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Novel Composites for Wing and Fuselage Applications

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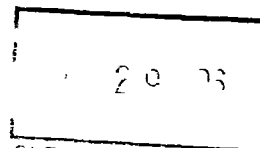*Northrop Grumman Corporation*
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FOREWORD

This Final Technical Report covers the work performed under Contract No. NAS1-18784 in Task 2 — Probabilsitic Analysis— from March 1991 to December 1993. The work was accomplished by the Northrop Grumman Corporation under the sponsorship of NASA Langley Research Center, Hampton, Virginia 23681-0001. Mr. H. Benson Dexter was the NASA LaRC Contracting Officer Technical Representative, and Dr. C. C. Chamis of NASA Le RC was the Technical Advisor.

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1 OBJECTIVES AND SUMMARY

1.1 PROGRAM OBJECTIVES

The overall objective of the Novel Composites for Wing and Fuselage Applications (NCWFA) program is to integrate innovative design concepts with cost-effective fabrication processes to develop damage tolerant structures that can perform at a design ultimate strain level of 6000 micro-inch/inch. The specific objectives are as follows:

1. Develop optimum wing design concepts that utilize high performance fiber architectures to achieve improved damage tolerance and durability, high notch strength, and increased out-of-plane load capability.

2. Develop integrally stiffened fuselage bulkhead concepts that minimize fabrication cost and eliminate skin/stiffener separation failure modes.

3. Explore textile processes such as automated weaving, knitting, and stitching to achieve affordable integral skin/stiffener structures.

4. Explore resin transfer molding processes and hybrid graphite/thermoplastic fiber forms for cost-effective fabrication of primary wing and fuselage structural components.

5. Conduct tests to validate structural performance and correlate test results with analytical predictions

6. Develop and verify probabilistic analysis methods for composite materials and structures.

1.1.1 Program Definition

This program is divided into the following six major tasks:

    Task 1 - Novel Wing Design Concepts
    Task 2 - Probabilistic Analysis

Task 3 - Cross-Stiffened Subcomponent

Task 4 - Design Guidelines/Analysis of Textile
Reinforced Composites

Task 5 - Integrally Woven Fuselage Panel - Common

Structural Test Component (CSTC)

Task 6 - Computational Methods TESTBED Validation.

1.1.2 Objectives of Task 2 — Probabilistic Analysis

THE NASA Lewis Research Center has been developing the IPACS (Integrated Probabilistic Assessment of Composite Structures) computer code for the probabilistic analysis of composite structures. Under the technical guidance of Dr. C. C. Chamis of NASA LeRC, Northrop Grumman's objectives for the probabilistic task consist of: (1) applying IPACS while it was in an evolving state to analyze the material and structural response of laminated composite structures, (2) validating IPACS by comparing its predictions with test results, (3) modifying IPACS to perform structural reliability analysis, and (4) making suggestions, from an industrial user's perspective, to the developers to help make IPACS more user-friendly, to improve its methodology, and to embody practical considerations.

1.2 SUMMARY

Probabilistic predictions based on the IPACS code are presented for the material and structural response of unnotched and notched, IM6/3501-6 Gr/Ep laminates. Comparisons of predicted and measured modulus and strength distributions are given for unnotched unidirectional, cross-ply and quasi-isotropic laminates. The predicted modulus distributions were found to correlate well with the test results for all three unnotched laminates. Correlations of strength distributions for the unnotched laminates are judged good for the unidirectional laminate and fair for the cross-ply laminate, whereas the

strength correlation for the quasi-isotropic laminate is judged poor because IPACS did not have a progressive failure capability at the time this work was performed. The report also presents probabilistic and structural reliability analysis predictions for the strain concentration factor (SCF) for an open-hole, quasi-isotropic laminate subjected to longitudinal tension. A special procedure was developed to adapt IPACS for the structural reliability analysis. The reliability results show the importance of identifying the most significant random variables upon which the SCF depends, and of having accurate scatter values for these variables. As part of the IPACS assessment effort, about 100 documented suggestions and comments, made from an industrial user's perspective, were transmitted to the developers to help make IPACS more user-friendly, to embody practical considerations, and to suggest improvements in the basic methodology.

It is noted that this report forms the basis of a paper entitled "Probabilistic & Structural Reliability Analysis of Laminated Composite Structures Based on the IPACS Code", which was presented at the 34th SDM Conference, held at LaJolla, California, April 18-21, 1994

2 INTRODUCTION

Traditional deterministic design methods do not directly account for uncertainties in the design variables (material properties, boundary conditions, etc.), which are treated as being single-valued. Uncertainties in the design variables are hidden under a blanket of factors-of-safety and, indirectly, so is the possibility of failure. However, real-world design variables are inherently random in nature, that is, each variable assumes a spectrum of values rather than being single-valued. Modern probabilistic design methods directly account for uncertainties in the design variables and their effects on the response variables (stress, etc.), which are also random. Furthermore, these methods recognize that there is a finite possibility of failure, p_f; indeed, design criteria can be established based on an acceptable prescribed value of p_f. Clear discussions of these basic probability notions are given by Haugen and Wirching (Ref. 1).

Probabilistic analysis methods are especially needed for composite materials which have more intrinsic variables than metals due to their heterogeneity, and are subjected to variability during manufacturing. Efficient probabilistic analysis of laminated composite structures may be performed using the IPACS (Integrated Probabilistic Assessment of Composite Structures) code (Ref. 2) probabilistic code recently developed by Sverdrup Technology, Inc. under a contract with NASA LeRC. Basically, IPACS synergistically combines the PICAN probabilistic code (Ref. 3) for material (point) response with the NESSUS probabilistic code (Ref. 4) for structural response (see Figure 1, adapted from Ref. 5 where this figure is described in detail). PICAN and NESSUS are based on ICAN (Ref. 6) and MHOST (Ref. 7), respectively, for performing deterministic analyses, and on FPI (Ref. 8) for rapidly conducting probabilistic analyses. The relationship between these various codes is depicted in Figure 2 .

The present probabilistic task, which was initiated when IPACS was still in an evolving state, involved assessing IPACS in addition to applying it to composite structures. The assessment effort, which loosely may be called "shaking out the code", resulted in much feedback and interaction between the industrial user and the developer, and constituted a significant portion of the total effort. About 100 documented suggestions and comments, made from an industrial user's perspective, were transmitted to the developers to help make IPACS more user-friendly, and to embody practical and theoretical computational considerations. We believe that this is a worthwhile way to proceed for assessing an evolving code that is nearing completion.

Typical results obtained from applying IPACS to predict material and structural responses are described in this report for IM6/3501-6 Gr/Ep unnotched and notched laminates. Specifically, the report compares IPACS predicted material response distributions with Northrop Grumman's test results for the unnotched specimens[*]. The report concludes with a discussion of structural reliability results for the notched laminate based on our adaptation of IPACS to perform reliability analysis.

2.1 The IPACS Code

The input to IPACS includes mean values, standard deviations and assumed distributions for the following material and structural design variables (equivalently called primitive variables or random variables herein): (1) 29 constituent (17 fiber and 12 matrix) properties, (2) 4 fabrication variables (ply thickness & orientation, fiber & void volume ratios), (3) geometry (coordinates of nodes), (4) boundary conditions (spring constants), (5) loads (pressures and nodal forces), and (6) environmental effects (nodal temperatures

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* Strictly speaking, these predicted distributions were obtained from PICAN before it was embedded in IPACS, however, for simplicity, we refer to all results presented here as being IPACS results

and moisture content). Table 1 lists the statistics of the input material design variables that are relevant to the analysis of the IM6/3501-6 Gr/Ep laminates considered here. All structural design variables were assumed to be deterministic.

Based on uncertainties in the design variables, IPACS computes means, standard deviations, probability density functions (PDF), cumulative distribution functions (CDF) and probabilistic sensitivity factors for (1) material properties at the ply and laminate levels and (2) structural responses (displacements, stresses, strains, buckling loads and frequencies) at the structures level. The distributions may be computed from three probabilistic methods: the primitive variable (PV) method, the hierarchy (HY) method, and the traditional Monte Carlo method. A primary difference between the PV and HY method is that the PV method uses the Fast Probability Integrator (FPI) for the probabilistic structural analysis at the structures level only, whereas the HY method uses FPI at the laminate level as well as at the structures level, as described more fully in the IPACS user's manual (Ref. 2). Analysis results in Figures 3 to 8 are based on the HY method, which was the only method available when those analyses were performed, and the remaining probabilistic figures are based on the PV method which, to us, appears to be more rigorous.

