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Applications of Direct Numerical Simulation of Turbulence in Second Order Closures

Tsan-Hsing Shih
*Institute for Computational Mechanics in Propulsion
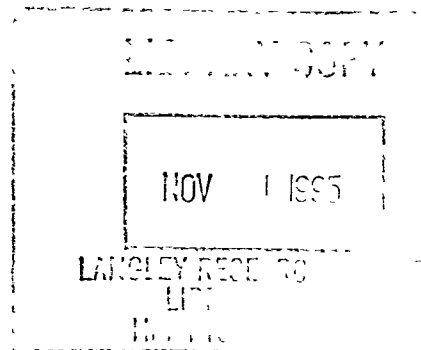and Center for Modeling of Turbulence and Transition
Cleveland, Ohio*

and

John L. Lumley
*Cornell University
Ithaca, New York*

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Tsan-Hsing Shih
Center for Modeling of Turbulence and Transition
Institute for Computational Mechanics in Propulsion
NASA Lewis Research Center, Cleveland, Ohio 44135

John L. Lumley
Cornell University, Ithaca, New York

Abstract

This paper discusses two methods of developing models for the rapid pressure-strain correlation term in the Reynolds stress transport equation using direct numerical simulation (DNS) data. One is a perturbation about isotropic turbulence, the other is a perturbation about two-component turbulence — an extremely anisotropic turbulence. A model based on the latter method is proposed and is found to be very promising when compared with DNS data and other models.

1. Introduction

Transport equation models for second moments such as the Reynolds stresses are gradually being accepted as advanced tools for studying complex turbulent flows. This is due to the fact that important turbulent physics can be represented by the terms in the second moment equations, e.g., the pressure-strain correlation tensor, the dissipation rate tensor, etc. The correct modeling of these terms will enable us to capture important turbulent physics and to improve the prediction of complex turbulent flows.

Over the past several years, considerable attention[1−7] has been directed towards the development of models for the pressure-strain correlation term which is considered to be one of the most important terms in the second moment equations. Many attractive models have been developed by using a variety of theories and principles, such as invariant theory, rapid distortion theory and realizability, etc. The present paper will concentrate on the rapid part of the pressure-strain correlation term. We will first describe some of these models and related theories. Then we will explore whether or not the intended behavior of the models is possible by examining the results of direct numerical simulations (DNS) of homogeneous turbulence[8]. By analysing these models using DNS data, we are able to identify the deficiencies of the models and the sources of the deficiencies. We are also able to identify the applicable range of flows for each model and provide the direction for improving the model's performance. For example, in the Launder, Reece and Rodi model[1], we are able to suggest a function form for their model coefficient C_2 which leads to good agreement with the most important shear component for all available DNS shear flows. In this paper, a rapid pressure-strain correlation model based on Shih and Lumley's method[8] has been developed using a perturbation series about a two-component turbulence state.

2. Modeling of the pressure-strain correlation

The Reynolds stress equation for homogeneous turbulence in terms of the anisotropy tensor b_{ij} can be written as

$$\frac{\partial}{\partial t} b_{ij} = \frac{\Pi_{ij}}{2k} - \frac{1}{2k}\left(\varepsilon_{ij} - \frac{2}{3}\varepsilon\delta_{ij}\right) - \frac{1}{3}(U_{i,j} + U_{j,i})$$

$$+ \frac{\varepsilon}{k} b_{ij} - \left(b_{ik}U_{j,k} + b_{jk}U_{i,k} - \frac{2}{3}b_{pq}U_{p,q}\delta_{ij}\right) + 2b_{ij}b_{pq}U_{p,q} \qquad (1)$$

where $b_{ij} = \overline{u_i u_j}/2k - \delta_{ij}/3$ is the anisotropy tensor, $U_{i,j}$ is the mean velocity gradient, k and ε are the turbulent kinetic energy and its dissipation rate. Π_{ij} and ε_{ij} are the pressure-strain correlation tensor and the dissipation rate tensor, respectively. The latter are the new unknown terms in the Reynolds stress equation and must be modeled. In this paper we only consider the rapid part of the pressure-stain correlation term $\Pi_{ij}^{(1)}$. Its exact expression is

$$\Pi_{ij}^{(1)} = 2U_{p,q}(X_{pjqi} + X_{piqj}) \tag{2}$$

where

$$X_{pjqi} = -\frac{1}{4\pi} \int \int \int_v \overline{[u_q(r)u_i(r')]}_{,pj} \frac{1}{|r - r'|} dv \tag{3}$$

2.1 Phenomenological modeling of the rapid pressure-strain correlation

Equation (1) indicates that the relationship between the pressure-strain correlation Π_{ij} and the anisotropy tensor b_{ij} should in general be nonlinear due to the nonlinear term $2b_{ij}b_{pq}U_{p,q}$. Equation (2) reveals that modeling of the rapid pressure-strain correlation requires a model of the fourth order rank tensor X_{pjqi}. In the literature, X_{pjqi} is assumed to be a function of b_{ij}. However, Eq. (1) indicates that X_{pjqi} should at least be a functional of b_{ij}, i.e., it should depend on the history of b_{ij} in the whole flow field. Therefore, the real problem is quite complicated. Here, as an engineering approximation, we will assume, like most other researchers, that X_{pjqi} is only a function of b_{ij} at local point in space and time. We notice that this assumption will at least lose the effect of dimensionality on X_{pjqi} and may cause the problem in modeling of rapid rotating turbulence as pointed out by Reynolds[9]. However, in practice, this assumption is quite appropriate for many turbulent shear flows. Furthermore, we expect that X_{pjqi} should at least be a quadratic function of b_{ij} because of the nonlinearity between Π_{ij} and b_{ij}. In fact, from a method of invariant theory of rational mechanics we may find that the most general tensorial function form for X_{pjqi} in terms of b_{ij} is[3,6]

$$\begin{aligned}
\frac{X_{pjqi}}{2k} =\ & \alpha_1 \delta_{qi}\delta_{pj} + \alpha_2(\delta_{pq}\delta_{ij} + \delta_{qj}\delta_{pi}) + \alpha_3\delta_{qi}b_{pj} + \alpha_4\delta_{pj}b_{qi} \\
& + \alpha_5(\delta_{pq}b_{ij} + \delta_{ij}b_{pq} + \delta_{jq}b_{pi} + \delta_{pi}b_{qj}) + \alpha_6\delta_{qi}b_{pj}^2 + \alpha_7\delta_{pj}b_{qi}^2 \\
& + \alpha_8(\delta_{pq}b_{ij}^2 + \delta_{ij}b_{pq}^2 + \delta_{jq}b_{pi}^2 + \delta_{pi}b_{qj}^2) + \alpha_9 b_{qi}b_{pj} \\
& + \alpha_{10}(b_{pq}b_{ij} + b_{qj}b_{pi}) + \alpha_{11}b_{qi}b_{pj}^2 + \alpha_{12}b_{pj}b_{qi}^2 \\
& + \alpha_{13}(b_{pq}b_{ij}^2 + b_{ij}b_{pq}^2 + b_{qj}b_{pi}^2 + b_{pi}b_{qj}^2) \\
& + \alpha_{14}b_{qi}^2 b_{pj}^2 + \alpha_{15}(b_{pq}^2 b_{ij}^2 + b_{qj}^2 b_{pi}^2)
\end{aligned} \tag{4}$$

Equation (4) already satisfies the symmetry properties of $X_{pjqi} = X_{jpqi}$ and $X_{pjqi} = X_{pjiq}$ required by Eq. (3). The number of coefficients in Eq. (4) can be reduced to nine by the following basic mathematical properties required also by Eq. (3):

$$X_{ppqi} = 2k(b_{qi} + \frac{1}{3}\delta_{qi}), \quad X_{pjqj} = 0. \tag{5}$$

When the term $(X_{pjqi} + X_{piqj})U_{p,q}$ is formed, the coefficients will be further reduced to seven as shown by Johansson and Hallbäck[7] because of the following tensorial identity relations[6,7]:

$$\left(b_{ip}^2\delta_{jq} + b_{jp}^2\delta_{iq} - b_{ij}b_{pq} + b_{iq}b_{jp} - b_{pq}^2\delta_{ij} - \frac{1}{2}II\delta_{ip}\delta_{jq} \right)S_{pq} = 0 \tag{6.1}$$

2

$$(b_{ip}^2 b_{jq} + b_{jp}^2 b_{iq} - b_{ij} b_{pq}^2 - b_{ij}^2 b_{pq} + \frac{1}{3} III \delta_{ip} \delta_{jq}) S_{pq} = 0 \qquad (6.2)$$

$$[\, b_{iq}^2 b_{jp}^2 + b_{jq}^2 b_{ip}^2 - 2 b_{ij}^2 b_{pq}^2 - II(b_{ij} b_{pq} - b_{iq} b_{jp})$$

$$+ \frac{2}{3} III(b_{ip} \delta_{jq} + b_{jp} \delta_{iq} - b_{pq} \delta_{ij}) \,] S_{pq} = 0 \qquad (6.3)$$

2.2 Truncated anisotropy power series models

The seven undetermined coefficients in Eq. (4) are in general functions of the invariants of b_{ij}: $II = -b_{ij} b_{ji}/2$ and $III = b_{ij} b_{jk} b_{ki}/3$. Because the absolute values of II and III are always less than unity, one may expand the coefficients in a power series of II and III[6,7]:

$$\alpha_i = C_i^{(1)} + C_i^{(2)} \, II + C_i^{(3)} \, III + C_i^{(4)} \, II^2$$

$$+ C_i^{(5)} \, II \, III + C_i^{(6)} \, III^2 + \cdots \quad (i = 1, 15) \qquad (7)$$

The coefficients in Eq. (7) are independent of II and III hence are considered to be constant. Therefore, if their values are determined by using a particular flow, then the values should be valid for other flows as well. That is, Eq. (7) (with an infinite number of terms) will provide us with a universal rapid pressure-strain correlation model. However, in practice, we must truncate the series to a certain power of II and III and use certain flows and constraints (such as homogeneous flows and realizability) to determine the coefficients in Eq. (7). Having done that, we then must verify if such determined coefficients are constant and are valid for other flows as well.

