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COMPUTER ALGORITHM FOR THE
RELIABILITY ANALYSIS OF LAMINATED
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CCARES - A COMPUTER ALGORITHM FOR THE RELIABILITY
ANALYSIS OF LAMINATED CMC COMPONENTS

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Introduction

Structural components produced from laminated CMC (ceramic matrix composite) materials are being considered for a broad range of aerospace applications that include various structural components for the national aerospace plane, the space shuttle main engine, and advanced gas turbines. Specifically, these applications include segmented engine liners, small missile engine turbine rotors, and exhaust nozzles. Use of these materials allows for improvements in fuel efficiency due to increased engine temperatures and pressures, which in turn generate more power and thrust. Furthermore, this class of materials offers significant potential for raising the thrust-to-weight ratio of gas turbine engines by tailoring directions of high specific reliability. The emerging composite systems, particularly those with silicon nitride or silicon carbide matrix, can compete with metals in many demanding applications. Laminated CMC prototypes have already demonstrated functional capabilities at temperatures approaching 1400 °C, which is well beyond the operational limits of most metallic materials.

Laminated CMC material systems have several mechanical characteristics which must be carefully considered in the design procedure. Focusing attention on an individual ply, the most deleterious of these characteristics are low strain tolerance, low fracture toughness, and a large variation in failure strength, especially in the material orientation transverse to the fiber direction. Thus, analyses of components fabricated from ceramic materials require a departure from the usual deterministic design philosophy (i.e., the factor of safety approach) prevalent in designing metallic structural components. Since failure will be dominated by the scatter in strength, statistical design approaches must be employed. Although the so-called size effect has been reported to be nonexistent in the fiber direction (see ref. 1), a unidirectional reinforced ply will exhibit decreasing bulk strength transverse to the fiber direction with increasing component volume. Thus, on the one hand the composite exhibits high reliability and little size effect in the fiber direction,

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and on the other, low reliability and bulk size effects in the transverse direction. This indicates that, in addition to capturing any inherent scatter in strength, analyses must account for material symmetry imposed by the reinforcement in a rational manner. Current computational structural mechanics must evolve to yield new design protocols that address these concerns. Test bed software programs are needed that incorporate stochastic design concepts that are user friendly, computationally efficient, and have flexible architectures that readily incorporate changes in design philosophy. The CCARES (Composite Ceramics Analysis and Reliability Evaluation of Structures) program is representative of an effort to fill this need. CCARES is a public domain computer algorithm, coupled to a general purpose finite element program, which predicts the fast fracture reliability of a structural component under multiaxial loading conditions. (See Figs. 1 and 2.)

Stochastic Design Approach

We begin by advocating the use of a weakest-link reliability theory in designing components even if most of the load in an individual ply is oriented in the fiber direction. Current analytical practice discretizes a component and uses finite-element methods to determine the state of stress throughout the component. It is assumed that failure is dependent on the stress state in a component such that deformations are not controlling design. Since failure may take place in any of the discrete volumes, it is useful to consider a component from a systems viewpoint. Two fundamental systems can be considered. These are the series system and the parallel system. A component comprised of discrete volumes is a series system if it is in a state of failure whenever one of the discrete volumes fails. This is also referred to as a weakest-link system. In a parallel system, failure of a single element does not dictate that the component has failed, since the remaining elements may be able to sustain the load through redistribution. Assuming that a laminated structure behaves in a weakest-link manner allows the calculation of a lower bound for component reliability (i.e., a conservative estimate of structural reliability). Thomas and Wetherhold (ref. 2) point out that this is consistent with predicting the probability of the occurrence of the first matrix crack in an individual ply (i.e., first-ply matrix cracking). For most applications the design failure stress for a laminated material will be taken to coincide with first matrix cracking. The reason for this is that matrix cracking usually allows high temperature oxidation of the fibers to take place leading to embrittlement of the composite.

