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James Reuther and Antony Jameson

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# **Supersonic Wing and Wing-Body Shape Optimization Using an Adjoint Formulation**

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# SUPERSONIC WING AND WING-BODY SHAPE OPTIMIZATION USING AN ADJOINT FORMULATION

**J. Reuther**

Research Institute for Advanced Computer Science  
Mail Stop 227-6  
NASA Ames Research Center  
Moffett Field, California 94035

**A. Jameson**

Department of Mechanical and Aerospace Engineering  
Princeton University  
Princeton, New Jersey 08544

## ABSTRACT

This paper describes the implementation of optimization techniques based on control theory for wing and wing-body design of supersonic configurations. The work represents an extension of our earlier research in which control theory is used to devise a design procedure that significantly reduces the computational cost by employing an adjoint equation. In previous studies it was shown that control theory could be used to devise transonic design methods for airfoils and wings in which the shape and the surrounding body-fitted mesh are both generated analytically, and the control is the mapping function [5, 6, 8]. The method has also been implemented for both transonic potential flows and transonic flows governed by the Euler equations using an alternative formulation which employs numerically generated grids, so that it can treat more general configurations [16, 9, 17]. Here results are presented for three-dimensional design cases subject to supersonic flows governed by the Euler equation.

## INTRODUCTION

Since the inception of CFD, researchers have sought not only accurate aerodynamic prediction methods for given configurations, but also design methods capable of creating new optimum configurations. Yet, while flow analysis can now be carried out over quite complex configurations using the Navier-Stokes equations with a high degree of confidence, direct CFD based design is still limited to simple two-dimensional and three-dimensional configurations, usually without the inclusion of viscous effects.

One approach to extend the existing CFD analysis capability to treat the design problem, but only at the price of heavy computational expense, is to use numerical optimization methods. The essence of these methods is very simple: a numerical optimization procedure is coupled directly to an existing CFD analysis algorithm. The numerical optimization procedure attempts to extremize a chosen aerodynamic measure of merit which is evaluated by the given CFD code. The configuration is systematically modified through user specified design variables. Most of these optimization procedures require the gradient of the the objective function with respect to changes in the design variables. The simplest method of obtaining

gradient information is by finite differences. In this technique, the gradient components are estimated by independently perturbing each design variable with a finite step, calculating the corresponding value of the objective function using CFD analysis, and forming the ratio of the differences. The gradient is used by the numerical optimization algorithm to calculate a search direction using steepest descent, conjugate gradient, or quasi-Newton techniques. After finding the minimum or maximum of the objective function along the search direction, the entire process is repeated until the gradient approaches zero and further improvement is impossible.

The use of numerical optimization for transonic aerodynamic shape design was pioneered by Hicks, Murman and Vanderplaats [4]. They applied the method to two-dimensional profile design subject to the potential flow equation. The method was quickly extended to wing design by Hicks and Henne [3]. More recently, in the work of Reuther, Cliff, Hicks and Van Dam, the method has proven to be successful for the design of supersonic wing-body transport configurations [15]. In all of these cases finite difference methods were used to obtain the required gradient information.

## FORMULATION OF THE DESIGN PROBLEM AS A CONTROL PROBLEM

In an attempt to reduce the computational expense of aerodynamic optimization recent works have focused on using control theory to provide inexpensive gradient information. The technique can be outlined as follows.

For flow about an airfoil or wing, the aerodynamic properties which define the cost function are functions of the flow-field variables ( $w$ ) and the physical location of the boundary, which may be represented by the function  $\mathcal{F}$ , say. Then

$$I = I(w, \mathcal{F})$$

and a change in  $\mathcal{F}$  results in a change

$$\delta I = \frac{\partial I^T}{\partial w} \delta w + \frac{\partial I^T}{\partial \mathcal{F}} \delta \mathcal{F} \quad (1)$$

in the cost function. Each term in (1), except for  $\delta w$ , can be easily obtained. Finite difference methods require a number of additional flow calculations equal to the number of design variables. Using

control theory, the governing equations of the flowfield are introduced as a constraint in such a way that the final evaluation of the gradient does not require multiple flow solutions. In order to achieve this,  $\delta w$  must be eliminated from (1). The governing equation  $R$  and its first variation express the dependence of  $w$  and  $\mathcal{F}$  within the flowfield domain  $D$ :

$$R(w, \mathcal{F}) = 0, \quad \delta R = \left[ \frac{\partial R}{\partial w} \right] \delta w + \left[ \frac{\partial R}{\partial \mathcal{F}} \right] \delta \mathcal{F} = 0. \quad (2)$$