3 RESULTS AND DISCUSSION

IPACS was used to predict probabilistic tensile modulus and strength distributions for longitudinally-loaded coupon specimens that were tested at room temperature as part of the Gr/Ep Control Surfaces Program. Predicted and measured cumulative distribution functions (CDF)[**] are compared here for the longitudinal tension modulus (EC11) and strength (SCXXT) of $[0]_8$ unidirectional, $[0/90]_{4s}$ cross-ply, and $[0/45/90/-45]_s$ quasi-isotropic laminates. The strength predictions were obtained from two failure criteria, one based on Chamis' combined-stress criterion (Ref. 9) and the other on the maximum uniaxial stress criterion. IPACS does not have a progressive failure criterion. Analyses were performed using the following analysis options: (1) a *linear response surface option* (MVFO, see Ref. 10) in which each response variable (e.g., modulus) is assumed to vary linearly with the random variables, and which corresponds to retaining only linear terms in a Taylor series expansion of the response variable about a suitable expansion point. The expansion point is taken to be the mean value of the random variables. The n + 1 coefficients in the linear relationship, where n is the number of random variables, are obtained by performing n + 1 deterministic solutions, one solution is based on the mean values of the random variables, and n solutions correspond to perturbing each of the n random variables in turn about its mean value; (2) a *quadratic response surface option* (MVSO, see Ref. 10) in which incomplete quadratic terms (no coupling) are retained in a polynomial representation of the response variable. Following a procedure similar to that of the linear option, the coefficients of the constant, linear, and quadratic terms in the polynomial representation of the response function are obtained from a least squares solution; and (3) the conventional *Monte Carlo method*, which is "exact" (within the

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[**] The value of the CDF value corresponding to a specified value of EC11, for instance, represents the probability that EC11 will be less than or equal to that value.

framework of the deterministic equations used in IPACS) for a sufficiently large number of simulations, but which is considerably more costly than the other two options. Based on considerations of turn-around time and memory limitations, it was found that 1000 simulations presently represents the practical maximum number of simulations that could be handled on Grumman's Cray computer, although it is possible to run IPACS for a larger number of simulations. All Monte Carlo results for the unnotched laminates are based on 1000 simulations. Comparison of these results with those for a smaller number of simulations reveals that 1000 simulations provides essentially converged results, except possibly in the left tail.

It is noted that in the original version of PICAN, the deterministic material response module of IPACS (see Figure 2), was based on the linear MVFO method. Grumman has recommended that two additional methods be included in IPACS to improve the accuracy of the probabilistic predictions: the quadratic MVSO method, which has been included, and the advanced mean-value (AMV) method, described below, which the IPACS developers plan on implementing. Thus, at the time the current work was performed only the MVFO and MVSO methods were available. However, the accuracy of the CDF predictions obtained from the MVFO and MVSO methods deteriorates with probability levels away from the mean, especially for response functions that are highly nonlinear functions of the random variables. The accuracy of the MVFO or MVSO results at a specified probability level can be improved by performing a response function update (or "move") based on the "most probable point" (Ref. 10) values of the random variables found by either of the two mean value methods. This updating procedure is referred to as the advanced mean-value (AMV) method (see Ref. 10). It is remarked that the current version of IPACS no longer provides the MVFO linear analysis method.

3.1 CORRELATION OF ANALYTICAL & EXPERIMENTAL STIFFNESS & STRENGTH RESULTS FOR UNNOTCHED LAMINATES (MATERIAL OR POINT RESPONSE)

3.1.1 Unidirectional Laminate

As already mentioned, IPACS requires input of fiber constituent properties. However, due to the practical difficulty of testing individual fibers, it is more practical to begin at the level of unidirectional tape tests and "backfigure" equivalent fiber properties using the micromechanics equations upon which IPACS is based. For example, longitudinal fiber modulus, E_{f11}, was computed from the experimental mean for the corresponding ply modulus, E_{ply11}, according to the rule-of-mixture's equation

$$E_{f11} = (E_{ply11} - k_m E_m)/k_f$$

where k_m and k_f are matrix and fiber volume ratios, respectively, and E_m is Young's modulus for the matrix. This and similarly determined constituent properties were then used in the probabilistic analysis of a number of laminates. Because of the backfiguring, linearly predicted and measured mean values must agree for the unidirectional laminate, as is evident from an examination of Figures 3 and 4 for the CDF's for EC11 and SCXXT, respectively.

Figure 3 shows that the predicted CDF's for EC11 based on the linear, quadratic and Monte Carlo analysis options are close and agree well with the test results for the unidirectional laminate. Note that the scale employed in this and subsequent similar figures exaggerates percent differences. For instance, the analysis predictions at the 50% probability level differ by only 0.3%. Figure 4 compares predicted and experimental CDF's for SCXXT (both strength criteria provide identical CDF's for this laminate). The close agreement in slopes of the CDF's in the vicinity of the median infers that the PDF's

would also agree well in this vicinity. At a CDF of 10% in the lower tail, which corresponds to a probability of survival of 90% used in the determination of B-basis allowables, the predicted value of SCXXT is only 3% higher than the experimental value.

3.1.2   Cross-ply Laminate

As with the unidirectional laminate, the three analysis methods predict virtually identical CDF's for the tension modulus EC11, as may be seen from Figure 5. Close examination of the figure discloses that the shapes of analytical and experimental CDF's are similar for the lower half of the CDF. IPACS underestimates the experimental mean value of EC11 by 3%. Strength CDF's for the cross-ply laminate are compared in Figure 6. We first discuss the analytical results before comparing them with the test results. The three leftmost curves are linear, quadratic and Monte Carlo predictions for SCXXT based on the first-ply combined-stress failure criterion, whereas the three rightmost curves correspond to analysis results predicted by the maximum uniaxial stress criterion. For the former criterion, the quadratic CDF agrees well with the Monte Carlo CDF, with deviations occurring in the tails. The shapes of the three analytical curves obtained from the maximum uniaxial stress criterion are similar, with the linear and Monte Carlo mean values differing by 2.9%. Intuitively, one expects that failure of a sixteen-ply cross-ply laminate with an equal number of zero and ninety degree plies should occur at a strength value that is slightly larger than one-half the strength value of the corresponding eight-ply unidirectional laminate, as is confirmed by the IPACS results through comparison of the rightmost curves in Figure 6 for the cross-ply laminate with the corresponding curves in Figure 4 for the unidirectional laminate. This expectation is also confirmed in Figure 6 by results obtained from Grumman's deterministic progressive failure code, STRX (Ref. 11), which uses a modified Hill-von Mises combined-stress criterion. The symbol with the solid triangle on the right of the plot is STRX's progressive failure prediction, which is plotted at the 50%

probability level and which is seen to agree with the IPACS results. It is also observed that the first-ply failure prediction obtained from STRX (solid circle on the left of the figure) is in good agreement with the IPACS results based on the combined-stress criterion (note that STRX and IPACS use different combined-stress criteria). Based on this discussion, one would expect the experimental mean strength for the cross-ply laminate to be about one-half that for the corresponding unidirectional laminate. However, comparison of test results in Figure 6 and Figure 4 reveals that the cross-ply laminate strength is less than half that of the unidirectional laminate. This anomaly in the test data is under study.

3.1.3 Quasi-isotropic Laminate

From the modulus results presented in Figure 7 it is seen that analytical and experimental distributions for EC11 agree well with respect to shape, and have mean values that differ by only 5%. Comparisons of strength results are given in Figure 8, which is similar to the previously described Figure 6 and, hence, some of the comments pertaining to Figure 6 apply to Figure 8 as well. The Monte Carlo prediction based on the maximum uniaxial stress failure criterion differs by 3% from the linear analysis prediction. Also shown in the figure are STRX deterministic first ply and laminate failure predictions. STRX's first ply failure prediction is in agreement with IPACS's combined-stress predictions. The laminate failure strength predicted by STRX agrees well with the test results and is less than IPACS's maximum uniaxial strength prediction. It is to be recalled that IPACS does not have a progressive failure capability.

3.1.4 Probabilistic Sensitivity Factors

There are a number of sensitivity measures that can be used to "screen" or reduce the number of random (design) variables and attendant tests. For *deterministic* structural

analysis, the most commonly used measure is the structural sensitivity, $\partial Z/\partial X_1$, which gives the change in a structural response variable, Z, due to changes in the random variables, X_i. This concept can be extended to the case of *probabilistic* structural analysis by use of "*probabilistic sensitivity factors*", α_i, which depend on *both* the structural sensitivity and the uncertainties in the random variables, as characterized by their standard deviations. The absolute values of α_i range between 0 and 1. Large values of α_i identify the most important random variables that have the greatest influence on the uncertainty of the response, and infer that it might be beneficial to obtain improved statistical data or to tighten design tolerances for these important variables. Conversely, small values of α_i identify the least important random variables and, hence, accurate statistical data is not needed for such variables. It is also noted that random variables with low structural sensitivity (weak structural variables), but with large uncertainties (scatter), may have probabilistic sensitivity factors that are more important than those for strong variables with small scatter. Thus, the probabilistic sensitivity factors provide designers and analysts with valuable information for making design improvements and for establishing test requirements.