A first order power truncation model — LRR model

If we decide to only keep the terms in Eq. (4) which are of up to order $O(b)$ (the norm of b_{ij}) and note that $II = O(b^2)$ and $III = O(b^3)$, then the coefficients $\alpha_6, \cdots, \alpha_{15}$ must be set to zero and in Eq. (7) we only need to keep the first term $C_i^{(1)}(i = 1, 5)$ which are constant. In this case, Eq. (4) becomes

$$\frac{X_{pjqi}}{2k} = \alpha_1 \delta_{qi} \delta_{pj} + \alpha_2(\delta_{pq} \delta_{ij} + \delta_{qj} \delta_{pi}) + \alpha_3 \delta_{qi} b_{pj} + \alpha_4 \delta_{pj} b_{qi}$$

$$+ \alpha_5(\delta_{pq} b_{ij} + \delta_{ij} b_{pq} + \delta_{jq} b_{pi} + \delta_{pi} b_{qj})$$

Using the properties in Eq. (5), there will be only one undetermined coefficient left and the corresponding rapid pressure-strain correlation model Eq. (2) becomes

$$\frac{\Pi_{ij}^{(1)}}{4\,k} = 0.2 S_{ij} + \frac{9 C_2 + 6}{22}(b_{ik} S_{jk} + b_{jk} S_{ik} - \frac{2}{3} \delta_{ij} b_{kl} S_{kl})$$

$$+ \frac{10 - 7 C_2}{22}(b_{ik} \Omega_{jk} + b_{jk} \Omega_{ik}) \qquad (8)$$

where $S_{ij} = \frac{1}{2}(U_{i,j} + U_{j,i})$ and $\Omega_{ij} = \frac{1}{2}(U_{i,j} - U_{j,i})$. Equation (8) is the Launder, Reece and Rodi (LRR) model[1]. The undetermined coefficient C_2 is taken to be constant and was set at a value of 0.4.

Now, we may use DNS data of turbulent shear flows[8] to examine the coefficient C_2 in Eq. (8). For homogeneous turbulent shear flows, there are only three independent components Π_{11}, Π_{22} and Π_{12}. From Eq. (8), each component of Π_{ij} can be used for determining C_2. Figure 1 shows the values of C_2 deduced from the DNS data of Π_{11}, Π_{22} and Π_{12} for the case of C128W. Apparently, C_2 is not a constant. Different component gives a different value of C_2. This situation is also ture

for other flows (C128U, C128V, and C128X). The above result indicates that the first order power truncation model can not be expected to work for all the turbulent components. However, Figure 1 shows that for the 1-2 component, $C_2 = 0.4$ is indeed a good approximation. This indicates that LRR model is probably good for two dimensional turbulent shear flows since the 1-2 component dominates the flow. However, if the flow is dominated by normal components, then we cannot in general expect LRR model to give a good prediction.

In order to improve the model performance, a natural way is to pursue a higher order power truncation.

A fourth order power truncation model — JH model

Now if the terms in Eq. (4) are retained up to order (b^4), then it will require the following truncated power series in Eq. (7):

$$\alpha_i = C_i^{(1)} + C_i^{(2)} \, II + C_i^{(3)} \, III + C_i^{(4)} \, II^2 \quad (i = 1, 2)$$

$$\alpha_i = C_i^{(1)} + C_i^{(2)} \, II + C_i^{(3)} \, III \quad (i = 3, 4, 5)$$

$$\alpha_i = C_i^{(1)} + C_i^{(2)} \, II \quad (i = 6, 7, 8, 9, 10)$$

$$\alpha_i = C_i^{(1)} \quad (i = 11, 12, 13, 14, 15)$$

In this truncation form, there are 32 "constants" $C_i^{(1)}, C_i^{(2)}, \cdots, C_i^{(4)}$ need to be determined. Using the constraints in Eq. (5) and (6), and the condition of realizability[3]:

$$U_{p,q} X_{p\alpha q\alpha} = 0 \quad \text{if} \quad \overline{u_\alpha^2} = 0, \tag{9}$$

Johansson and Hallbäck[7] found that there are only four undetermined constants left and the rapid pressure-strain correlation model becomes

$$
\begin{aligned}
\frac{\Pi_{ij}^{(1)}}{4\,k} =\;& \frac{Q_1}{4} S_{ij} + \frac{Q_2}{2}(b_{ik}S_{jk} + b_{jk}S_{ik} - \frac{2}{3}\delta_{ij}b_{kl}S_{kl}) \\
& + Q_3 b_{kl}S_{kl}b_{ij} + Q_4(b_{kj}b_{li}S_{kl} - \frac{1}{3}b_{kl}^2 S_{kl}\delta_{ij}) \\
& + 2\,Q_5 b_{kl}^2 S_{kl}b_{ij} + (2Q_5 b_{kl}S_{kl} + 4Q_6 b_{kl}^2 S_{kl})(b_{ij}^2 + \frac{2}{3}II\delta_{ij}) \\
& + \frac{Q_7}{2}(b_{ik}\Omega_{kj} + b_{jk}\Omega_{ki}) + Q_8(b_{jk}^2 \Omega_{ki} + b_{ik}^2 \Omega_{kj}) \\
& + 2Q_9(b_{jk}^2 b_{il}\Omega_{kl} + b_{ik}^2 b_{jl}\Omega_{kl})
\end{aligned}
\tag{10}
$$

where

$$Q_1 = \frac{4}{5} + \frac{16}{5}(4B_2 + 15B_3)\,II - \frac{48}{5}B_5\,III - \frac{304}{55}B_6\,II^2$$

$$Q_2 = -12B_1 + 4B_5\,II - 12B_6\,III$$

$$Q_3 = -8B_2 + 36B_3 - \frac{28}{11}B_6\,II$$

$$Q_4 = 96B_2 - 36B_3 + \frac{28}{11}B_6\,II$$

$$Q_7 = -\frac{4}{3} - \frac{28}{3}B_1 - \frac{4}{3}(2B_4 - B_5)\,II - 4B_6\,III$$

$$Q_8 = -16B_2 + 28B_3 - \frac{12}{11}B_6\,II$$

$$Q_5 = B_5, \quad Q_6 = B_6, \quad Q_9 = B_4$$

and

$$B_1 = \beta_1 - 8\beta_4 II + 24\beta_7 III, \quad B_2 = \beta_2 - 8\beta_8 II$$
$$B_3 = \beta_3 - 8\beta_9 II, \quad B_4 = \beta_5, \quad B_5 = \beta_6, \quad B_6 = \beta_{10}$$

and

$$\beta_4 = -\frac{3}{160}(3 + 60\beta_1 + 48\beta_2 - 40\beta_3), \quad \beta_5 = -\frac{3}{2} - 132\beta_2 + \frac{2}{3}\beta_{10}$$

$$\beta_6 = \frac{3}{2} + 60(\beta_2 + \beta_3), \quad \beta_7 = \frac{9}{8}\beta_1 - \frac{9}{4}(\beta_2 + \beta_3)$$

$$\beta_8 = -\frac{3}{88}(\frac{3}{8} + 21\beta_2 + 10\beta_3 + \frac{1}{18}\beta_{10}), \quad \beta_9 = -\frac{9}{220}(\frac{3}{8} + 21\beta_2 + 10\beta_3)$$

The coefficients $\beta_1, \beta_2, \beta_3$ and β_{10} are undetermined constants. To determine these constants, Johansson and Hallbäck[7] used rapid distortion theory to analyze an irrotational strain imposed on initially isotropic turbulence and a pure rotation imposed on anisotropic turbulence . They suggested that

$$\beta_1 = -\frac{1}{7}, \quad \beta_2 = 0.0295, \quad \beta_3 = -0.0484, \quad \beta_{10} = 14$$

This fourth order power truncation model Eq. (10) was shown to be in excellent agreement with irrotational RDT results. The result indicates that for the RDT irrotational flows, the fourth order power truncation is appropriate and the truncation error is quite small. This model is also in excellent agreement with DNS data of axisymmetric contraction flows shown in Figure 8.

Now let us use DNS data of turbulent shear flows[8] to examine the "constants" of $\beta_1, \beta_2, \beta_3$ and β_{10}. Since only three components of II_{ij} are independent, Eq. (10) can determine only three coefficients. We set $\beta_1 = -1/7$ and use the DNS data of II_{11}, II_{22} and II_{12} to deduce the coefficients β_2, β_3 and β_{10}. Figures 2 shows the values of β_2, β_3 and β_{10} deduced from the DNS flows of C128U, C128V, C128W and C128X. Apparently, these coefficients are still not constant. If we believe that the DNS data are realistic and that the formulation of Eq. (10) is correct, then the above result indicates that the fourth order power truncation still have too large of a truncation error for the flows considered above. No constant values of these coefficients can ensure that the model will have good performance. To overcome this problem a much higher order truncation then must be retained to reduce the truncation error. However, at the present time, it is not clear what order of truncation is needed for a realistic turbulent flow. In addition, it is noticed that the values of β_2, β_3 and β_{10} deduced from RDT results[7] are very different from those deduced from DNS flows[8]. This inconsistency raises a question as to which of these flows are more realistic and can be considered as benchmark for turbulence model development?

2.3 Truncated F power series model

Here we discuss another truncation method which is a perturbation about the two-component (2-C) turbulence state. The deviation from the 2-C state can be measured by a parameter F which was introduced by Lumley[2]:

$$F = 1 + 9\ II + 27\ III \tag{10}$$

From the definition, F will vanish if turbulence is 2-C (in which the turbulence becomes extremely anisotropic) and F will equal unity if the turbulence is isotropic For a realizable turbulence, F is always positive and varies between zero and one.