Next, introducing a Lagrange multiplier  $\psi$ , we have

$$\begin{aligned} \delta I &= \frac{\partial I^T}{\partial w} \delta w + \frac{\partial I^T}{\partial \mathcal{F}} \delta \mathcal{F} - \psi^T \left( \left[ \frac{\partial R}{\partial w} \right] \delta w + \left[ \frac{\partial R}{\partial \mathcal{F}} \right] \delta \mathcal{F} \right) \\ &= \left\{ \frac{\partial I^T}{\partial w} - \psi^T \left[ \frac{\partial R}{\partial w} \right] \right\} \delta w + \left\{ \frac{\partial I^T}{\partial \mathcal{F}} - \psi^T \left[ \frac{\partial R}{\partial \mathcal{F}} \right] \right\} \delta \mathcal{F} \end{aligned}$$

Choosing  $\psi$  to satisfy the adjoint equation

$$\left[ \frac{\partial R}{\partial w} \right]^T \psi = \frac{\partial I}{\partial w} \quad (3)$$

the first term is eliminated, and we find that the desired gradient is given by

$$\mathcal{G}^T = \frac{\partial I^T}{\partial \mathcal{F}} - \psi^T \left[ \frac{\partial R}{\partial \mathcal{F}} \right]. \quad (4)$$

Since (4) is independent of  $\delta w$ , the gradient of  $I$  with respect to an arbitrary number of design variables can be determined without the need for additional flow-field evaluations. The main cost is in solving the adjoint equation (3). In general, the adjoint problem is about as complex as a flow solution. If the number of design variables is large, the cost differential between one adjoint solution and the large number of flowfield evaluations required to determine the gradient by finite differencing becomes compelling. Once equation (4) is obtained,  $\mathcal{G}$  can be fed into any numerical optimization algorithm to obtain an improved design.

## ISSUES OF IMPORTANCE FOR DESIGN PROBLEMS

The development of aerodynamic design procedures that employ an adjoint equation formulation is currently being investigated by many researchers. These methods promise to allow computational fluid dynamics methods to become true aerodynamic design methods. References [1, 2, 10, 13, 12, 11, 14, 19] represent a partial list of recent works that reflect this developing field. However, as is the case in any new research field, many questions remain. Probably the most salient issues of concern are the following:

1. Discrete vs. continuous sensitivities
2. Choice of optimization procedures
3. Treatment of geometric and aerodynamic constraints
4. The level of coupling between design and analysis
5. The parameterization of the design space

These topics still require intensive investigation in the up coming years. With regards to the first item, it is historically interesting that Jameson in 1988 [5], first developed the equations necessary for a continuous sensitivity approach to treat the design of airfoils and

wings subject to transonic flows. By continuous sensitivities it is implied that the steps represented by equations (1-4) are applied to the governing differential equations. The resulting adjoint differential equations with the appropriate boundary conditions may then be discretized and solved in a manner similar to that used for the flow solver. One may alternatively derive a set of discrete adjoint equations directly from the discrete approximation to the flow equations by following the procedure outlined in equations (1-4). The resulting discrete adjoint equations are one of the possible discretizations of the continuous adjoint equations. This alternative is mentioned in [6], but was not adopted in that work because of the complexity of the resulting discrete adjoint system. The approach has found favor in the work of Taylor et al. [13, 10] and Baysal et al. [1, 2].

It seems that both alternatives have some advantages. The continuous approach gives the researcher some hope for an intuitive understanding of the adjoint system and its related boundary conditions. The discrete approach, in theory, maintains perfect algebraic consistency at the discrete level. If properly implemented it will give gradients which closely match those obtained through finite differences. The continuous formulation produces slightly inaccurate gradients due to differences in the discretization. However, these inaccuracies must vanish as the mesh width is reduced.

The discrete adjoint equations are a linear system, whether derived directly by the discrete approach or by discretization of the continuous adjoint equation. The size and complexity of the system, however, makes the use of direct solution methods unrealistic for all but the smallest problems. The application of the continuous sensitivity analysis fosters the easy recycling of the flow solution algorithm for the solution of the adjoint equations, since the steps applied to the original governing differential equations can be duplicated for the adjoint differential equations. When the discrete approach is used, the adjoint equations have a complexity which makes it hard to find decompositions to facilitate their solution unless the structure from the continuous adjoint is used as a guide. The discrete method is subject, moreover, to the difficulty that the discrete flow equations often contain nonlinear flux limiting functions which are not differentiable. It also limits the flexibility to use adaptive discretization techniques with order and mesh refinement, such as the  $h-p$  method, because the adjoint discretization is fixed by the flow discretization.