The probabilistic sensitivity factors are given by (see Refs. 12, 13 for details)

$$\alpha_1 = \left| \left(\frac{\partial Z}{\partial X_1} \right)_{\{X\}^*} \right| \sigma_1$$

in which the derivatives are evaluated at the "most probable point", $\{X\}^*$, described below, σ_1 is the standard deviation of the random variable X_i, and the α_i are normalized such that

$$\sum (\alpha_i)^2 = 1$$

It is worth mentioning that the probabilistic sensitivity factors are a natural by-product of second-moment probabilistic methods (Ref. 13) used to determine probability estimates for a specified value of the response variable. In the probability space of the reduced random variables, it can be shown that the α_i are the direction cosines (as implied by the above equation) of a minimum-distance vector (safety index) from the origin to a point on the joint probability density function corresponding to the limit state $Z = Z_0$, where Z_0 is the specified value of the response variable. This point closest to the origin is called the most probable point, or "design point", because it gives the values of the random variables that are the ones most likely to occur for the specified value Z_0 (see Ref. 12).

Figure 9 gives α_i for one of the material response variables, the longitudinal modulus, EC11, for a 24 ply, quasi-isotropic, IM6/3501-6 Gr/Ep laminate (this laminate has a different number of plies from the one considered previously). EC11 was assumed to vary with the following nine random variables with specified statistics (mean, standard deviation, distribution): fiber longitudinal and transverse moduli, fiber density and volume ratio, matrix Young's modulus and density, void volume ratio, ply orientation and thickness. Figure 9 shows that fiber longitudinal modulus and fiber volume ratio are the dominant random variables for EC11. It is worth noting that IPACS also determines α_i at the structural response level.

3.2 ANALYTICAL RESULTS FOR NOTCHED LAMINATES

3.2.1 Probabilistic Analysis of an Open-hole Specimen

The open-hole specimen considered has been proposed as an industry standard by the Composite Materials Characterization, Inc., a national consortium of which Grumman is a member. The 24 ply, $[\pm45/90/0]_{3s}$, quasi-isotropic specimen is 12.0" long between

tabs, 1.5" wide, and has a 1/4" hole located at 5" from one end. Deterministic results for the axial strain concentration factor (SCF) are given in Figure 10, which also displays the finite element model. The figure shows that a finer mesh is used in the region containing the *critical* point, a_0, where failure analysis is performed according to Grumman's failure analysis procedure, and a coarser mesh is used outside this region to reduce computer time. The location of the point a_0 is determined experimentally from longitudinal tension coupon tests on open- and filled-hole specimens including countersunk holes, and represents the distance over which the material must be critically stressed in order to find a sufficient flaw size to initiate failure (see Ref. 14). IPACS results for the axial strain concentration factor in the region of interest are in excellent agreement with predictions obtained from Grumman's boundary element method (BEM) code. It appears that discrepancies in the analytical predictions outside of this region are due to the relatively coarse mesh used in the IPACS finite element model. Also shown in the figure is Savin's closed-form solution, SCF = 3, for an infinite, quasi-isotropic plate. The IPACS solution for the finite width plate (width/diameter = 6) is slightly higher, as expected. IPACS probabilistic predictions for the axial strain concentration factor (SCF) are displayed in Figure 11. The two IPACS curves in the figure are CDF's determined from: (1) the primitive variable based method (PV) option in IPACS, which uses Wu's Fast Probability Integration (FPI) algorithm (Ref. 8), and (2) the conventional Monte Carlo method using 250 simulations. As mentioned earlier, the PV method is based on the simulation of uncertainties at the structural response level (i.e., the SCF) directly in terms of uncertainties in the fiber, matrix and fabrication related variables at the lowest (primitive) level. The nine primitive variables are fiber longitudinal and transverse moduli, fiber density and volume ratio, matrix Young's modulus, void volume ratio, and ply orientation and thickness. Each variable, such as matrix modulus, in each of the 24 plies is assumed to be fully correlated from ply-to-ply to reduce the total number of independent random variables and attendant run time and output. Examination of Figure 11 reveals that the CDF based on the PV method agrees well with

the Monte Carlo CDF. It is noted that the PV method requires 4(9)+1 = 37 "simulations" (deterministic finite element solutions) corresponding to perturbing each of the nine random variable by ± one and ± two standard deviations, in addition to performing the deterministic finite element solution at the mean. These finite elements solutions are needed to evaluate the coefficients in a quadratic representation of the strain concentration factor in terms of the primitive variables. In contrast, 250 Monte Carlo simulations were used to obtain a CDF that appears to be converged or nearly converged, as may be deduced through comparison of results, not shown, for 100 and 250 simulations. However, the PV method of analysis was found to be almost an order of magnitude faster than the Monte Carlo solution. Besides, the PV method provides the important probabilistic sensitivity factors (as does the HY method).

3.2.2 Structural Reliability Analysis of an Open-hole Specimen

The general objective of structural reliability analysis is to obtain the probability of failure given by

$$p_f = P[g(X) \leq 0] = P[(R(X) - S(X)) \leq 0]$$

In other words, the structure will be considered to have failed if its "resistance" or "strength", $R(X)$, is less than its "stress" or "applied load effect", $S(X)$. In the above equation, $g(X) = R(X) - S(X)$ is termed the response function or limit state function, and X is a vector of primitive random variables. For the specific case of the open-hole specimen considered here, the "stress", $S(X)$, is taken to be the axial strain, $\varepsilon_{x,ao}$, at the critical location, a_o, where failure analysis is performed, as described above. The "strength", $R(X)$, is the failure strain of an unnotched, unidirectional laminate made from the same Gr/Ep material as that of the open-hole specimen. This is the strain that is used at

a_0 according to our failure analysis method. These definitions of R, S and g are summarized in Table 2, which contains other pertinent information.

A procedure has been formulated and implemented that permits IPACS to be used to predict structural reliability for open-hole specimens under longitudinal tension loading. The procedure (described in detail in Appendix A) involves (1) modifying the IPACS code to create output files containing previously generated IPACS data, (2) post-processing this data outside of IPACS, and (3) executing FPI outside of IPACS to determine the probability of failure, p_f (or reliability = 1-p_f). This procedure was applied to a different open-hole specimen than the one considered previously. The 24 ply, $[\pm 45/90/0_2/\pm 45/0_2/\pm 45/0]_s$ open-hole specimen is 5.50" long between tabs, 1.00" wide, 0.127" thick, and has a 3/16" centrally placed hole. Three open-hole specimens with conventional holes were tested and the corresponding failure loads were recorded. The average of the three failure loads, denoted as P_{fail}, was applied to the IPACS model of the open-hole specimen. The following primitive variables were selected: fiber longitudinal, transverse and shear moduli, and fiber Poisson ratio; matrix Young's modulus and Poisson ratio; fiber and void volume ratios; and misalignment angle for each of the 24 plies. The fiber and matrix properties as well as the two volume ratios were assumed to be fully correlated in each of the 24 plies to reduce the total number of independent random variables and attendant run time and output. The 24 misalignment angles were assumed to be fully uncorrelated to reflect the actual laminate layup process.

As indicated by the last line in Table 2, the probability of failure, p_f, was computed for a number of different values of the dimensionless load levels, λ, where $\lambda = P/P_{fail}$. The results so obtained are displayed in Figure 12, in which the open-circle symbols define the load levels employed and in which normal probability axes are used to more clearly see the probability of failure predictions in the important left tail region. These results were

obtained using the Monte Carlo option in FPI. A number of spot-checks were also made using the Advanced First Order Reliability Method option of FPI, and good correlation was found in a comparison of values of p_f predicted by the two analysis methods. The average test failure load and a design allowable load are indicated on the figure. Following industrial folklore for the present case in which there aren't enough test results to obtain a B-basis allowable, we arbitrarily take the design allowable load to be 80% of the average test failure load. Assuming the structure is designed up to the design allowable load, which corresponds to a zero margin of safety, we see that the probability of failure is 20 times in a million. Thus, we see the payoff in performing probabilistic analyses: we obtain additional important information —the probability of failure— that analysts and designers can use to help assess the adequacy of a design. Alternatively, if a probability of failure level is specified according to a reliability design criterion, then Figure 12 can be used to determined the corresponding load level.

Figure 12 shows that p_f varies linearly with the dimensionless load level over the range of probability levels considered, thereby indicating a normal distribution[*]. Thus, it is straightforward to determine the following statistics from the figure: mean value μ = 10.92 kips, standard deviation σ = 0.585 kips, and coefficient of variation COV = 5.4%. These statistics are employed in Figure 13, which is a re-plot of Figure 12 using linear scales, to obtain the vertical bands that represent the mean and $\pm$ one-σ and $\pm$ two-σ deviations from the mean. The width of each one-σ band, expressed as a percent difference relative to the mean, is the COV = 5.4%. Thus, the figure reveals the encouraging result that the three test failure loads fall all within a one-σ band.