In modeling the rapid pressure-strain correlation term, the focus should be directed towards situations where the turbulence anisotropy is strong ($F << 1$) rather than cases where the anisotropy is weak. This is because the Reynolds stress transport equation mainly emphasizes the effect of

anisotropy. For a flow with strong shear, F could be driven to zero while b is about $1/3$. Based on the above consideration, we propose to expand the coefficients in Eq. (4) in a power series of F:

$$\alpha_i = c_i^{(0)}(III) + c_i^{(1)}(III)\,F + c_i^{(2)}(III)\,F^2 + \cdots \quad (i = 1, 15) \tag{12}$$

where the coefficients $c_i^{(0)}, c_i^{(1)}, c_i^{(2)}, \cdots$ are functions of the third invariant III of b_{ij} and they can be expanded in a power series of III:

$$c_i^{(j)}(III) = c_{i,0}^{(j)} + c_{i,1}^{(j)}\,III + c_{i,2}^{(j)}\,III^2 + \cdots \tag{12a}$$

For small III, as a first order approximation they can be considered as constant. We will examine the behavior of the corresponding rapid pressure-strain model.

Let us now start with the most general form, Eq. (4). Using the conditions in Eq. (5) we may obtain six relations between the fifteen coefficients which will reduce the number of coefficients to nine. As a result, we obtain

$$\alpha_1 = \frac{2}{15} + \frac{II}{5}(4\alpha_6 - 2\alpha_8) + \frac{III}{5}(\alpha_{11} + a_{12} - 6a_{13})$$

$$\alpha_2 = -\frac{1}{30} - \frac{II}{5}(\alpha_6 - 3\alpha_8) - \frac{III}{10}(3\alpha_{11} + 3\alpha_{12} + 2\alpha_{13})$$

$$\alpha_3 = -\frac{1}{3} - \frac{11}{3}\alpha_5 + \frac{II}{3}(\alpha_{11} + 3\alpha_{12} + 8\alpha_{13}) - \frac{III}{3}(3\alpha_{14} + \alpha_{15})$$

$$\alpha_4 = \frac{1}{3} - \frac{4}{3}\alpha_5 + \frac{2II}{3}(\alpha_{11} + 2\alpha_{13}) - \frac{2}{3}III\alpha_{15}$$

$$\alpha_7 = -\frac{2}{3}\alpha_{10} - \frac{4}{3}\alpha_8 + \frac{2II}{3}(\alpha_{14} + \alpha_{15})$$

$$\alpha_9 = -\frac{11}{3}\alpha_8 - \frac{1}{3}\alpha_{10} - \alpha_6 + \frac{II}{3}(\alpha_{14} + 7\alpha_{15}) \tag{13}$$

Equation (13) will ensure the rapid pressure-strain correlation model to satisfy the basic mathematical conditions in Eq. (5) while the values of the nine coefficients: α_5, α_6, α_8, α_{10}, α_{11}, α_{12}, α_{13}, α_{14} and α_{15} can be arbitrary. In addition, the realizability condition of Eq. (9) will provide three additional relations for the $\alpha's$ at the 2-C state, i.e., $F = 0$. They lead to

$$\alpha_5 = -\frac{1}{10} + \frac{1}{30}(\alpha_{11} + \alpha_{12} + 4\alpha_{13} - 10\alpha_{15}/9)$$
$$+ \frac{II}{10}(3\alpha_{11} + 3\alpha_{12} + 8\alpha_{13} - 2\alpha_{15} - 6\alpha_6)$$

$$\alpha_8 = \frac{1}{3}\alpha_{13} - \frac{1}{9}\alpha_{15}$$

$$\alpha_{10} = -\frac{3}{10} + \frac{1}{90}(9\alpha_{11} + 9\alpha_{12} + 66\alpha_{13} - 10\alpha_{15})$$
$$+ \frac{II}{10}(9\alpha_{11} + 9\alpha_{12} + 24\alpha_{13} - 6\alpha_{15} - 18\alpha_6) \tag{14}$$

Equation (14) is valid only at the 2-C turbulence state and will ensure the rapid pressure-strain correlation model satisfies the necessary realizability condition of Eq. (9). Therefore, we conclude that Eqs (13) and (14) will ensure the model satisfies the conditions in both Eq. (5) and Eq. (9) with arbitrary values of the six coefficients: $\alpha_6, \alpha_{11}, \alpha_{12}, \alpha_{13}, \alpha_{14}$ and α_{15} This will allow us to set any of these coefficients to zero in order to simplify the model while maintaining its basic properties described by Eqs (5) and (9)

6

Tensorially quadratic model

Here we will set all the six undetermined coefficients $\alpha_6, \cdots, \alpha_{15}$ to zero to obtain the most simplified model. The necessity of retaining these coefficients will be left for a future study. Under this consideration, Eq. (13) becomes

$$\alpha_1 = \frac{2}{15} - \frac{2\,II}{5}\alpha_8, \quad \alpha_2 = -\frac{1}{30} + \frac{3\,II}{5}\alpha_8, \quad \alpha_3 = -\frac{1}{3} - \frac{11}{3}\alpha_5,$$

$$\alpha_4 = \frac{1}{3} - \frac{4}{3}\alpha_5, \quad \alpha_7 = -\frac{2}{3}\alpha_{10} - \frac{4}{3}\alpha_8, \quad \alpha_9 = -\frac{11}{3}\alpha_8 - \frac{1}{3}\alpha_{10}. \tag{15}$$

and Eq. (14) becomes

$$\alpha_5 = -\frac{1}{10}, \quad \alpha_8 = 0, \quad \alpha_{10} = -\frac{3}{10} \quad \text{if} \quad F = 0 \tag{16}$$

For $F \neq 0$, the coefficients α_5, α_8 and α_{10} could in general be a function of F and III. We propose to expand these coefficients in a power series of F and determine them by using DNS data of turbulent shear flows[8]. Figure 3 shows the values of the α_5, α_8 and α_{10} versus the parameter F corresponding to the DNS data The scatter is small that indicates that for the above turbulent shear flows the III is small such that the coefficients $c_i^{(j)}$ in Eq. (12) are approximately constant. Finally, the coefficients α_5, α_8 and α_{10} can be approximately expressed by a truncated fourth order power series of F:

$$\alpha_5 = -0.1 - 0.5\,F + 1.57\,F^2 - 2.22\,F^3 + 1.07\,F^4$$

$$\alpha_8 = -1.6\,F + 4.69\,F^2 - 5.92\,F^3 + 2.62\,F^4 \tag{17}$$

$$\alpha_{10} = -\frac{3}{2}\,(0.2 + 2.59\,F - 7.4\,F^2 + 8.86\,F^3 - 3.73\,F^4)$$

Using Eq. (15) the rapid pressure-strain correlation model becomes

$$\frac{\Pi_{ij}^{(1)}}{2q^2} = \frac{1}{5}S_{ij} - 3\alpha_5(b_{ik}S_{jk} + b_{jk}S_{ik} - \frac{2}{3}\delta_{ij}b_{kl}S_{kl})$$

$$+ \alpha_8(2b_{kl}^2 S_{kl}\delta_{ij} + \frac{2}{3}b_{ki}^2 S_{kj} + \frac{2}{3}b_{kj}^2 S_{ki} - \frac{22}{3}b_{ki}b_{lj}S_{kl} + \frac{2}{5}II\,S_{ij})$$

$$- \frac{2}{3}\alpha_{10}(b_{il}^2 S_{jl} + b_{jl}^2 S_{il} - 2b_{kj}b_{li}S_{kl} - 3b_{ij}b_{kl}S_{kl})$$

$$+ \frac{1}{3}(2 + 7\alpha_5)(b_{ik}\Omega_{jk} + b_{jk}\Omega_{ik}) - (\frac{4}{3}\alpha_8 + \frac{2}{3}\alpha_{10})(b_{il}^2\Omega_{jl} + b_{jl}^2\Omega_{il}) \tag{18}$$

Figures 4, 5, 6 and 7 show the direct comparisons between different rapid models and DNS data. As shown in figures, the models based on the truncated F power series are very encouraging. However, it should be pointed out that the model (18) with Eq. (17) is good for the cases where $|III|$ is small. For the cases where $|III|$ is not negligible, for example, a turbulence undergoes a rapid axisymmetric expansion, the coefficients in Eq. (17) must depend on the third invariant III. This explains the poor comparison between the model and DNS data[10] for axisymmetric strain and plain strain turbulence shown in Figures 8, 9 and 10.

Improvement of LRR model

The Launder, Reece and Rodi's model, Eq. (8), is tensorially linear in b_{ij} which satisfies the basic conditions in Eq. (5), but does not satisfy the realizability condition in Eq. (9). In this model, there is only one undetermined coefficient C_2 which is shown to be not a constant. We

may consider it to be a function of F and then use DNS data to determine its function form. For turbulent shear flows, the most important component is the 1-2 component. Therefore, we use DNS data of Π_{12} to deduce the coefficient C_2. A simple quadratic function is found as follows

$$C_2 = 0.38 - 0.6\,F + 0.5\,F^2$$

The LRR model with the new coefficient C_2 leads to excellent agreement with all DNS data for the 1-2 component (see Figures 4, 5, 6, 7). However, other components are unable to be predicted very well. For comparison, another tensorially linear model in b_{ij} proposed by Speziale, Sarkar and Gatski[11] (SSG) is also included in Figures 4-10. The behavior of the rapid part of SSG model is similar to LRR model.