It turns out that the determination of items (2-4) in the above list strongly hinge on the choice for (5). In Jameson's first works in the area [5, 6, 8], every surface mesh point was used as a design variable. In three-dimensional wing design cases this led to as many as 4224 design variables [8]. The use of the adjoint method eliminated the unacceptable costs that such a large number of design variables would incur for traditional finite difference methods. If the approach were extended to treat complete aircraft configurations, at least tens of thousands of design variables would be necessary. Such a large number of design variables precludes the use of descent algorithms such as Newton or quasi-Newton approaches simply because of the high cost of matrix operations for such methods. The limitation of using a simple descent procedure, such as steepest descent, has the consequence that significant errors can be tolerated initially in the gradient evaluation. Such methods therefore favor tighter coupling of the flow solver, the adjoint solver, and the overall design problem to accelerate convergence. Ta'asan et al. [11], have taken advantage of this by formulating the design problem as a one shot procedure where all three systems are advanced simultaneously. Choosing such

a design space suffers from admitting poorly conditioned design problems. This is best exemplified by the case where only one point on the surface of an airfoil is moved, resulting in a highly nonlinear design response. In his original work Jameson [6, 7] addressed this poor conditioning of the design problem by smoothing the control (surface shape) and thus removing the problematic high frequency content from the advancing solution shape.

Hicks and others [4, 3, 15] have in the past parameterized the design space using sets of smooth functions that perturb the initial geometry. By using such a parameterization it is possible to work with considerably fewer design variables than the choice of every mesh point. And since Hick's original works exclusively used finite difference gradients, the inherent limit to a small number of design variables allowed for the use of more efficient search strategies.

When distributed over the entire chord on both upper and lower surfaces, such analytic perturbation functions admit a large possible design space. They can be chosen such that symmetry, thickness, or volume can be explicitly constrained, thus avoiding the use of expensive constrained optimization algorithms to address geometric constraints. However, such a choice of design variables does not guarantee that a solution, for example, of the inverse problem for a realizable target pressure distribution will necessarily be attained. Finally, they have one last advantage over using the mesh points; there is no need to smooth the resulting solutions as the design proceeds when using Hicks-Henne functions, since by construction higher frequencies are not admitted and thus the design spaces are naturally well posed.

### THREE-DIMENSIONAL DESIGN USING THE EULER EQUATIONS

The application of control theory to aerodynamic design problems is illustrated by treating the case of three-dimensional wing design, using the inviscid Euler equations as the mathematical model for compressible flow. In this case it proves convenient to denote the Cartesian coordinates and velocity components by  $x_1, x_2, x_3$  and  $u_1, u_2, u_3$ . The three-dimensional Euler equations may be written as

$$\frac{\partial w}{\partial t} + \frac{\partial f_i}{\partial x_i} = 0 \quad \text{in } D, \quad (5)$$

where

$$w = \begin{pmatrix} \rho \\ \rho u_1 \\ \rho u_2 \\ \rho u_3 \\ \rho E \end{pmatrix}, \quad f_i = \begin{pmatrix} \rho u_i \\ \rho u_i u_1 + p \delta_{i1} \\ \rho u_i u_2 + p \delta_{i2} \\ \rho u_i u_3 + p \delta_{i3} \\ \rho u_i H \end{pmatrix} \quad (6)$$

and  $\delta_{ij}$  is the Kronecker delta function. Also,

$$p = (\gamma - 1) \rho \left\{ E - \frac{1}{2} (u_i^2) \right\}, \quad (7)$$

and

$$\rho H = \rho E + p \quad (8)$$

where  $\gamma$  is the ratio of the specific heats. Consider a transformation to coordinates  $\xi_1, \xi_2, \xi_3$  where

$$K_{ij} = \left[ \frac{\partial x_i}{\partial \xi_j} \right], \quad J = \det(H), \quad K_{ij}^{-1} = \left[ \frac{\partial \xi_i}{\partial x_j} \right].$$

Introduce contravariant velocity components as

$$\begin{pmatrix} U_1 \\ U_2 \\ U_3 \end{pmatrix} = K^{-1} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$$

The Euler equations can now be written as

$$\frac{\partial W}{\partial t} + \frac{\partial F_i}{\partial \xi_i} = 0 \quad \text{in } D, \quad (9)$$

with

$$W = J \begin{pmatrix} \rho \\ \rho u_1 \\ \rho u_2 \\ \rho u_3 \\ \rho E \end{pmatrix}, \quad F_i = J \begin{pmatrix} \rho U_i \\ \rho U_i u_1 + \frac{\partial \xi_i}{\partial x_1} p \\ \rho U_i u_2 + \frac{\partial \xi_i}{\partial x_2} p \\ \rho U_i u_3 + \frac{\partial \xi_i}{\partial x_3} p \\ \rho U_i H \end{pmatrix}. \quad (10)$$

Assume now that designs are limited to wing and wing-body configurations using a C-H topology mesh. If a body is present it is assumed to exist as part of the face containing the symmetry plane. Thus the new computational coordinate system conforms to the wing in such a way that the wing surface  $B_W$  is represented by  $\xi_2 = 0$  and the body and symmetry plane  $B_B$  conform such that  $\xi_3 = 0$ . Then the flow is determined as the steady state solution of equation (9) subject to the flow tangency conditions

$$U_2 = 0 \quad \text{on } B_W, \quad U_3 = 0 \quad \text{on } B_B. \quad (11)$$

At the far field boundary  $B_F$ , freestream conditions are specified for incoming waves, while outgoing waves are determined by the solution.