A brief parametric study was performed to determine the effect of scatter in an important random variable, fiber tension strength, S_{fT}, on the predicted probability of

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[*] Strictly speaking, this is a "pseudo-distribution" because the abscissa is deterministic

failure (see Figure 14). The two curves in the figure[+] correspond to two values of the coefficient of variation (COV) for S_{fT}: COV = 4.7%, also used in Figures 12 and 13, and COV = 0%, corresponding to a bounding solution in which S_{fT} is treated as being deterministic. Examination of Figure 14 shows the importance of identifying significant random variables (the previously discussed probabilistic sensitivity factors are especially useful in this respect), and of having accurate scatter values for these variables. For instance, arbitrarily taking the dimensionless load level λ =.95, which corresponds to the last computational point, we observe that treating S_{fT} as being deterministic would imply that the probability of failure is about one chance in one thousand, whereas more properly treating S_{fT} as being random results in failure occurring eight times in one hundred.

[+] Note that the two curves cross at P/P_{fail} = 1.026, and not at unity, because R_{mean}/S_{mean} = 1.026 (see Table 2)

4 CONCLUDING REMARKS & RECOMMENDATIONS

The major conclusions of this report are: (1) Good correlation was found in a comparison of predicted and measured longitudinal modulus distributions for the unnotched unidirectional, cross-ply and quasi-isotropic laminates, (2) Correlations of strength distributions for the unnotched laminates are judged good for the unidirectional laminate and fair for the cross-ply laminate, whereas the strength correlation for the quasi-isotropic laminate is judged poor because IPACS did not have a progressive failure capability at the time the present work was performed, (3) For the cross-ply and quasi-isotropic laminates, the linear response surface representation is accurate for the prediction of the modulus distributions, EC11, and inaccurate for the prediction of strength distributions, SCXXT, based on the combined-stress criterion. This is because SCXXT varies more nonlinearly with the random variables than does EC11. (4) It is very important to identify the most significant random variables upon which the response depends, and of having accurate scatter values for these variables, and (5) IPACS presently provides a powerful tool for the accurate and reasonably fast probabilistic analysis of laminated composite structures. However, for IPACS to realize its full potential, it is recommended that IPACS incorporate the following:[#]

- a progressive failure criterion

- a fully automated structural reliability capability for components

- a system reliability capability

- a "move" or " update" (AMV method, see Ref. 10) option, which is needed for accurate probabilistic and structural reliability predictions in the important tail regions

- an expanded element library (presently only one element is available)

[#] These recommendations were made during the time when this work was performed. Since then, some of the recommendations may have been incorporated into IPACS by the developers

• probabilistic postbuckling and geometric nonlinear analysis capabilities.

• an option of inputting data at the ply level instead of at the constituent level because, in many cases, reliable statistical data is only available at this level, and because it is very time consuming to obtain and interpret constituent properties obtained from vendors. It would then be necessary to provide a method for uncorrelating the various input ply properties, because FPI, the probabilistic analyzer module of IPACS, requires statistically independent input random variables.

Finally, It is also noted that because this work was initiated when IPACS was still in an evolving state, many documented suggestions and comments, made from an industrial user's perspective, were transmitted to the developers to help make IPACS more user-friendly, and to embody practical and theoretical computational considerations. This part of the validation process which, loosely, may be called "shaking out the code", resulted in much feedback and interaction between the industrial user and the developer, and constituted a significant portion of the total effort. We believe that this is a worthwhile way to proceed for validating an evolving code that is nearing completion.

APPENDIX A

PROCEDURE FOR ADAPTING IPACS TO PERFORM STRUCTURAL RELIABILITY ANALYSIS FOR OPEN-HOLE SPECIMENS

A.1 GENERAL STRUCTURAL RELIABILITY CONSIDERATIONS

We first use general reliability notation and concepts before specializing them to the open-hole coupon problem. Only component structural reliability involving a single mode of failure is considered. The component will be considered to have failed if its "resistance" or "strength", R, is less than its "stress", S (Ref. 15). The probability of "failure", p_f, can be expressed as

$$p_f = P[R(X) \leq S(Y)]$$
$$= P[R(X) - S(Y) \leq 0]$$
$$= P[g(X,Y) \leq 0]$$

where

$$g(X,Y) = R(X) - S(Y)$$

In these equations, P is the probability operator (P[E] is the probability of event E occurring); X and Y are vectors of random variables for R and S, respectively, with X and Y generally having some common variables; and $g(X,Y) = R(X) - S(Y)$ is termed the response function (or limit state function). General methods for solving these equations are described in Refs. 15 and 16 and elsewhere.

A.2 Structural Reliability Procedure for the Open-hole Specimen

We now illustrate the proposed procedure for the specific case of an open-hole specimen made from a multi-directional Gr/Ep laminate. For this case, the "stress", $S(Y)$, is taken to be the axial strain, $\varepsilon_{x,ao}$, at the critical location, a_0, where failure analysis is performed, as described in Section 3.3.1. The "strength", $R(X)$, is the allowable strain of an unnotched, unidirectional laminate made from the same Gr/Ep material as that of the open-hole specimen. This is the strain that is used at a_0 according to our failure analysis method.

The primitive variable (PV) method described in the IPACS's user's manual (Ref. 2) is well suited for the proposed structural reliability procedure, and only this method is considered henceforth. Then, under the simplifying assumption (which could be relaxed) that there are no random variables at the structural level (i.e., the only random variables are constituent properties and fabrication variables), it can be shown that R and S have the same random variables $\mathbf{Y} = \mathbf{X}$. The structural reliability procedure based on this assumption is described by the steps given below. The procedure uses the FPI (Fast Probability Integration) method, Refs. 8 and 16, which requires that the relationship between each response function ($R(X)$ or $S(X)$) with X be given by an explicit closed-form expression. Specifically, the closed-form expression is taken to be a quadratic polynomial using the response surface approach (Ref. 16).

•Step 1 Obtain Quadratic Response Surface Representation for $S(X)$

For a specified value of the applied load, P, IPACS is run to obtain the following (incomplete) quadratic response surface representation for $S(X)$, the axial strain, $\varepsilon_{x,ao}$, at the critical location, a_0.

$$S(X) = \varepsilon_{x,ao} = a_o + \sum_{i=1}^{n} a_i X_i + \sum_{i=1}^{n} b_i X_i^2$$

where n is the number of random variables. The 2n+1 coefficients (a_o, a_i, b_i) in this equation are obtained by performing 4n+1 deterministic solutions using MHOST, the finite element deterministic analyzer of IPACS: one solution is based on the mean values of the random variables $X = \{X_i\}$, and 4n solutions correspond to perturbing each of the n random variables four times (± 1 standard deviation, ± 2 standard deviations) in turn about its mean value. The coefficients of the constant, linear, and quadratic terms in the above polynomial representation are determined from a least squares solution for this over-described system.

•Step 2 Obtain Quadratic Response Surface Representation for SCXXT and EC11

For this problem, R(X) is the failure strain for a longitudinally loaded, unnotched, unidirectional laminate made from the same Gr/Ep material as that of the open-hole specimen. Now, this strain is not computed directly by IPACS. However, it may be determined from the ratio of the longitudinal tensile strength, SCXXT, and the corresponding modulus, EC11, both of which are computed by IPACS and both of which may therefore be expressed by

$$SCXXT = c_o + \sum_{i=1}^{n} c_i X_i + \sum_{i=1}^{n} d_i X_i^2$$

$$EC11 = e_o + \sum_{i=1}^{n} e_i X_i + \sum_{i=1}^{n} f_i X_i^2$$

The coefficients in these equations are obtained as described above for S(X) in Step 1, except that the deterministic analyses are performed by the ICAN module of IPACS and not by MHOST.

•Step 3 Determine the Limit State Function

$$g(X) = R(X) - S(X) = \frac{SCXXT}{EC11} - \varepsilon_{x,ao} =$$

$$= \frac{(c_0 + \sum_{i=1}^{n} c_i X_i + \sum_{i=1}^{n} d_i X_i^2)}{e_0 + \sum_{i=1}^{n} e_i X_i + \sum_{i=1}^{n} f_i X_i^2} - (a_0 + \sum_{i=1}^{n} a_i X_i + \sum_{i=1}^{n} b_i X_i^2)$$

•Step 4 Compute the Probability of Failure

Now that we have an explicit expression for the response function g(X) above, we use FPI to obtain the probability of failure; namely

$$p_f = P[g(X) \leq 0] = P[(R(X) - S(X)) \leq 0]$$

corresponding to the specified value of the applied load P.