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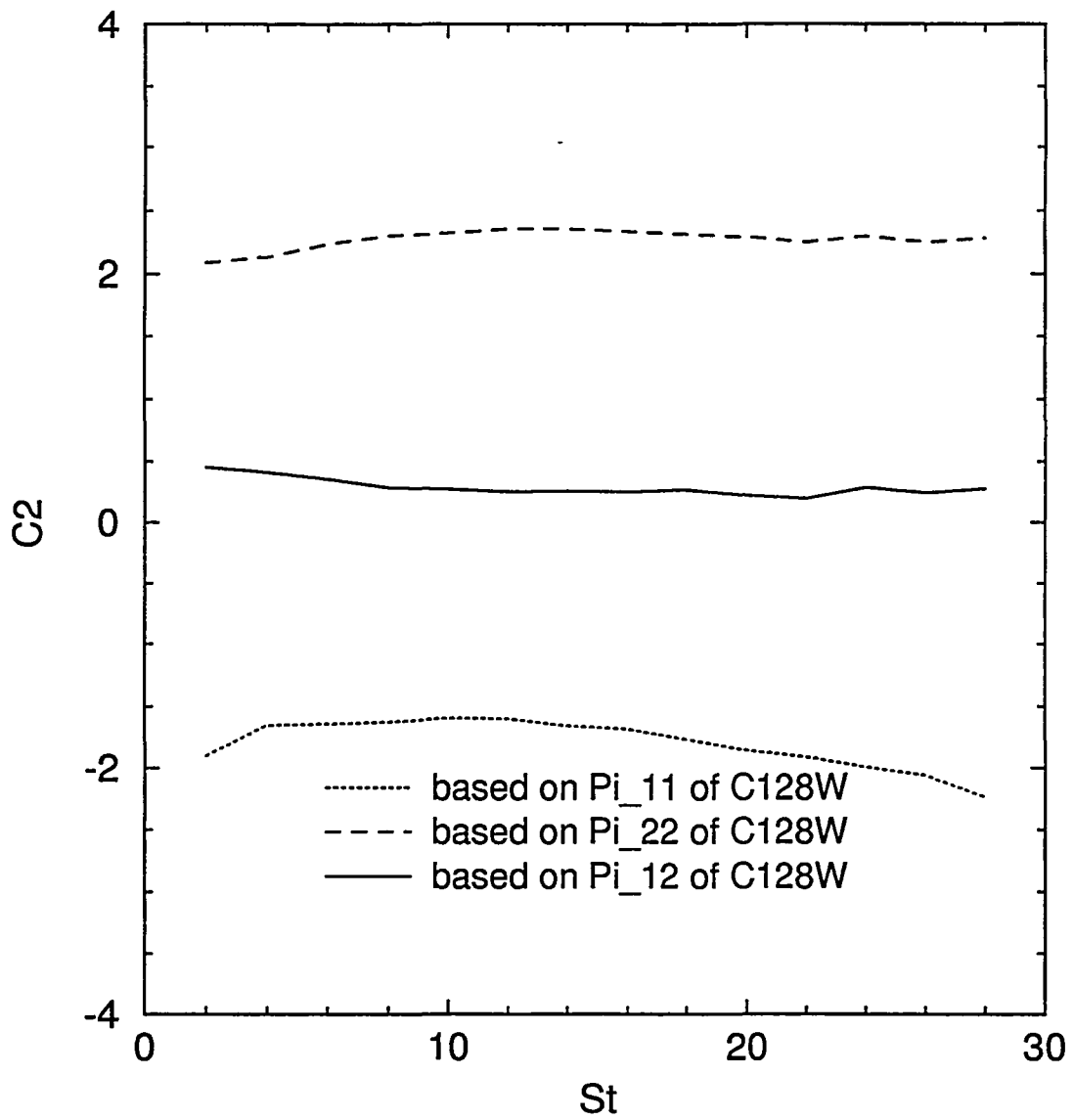
Figure 1. The model constant C2 of LRR model
deduced from the DNS of turbulent shear flow C128W.
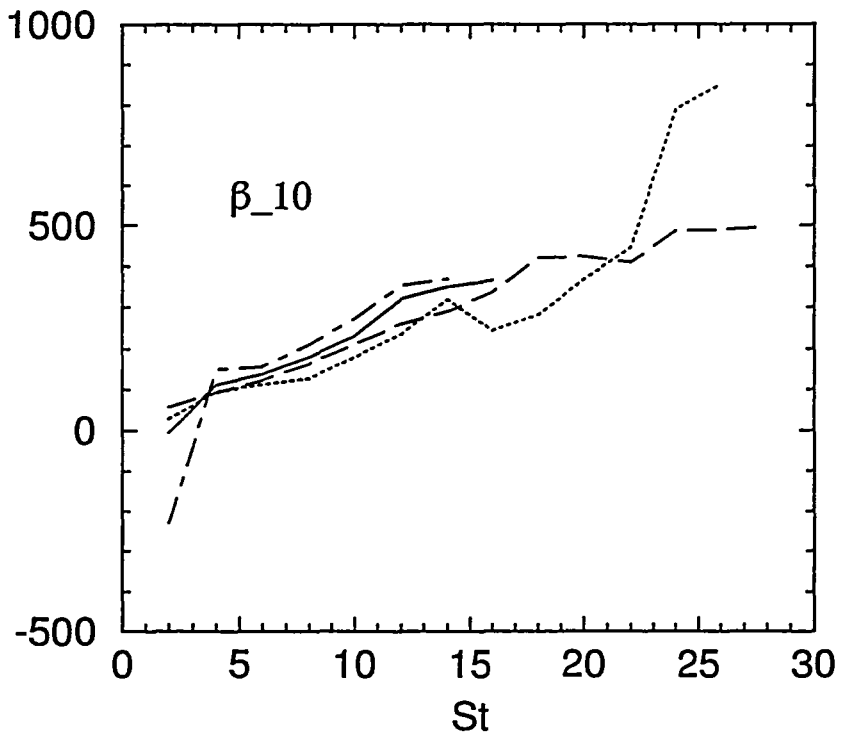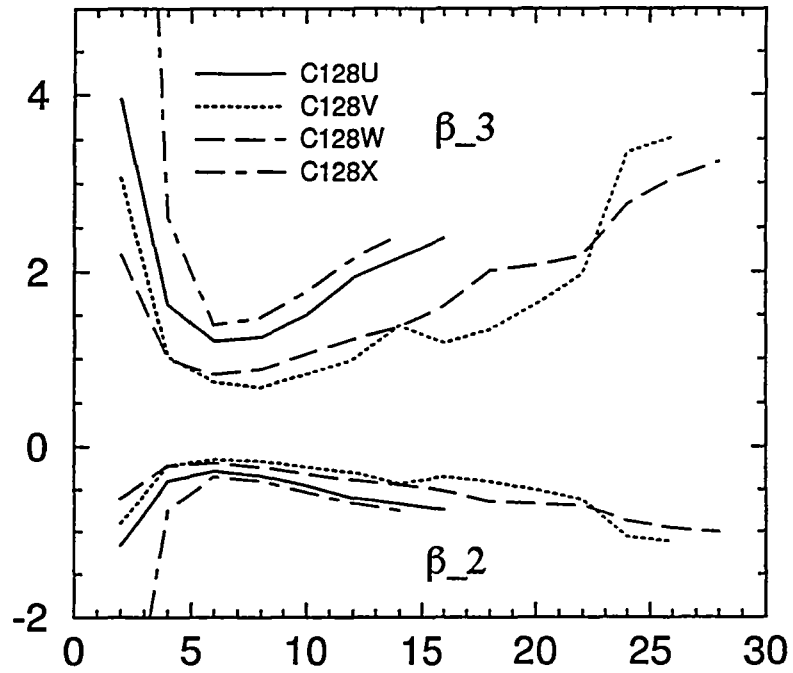
Figure 2. The model constants b_2, b_3 and b_{10} in JH model deduced from the DNS data of turbulent shear flows.
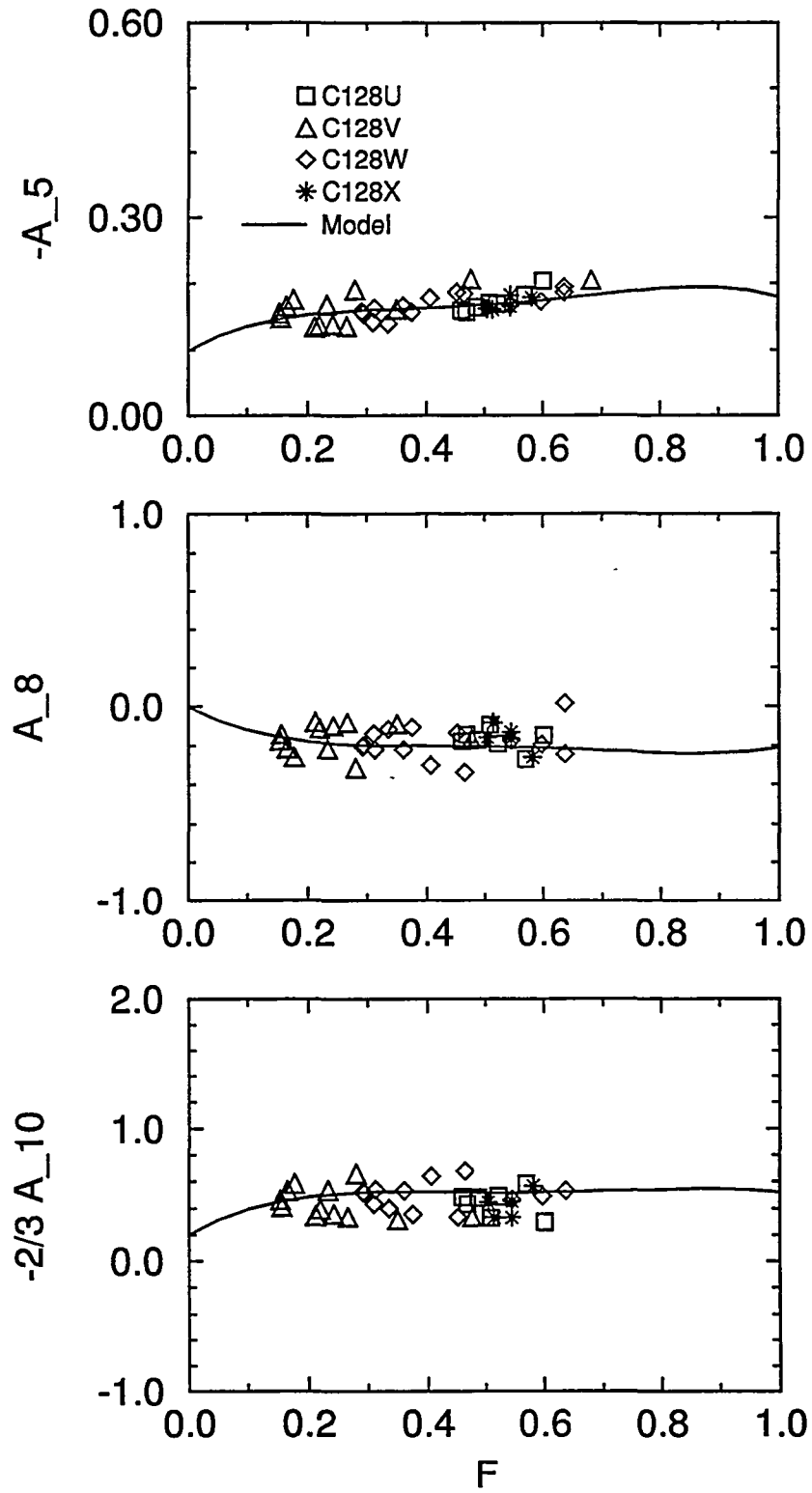
