a driver chamber of 60x5.2 in. and a two part driven chamber with a test section of 60x5.2 in. and an extension tube of 76x5.2 in. Aluminized mylar sheets of two different thicknesses were used as the material for the diaphragm which separates the driven chamber from the driver chamber. Compressed air is supplied to the driver chamber from a reservoir, where the relative humidity of the air could be controlled by setting the temperature in the reservoir at 70°F.

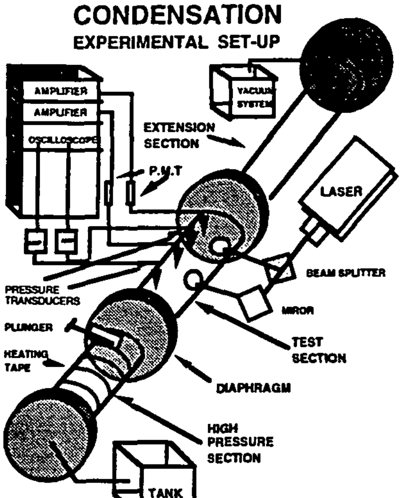

Fig.1

A heating tape is wrapped around the driver chamber, and a thermocouple probe is used in the driver chamber to measure the initial temperature. The test section has three stations for optical diagnostics, two of which consist of four perpendicular glass ports, the other station has two in-line glass ports. An Argon laser is used for the right angle scattering through two of the three optical stations. Two pressure transducers are mounted on either side of the optical station . The static pressure and the right angle scatter signals are recorded on the four channel 500-MHz Tektronix digital oscilloscope. All of the signals are stored on disks. The optics for receiving the scattered signal from the droplets include a natural density filter; the received signals are transmitted to the photomultiplier tubes using the fiber optics and also stored on the oscilloscope just mentioned.

Appropriate analytical and calibration procedures are used so as to provide continuous recordings of droplet size and density at the measuring stations. Off-line reconstructions are performed, using appropriate computer software and hardware, so as to provide histories in the droplet frame of reference using the velocity calculated from the assumptions of one-dimensional shock tube flow and confirmed by direct LDV. From these results, we obtain droplet growth rate data and a droplet growth rate model which will determine the roles of temperature and suspensions in the evolution of droplets.

We know that, in a pressure driven shock tube, at a shock wave velocity of Mach number of 1.8 the contact surface is fully turbulent.[10] The shock wave velocity is fixed (M=1.8), by maintaining the pressure ratio (P_4/P_1) constant. We changed Reynolds numbers by changing P_1 and P_4 at (P_4/P_1) constant. After each run, we measured the shock wave velocity using the quartz pressure transducers.

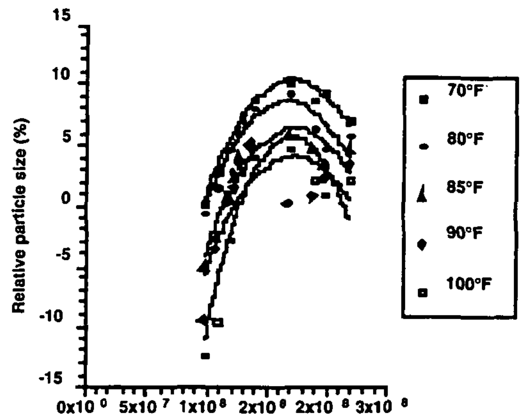

Figure 2

The scatter signals from the two photomultipliers were converted to the particle size by using Raleigh's scattering theory.[11] The intensity (in voltage) of the scattered wave at the distance r from the origin of axes to the particle is given by Rayleigh formula:

$$I=\frac{8\pi^4 a^6}{r^2\lambda^4}\left|\frac{m^2-1}{m^2+1}\right|^2(1+\cos\theta) \qquad (1)$$

where a is the particle 's radius ,m is a complex relative refraction index, λ is the wave length in the medium. For simplicity in making comparisons, we used the relative particle size respect to the first particle. Hence the relative particle size is given by $\frac{a}{a_0}=\frac{I}{I_0}$

The influence of the turbulence on the particle size is shown on the Fig.2, where we plotted the relative particle size change against Reynolds number. The relative particle size change is given by: $(\frac{a_{1j}-a_{10}}{a_{10}})$. where a is the particle radius, the first index refers to the port number (one or two) and the second one is the

[10]Johnson, J. A. ,Santiago ,J.and Jones ,W. R. "Driver Gas Flow with Fluctuations", Journal of Physics D : Applied Physics,Vol.13 , 1980 pp.1413-1425.
[11]Kerker,M." The Scattering Of Light." Academic Press, New York,1969, Chap.13

index for the Reynolds number. The direct explicit connection between turbulence and Reynolds number and the role of this connection in nonequilibrium phenomena have already been shown.[12] Fig. 2 shows that the particle size increases as the temperature decreases at fixed Reynolds number. The data in Fig.2 are results from the first port. The same results are available from the second port.

We have determined the relative particle size change per unit time as function of temperature and Reynolds number. The relative particle size change per unit time is given by $(\frac{a_{2j} - a_{1j}}{a_{10}t})$ where t is the time required for the contact surface to travel from the port one to the port two. With these measurements, we have found a new clear dependence of droplet size on the strength of the turbulence. As Fig, 2 above shows, increasing Reynolds number produces an increase in the relative sizes of the droplets in a manner which changes with temperature and changes with Reynolds number according to a standard strong coherence model. Furthermore, when the rate of droplet growth is measured by observing the change in droplet size at two locations, we find that the rate of change of droplet size is also dependent on the Reynolds number. (See Fig. 3.)

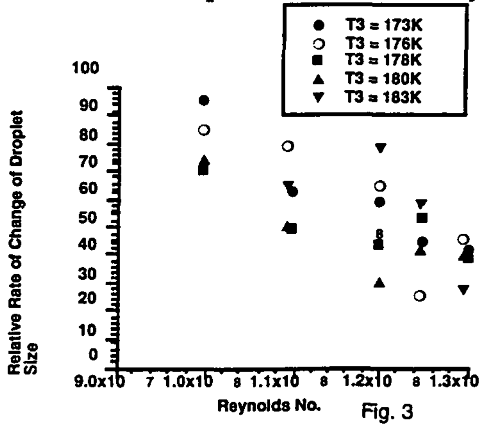

Fig. 3

We show a temperature sensitivity in these results also. We expect, with this motivation and these results, to extend our investigations to low temperature regimes; we are particularly interested in the pressure and temperature condi-

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12 Johnson, J. A. and Ramaiah , R. I. L., "Reduced Molecular Chaos and Flow Instability " Stability in the Mechanics of Continua , edited by F. H. Schroeder, Spring-Verlag , Berlin , 1982 ,pp. 318-329.

tions appropriate to sublimation. Under these conditions we expect to determine the role of both turbulence and temperature in the evolution of ice crystals, appropriate to high altitude jet trails and to the onset of icing on the wings of airplanes.

V. Bibliography for NAG 2-921

R. Hazoumé, J. A. Johnson III, "Condensate Droplet Growth with Temperature Fluctuations," (submitted to Physics Letters)

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