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(NASA-CR-199951) A TWO DIMENSIONAL INTERFACE ELEMENT FOR COUPLING OF INDEPENDENTLY MODELED THREE DIMENSIONAL FINITE ELEMENT MESHES AND EXTENSIONS TO DYNAMIC AND NON-LINEAR REGIMES (Old Dominion Univ.) 18 p

A TWO DIMENSIONAL INTERFACE ELEMENT FOR COUPLING OF INDEPENDENTLY MODELED THREE DIMENSIONAL FINITE ELEMENT MESHES AND EXTENSIONS TO DYNAMIC AND NON-LINEAR REGIMES

By

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Performance Report (Summary Report)

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Status Report for NASA Grant NAG1-1670

I. Work Completed

The proposal for Grant NAG1-1670 was written for a three year period (please see the Duration Section in the proposal, a copy of which is attached). However, the grant was budgeted only for the first year. Therefore, the work reported here pertains only to this first year and not the entire work stated in the proposal. In this first year phase of the grant the following has been completed.

A. In the course of studying and testing the 1-D interface element (as a prelude to the 2-D work) errors were encountered in the code for built-up structures and curved interfaces. These errors were corrected and the benchmark Boeing composite crown panel was analyzed successfully.

B. Studied the NURBS (Non-Uniform Rational B-Splines), cubic splines, and cubic B-splines and concluded that the cubic B-splines are the state of the art and best suite our application.

C. When analyzing the Boeing crown panel, it was observed that coordinate data points representing the interface contained small random error (noise) which in turn caused some noise in the results. This noisy coordinate data will also be present in the 2-D interface, perhaps causing more severe error in the results. To construct a smooth function from the noisy data a least squares approach combined with cubic B-splines for the planar curve in the 1-D case was developed and tested with success. The work on planar curves should be generalizable if need for space curves arises in the future. The theoretical formulation is designed such that the 2-D surface representation would be realized as a tensor product of the 1-D work, thus capitalizing on the 1-D work.

D. Following the successful formulation of a discontinuous 1-D beam element, preliminary investigations for the formulation of discontinuous 2-D shell and 3-D solid elements were conducted. It turns out that the 2-D hybrid shell elements can be reformulated (some programming is required) to account for discontinuity across the shell element surface (applicable in the cross-surface method mentioned in the proposal). However, for discontinuous loading on the shell element edges and on the 3-D solid surfaces, although a formulation is workable using classical elasticity solutions, the formulation is not as straight forward

and the accuracy of the results is not as clear. More investigation is necessary for these cases.

Some key features of the technology developed so far are summarized below.

1. The basis functions are piecewise cubic polynomials defined over the entire domain and are continuous through the second derivative.

2. The basis functions are of compact support, i.e., they are non-zero over only a 5-knot consecutive subset of knots. This implies that solution matrices derived for fitting data are pentadiagonal.

3. The knot set and corresponding basis functions are defined independently of the data ordinates. There is an algorithm for automatic knot distribution definition which provides nearly equal number density of data points in the knot partition domain.

4. The data can be given in local or global coordinates, but the fitting takes place locally for more accurate data representation.

A sketch of a typical 1-D interface between incompatible grids is given in diagram A. Examples of the 1-D "noisy data" representation capability are shown in Figures 1-6. Figure 1 shows basis functions for a non-uniform knot set of nine knots partitioning the domain [0,1]. Basis function number four is shown alone for clarity in Figure 2 plotted over the entire domain. The region of compact support is clearly seen. In Figure 3 is plotted a circular arc section of 10% maximum height together with the least-squares fit using the previously shown 9-knot basis functions. The vertical scale is exaggerated by a factor of about 10, and the excellent quality of representation can be seen. The same circular arc data with considerable random error corrupting the points is shown in Figure 4, together with the least-squares spline fit. Again, the scale is exaggerated and the basis splines represent the data nicely.

Figures 5 and 6 show circular arc augments of 90 degrees and 135 degrees in a global coordinate system. The data are fitted locally with the knot set spanning the chords. This technique of local fitting generally assures a single-valued function and improved accuracy.

II. Work in Progress and Planned

A. Two Dimensional Interface Representation

   1. An algorithm is being developed to select a local coordinate system to serve as a 2-D parameterization space and projection plane. Various issues and options for this algorithm are currently being studied and assessed.

   2. The 1-D knot partitioning and distribution algorithm is being generalized for 2-D partitioning and knot selection in the local projection plane. The algorithm will be such that nearly equal number density of projected points occur in each knot-partition cell.

   3. Development of 2-D basis functions as tensor products if 1-D basis functions.

   4. Algorithm for least-squares representation of the interface using 2-D basis functions.

B. Two-Dimensional Interface Element

   1. Formulate equations coupling displacements and tractions using a hybrid variational method similar to the 1-D formulation.

   2. Generalized stiffness matrices resulting from the above mentioned formulation contain sub-matrices referred to as the M and G matrices. These submatrices are obtained by integration, over the interface, of quantities related to traction and displacement. These integrals are to be computed numerically using the bi-cubic B-splines as interpolation functions for integration.

C. The formulation and implementation of consistent load calculations for the 3-D hybrid solid elements in COMET-AR. These load calculations are necessary to enable the use of the hybrid solid elements in COMET-AR in conjunction with the 2-D interface element.

D. Benchmarking and Demonstration Applications

1. The 2-D interface technology is expected to be verified by solving a beam with a 2-D planar interface and comparing to existing solutions.