Figure 3. The model coefficients α_5, α_8 and α_{10} in the present model deduced from the DNS data of turbulent shear flows.
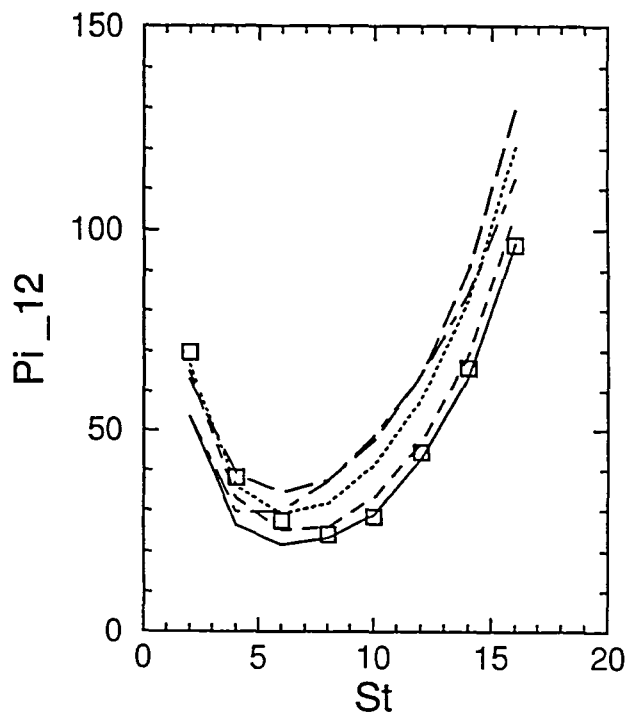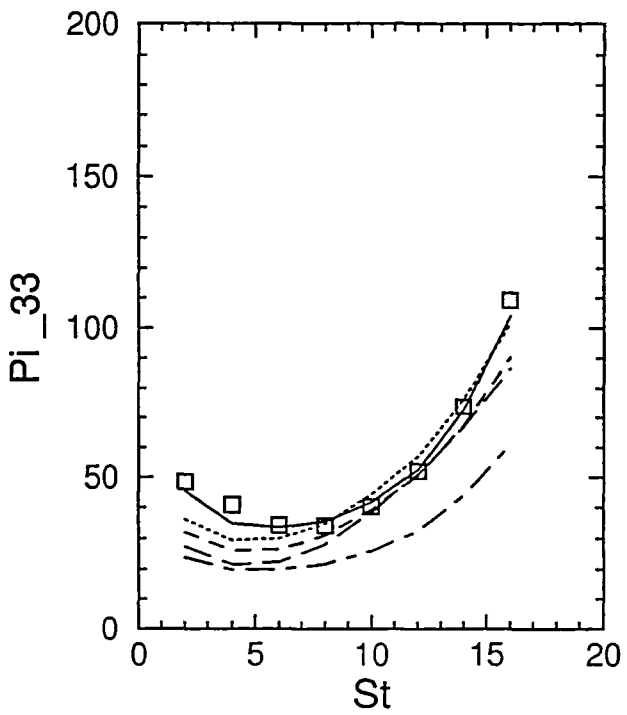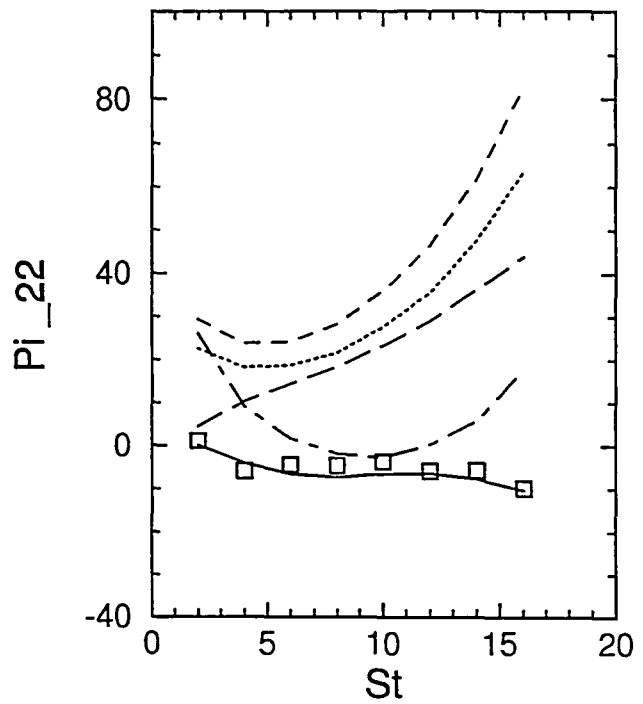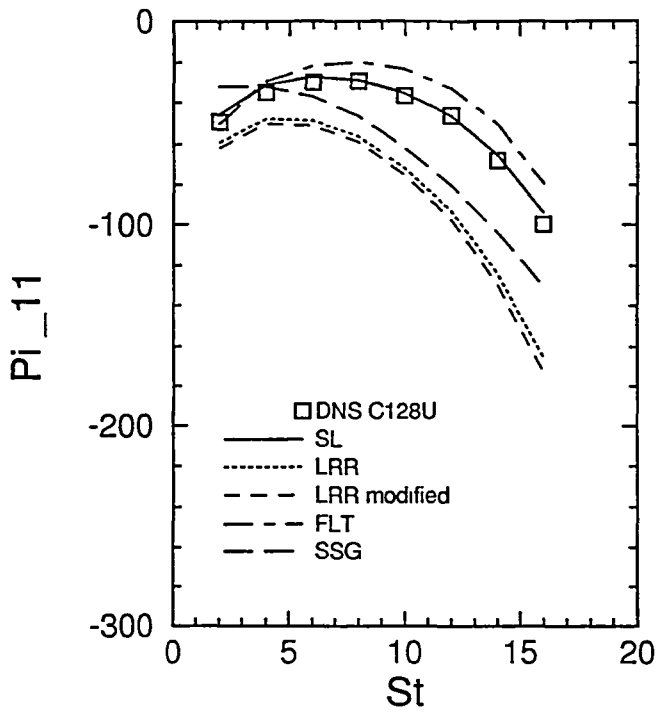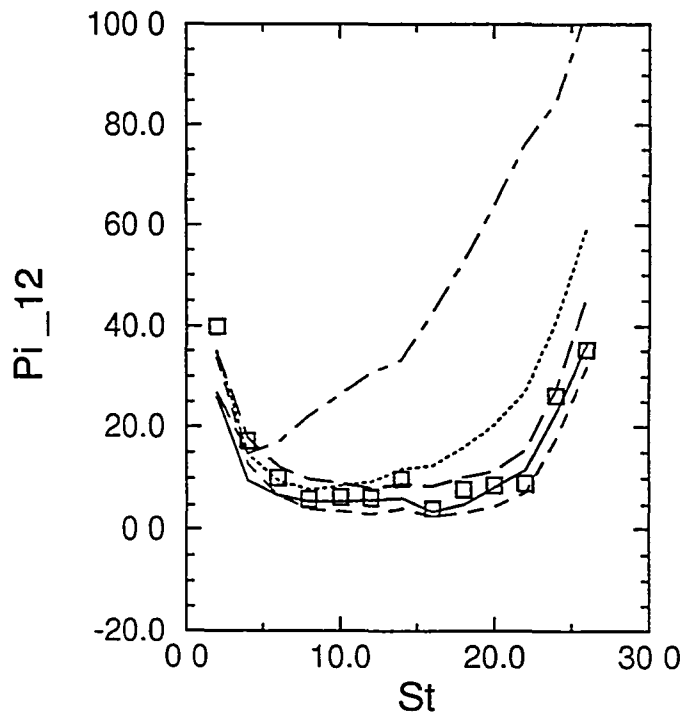
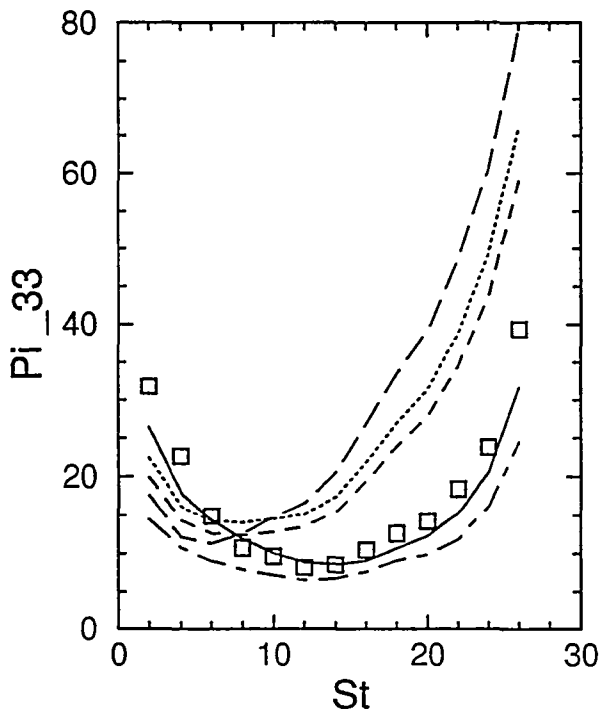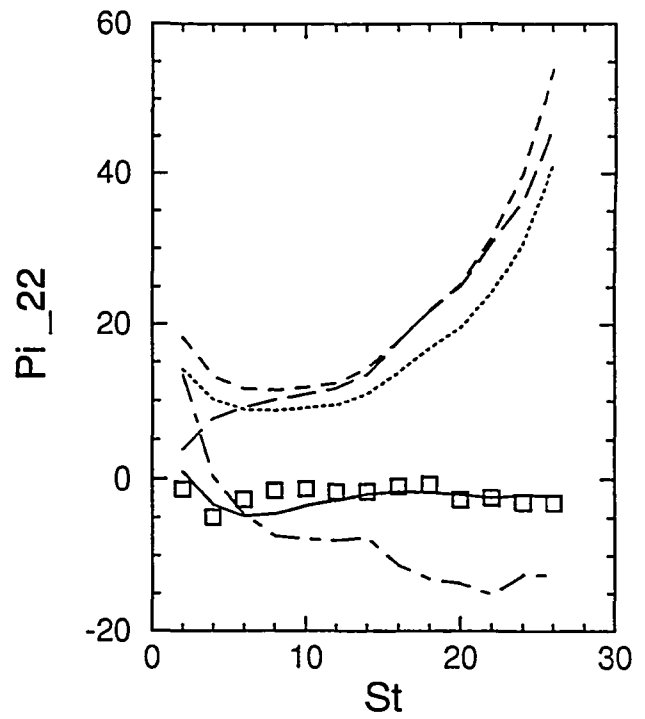