The above steps can be repeated for different values of P to trace out the curve of p_f vs. P. Advantage can be taken of linearity to reduce the number of computations.

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TABLE 1 DESCRIPTION & STATISTICS OF INPUT RANDOM VARIABLES USED IN IPACS ANALYSES FOR THE MATERIAL RESPONSE OF IM6/3501-6 GR/EP LAMINATES

	VARIABLE	DESCRIPTION	MEAN μ	COEF. OF VARIATION COV (%)	STANDARD DEVIATION σ	DISTRIBUTION
IM6 FIBER PROPERTIES	Ef11	Modulus in fiber direction (Msi)	37.89	2.44	0.925	Normal
	Ef22	Modulus in transverse direction (Msi)	2.13	2.44	0.0459	Normal
	Gf12	In-plane Shear Modulus (Msi)	2.25	2.44	0.0325	Normal
	Gf23	Out-of-plane Shear Modulus (Msi)	0.85	2.44	0.0207	Normal
	v12	In-plane Poisson's Ratio	0.356	2.44	0.0087	Normal
	v23	Out-of-plane Poisson's Ratio	0.267	2.44	0.00651	Normal
	SfT	Longitudinal Tension Strength (ksi)	591	4.74	28.01	Weibull
	SfC	Longitudinal Compression Strength (ksi)	323	4.74	11.85	Weibull
	ρ	Weight Density (lb/in.**3)	0.0631	1.27	0.000802	Normal
	Nf	Number of Fibers Per End	12000	—	—	Deterministic
	df	Fiber Diameter (in.)	0.0002	5.00	0.00001	Normal
3501-6 MATRIX PROPERTIES	Em	Modulus (Msi)	0.62	3.42	0.0212	Normal
	Gm	Shear Modulus (Msi)	0.23	3.42	0.00786	Normal
	v	Poisson's Ratio	0.34	3.42	0.0116	Normal
	SmT	Tension Strength (ksi)	11.27	16	1.327	Weibull
	SmC	Compression Strength (ksi)	55.5	8	8.84	Weibull
	SmS	In-Plane Shear Strength (ksi)	23.29	3	2.96	Weibull
	ρ	Weight Density (lb/in.**3)	0.0457	1	0.000457	Normal
SUBPLY FABRICATION PROPERTIES	kf	Fiber Volume Ratio	0.61	2.46	0.0151	Normal
	kv	Void Volume Ratio	0.012	5	0.0006	Gamma
	$\Delta\theta$	Subply Misalignment Angle (degrees)	0	—	0.093	Normal
PLY FABRICATION PROPERTIES	$\Delta\theta$	Ply Misalignment Angle (degrees)	0	—	0.33	Normal
	tp	Nominal Ply thickness (in.)	0.0053	5	0.000265	Normal

SHADED VALUES ARE BASED ON CONSTITUTENT TESTS.

TABLE 2 STRUCTURAL RELIABILITY PREDICTION FOR IM6/3501-6 Gr/Ep OPEN-HOLE SPECIMEN: DEFINITIONS OF R, S, g

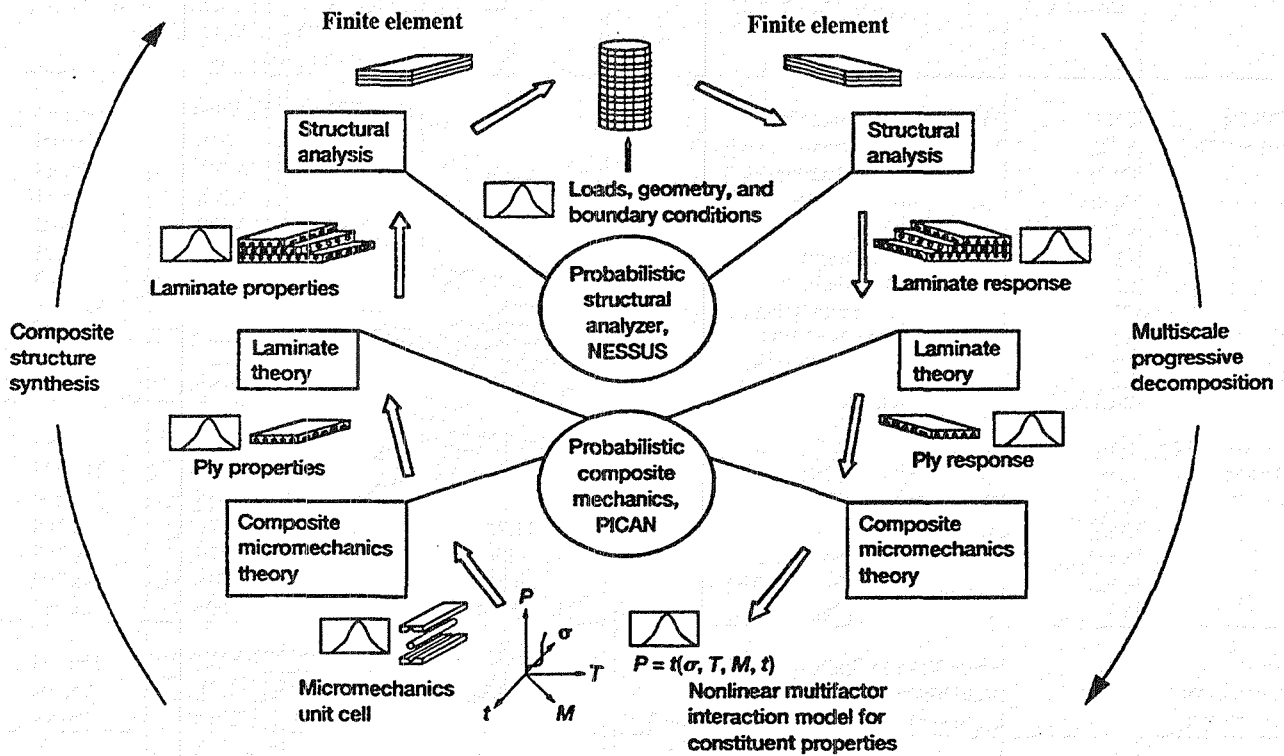| ITEM	R	S
STRUCTURE	$P \leftarrow$ [ ] $\rightarrow P$ 8-PLY UNIDIRECTIONAL LAMINATE	$P \leftarrow$ [a_0 O ε_{x,a_0}] $\rightarrow P$ 24-PLY MULTI-DIRECTIONAL LAMINATE
ANALYSIS METHOD	IPACS (MATERIAL RESPONSE)	IPACS (STRUCTURAL RESPONSE)
LOAD LEVEL, P	IRRELEVANT	P_{fail} = AVG. OF EXPERIMENTAL FAILURE LOADS
RESPONSE VARIABLES	SCXXT (LONGITUDINAL TENSION STRENGTH) EC11 (LONGITUDINAL MODULUS) $R = \varepsilon_{x,strength} = \dfrac{SCXXT}{EC11}$ $R_{mean} = 15500 \ \mu in./in.$	ε_{x,a_0} (AXIAL STRAIN AT POINT a_0) $S = (\varepsilon_{x,a_0})_{P=P_{fail}}$ $S_{mean} = 15100 \ \mu in./in.$ at $P = P_{fail}$
LIMIT STATE FUNCTION AT $P = P_{fail}$	$g(X) = R - S = \dfrac{SCXXT}{EC11} - (\varepsilon_{x,a_0})_{P=P_{fail}}$	
LIMIT STATE FUNCTION FOR ARBITRARY P	$g(X) = \dfrac{SCXXT}{EC11} - \lambda(\varepsilon_{x,a_0})_{P=P_{fail}}, \quad \lambda = P/P_{fail}$	
PROBABILITY OF FAILURE	$p_f = Prob\left[\dfrac{SCXXT}{EC11} - \lambda(\varepsilon_{x,a_0})_{P=P_{fail}} \leq 0\right]$, FOR DIFFERENT VALUES OF λ	

Finite element   **Finite element**

Structural analysis   Structural analysis

Loads, geometry, and boundary conditions

Laminate properties   Laminate response

Composite structure synthesis   Multiscale progressive decomposition

Probabilistic structural analyzer, NESSUS

Laminate theory   Laminate theory

Ply properties   Ply response

Probabilistic composite mechanics, PICAN

Composite micromechanics theory   Composite micromechanics theory

Micromechanics unit cell

P
σ
T
t
M

$P = t(\sigma, T, M, t)$

Nonlinear multifactor interaction model for constituent properties

FIGURE 1 IPACS SYNERGISTICALLY COMBINES THE PICAN PROBABILISTIC CODE FOR MATERIAL (POINT) RESPONSE WITH THE NESSUS PROBABILISTIC CODE FOR STRUCTURAL RESPONSE.