2. A second problem of suitable complexity and having a curved interface is to be solved. The exact features of the proposed problem have not been decided upon, but an example of the type of problem envisioned is a ball and socket for two or more interfaces resulting from relative movement between the ball and socket. The envisioned methodology will be restricted to movement increments at least one grid line in one of the grid directions. A pictorial sketch of an expected typical 2-D interface is shown in diagram B.
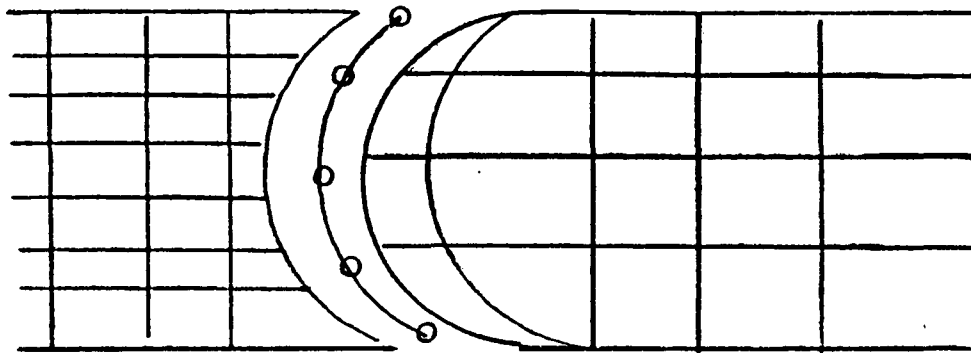
Diagram A: Curved 1–D Interface
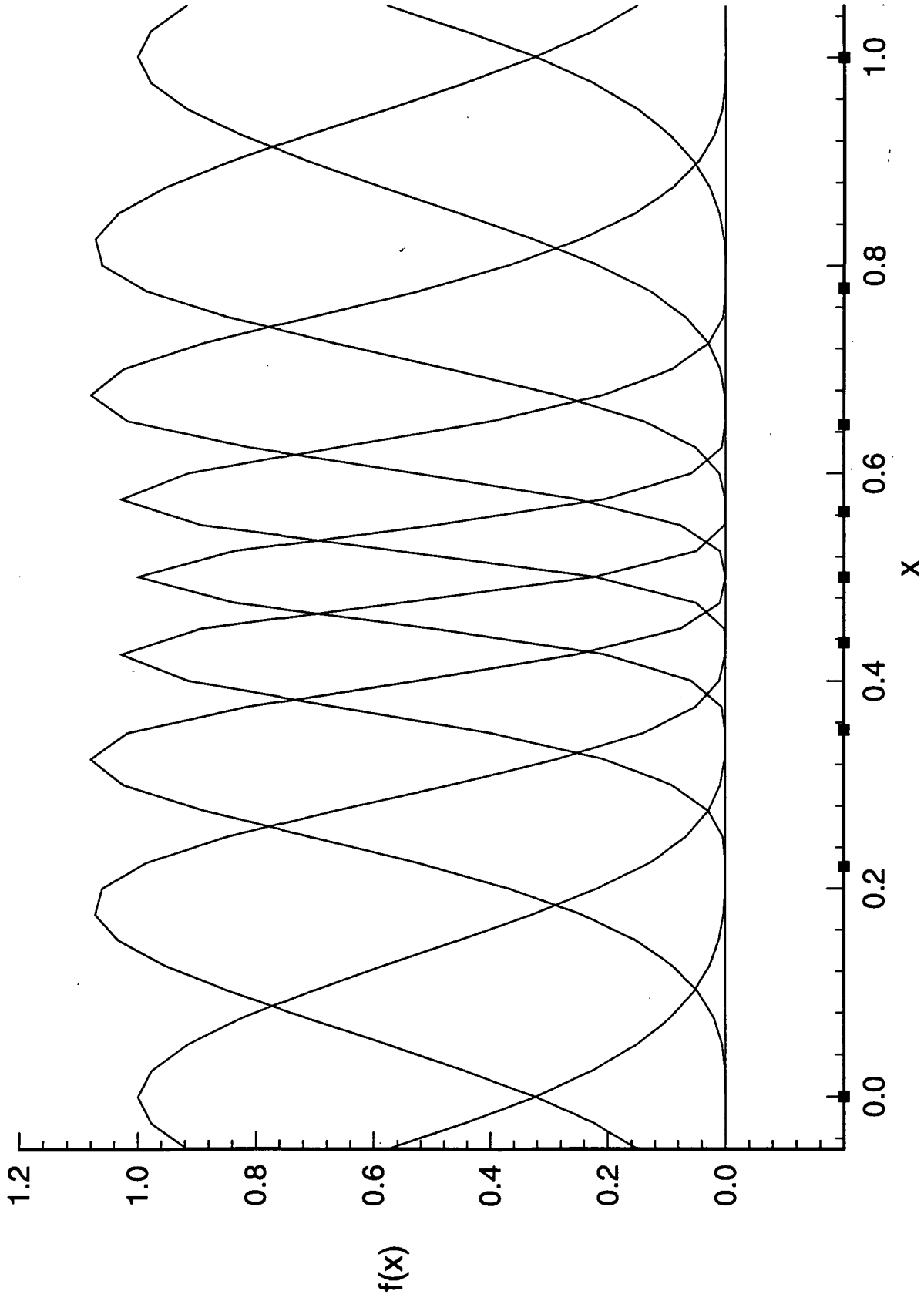
Figure 1: BASIS FUNCTIONS FOR 9 KNOTS