Next, we address the righteousness of applying weakest-link theory to a material that in some sense does not exhibit size effects. In general, the mean strength of a sample population representing test specimens subject to uniaxial tension is obtained by integrating the probability of survival (P_S) with respect to the applied tensile stress (see Fig. 3). The form of P_S depends on the probability density function that best represents the failure data and on whether the structural component acts as a parallel or series system. Adopting a Weibull probability density function and assuming a weakest-link system (a conservative assumption) give a specific form to the integral. The closed-form solution of the integral depends on stress, volume, the Weibull parameters, and the gamma function. Following an argument originally proposed by Jayatilaka (ref. 3), two uniaxial tensile specimen populations with distinctly different specimen volumes will yield different mean strengths. The ratio of the mean strengths depends only on the specimen volume and the Weibull modulus (see Fig. 4). As the Weibull modulus increases, the ratio of the mean strengths approaches unity. This represents the situation where the material exhibits no size effect (even though the distribution of failure strength

may be represented by a Weibull probability density function). From a practical standpoint, doubling the specimen size of a material whose Weibull modulus is 14 would yield a 5-percent difference in the mean failure strengths of the two populations. We have expectations that the Weibull modulus associated with the strength of CMC materials in the fiber direction will be in excess of 14. Thus, a difference of less than 5 percent in the mean failure strengths should be well within experimental error. The overall point is that weakest-link theory allows for diminishing size effects through an increasing Weibull modulus.

Reliability Model

The ongoing metamorphosis of ceramic material systems and the lack of standardized design data have in the past tended to minimize the emphasis on modeling. Many structural components fabricated from ceramic materials were designed by "trial and error," since emphasis was placed on demonstrating feasibility rather than fully understanding the processes controlling behavior. This is understandable during periods of rapid improvements in material properties for any system. Current research combines experimental investigation of failure mechanisms with the development of failure models. This facilitates improvements in processing and allows one to gain insight and intuition prior to constructing multiaxial failure theories that in some respect reflect the appropriate microstructural behavior. However, there is a philosophical line of demarcation that separates analytical schools of thought for ceramic composites into two categories. These are microstructural methods (usually based on principles of fracture mechanics) and phenomenological methods. In the first group one designs the material in the sense that the constituents are distinct structural components, and the composite ply (or lamina) is considered a structure in its own right. The second school of thought represents the ply (or lamina) as a homogenized material with properties that are determined from a finite number of well-thought-out phenomenological experiments.

We currently espouse the philosophy that one would use the former for the design of the material (i.e., analyze the composite) and use the latter to design components with the material (i.e., analyze structural components). There are practical reasons for initially adopting this viewpoint. We fully recognize that the failure characteristics of these composites are controlled by a number of local phenomena including matrix cracking, debonding and slipping between matrix and fibers, and fiber breakage, all of which strongly interact. A top-down approach (i.e., first proposing design models at the ply level) will initially establish viable and working design protocols. In the future, the intent is to possibly extend analytical methods to the constituent level in a rational and practical manner. However, initially adopting the bottom-up approach allows for the possibility of becoming mired in detail (experimental and analytical) when multiaxial analyses are conducted at the constituent level. Analytical methods based on principles of fracture mechanics for laminated ceramic composite systems are emerging, yet mature design concepts are limited due to the lack of coherent mixed mode fracture criterion and, in most cases, the lack of an observable macrocrack.

Material science research is presently concerned with improving and predicting composite tensile strength in the fiber direction, which addresses the upper bound in a structural design problem. Typical stress/strain curves of unidirectional systems are bilinear when loaded along the fiber direction, with a distinct breakpoint that usually corresponds to transverse matrix cracking. Since monolithic ceramics are much stronger in compression than in tension, fibers are incorporated to mitigate tensile failure by bridging inherent matrix flaws. Conversely, a tensile load applied transverse to the fiber direction results in failure behavior similar to a

monolithic ceramic, which represents the lower bound of composite strength. Multi-axial design methods must be capable of predicting these two bounds. A number of phenomenological theories exist that treat unidirectional composites as homogenized, anisotropic materials. Upper and lower bounds for strength are stipulated in the fiber direction and the direction transverse to the fiber. These macroscopic criteria represent modeling techniques that have been adapted from existing technologies in other material systems and are familiar to design engineers experienced in dealing with polymer matrix composites.