IPACS

MATERIAL RESPONSE   STRUCTURAL RESPONSE

PICAN   NESSUS

ICAN   FPI   FPI   MHOST

PROBABILISTIC INTEGRATOR

DETERMINISTIC ANALYZER
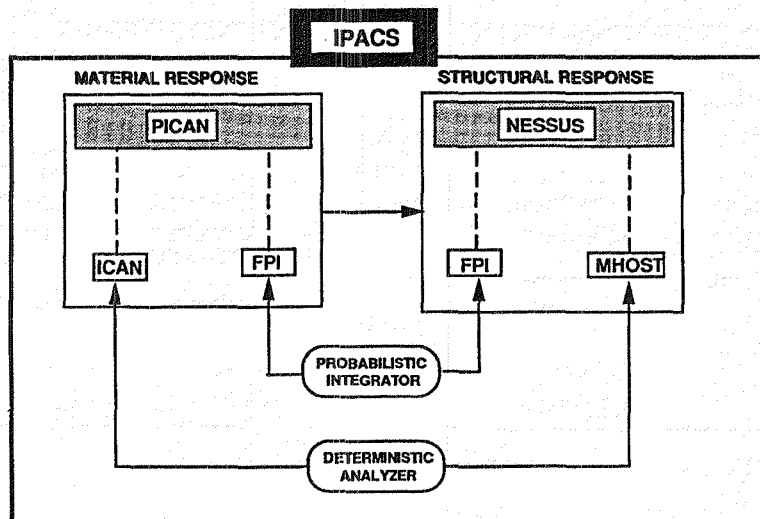
FIGURE 2 CODES UPON WHICH IPACS IS BASED
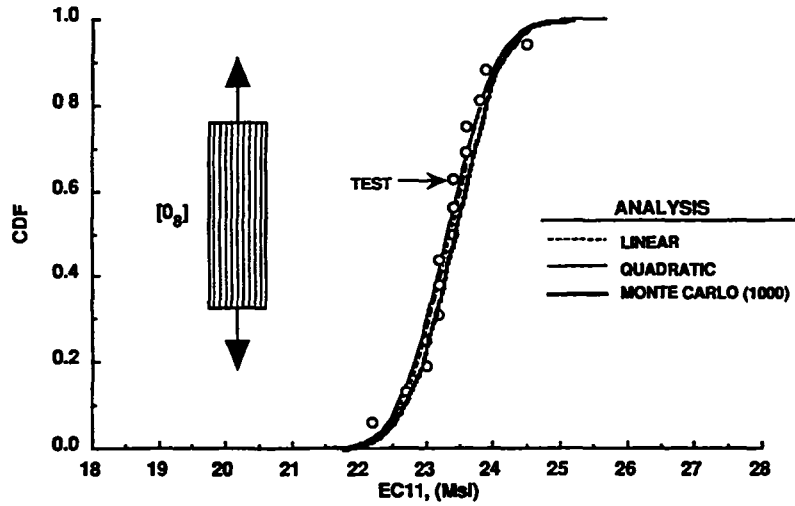
FIGURE 3   PROBABILITY DISTRIBUTION FOR LONGITUDINAL MODULUS OF
ELASTICITY OF UNIDIRECTIONAL IM6/3501-6 Gr/Ep TEST COUPONS.
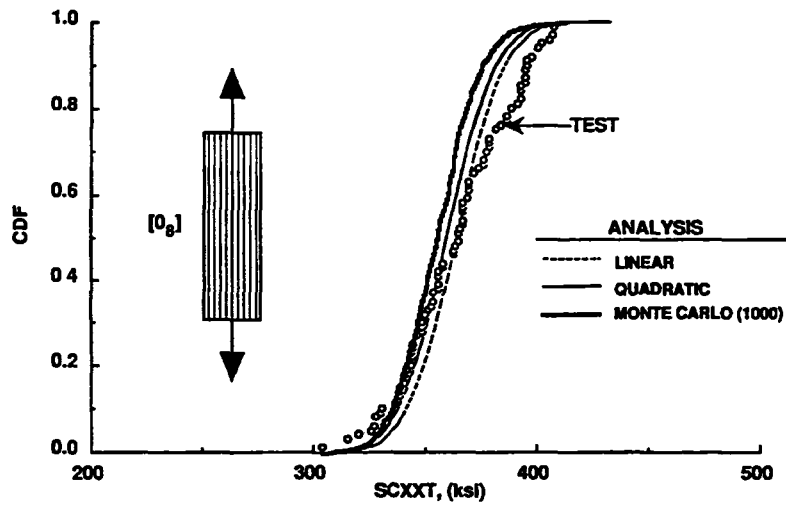
FIGURE 4   PROBABILITY DISTRIBUTION FOR LONGITUDINAL TENSION
STRENGTH OF UNIDIRECTIONAL IM6/3501-6 Gr/Ep TEST COUPONS.
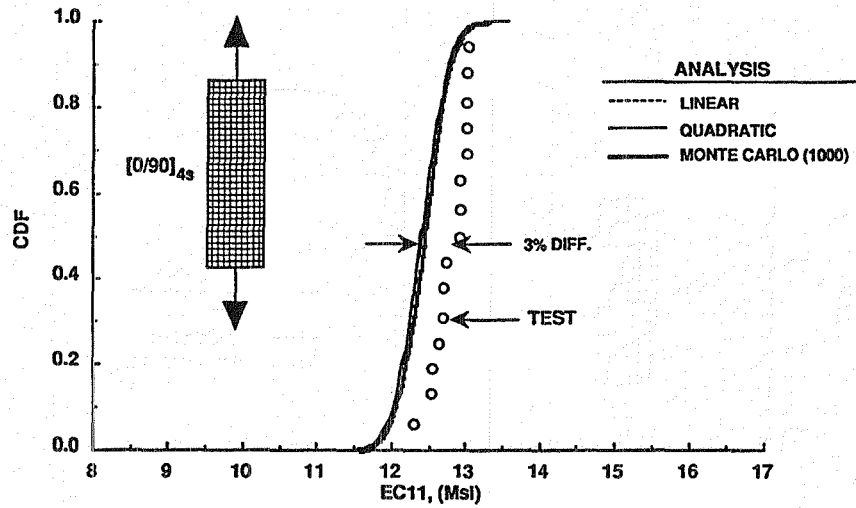
FIGURE 5   PROBABILITY DISTRIBUTION FOR LONGITUDINAL MODULUS OF ELASTICITY OF CROSS-PLY IM6/3501-6 Gr/Ep TEST COUPONS.
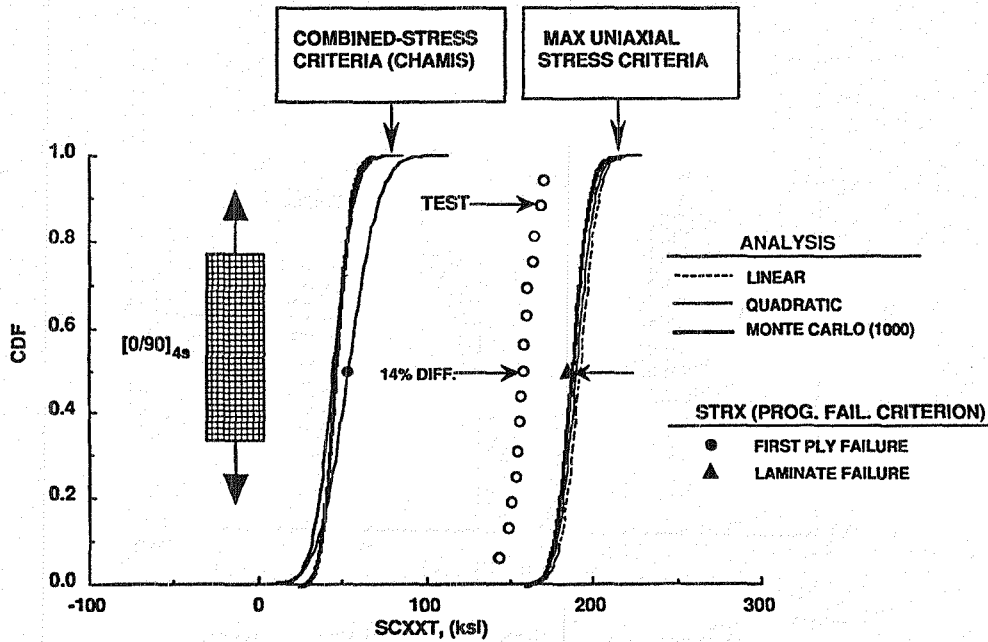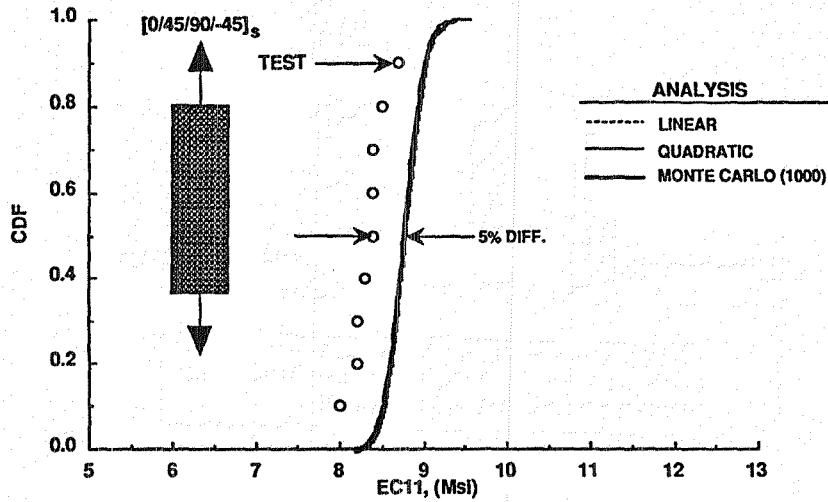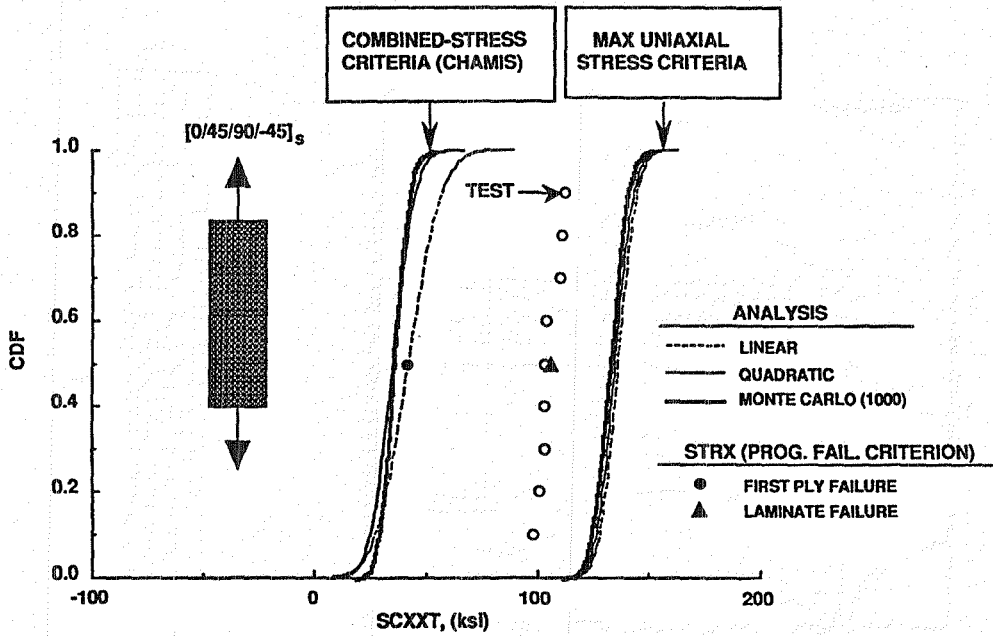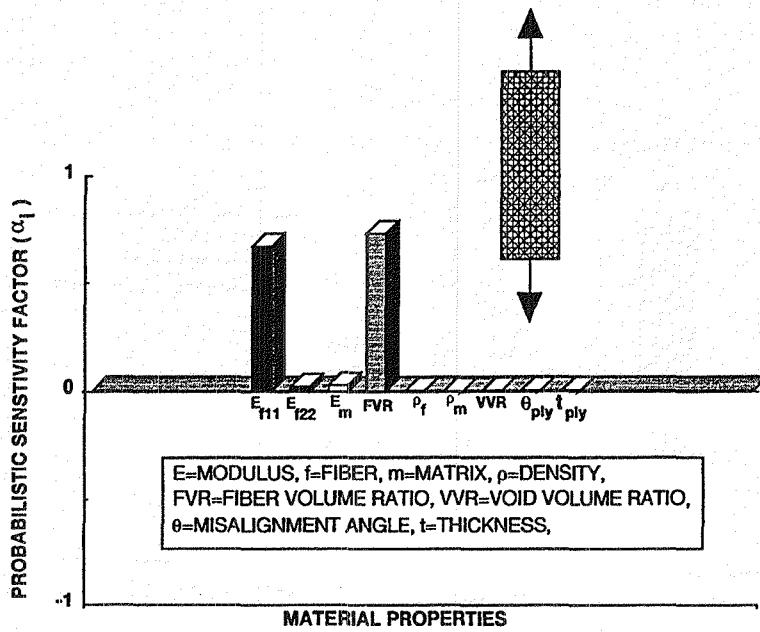