In our previous works, the illustrative problem most often used specified the cost function as the difference between the current and some desired pressure distribution. However for supersonic flows where pressure distributions for optimum aerodynamic performance are less clearly understood than for transonic flows, it is advantageous to consider the use of drag as the cost function.

$$\begin{aligned} I &= C_{D_W} + C_{D_B} \\ &= (C_{A_W} + C_{A_B}) \cos \alpha + (C_{N_W} + C_{N_B}) \sin \alpha \\ &= \frac{1}{S_{\text{ref}}} \iint_{B_W} C_P (S_x \cos \alpha + S_y \sin \alpha) d\xi_1 d\xi_3 \\ &\quad + \frac{1}{S_{\text{ref}}} \iint_{B_B} C_P (S_x \cos \alpha + S_y \sin \alpha) d\xi_1 d\xi_2, \end{aligned}$$

where  $S_x$  and  $S_y$  define projected surface areas, and  $S_{\text{ref}}$  is the reference area. The design problem is now treated as a control problem where the control function is the wing shape, which is to be chosen to minimize  $I$  subject to the constraints defined by the flow equations (5–10). A variation in the shape will cause a variation  $\delta p$  in the pressure and consequently a variation in the cost function

$$\begin{aligned} \delta I &= \delta C_A \cos \alpha + \delta C_N \sin \alpha \\ &\quad + \{-C_A \sin \alpha + C_N \cos \alpha\} \delta \alpha \\ &\quad + \left\{ \frac{\partial C_A}{\partial \alpha} \cos \alpha + \frac{\partial C_N}{\partial \alpha} \sin \alpha \right\} \delta \alpha. \end{aligned}$$

where  $\delta C_A$  and  $\delta C_N$  the variation due to changes in the design parameters with  $\alpha$  fixed. To treat the interesting problem of practical supersonic design wave drag must be minimized at a fixed lift coefficient. Thus an additional constraint is given by

$$\delta C_L = 0 = \delta C_{L_W} + \delta C_{L_B},$$

which gives

$$\begin{aligned} & \delta C_N \cos \alpha - \delta C_A \sin \alpha \\ + & \{-C_N \sin \alpha - C_A \cos \alpha\} \delta \alpha \\ + & \left\{ \frac{\partial C_N}{\partial \alpha} \cos \alpha - \frac{\partial C_A}{\partial \alpha} \sin \alpha \right\} \delta \alpha = 0 \end{aligned}$$

Combining these two expressions to eliminate  $\delta \alpha$  gives

$$\begin{aligned} \delta I &= \delta C_A \cos \alpha + \delta C_N \sin \alpha \\ &+ \Omega \{\delta C_N \cos \alpha - \delta C_A \sin \alpha\}, \end{aligned} \quad (12)$$

where  $\Omega$  is given by

$$\Omega = \frac{(-C_L + \frac{\partial C_A}{\partial \alpha} \cos \alpha + \frac{\partial C_N}{\partial \alpha} \sin \alpha)}{(C_D + \frac{\partial C_N}{\partial \alpha} \cos \alpha - \frac{\partial C_A}{\partial \alpha} \sin \alpha)}.$$

Since  $p$  depends on  $w$  through the equation of state (7-8), the variation  $\delta p$  can be determined from the variation  $\delta w$ . If a fixed computational domain is used the variations in the shape result in variations in the mapping derivatives. Define the Jacobian matrices

$$A_i = \frac{\partial f_i}{\partial w}, \quad C_i = JK_i^{-1} A_j. \quad (13)$$

Then the equation for  $\delta w$  in the steady state becomes

$$\frac{\partial}{\partial \xi_i} (\delta F_i) = 0,$$

where

$$\delta F_i = C_i \delta w + \delta \left( J \frac{\partial \xi_i}{\partial x_j} \right) f_j.$$