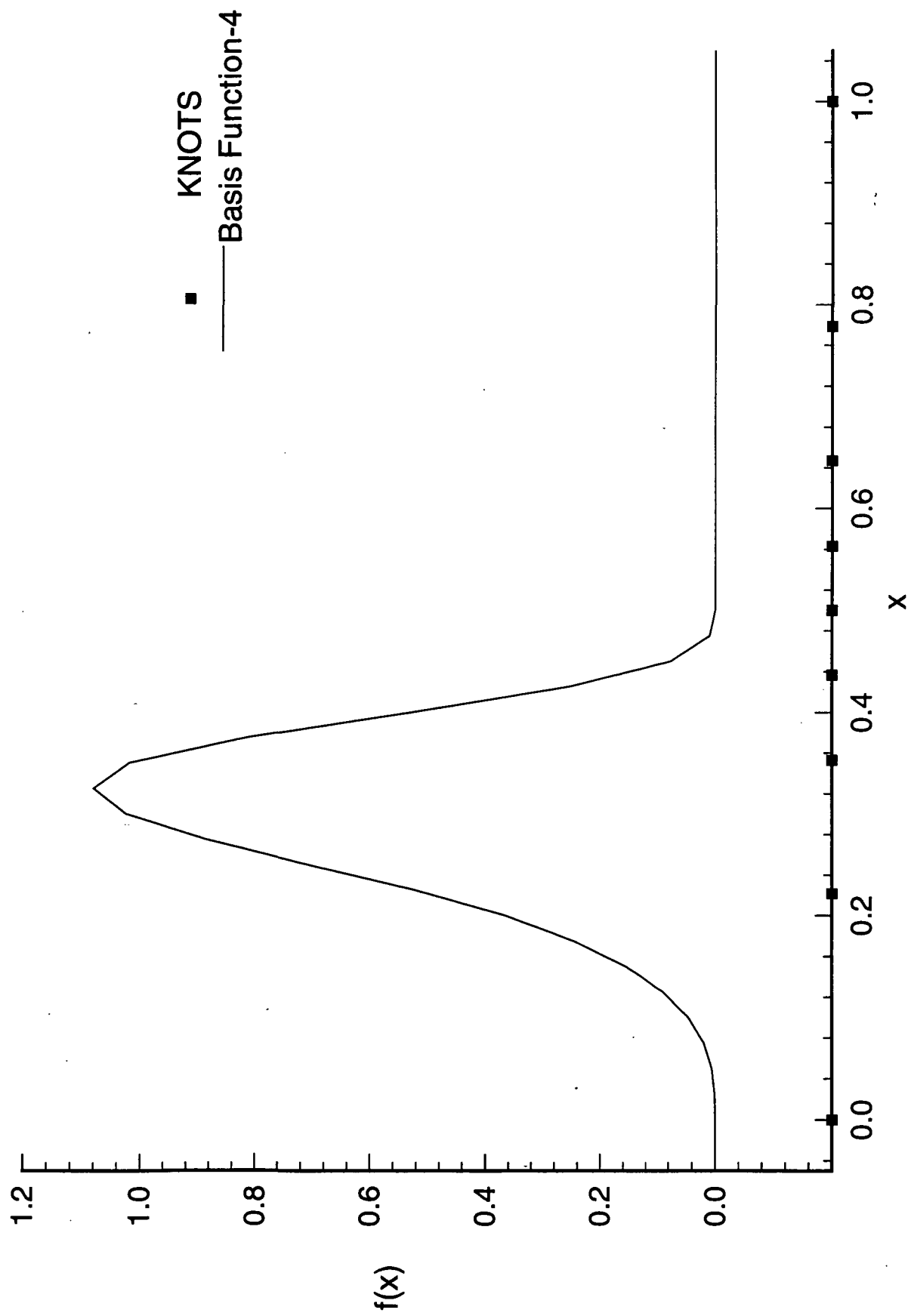Figure 2: BASIS FUNCTION NUMBER 4 (9 KNOTS)