The current version of CCARES uses a macroscopic model proposed by Thomas and Wetherhold (ref. 2) to analytically describe reliability limits of the ply, thereby enabling one to calculate reliability of the laminated component (see Fig. 5). A unidirectional ply is considered a two-dimensional structure, assumed to have three basic strengths (or failure modes). They include a strength capacity related to normal loads in the fiber direction, a strength capacity related to normal loads in the direction transverse to the fiber direction, and an in-plane shear strength. It is assumed that the random variables representing strength act independently in producing failure (i.e., a noninteractive theory). In addition, each ply is discretized into individual sub-ply volumes. For reasons discussed in the previous section we assume that failure of a ply is governed by its weakest link (or sub-ply volume). Under this assumption, events leading to failure of a given link do not affect other links (see e.g., ref. 4); thus the probability of first-ply failure is given by the expression found in Fig. 6. Figures 7 and 8 show applications for a thin-wall pressure vessel and a rectangular plate with a hole.

Future Directions

Ceramic material systems will play a significant role in future aerospace, elevated-temperature applications. To this end, there are a number of issues that must be addressed by the structural mechanics research community (Fig. 9). One important aspect (see ref. 5) is the effect of R-curve behavior. Localized energy dissipation takes place within a process zone around a crack tip. The resistance to crack growth that develops results in increasing damage tolerance. In theoretical terms, R-curve behavior will modify the stochastic parameters associated with each random strength variable with crack extension. We intend to pursue this concept in the near term. Also, total failure of an individual ply effectively reduces the overall laminate stiffness. This causes local redistribution of the load to adjacent layers. In addition, delamination between plies will relax the constraining effects among layers, allowing in-plane strains to vary in a stepwise fashion within a laminate. These effects require the development of rational load redistribution schemes. It is apparent that the utilization of ceramics as structural components in harsh service environments requires thoughtful consideration of reliability degradation due to time-dependent phenomena. Thus, issues germane to component life such as cyclic fatigue and creep behavior must be addressed analytically. Computational strategies are needed that extend current methods of analysis involving subcritical crack or damage growth and creep rupture to laminated CMC materials that are subject to multi-axial states of stress.

In closing, we recognize that when failure is less sensitive to imperfections in the material, stochastic methods may not be that essential. Yet, trends in design protocols are moving in the direction of probabilistic analyses (even for metals) and away from the simplistic safety factor approach. In this sense brittle ceramics will serve as model materials in the study and development of reliability methods that will act as the basis of future design codes.

References

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Objective

Develop and refine analytical methods and computer codes for predicting

- **Fast fracture**
- **Life**

for laminated ceramic composites used as structural components in aerospace applications that entail high-temperature environments.

Summary of Progress to Date

- Preliminary version of CCARES (*Composite Ceramics Analysis and Reliability Evaluation of Structures*) is available. This computer algorithm has been coupled to MSC/NASTRAN laminate analysis.
- Currently the analysis is conducted at the ply level. This approach allows the design engineer to incorporate ply geometry and orientation in an analysis. Probability of first-ply-failure (FPF) is computed.
- The analysis allows three failure modes to emerge within the ply: failure in the fiber direction; failure transverse to the fiber direction; and in-plane shear failure.
- Ply strength associated with each failure mode is treated as a random variable with an assumed Weibull distribution (three-parameter formulation).
- Nonlinear regression analysis is used to estimate the Weibull parameters for each ply strength from experimental failure data.

3rd HITEMP

Fig. 2

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Size Effect—Uniaxial Tension

In general, the mean strength of a sample population representing test specimens subject to uniaxial tension is given by

$$\bar{\sigma} = \int_0^{\infty} P_s \, d\sigma$$

Adopting a Weibull distribution for the PDF and a weakest-link approach to reliability give

$$P_s = \exp \left[- \left(\frac{\sigma}{\beta} \right)^\alpha V \right]$$

Here α and β are Weibull parameters; V is the volume of the gage section of the specimen. Using this particular expression for P_s in the integral above yields the following closed-form solution:

$$\bar{\sigma} = \frac{\beta}{(V)^{1/\alpha}} \Gamma \left(1 + \frac{1}{\alpha} \right)$$