FIGURE 6   PROBABILITY DISTRIBUTION FOR LONGITUDINAL TENSION STRENGTH OF CROSS-PLY IM6/3501-6 Gr/Ep TEST COUPONS.

FIGURE 7   PROBABILITY DISTRIBUTION FOR LONGITUDINAL MODULUS OF
ELASTICITY OF [0/45/90/-45]$_s$, QUASI-ISOTROPIC, IM6/3501-6 Gr/Ep
TEST COUPONS.

FIGURE 8   PROBABILITY DISTRIBUTION FOR LONGITUDINAL TENSION
STRENGTH OF [0/45/90/-45]$_s$, QUASI-ISOTROPIC, IM6/3501-6 Gr/Ep
TEST COUPONS.

FIGURE 9 PROBABILISTIC SENSITIVITIES FOR LONGITUDINAL MODULUS OF ELASTICITY OF [45/-45/90/0]3s, IM6/3501-6 Gr/Ep TAPE COUPON SPECIMEN
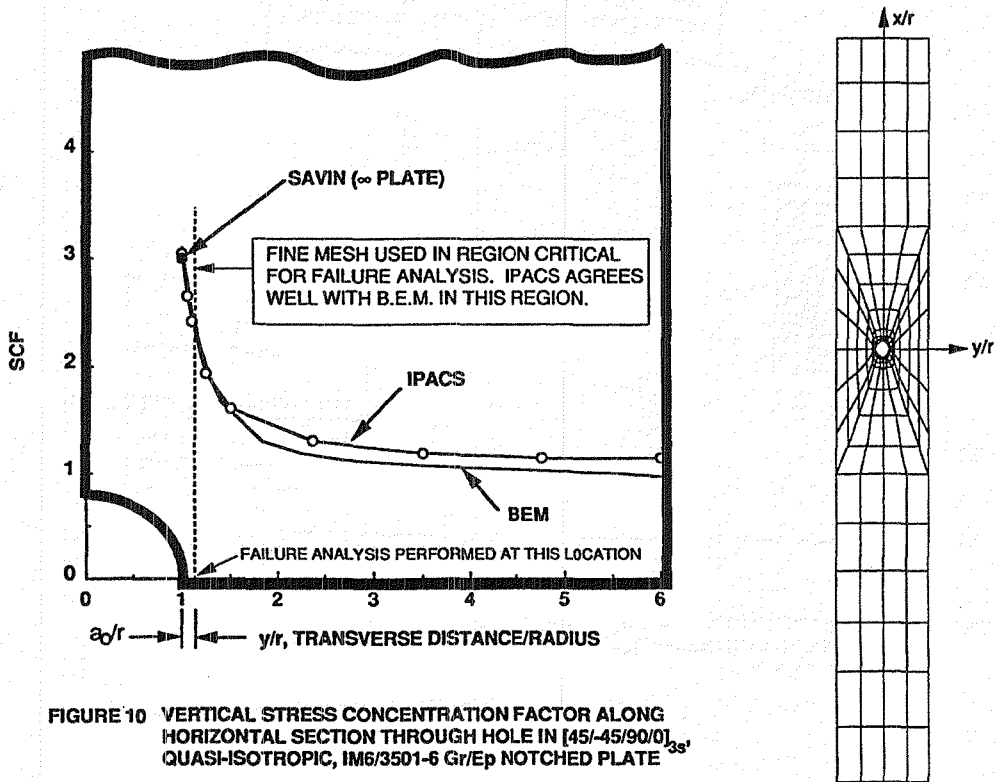