Figure 3: LEAST-SQUARES FIT TO 10 % MAX HEIGHT CIRCULAR ARC
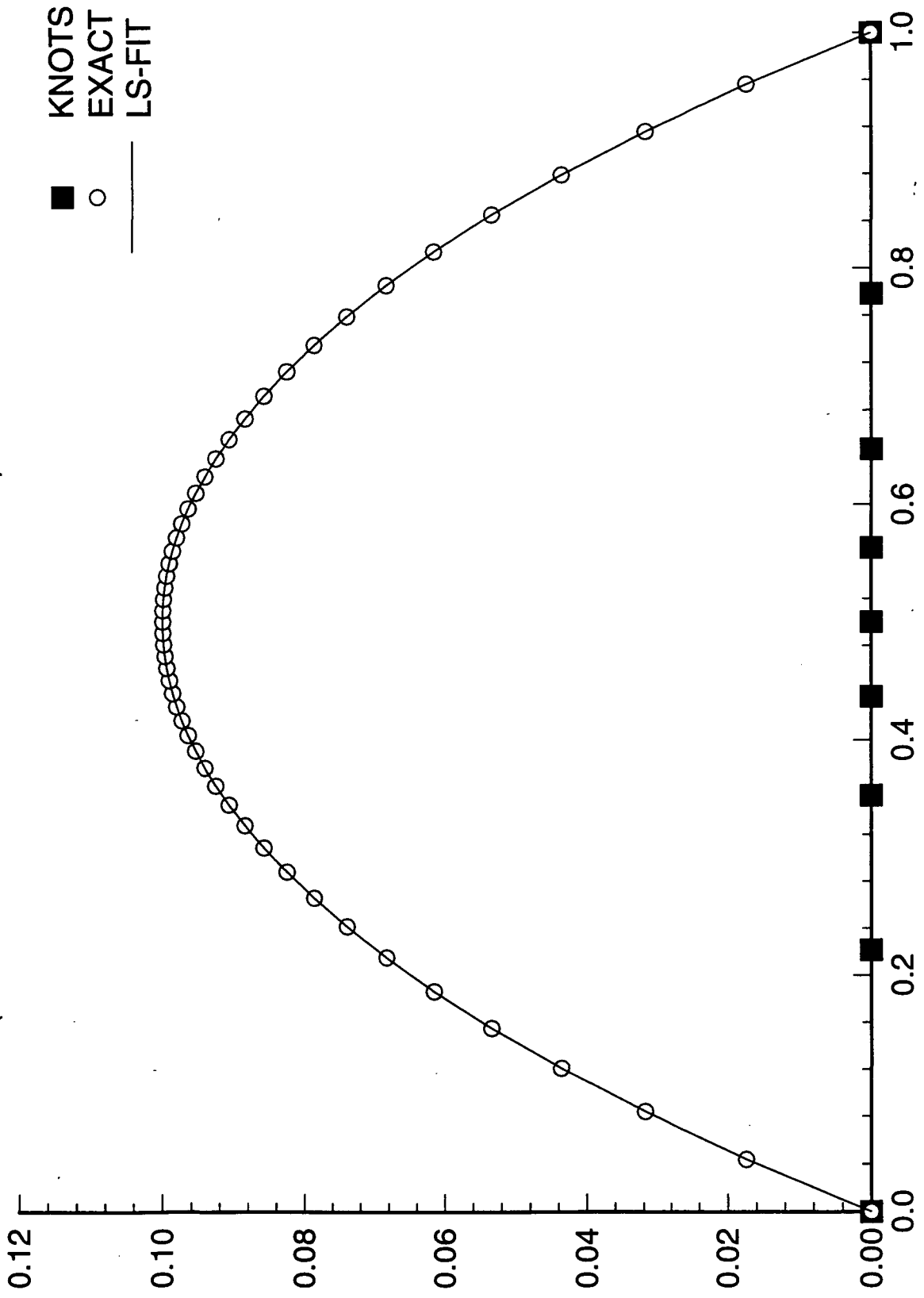(EXAGGERATED VERTICAL SCALE)

KNOTS
EXACT
LS-FIT

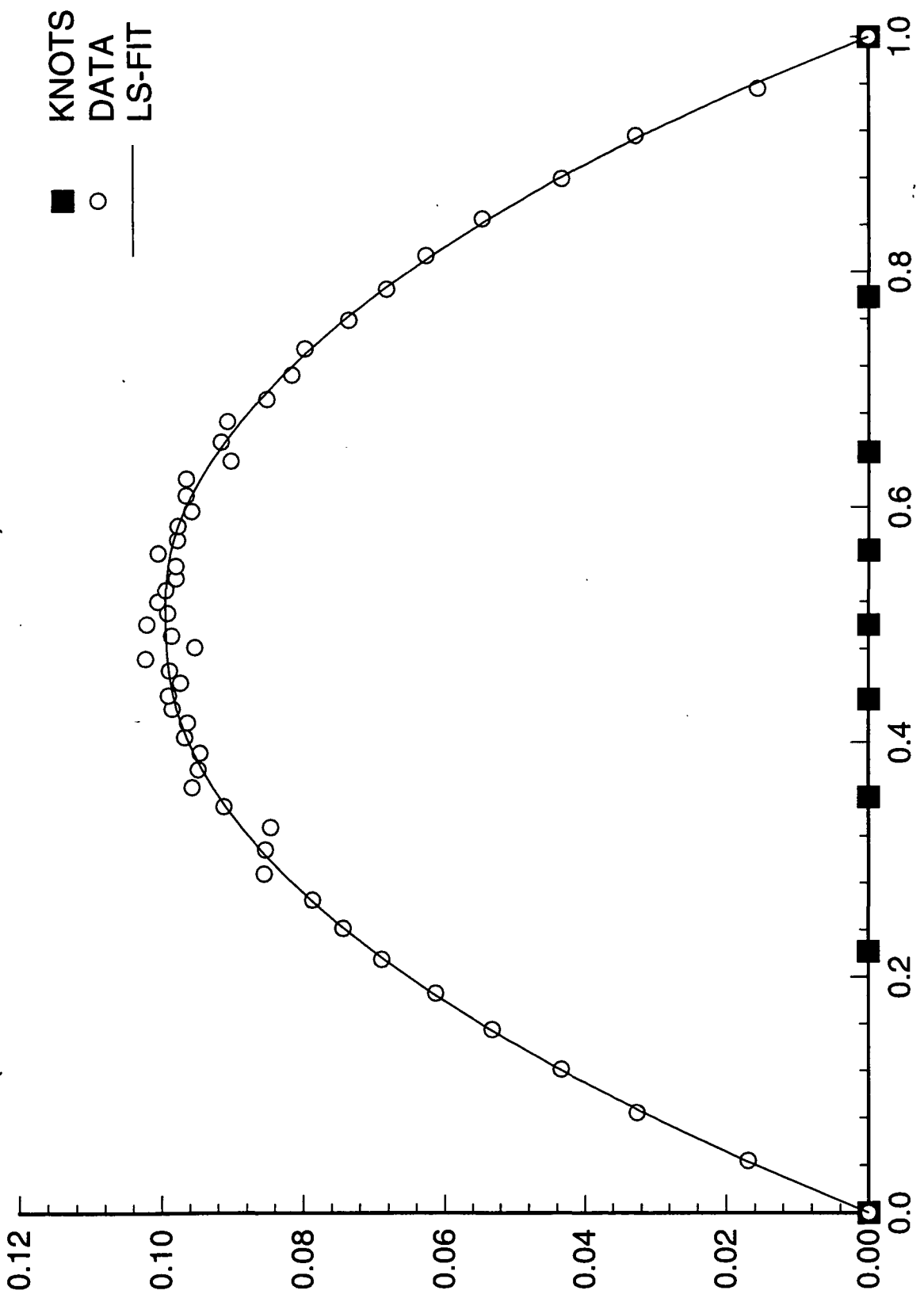Figure 4: LEAST-SQUARES FIT TO 10 % MAX HEIGHT CIRCULAR ARC - NOISY DATA
(EXAGGERATED VERTICAL SCALE)