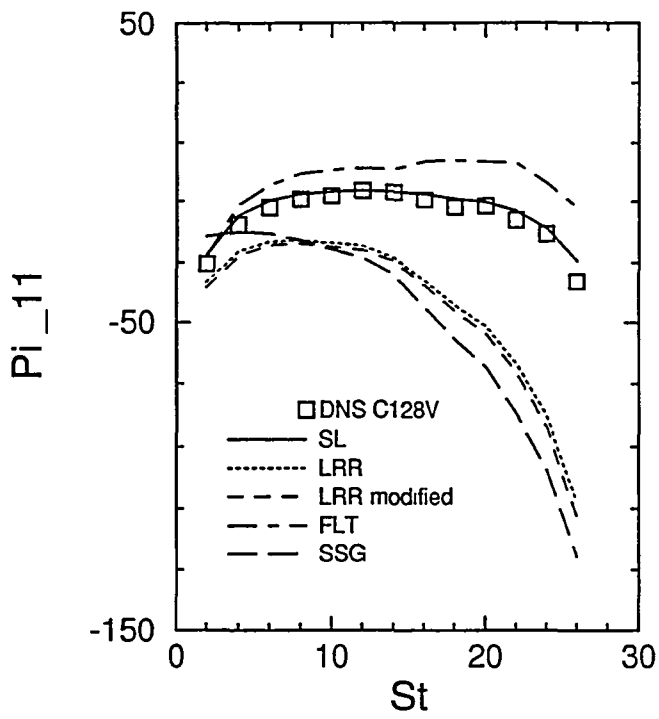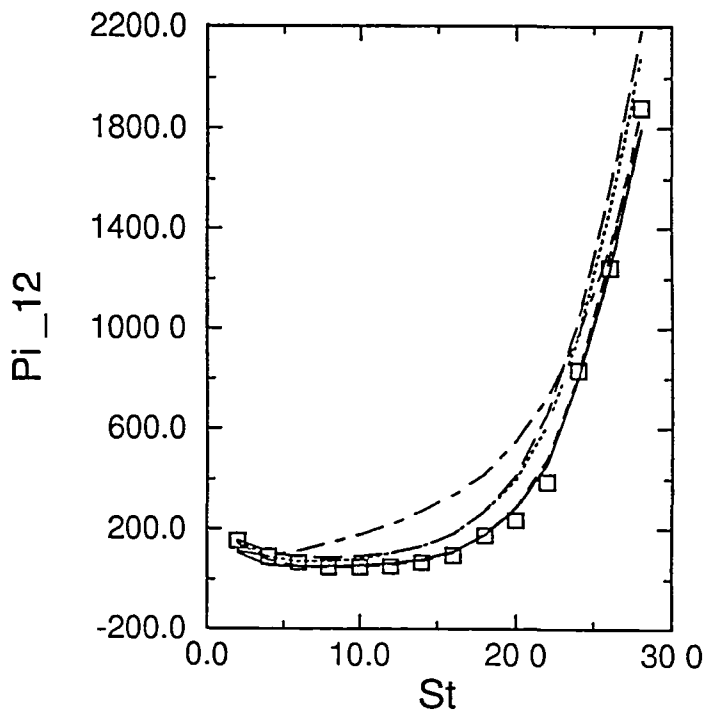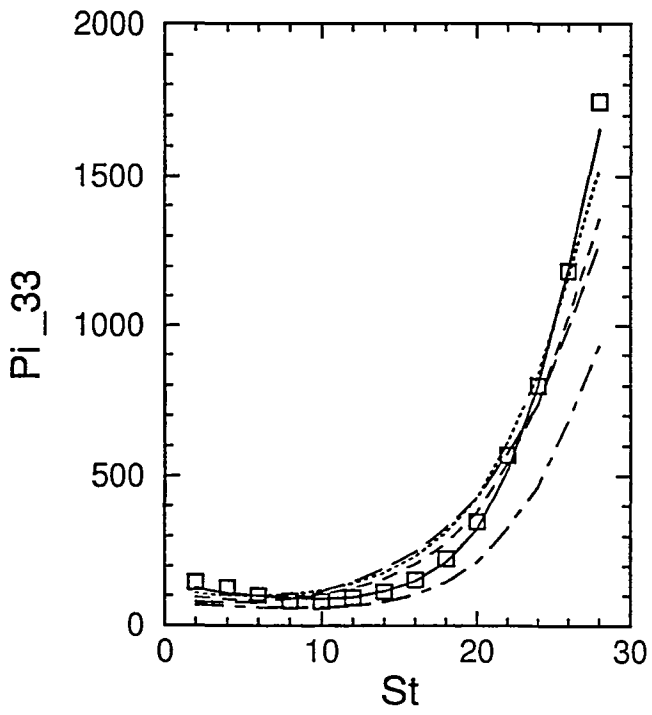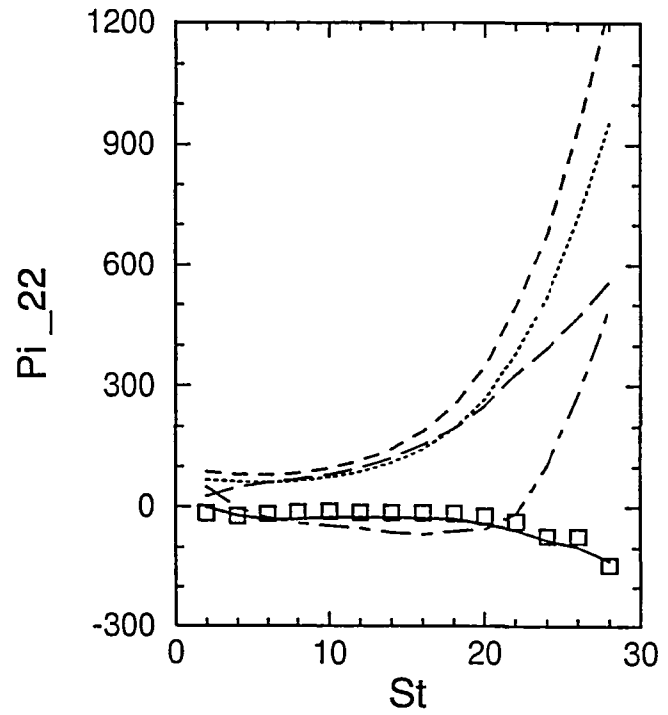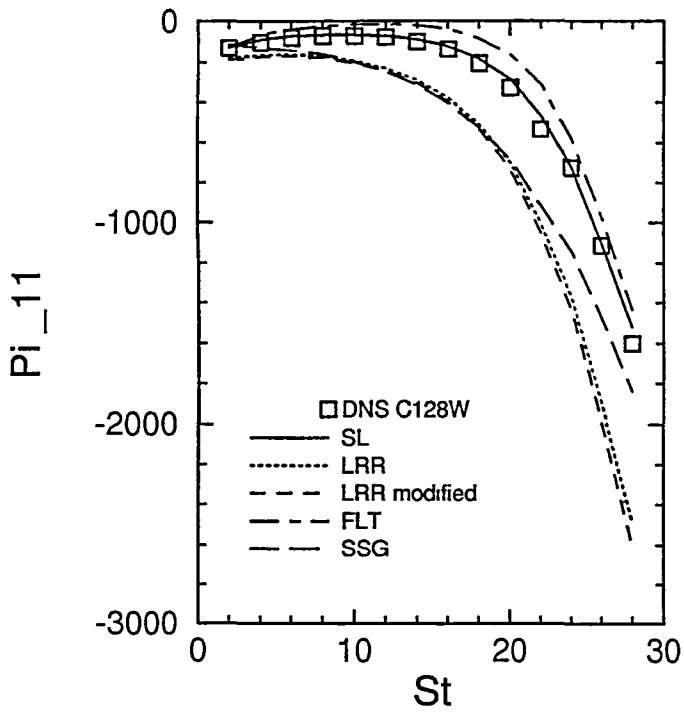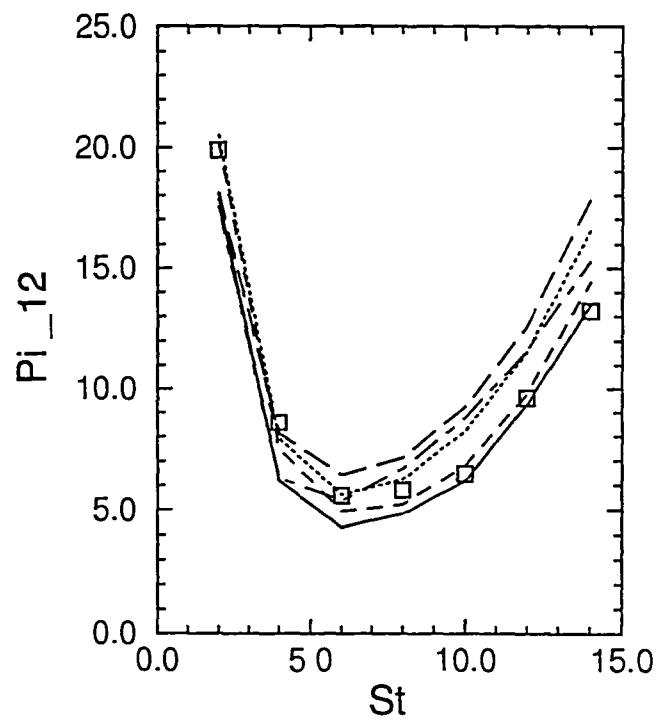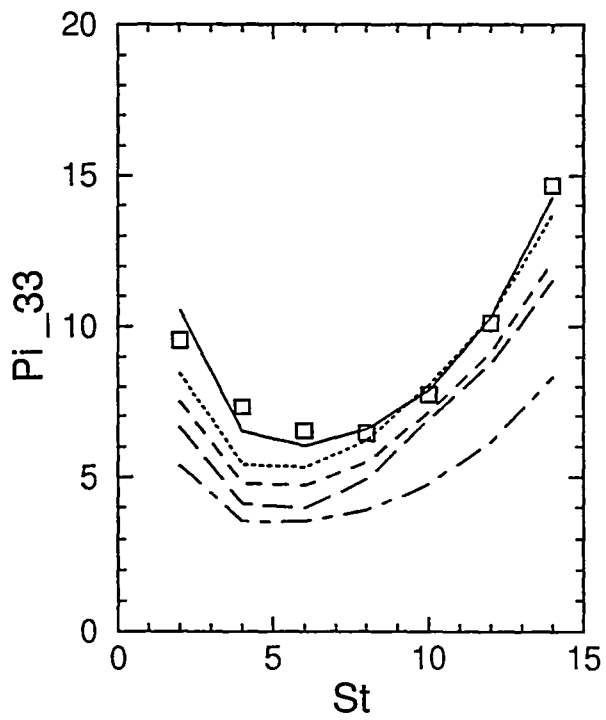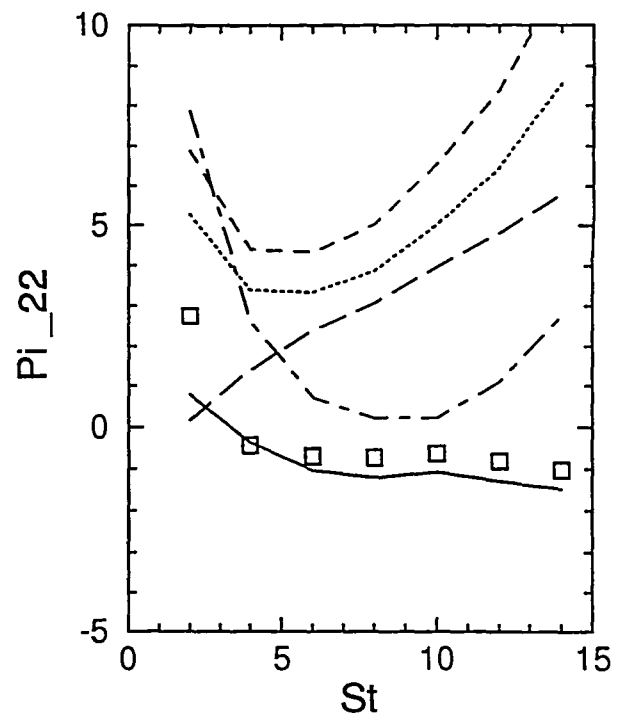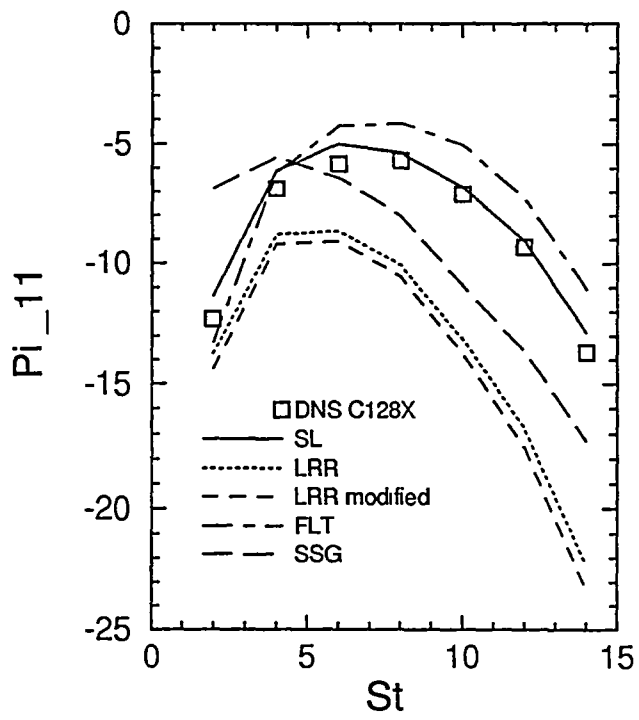Figure 4 Direct comparison betwenn the rapid models and the DNS data of C128U