FIGURE 10 VERTICAL STRESS CONCENTRATION FACTOR ALONG HORIZONTAL SECTION THROUGH HOLE IN [45/-45/90/0]3s, QUASI-ISOTROPIC, IM6/3501-6 Gr/Ep NOTCHED PLATE
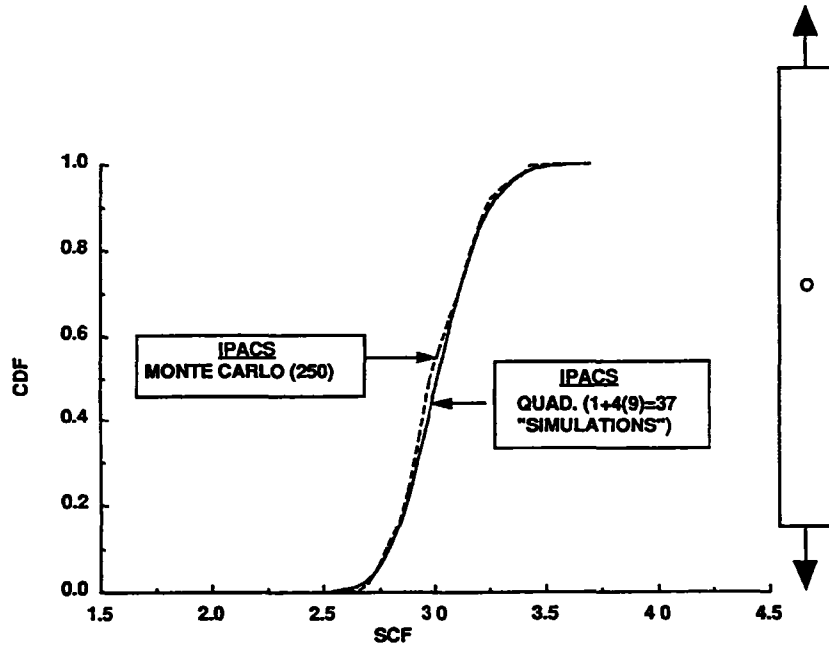
FIGURE 11   PROBABILITY DISTRIBUTION FOR VERTICAL STRAIN
             CONCENTRATION FACTOR AT EDGE OF HOLE FOR
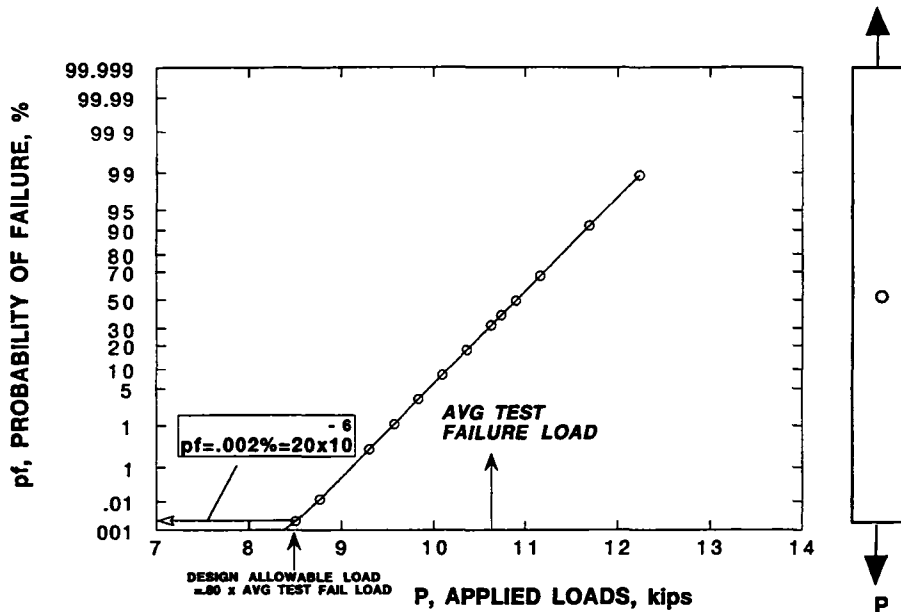   [±45/90/0]₃ₛ, IM6/3501-6 Gr/Ep TAPE COUPON SPECIMEN

FIGURE 12 PROBABILITY OF FAILURE PREDICTION FOR [±45, 90, 0 ,±45, 0 ,±45, 0]ₛ,
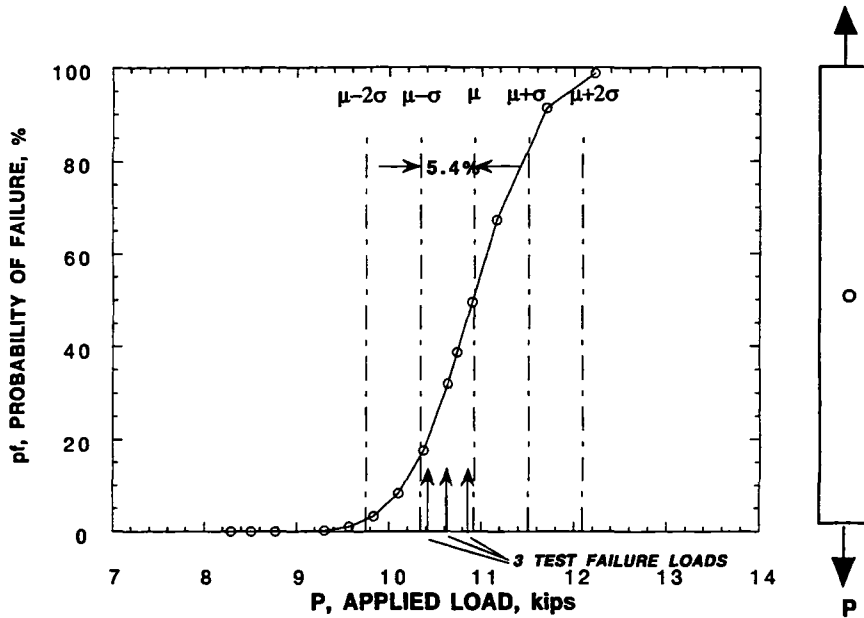   IM6/3501-6 Gr/Ep OPEN-HOLE SPECIMEN, PLOTTED ON NORMAL
           PROBABILITY AXES

FIGURE 13 PROBABILITY OF FAILURE PREDICTION FOR [±45, 90, 0 ,±45, 0 ,±45, 0]$_s$, IM6/3501-6 Gr/Ep OPEN-HOLE SPECIMEN, PLOTTED ON LINEAR AXES
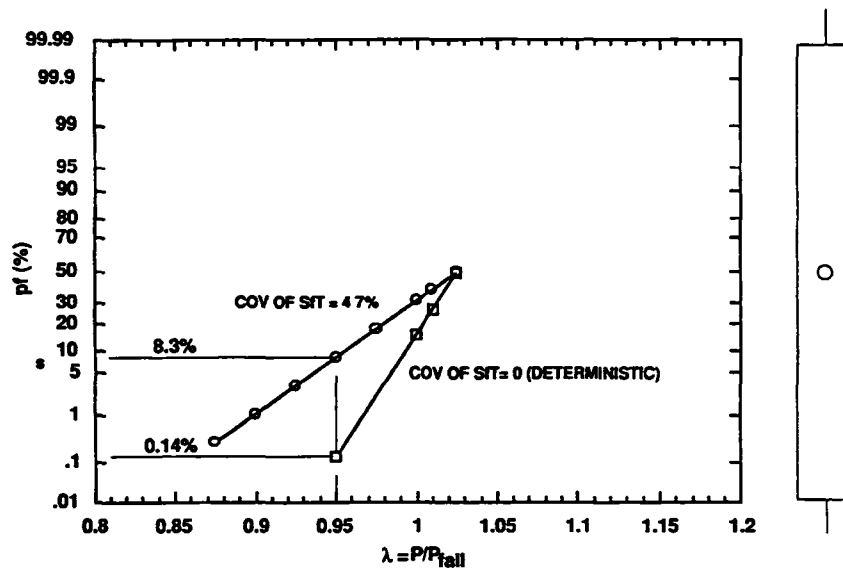

FIGURE 14 EFFECT OF SCATTER IN THE FIBER TENSION STRENGTH ON THE PROBABILITY OF FAILURE PREDICTION FOR [±45, 90, 0$_2$, ±45, 0$_2$,±45, 0]$_s$, IM6/3501-6 Gr/Ep OPEN-HOLE SPECIMEN, PLOTTED ON NORMAL PROBABILITY AXES

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13 ABSTRACT (Maximum 200 words)

Probabilistic predictions based on the IPACS code are presented for the material and structural response of unnotched and notched, IM6/3501–6 Gr/Ep laminates. Comparisons of predicted and measured modulus and strength distributions are given for unnotched unidirectional, cross-ply and quasi-isotropic laminates. The predicted modulus distributions were found to correlate well with the test results for all three unnotched laminates. Correlations of strength distributions for the unnotched laminates are judged good for the unidirectional laminate and fair for the cross-ply laminate, whereas the strength correlation for the quasi-isotropic laminate is judged poor because IPACS did not have a progressive failure capability at the time this work was performed The report also presents probabilistic and structural reliability analysis predictions for the strain concentration factor (SCF) for an open-hole, quasi-isotropic laminate subjected to longitudinal tension. A special procedure was developed to adapt IPACS for the structural reliability analysis. The reliability results show the importance of identifying the most significant random variables upon which the SCF depends, and of having accurate scatter values for these variables.

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