Figure 5: LOCAL COORDINATE SYSTEM REPRESENTATION
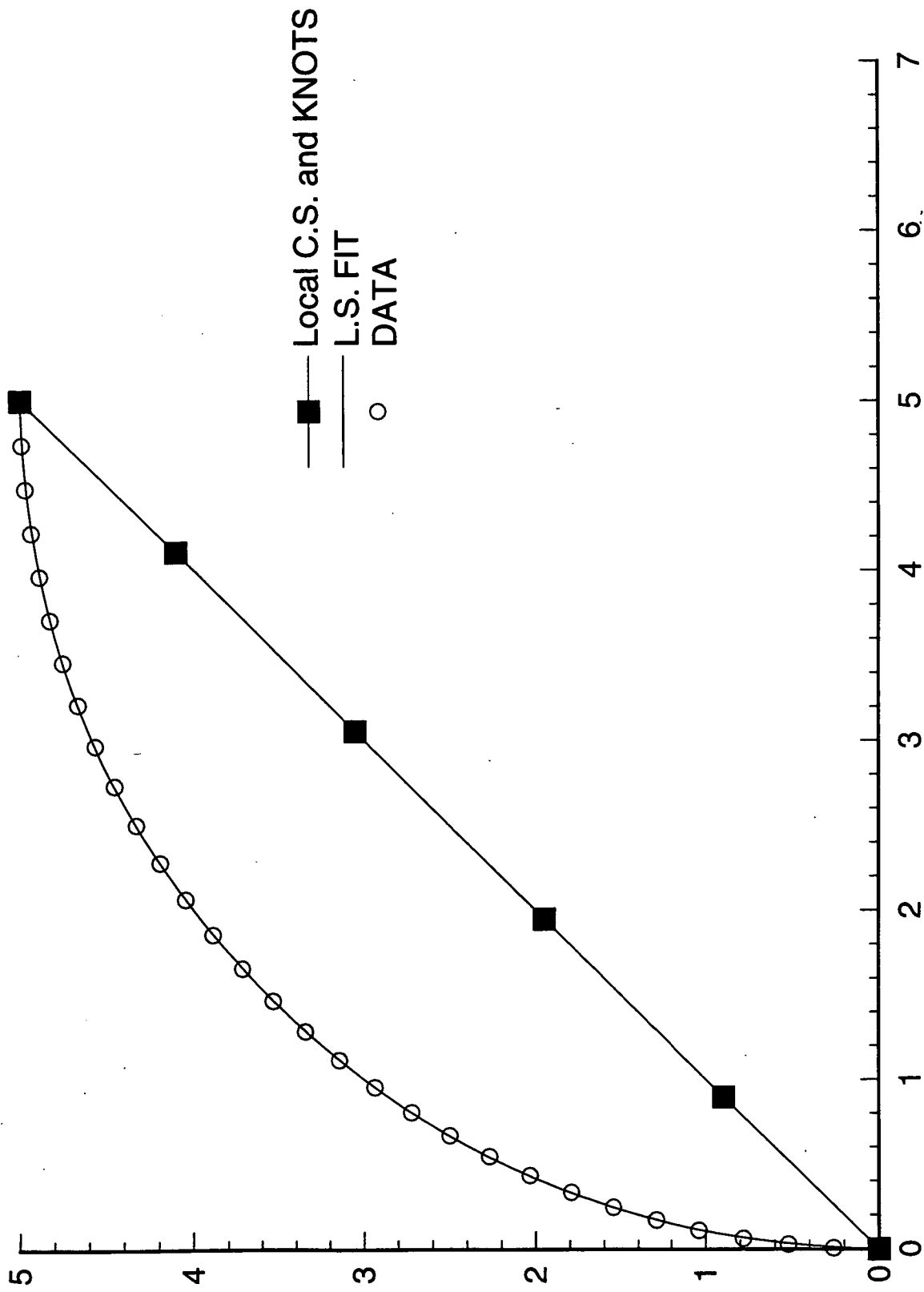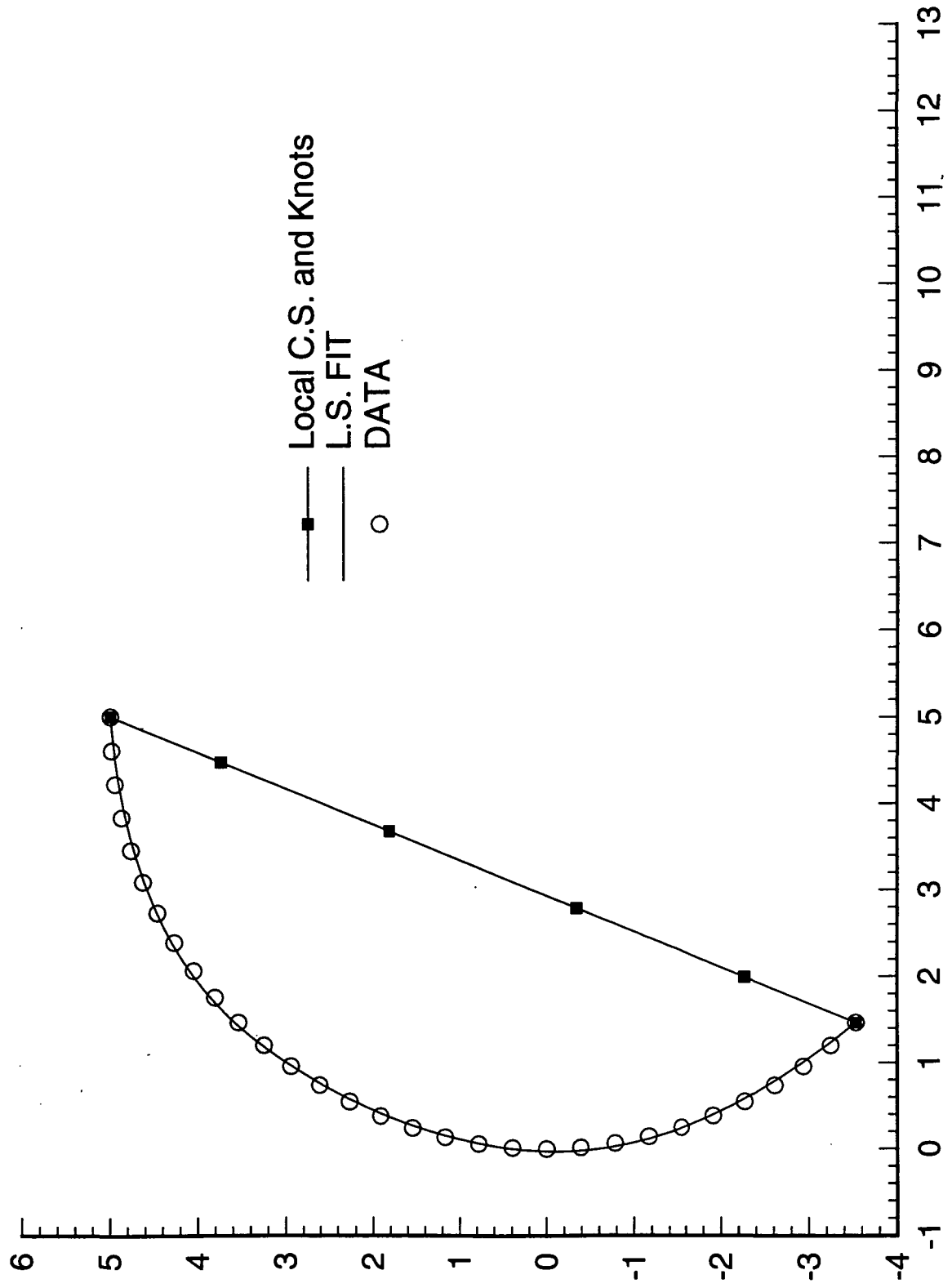90 DEGREE CIRCULAR ARC - RADIUS 5