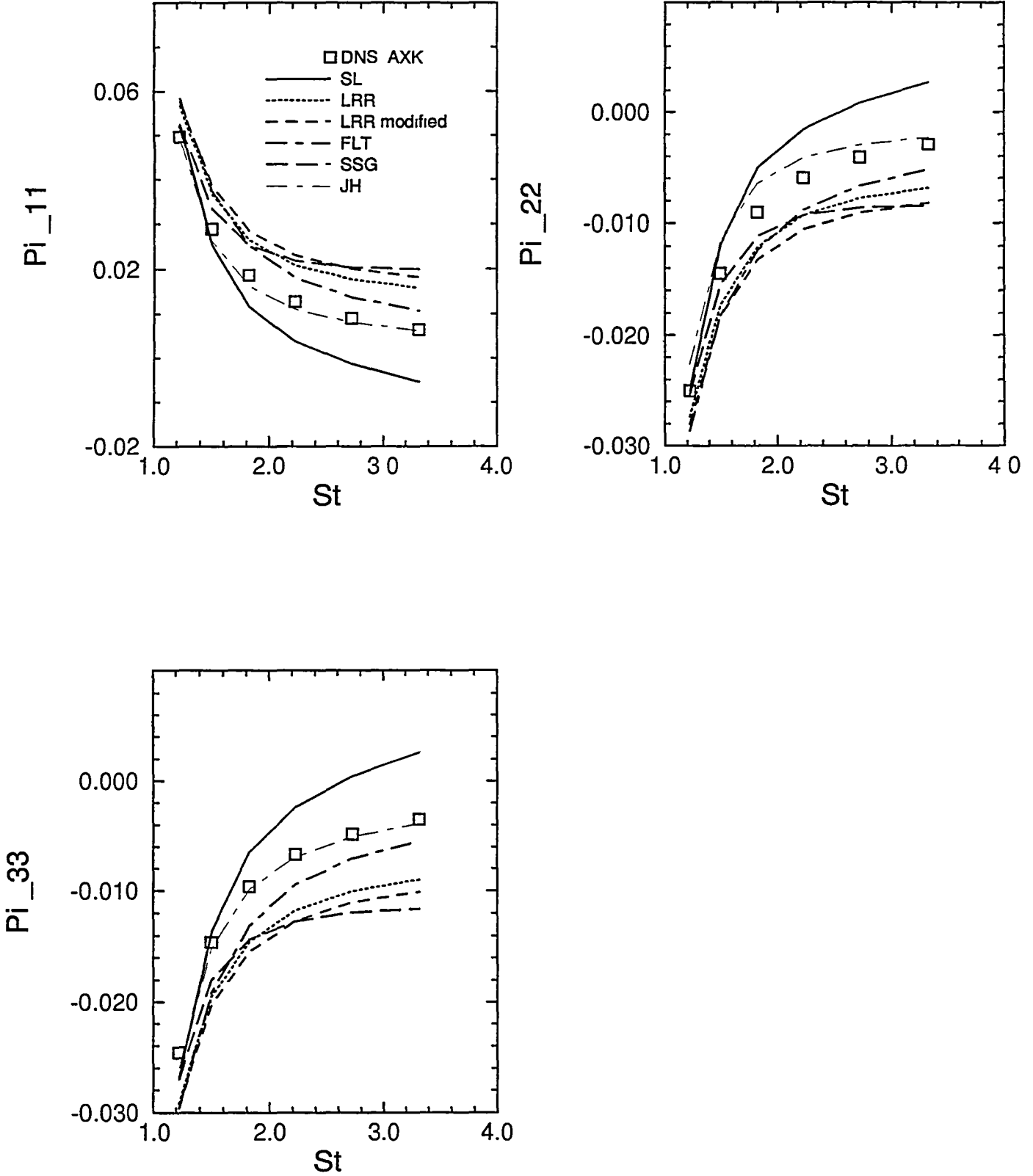Figure 5 Direct comparison betwenn the rapid models and the DNS data of C128V.