Now, multiplying by a vector co-state variable  $\psi$ , assuming the result is differentiable, and integrating by parts over the domain

$$\int_D \left( \frac{\partial \psi^T}{\partial \xi_i} \delta F_i \right) d\xi_j = \int_B (\bar{n}_i \psi^T \delta F_i) d\xi_j, \quad (14)$$

where  $\bar{n}_i$  are components of a unit vector normal to the boundary. The variation in the cost function can also be expressed in terms of  $\delta p$  after (12 and 14) are summed to give,

$$\begin{aligned} \delta I &= \frac{1}{\frac{1}{2} \gamma M_\infty^2 S_{\text{ref}}} \iint_{B_W} \delta p \{ (S_x \cos \alpha + S_y \sin \alpha) \\ &+ \Omega (S_y \cos \alpha - S_x \sin \alpha) \} d\xi_1 d\xi_3 \\ + & \frac{1}{\frac{1}{2} \gamma M_\infty^2 S_{\text{ref}}} \iint_{B_B} \delta p \{ (S_x \cos \alpha + S_y \sin \alpha) \\ &+ \Omega (S_y \cos \alpha - S_x \sin \alpha) \} d\xi_1 d\xi_2 \\ + & \frac{1}{S_{\text{ref}}} \iint_{B_W} C_p \{ (\delta S_x \cos \alpha + \delta S_y \sin \alpha) \\ &+ \Omega (\delta S_y \cos \alpha - \delta S_x \sin \alpha) \} d\xi_1 d\xi_3 \end{aligned}$$

$$\begin{aligned} + & \frac{1}{S_{\text{ref}}} \iint_{B_B} C_p \{ (\delta S_x \cos \alpha + \delta S_y \sin \alpha) \\ &+ \Omega (\delta S_y \cos \alpha - \delta S_x \sin \alpha) \} d\xi_1 d\xi_2 \\ - & \int_D \left( \frac{\partial \psi^T}{\partial \xi_i} \delta F_i \right) d\xi_j + \int_B (\bar{n}_i \psi^T \delta F_i) d\xi_j. \end{aligned} \quad (15)$$

On the wing surface  $B_W$ ,  $\bar{n}_1 = \bar{n}_3 = 0$  and on the body and symmetry plane  $B_B$ ,  $\bar{n}_1 = \bar{n}_2 = 0$ . It follows from equation (11) that

$$\begin{aligned} \delta F_2 &= J \begin{Bmatrix} 0 \\ \frac{\partial \xi_2}{\partial x_1} \delta p \\ \frac{\partial \xi_2}{\partial x_2} \delta p \\ \frac{\partial \xi_2}{\partial x_3} \delta p \\ 0 \end{Bmatrix} + p \begin{Bmatrix} 0 \\ \delta \left( J \frac{\partial \xi_2}{\partial x_1} \right) \\ \delta \left( J \frac{\partial \xi_2}{\partial x_2} \right) \\ \delta \left( J \frac{\partial \xi_2}{\partial x_3} \right) \\ 0 \end{Bmatrix} \text{ on } B_W \\ \delta F_3 &= J \begin{Bmatrix} 0 \\ \frac{\partial \xi_3}{\partial x_1} \delta p \\ \frac{\partial \xi_3}{\partial x_2} \delta p \\ \frac{\partial \xi_3}{\partial x_3} \delta p \\ 0 \end{Bmatrix} + p \begin{Bmatrix} 0 \\ \delta \left( J \frac{\partial \xi_3}{\partial x_1} \right) \\ \delta \left( J \frac{\partial \xi_3}{\partial x_2} \right) \\ \delta \left( J \frac{\partial \xi_3}{\partial x_3} \right) \\ 0 \end{Bmatrix} \text{ on } B_B. \end{aligned} \quad (16)$$

Suppose now that  $\psi$  is the steady state solution of the adjoint equation

$$\frac{\partial \psi}{\partial t} - C_i^T \frac{\partial \psi}{\partial \xi_i} = 0 \text{ in } D. \quad (17)$$

For supersonic flows the choice of boundary conditions at the outer domain can be developed from physical intuition as well as mathematics. For a given geometry, say a wing, a change in the surface at any particular point,  $\mathcal{P}$ , will incur changes in the flowfield and hence the performance in the area defined by the Mach cone originating at  $\mathcal{P}$ . Similarly, it is possible to determine the region over which surface changes effect the flow condition at a given point. This region would also form a cone that would point roughly in the opposite direction, depending on local conditions, as the Mach cone. It is the solution of this reverse problem that the adjoint represents. The contribution to say drag, at a given point is influenced by changes to the surface at all points within the reverse cone. The correct farfield boundary conditions for the adjoint equation that are consistent with this reversed character are,

$$\begin{aligned} \psi_{1-5} &= 0 \text{ at the exit} \\ \psi_{1-5} &\text{ extrapolated from the interior at the entrance} \end{aligned}$$

Then if the coordinate transformation is such that  $\delta (JK^{-1})$  is negligible in the far field, the last integral in (15) reduces to

$$- \iint_{B_W} \psi^T \delta F_2 d\xi_1 d\xi_3 - \iint_{B_B} \psi^T \delta F_3 d\xi_1 d\xi_2. \quad (18)$$