Local C.S. and KNOTS
L.S. FIT
DATA

Figure 6: LOCAL COORDINATE SYSTEM REPRESENTATION
135 DEGREE CIRCULAR ARC - RADIUS 5

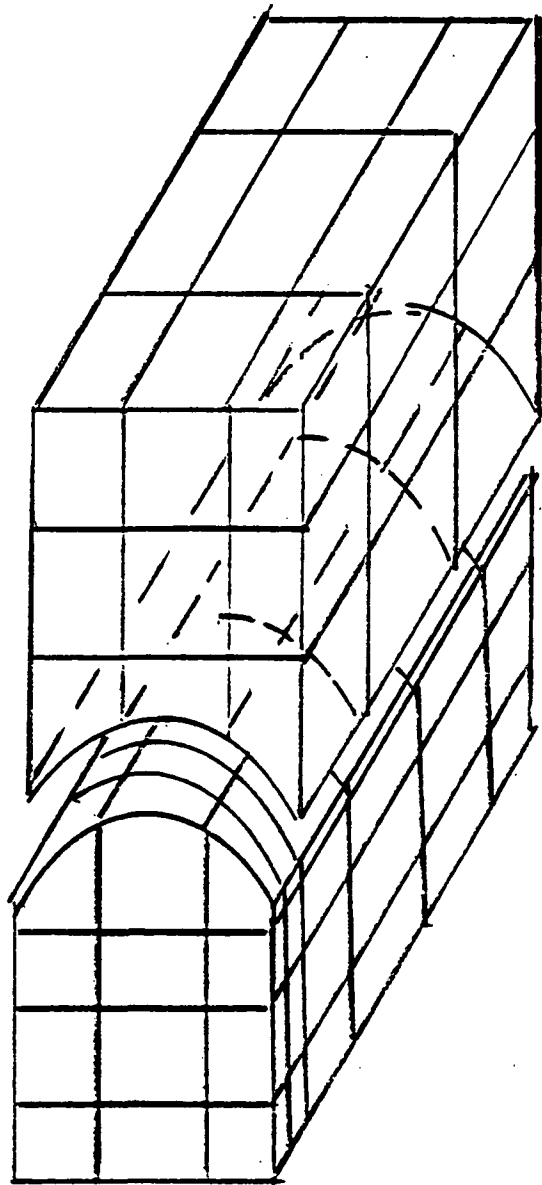Local C.S. and Knots
L.S. FIT
DATA

Duagram B: Curved 2–D Interface

A Two Dimensional Interface Element for Coupling of Independently Modeled Three Dimensional Finite Element Meshes and Extensions to Dynamic and Non-Linear Regimes