Figure 6. Direct comparison betwenn the rapid models and the DNS data of C128W.

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Figure 7. Direct comparison betwenn the rapid models and the DNS data of C128X.

Figure 8. Direct comparison betwenn the rapid models and
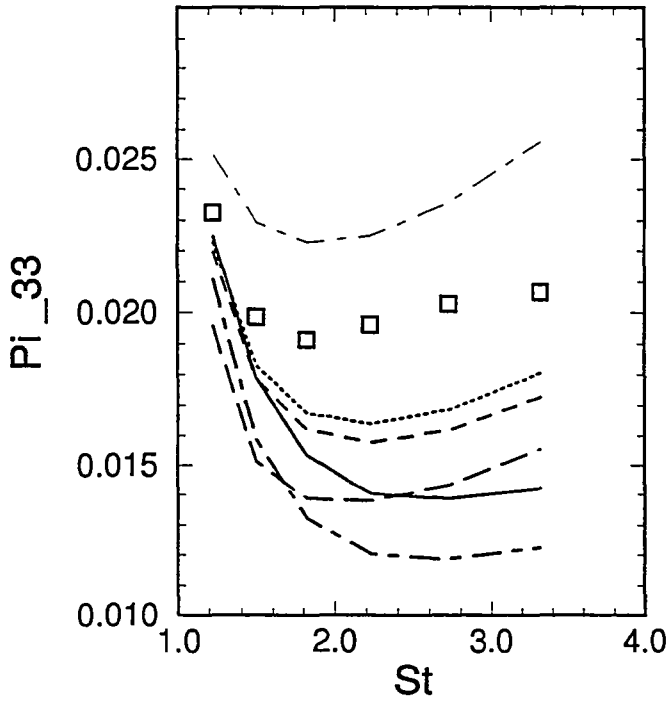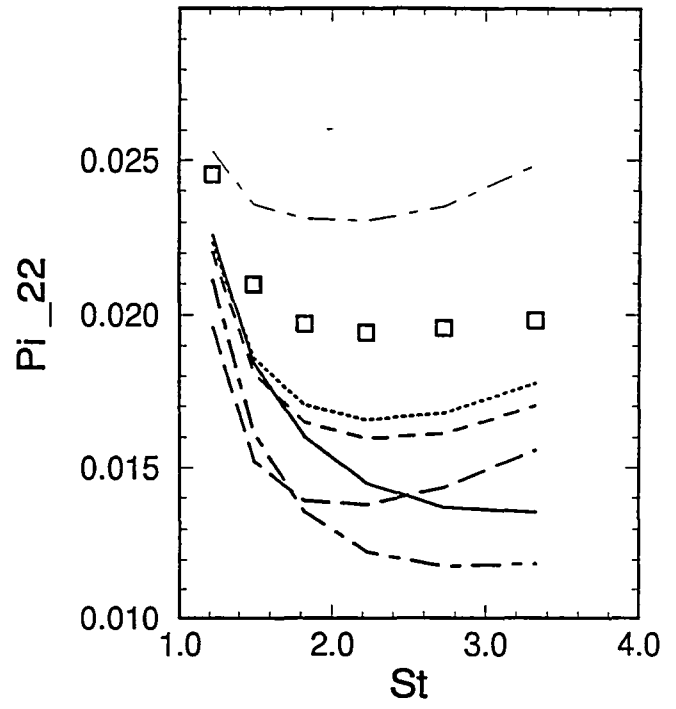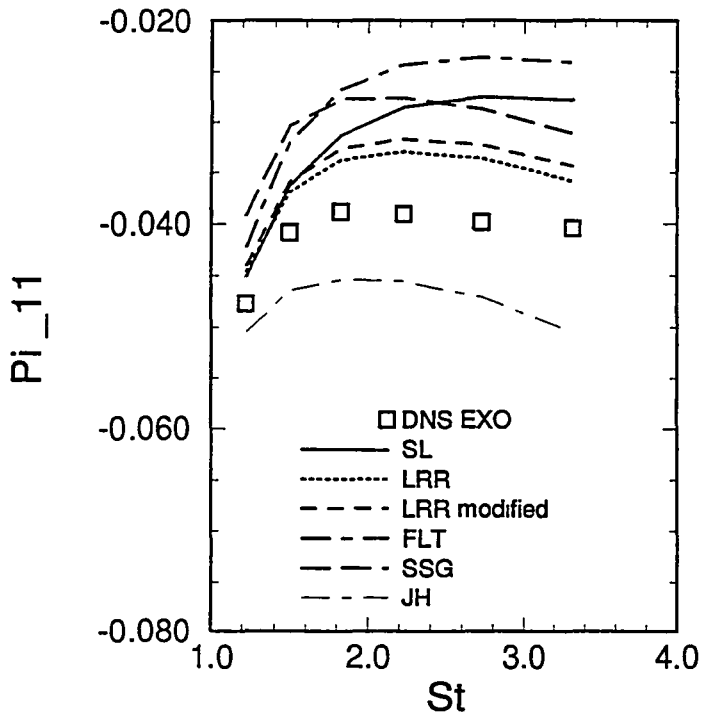the the DNS data of the axisymmetric contraction flow AXK.

Figure 9. Direct comparison betwenn the rapid models and
the DNS data of the axisymmetric expansion flow EXO.

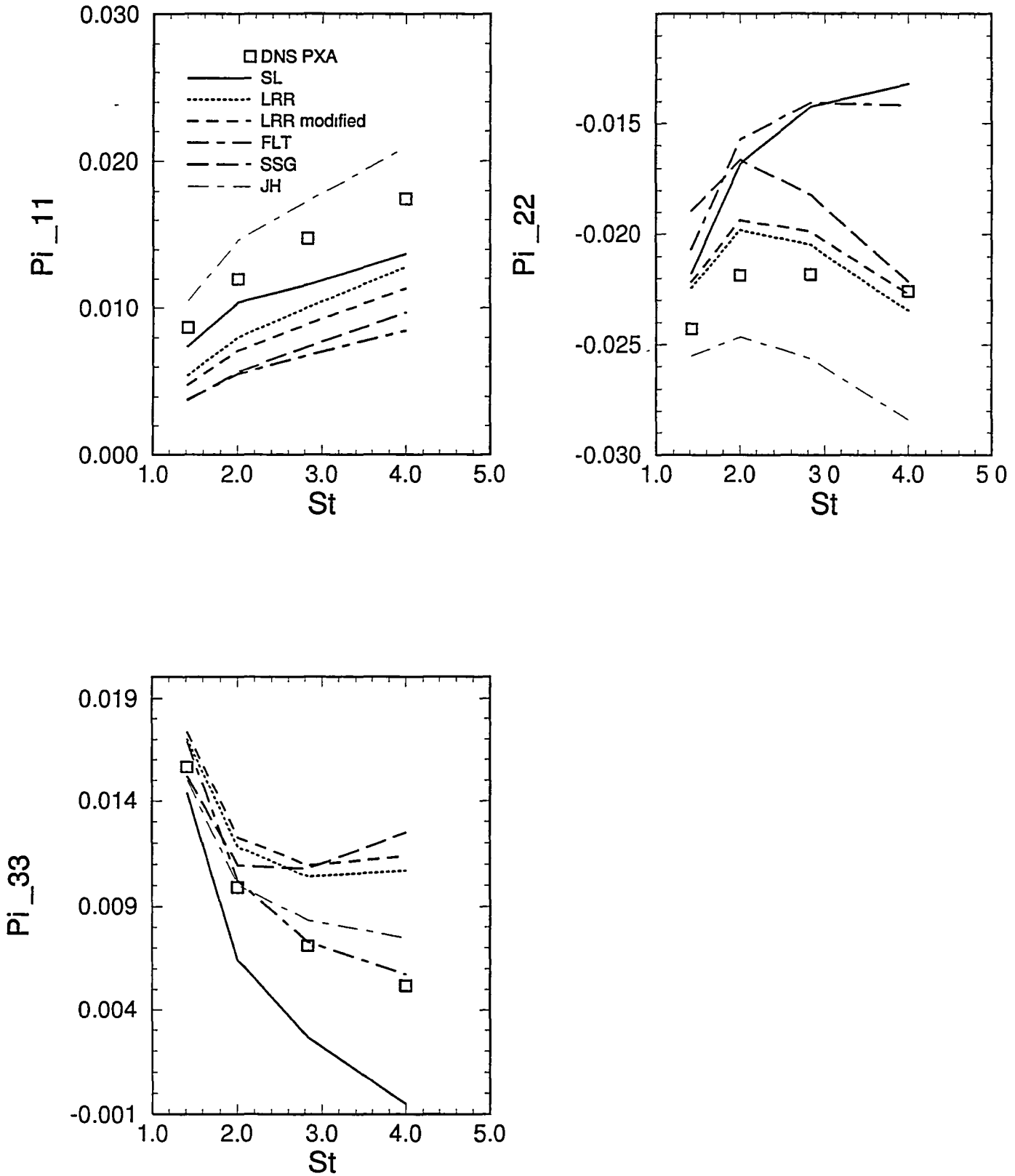Figure 10. Direct comparison betwenn the rapid models and
the DNS data of the plane strain flow PXA.

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13. ABSTRACT (Maximum 200 words)

This paper discusses two methods of developing models for the rapid pressure-strain correlation term in the Reynolds stress transport equation using direct numerical simulation (DNS) data. One is a perturbation about isotropic turbulence, the other is perturbation about two-component turbulence–an extremely anisotropic turbulence. A model based on the latter method is proposed and is found to be very promising when compared with DNS data and other models.

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