Thus by letting  $\psi$  satisfy the boundary conditions,

$$J \left( \psi_2 \frac{\partial \xi_2}{\partial x_1} + \psi_3 \frac{\partial \xi_2}{\partial x_2} + \psi_4 \frac{\partial \xi_2}{\partial x_3} \right) = Q \text{ on } B_W \text{ and } B_B, \quad (19)$$

where

$$\mathcal{Q} = \frac{1}{\frac{1}{2}\gamma M_\infty^2 S_{\text{ref}}} \left\{ (S_x \cos \alpha + S_y \sin \alpha) + \Omega (S_y \cos \alpha - S_x \sin \alpha) \right\}$$

We find after integrating by parts again that

$$\begin{aligned} \delta I = & \frac{1}{S_{\text{ref}}} \iint_{B_W} C_p \left\{ (\delta S_x \cos \alpha + \delta S_y \sin \alpha) \right. \\ & \left. + \Omega (\delta S_y \cos \alpha - \delta S_x \sin \alpha) \right\} d\xi_1 d\xi_3 \\ & + \frac{1}{S_{\text{ref}}} \iint_{B_B} C_p \left\{ (\delta S_x \cos \alpha + \delta S_y \sin \alpha) \right. \\ & \left. + \Omega (\delta S_y \cos \alpha - \delta S_x \sin \alpha) \right\} d\xi_1 d\xi_2 \\ & + \int_D \psi^T \frac{\partial}{\partial \xi_i} (\delta G_{ij} f_j) d\xi_k \end{aligned} \quad (20)$$

where  $G_{ij} = JK_{ij}^{-1}$ .

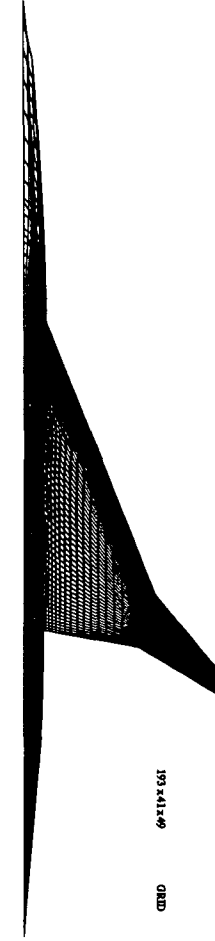
In our past works it was shown that the remaining variational terms in (20) that are related to mesh and surface deformation, can be obtained through either an analytic mapping procedure [6, 8] or an analytic mesh perturbation procedure [16, 9, 17]. The entire design procedure is outlined as follows:

1. Solve the flowfield governing equations (5-10).
2. Solve the adjoint equations (17) subject to the boundary condition (19).
3. Determine the mesh variational terms from analytic mesh perturbation relationships.
4. Construct  $\delta I$  for each design variable to form the gradient by (20).
5. Calculate the search direction based on a quasi-Newton algorithm and perform a line search.
6. Return to (1).

The basic method here builds on that used in reference [17] with the proper extensions to treat supersonic flow cases and more involved objective functions. Both the flow and adjoint solvers use an explicit multi-stage Runge-Kutta like algorithm accelerated by residual smoothing and multigridding. Design variables were chosen as a set of Hicks-Henne functions that tend to eliminate the presence of high frequency response in the design space [18].

## NUMERICAL TEST CASES AND CONCLUSIONS

Two cases of design in the presence of supersonic flow are examined. In the first case a wing alone design is carried out on a cranked delta plan form shown in Figure (1), with the body removed. The initial wing is defined as a flat, untwisted, and uncambered wing with a thickness distribution varying from 4.25 % at the root to 2.40 % at the tip. The outboard wing panel is swept ahead of the Mach cone at the cruise Mach number of 2.4 and thus has a sharp leading edge. The inboard subsonic portion of the wing retains a blunt leading edge to maintain low speed performance. The design variables



1a: Wing-Body configuration

Figure 1: C-H Mesh Topology 193x41x49.

consisted of 90 Hicks-Henne functions and 9 twist variables placed over the entire span and in such a way as to keep thickness constant. The design problem involved minimizing total inviscid drag while keeping  $C_L$  constant at a value of 0.12. The flow solver was run in the constant lift mode by using an outer iteration to modify  $\alpha$ . The necessary  $\frac{\partial C_A}{\partial \alpha}$  and  $\frac{\partial C_N}{\partial \alpha}$  terms used in the construction of the adjoint boundary condition are estimated by running two initial flow solutions at slightly different values of  $\alpha$ . Figure (2) shows both the original and final airfoil sections and pressures at four representative stations. It is seen from these figures that the loading has shifted inboard while the shock on the leading edge inboard region has been eliminated. There is also a trend to reduce the overall levels of minimum pressure across the entire upper surface. The overall inviscid drag was reduced by 6.33% in 25 iterations, from  $C_D = 0.01121$  to 0.01050.