Background

A newly developed one dimensional interface element[1-4] was shown to be a successful method for coupling of independently modeled two dimensional (e.g., plate and shell) finite element meshes. The use of the interface element eliminates transition modeling and greatly simplifies the modeling efforts. As such this interface element may be used for effective modeling of aerospace structures by connecting independently modeled substructures as well as independently modeled global and local subdomains.

This interface element was based on the hybrid variational principles and was shown to be able to recover the constant and linear states of strain and curvature with no error (provided that the finite elements used in the analysis are capable of doing the same). The exact recovery of these states of strain and curvature is essential if one hopes to converge to the correct solution as the associated finite element meshes and the interface elements are refined. Subsequently these tests may be considered as patch tests for the interface element. The interface element based on the hybrid variational principle possesses the following four inherent properties. (1) The balance of energy is preserved across the interface (i.e. the constraint integrals introduced into the formulation do not contribute to the energy of the system). (2) Equilibrium is maintained across the interface (i.e., the sum of the forces at the interface is zero). (3) Displacement compatibility is preserved, in a variational sense, across the interface. (4) The forces at the interface are redistributed among various substructures in a variational sense that is consistent with the distribution of forces over the finite elements at the interface from each substructure.

The first two properties are satisfied by any method that produces a correct result on a global scale. The last two properties, however, are not satisfied by most methods and as a result a high level of noise is usually introduced into the solution details (such as stresses) particularly near the interface region. In these methods, the property (3) above, is satisfied only at discrete points and as a result the forces at the interface are redistributed among various substructures in a manner that is not consistent with the distribution of forces over the finite elements at the interface from each substructure. These forces, however, are in an overall equilibrium (i.e., property (2) above) but do not have the "correct" distribution. The hybrid variational formulation, on the other hand, provides the "correct" distribution of forces along the edge of each substructure at the interface. This is the key to the success of the hybrid variational formulation.

Also, the 1-D hybrid variational interface element formulation has recently been extended to develop an enhanced interface element that possesses cross surface capabilities[5]. While the original interface element is placed along substructure edges, this new enhanced interface element may be placed over a surface crossing the element sides. While the original element simplifies modeling and avoids transition modeling in substructuring and global/local analysis, this latter enhancement simplifies modeling and enables the user to perform component substructuring. That is, the components may first be modeled independently and then assembled together and not necessarily along their edges. However, this latter element, although successful, needs some more work before becoming fully operational.

The interface element may also be utilized to perform "substructure adaptive refinement". The major difficulty with the current adaptive refinement strategies is that they create distorted meshes which, in turn, deteriorate the performance of the underlying finite elements and create the so called mesh locking phenomenon. With the use of interface elements, however, it is natural to perform uniform refinement in each subdomain such that the underlying finite elements are not distorted beyond their original shapes and, hence, the performance of the finite elements are not deteriorated. This strategy may be referred to as a "adaptive-uniform refinement technique."

The major drawback of the interface element technology is that the final system matrix, although nonsingular, is an indefinite matrix. As such, a unique solution exists for the problem at hand; however, conventional positive definite fast solvers cannot be used for this method, and regular Gaussian elimination solvers are too slow and require a large amount of memory. There are two tentative remedies to this difficulty. 1) To develop a special sparse solver with full pivoting (single and double rows and columns) with out-of-core capability such as the one in the Boeing extended mathematical library and 2) to partition the system matrix and manipulate it to create smaller size positive definite matrices that can be solved using the available fast solvers.

Proposed Research

Since the one dimensional interface element has been a very successful method for coupling of shell structures, it is proposed herein to explore the possibilities of coupling of three dimensional finite element subdomains using a two dimensional interface element. This two dimensional interface element does not exist and its development requires research. Some type of surface spline has to be used for the development of this element in order to accurately match the surfaces of the independently modeled 3-D solids. Surface splines in conjunction with global/local analysis have already been used[6,7] and the technology is available to us with little effort. However, the performance of these surface splines in this kind of application has to be assessed and most likely the NURBS (Non-Uniform Rational

B-Splines) surface design methods will have to be formulated and programmed in the development of this interface element. However, this requires some research and it is not as straight forward as the surface splines that have been used in conjunction with the global/local analyses. The Lagrange multipliers interpolation used in the 1-D interface element also has to be extended to the 2-D version. This also requires some research regarding the type and order of approximations used to ensure numerical stability and rank sufficiency of the final system matrix. This element would be very useful for 3-D/3-D and 2-D/3-D substructuring and global/local analysis in conducting local detail stress analysis of primary structural components.

Also, since the enhanced version of the 1-D interface element for component substructuring has been successful, it is proposed herein that this concept to be also extended to 3-D structural applications which makes modeling of complicated structures very simple by independent 3-D component substructuring.