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Fig. 3

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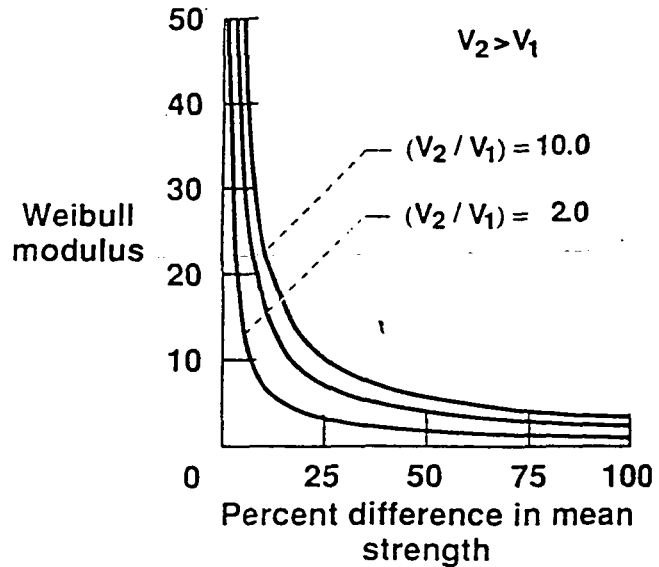
The mean uniaxial tensile strengths of two populations with volumes V_1 and V_2 are

$$\bar{\sigma}_1 = \frac{\beta}{(V_1)^{1/\alpha}} \Gamma\left(1 + \frac{1}{\alpha}\right) \quad \bar{\sigma}_2 = \frac{\beta}{(V_2)^{1/\alpha}} \Gamma\left(1 + \frac{1}{\alpha}\right)$$

The ratio of mean strengths is

$$\frac{\bar{\sigma}_1}{\bar{\sigma}_2} = \left(\frac{V_2}{V_1}\right)^{1/\alpha}$$

In the limit as α approaches infinity, the ratio of mean strengths approaches unity.



3rd HITEMP

Fig. 4

CD-90-48311

Reliability Analysis of Laminated CMC Components

We adopt a phenomenological approach and assume the overall strength of a ply is governed by the strength of its weakest link.

Taking

$$R_i = \text{reliability of } i^{\text{th}} \text{ ply}$$

$$= \exp\left(-\int_V \psi \, dV\right)$$

where

$$\psi = \text{failure function per unit ply volume}$$

$$= \left(\frac{\langle \sigma_1 - \gamma_1 \rangle}{\beta_1}\right)^{\alpha_1} + \left(\frac{\langle \tau_{12} - \gamma_2 \rangle}{\beta_2}\right)^{\alpha_2} + \left(\frac{\langle \sigma_2 - \gamma_3 \rangle}{\beta_3}\right)^{\alpha_3} \\ + \left(\frac{\langle (-1)(\sigma_1 + \gamma_4) \rangle}{\beta_4}\right) + \left(\frac{\langle (-1)(\sigma_2 + \gamma_5) \rangle}{\beta_5}\right)$$

The α 's, β 's, and γ 's correspond to the Weibull shape, scale, and threshold parameters.

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Fig. 5

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Note that

σ_1, σ_2 in-plane normal stresses

τ_{12} in-plane shear stress

$$\langle x \rangle = x \cdot u[x] = \begin{cases} x & x > 0 \\ 0 & x \leq 0 \end{cases}$$

The probability of first-ply failure for the laminate is given by

$$P_{\text{fpf}} = 1 - \prod_{i=1}^n R_i$$

where n is the number of plies.