In the second design example the complete wing-body configuration illustrated in Figure (1) was optimized. The configuration started with the same flat, uncambered, untwisted wing as that used in the wing alone design. In this case 121 design variables were used, consisting of 109 Hicks-Henne functions and 12 twist vari-

ables distributed over the wing. The fuselage was fixed during the design process. Lift was once again fixed at  $C_L = .12$  by iterating on alpha. Figure (3) shows both the original and final airfoil sections and pressures at four stations. The overall inviscid drag was reduced by 6.07% in 8 iterations, from  $C_D = 0.01070$  to 0.01005. The figures show that the same trend to reduce the minimum upper surface pressure is again seen for this wing-body design. The strong shock seen at the most inboard station of the initial design has been completely eliminated by the final design. Unlike the wing alone case the loading distribution is not drastically effected. Finally a small upper surface low pressure spike is retained in the final design at the leading edge and over the blunt leading edge. This spike has been seen in other design attempts of similar planforms indicating the possible advantage of a leading edge thrust component.

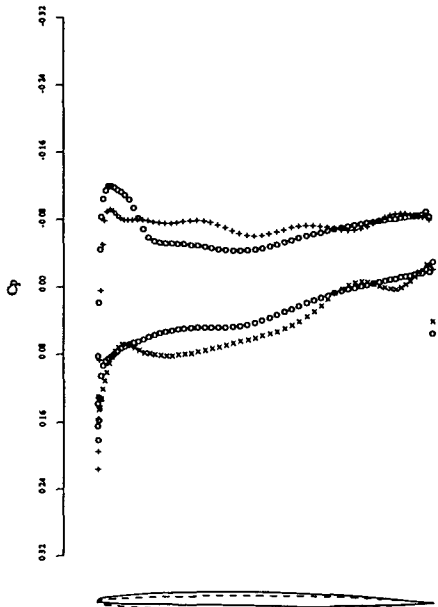
These results are comparable to those that could be obtained by previous methods using finite differences to calculate the gradients [15], and it appears that there is little room for further improvement at this flight condition with the given planform.

In this research the development of an aerodynamic shape optimization method has been extended to treat the design of wing and wing-body configurations subject to supersonic flows. The method is roughly 30 times more efficient than comparable finite difference methods due to the use of an adjoint solver to obtain the gradients. The clear limitation in these current results is the reliance on a single structured block for both the state and co-state fields. In the future our group intends not only to extend the method to treat both multiblock and unstructured meshes, but also to implement a Navier-Stokes version.

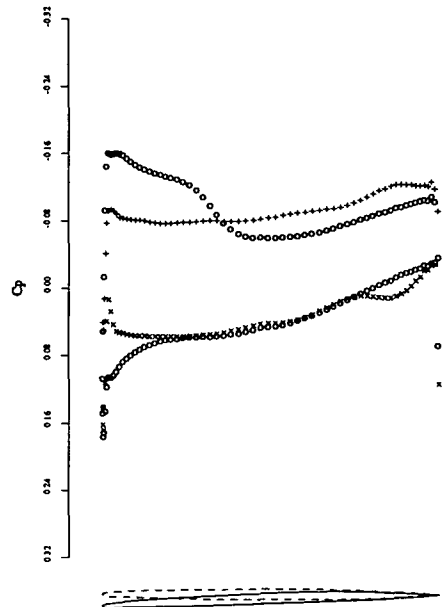
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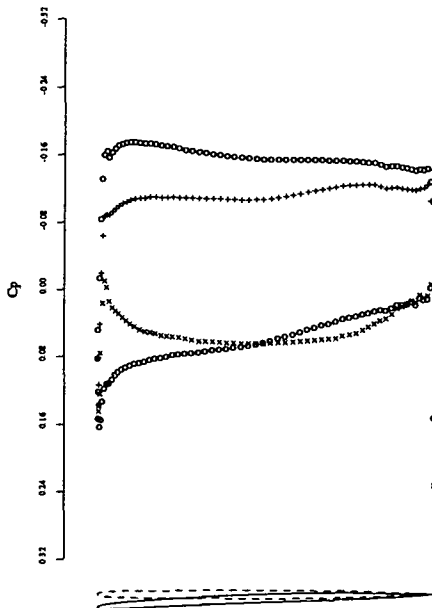