It is also proposed that the extension of the 1-D interface element to dynamic and nonlinear regimes be studied. These seem to be natural extensions for the successful 1-D interface element. Furthermore, it is proposed to study the extension of the 2-D interface element to dynamic and nonlinear regimes as well.

Finally, the issues discussed earlier in the background section regarding an efficient equation solver will be studied and the two possibilities discussed there will be explored to devise a satisfactory and workable solution method.

Once the formulation and implementation is complete, several test cases will be studied to ensure accuracy and correctness of the formulation. These test cases will include various boundary conditions and loadings.

Personnel and Resources
Dr. Aminpour will be the principal investigator. He will work with two of his graduate students to accomplish this research. NASA computing facilities will be used and some time will be spent at NASA LaRC during the period of performance.

Duration
The above research is proposed to be performed over a period of three years (October 1, 1994 through September 30, 1997) by the principal investigator on a part time basis during the academic year and 100% of his time during the summer and two graduate students at 50% of their time throughout the period of performance.

References
1. **M. A. Aminpour, J. B. Ransom, and S. L. McCleary,** "Coupled Analysis of Independently Modeled Finite Element Subdomains", the 33rd AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference, Dallas, TX, April 13-15, 1992, part 1, Structures I, pp. 109-120, 1992.

2. M. A. Aminpour, S. L. McCleary, J. B. Ransom, and J. M. Housner, "A Global/Local Analysis Method for Treating Details in Structural Design", ASME AMD-Vol. 157, Adaptive, Multilevel, and Hierarchical Computational Strategies, edited by A. K. Noor, pp. 119-137, 1992.

3. J. B. Ransom, S. L. McCleary, and M. A. Aminpour, "A New Interface Element for Connecting Independently Modeled Substructures", the 34th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, La Jolla, CA, April 19-22, AIAA/ASME Adaptive Structures Forum, Part 3, pp. 1693-1703, 1993.

4. M. A. Aminpour, T. Krishnamurthy, S. L. McCleary, and M. A. Baddourah, "Application of a new Interface Element to the Global/Local Analysis of a Boeing Composite Crown Panel", Fourth NASA/DoD Advanced Composites Technology Conference, Salt Lake City, UT, June 7-11, 1993, CA, NASA CP–3229, compiled by J. G. Davis, Jr., J. E. Gardner, and M. B. Dow, Volume I, Part 2, pp. 773-788, 1992.

5. C. G. Dávila and M. A. Aminpour, "An Interface Element for Coupling Built-up Structural Subdomains", AIAA paper 94-1336, to be presented at the 35th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference to be held at Hilton Head, SC, April 18-21, 1994.

6. J. B. Ransom, S. L. McCleary, M. A. Aminpour, and N. F. Knight, Jr., "Computational Methods for Multilevel Analysis of Composite Structures", NASA TM-107591, 1992.

7. J. B. Ransom and N. F. Knight, Jr., "Global/Local Stress Analysis of Composite Panels", Computers and Structures, Vol. 37, No. 4, pp. 375-395, 1990.

Consistent Load Calculations for 3-D Hybrid Solid Elements in COMET-AR and Investigation of 2-D and 3-D Discontinuous Finite Elements to Be Used with Interface Elements Supplemental Task No. 1 for NAG1-1670

Supplemental Task 1
Supplemental task 1 will focus on the following two topics.

(i) The formulation and implementation of consistent load calculations for the three-dimensional hybrid solid elements in COMET-AR. These load calculations are necessary to enable the use of the solid elements in conjunction with the 2-D interface element under development. This 2-D interface element, like it's 1-D counterpart, is based on the hybrid variational principle[1-3] and is used for connecting independently modeled three-dimensional substructures.

(ii) When connecting independently modeled substructures, the elements on the boundary of one substructure may cut across the faces of the elements on the boundary of other substructures creating a stress discontinuity in those elements. Current finite elements are not capable of dealing with these internal discontinuities. In references 4 and 5, two different methods were devised to deal with this situation for the 1-D interface element. Both methods resulted in satisfactory results. However, these methods are somewhat awkward to implement and the extension of these methods to the 2-D interface element is not very promising. It is proposed that a preliminary investigation into the formulation of discontinuous 2-D shell and 3-D solid elements be conducted. A formulation of a discontinuous 1-D beam element by the Principal Investigator has already been developed with very satisfactory results.

Personnel and Resources
Dr. Aminpour will be the Principal Investigator. He will work with one of his graduate students to accomplish this research. NASA computing facilities will be used and some time will be spent at NASA LaRC during the period of performance.

Duration
The above research is proposed to be performed over the period September 1, 1995 through January 11, 1996 by the principal investigator on a part time basis and one graduate student at 50% of his/her time throughout the period of performance.

References
1. M. A. Aminpour, J. B. Ransom, and S. L. McCleary, "Coupled Analysis of Independently Modeled Finite Element Subdomains", the 33rd AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference, Dallas, TX, April 13-15, 1992, Part 1, Structures I, pp. 109-120, 1992.

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4. C. G. Dávila and M. A. Aminpour, "An Interface Element for Coupling Built-up Structural Subdomains", the 35th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, Hilton Head, SC, April 18-21, 1994, Part 1, pp. 244-252, AIAA paper 94-1336.

5. J. M. Housner, M. A. Aminpour, C. G. Dávila, J. E. Schiermeier, W. J. Stroud, J. B. Ransom, and R. E. Gillian, "An Interface Element for Global/Local and Substructuring Analysis", MSC World Users' Conference Proceedings 95, Los Angeles, CA, May 8-12, 1995.