3rd HITEMP

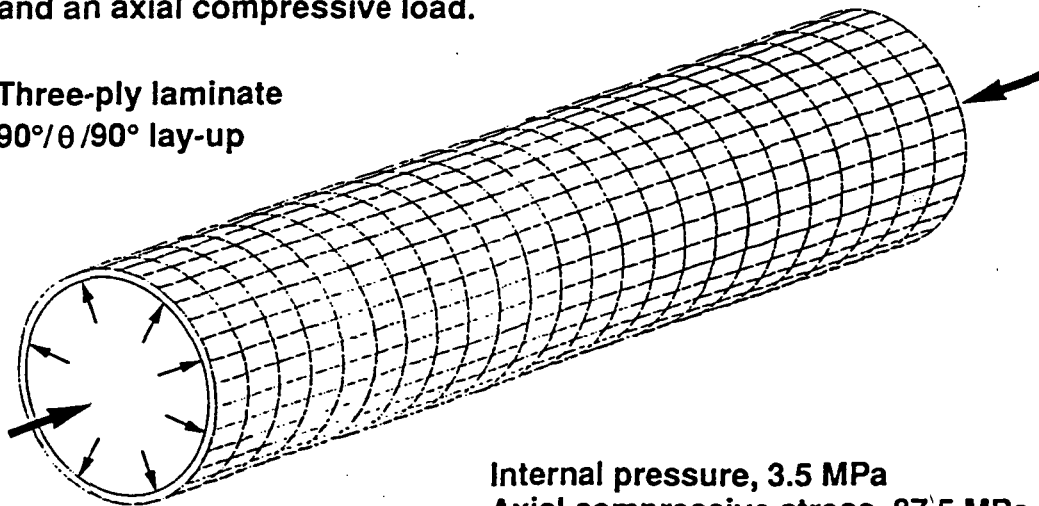
Fig. 6

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Application – Thin-Wall Pressure Vessel

The concept of laminate reliability is illustrated in the following application where a thin-wall tube is subjected to an internal pressure and an axial compressive load.

Three-ply laminate
90°/θ/90° lay-up



Internal pressure, 3.5 MPa
Axial compressive stress, 87.5 MPa

Angle θ is measured relative to the longitudinal axis.

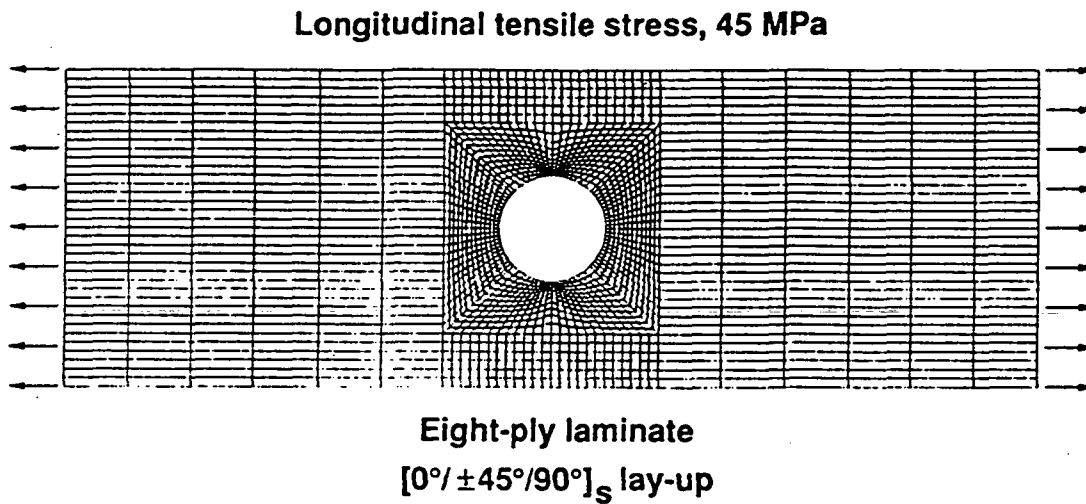
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Fig. 7

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Application – Rectangular Plate with Hole

A more complex structure is considered next. The presence of a hole introduces local stress concentrations that affect the distribution of risk of rupture intensity in each ply.



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Fig. 8

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Future Directions

- Load-sharing rules must be adopted so that the probability of second-ply-failure, third-ply-failure,...., can be computed.
- Probability of failure due to delamination must be addressed.
- Ply strength database must be established for each assumed mode of failure (possible collaboration with the Materials Division).
- Consideration should be given to conducting reliability analysis at the constituent level.
- Time-dependent reliability must be addressed.

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Fig. 9