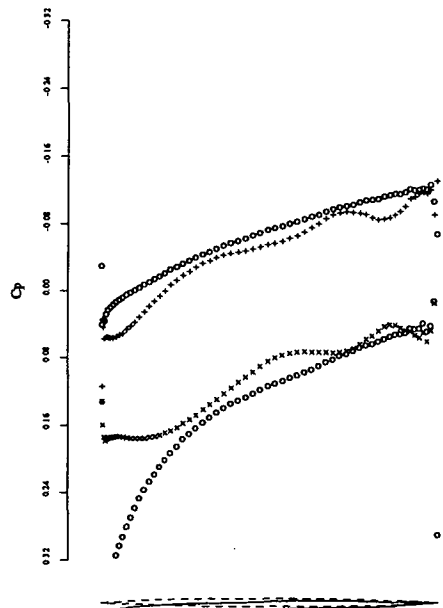
2a: span station  $z = 0.172$



2b: span station  $z = 0.391$

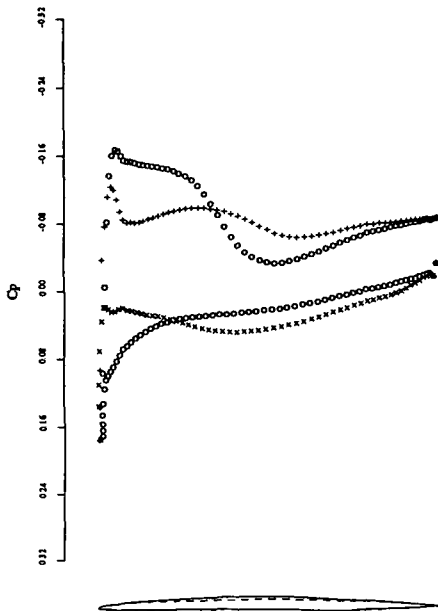


2c: span station  $z = 0.609$

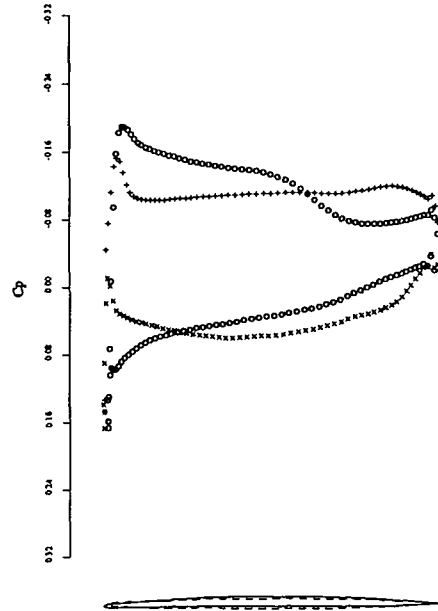


2d: span station  $z = 0.828$

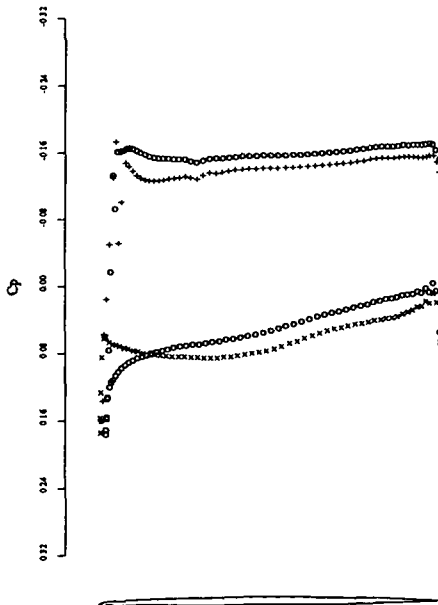
**Figure 2: SYN87 Wing Alone Design.**  
 $M = 2.4, C_L = 0.120, \text{Initial } C_D = 0.01121, \text{Final } C_D = 0.01050$   
 90 Hicks-Henne variables, 9 twist variables.  
 —, o Initial Wing  
 ---, + Design after 25 iterations.



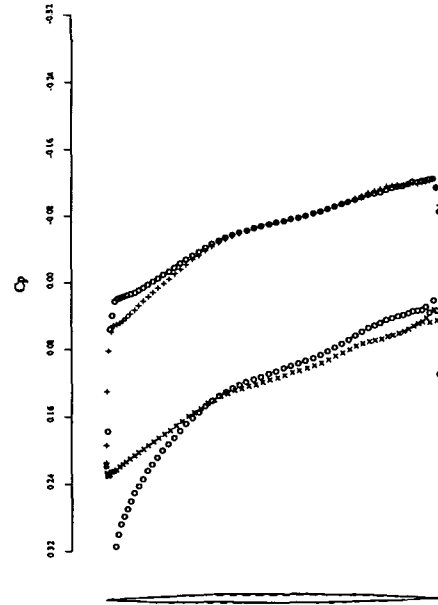
3a: span station  $z = 0.252$



3b: span station  $z = 0.441$



3c: span station  $z = 0.653$



3d: span station  $z = 0.842$

**Figure 3: SYN87 Wing-Body Design.**  
 $M = 2.4, C_L = 0.120, \text{Initial } C_D = 0.01070, \text{Final } C_D = 0.01005$   
 109 Hicks-Henne variables, 12 twist variables.  
 —, o Initial Wing  
 ---, + Design after 8 iterations.



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