HIGHER–ORDER SQUEEZING
IN A BOSON COUPLED TWO–MODE SYSTEM

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Abstract

We consider a model for nondegenerate cavity fields interacting through an intervening Boson field. The quantum correlations introduced in this manner are manifest through their higher–order correlation functions where a type of squeezed state is identified.

1 Introduction

Squeezed state generation of electromagnetic fields provides a means of reducing uncertainty in one electric field quadrature at the expense of a larger uncertainty in its conjugate partner [1, 2]. It is one realization of nonclassical states (ideally, minimum uncertainty states) that has received wide attention. Ordinarily, in single or multi–mode squeezing, the fluctuations of linear combinations of the field operators are considered [1]; however, Hillery [3] introduced quadratic combinations of the field operators as a type of higher–order squeezing [4]. The higher–order combinations are examined to help elucidate the nature of the phase space occupied by the squeezed states.

We consider a two–mode model originally developed to study stimulated Raman scattering [5, 6]. In a cavity environment the model has features of amplifiers [7, 8] in which quantum states are rendered macroscopic and therefore, classically measurable, while at the same time the fields retain some quantum mechanical correlations. The introduction of both Stokes and anti–Stokes fields indirectly coupled through a Boson field, whose origin stems either from phonons or weak atomic excitation of the medium, is an interesting two–mode quantum system. It differs from several previous two–mode systems, eg. [1, 8, 9], because the two modes are coupled through the intermediate field that acts like a reservoir.

The emphasis of this paper is placed on higher–order squeezing found in the fields because squeezing of the linear combinations of the operators is not present in this model. A more complete discussion of the results can be found in [10]. The type of higher–order squeezing found is in the variance of the variables defined by Hillery, so–called sum or difference squeezing variables; they are used to infer that quantum correlations exist between the electromagnetic fields and the Boson fields.

2 Model

We investigate the model Hamiltonian for a stimulated Raman scattering process with undepleted laser field e_L, which can be treated classically. The fields in the interaction are the Stokes field, subscript S, and anti–Stokes field, subscript A, that are coupled through a Boson field with multiple modes [5, 6]:

$$\mathcal{H} = \hbar\omega_S a_S^\dagger a_S + \hbar\omega_A a_A^\dagger a_A + \sum_l \hbar\omega_{Bl} a_{Bl}^\dagger a_{Bl} - \sum_l (\hbar g_l e_L a_S^\dagger a_{Bl}^\dagger + \hbar\kappa^* e_L a_A^\dagger a_{Bl} + \text{h.c.}). \tag{1}$$

This model has a bath of Bosons, eg., phonons that have excitation energies spread over a range of frequencies. In this model the Bosons are responsible for coupling the electromagnetic fields and for introducing damping, as well.

In order to calculate various moments we determine the characteristic function of the operators in normal–ordered form. The normal characteristic function after reducing the intermediate reservoir in the dynamical equations is expressed as an average over an initial distribution of complex amplitudes $\{\xi_S, \xi_A\}$, which is the coherent-state representation for the initial field operators,

$$C_N(\beta_S, \beta_A, t) = \left\langle e^{-B_S(t)|\beta_S|^2 - B_A(t)|\beta_A|^2 + [D_{SA}(t)\beta_S^*\beta_A^* + c.c.] + [\beta_S\xi_S^*(t) + \beta_A\xi_A^*(t) - c.c.]} \right\rangle, \tag{2}$$

where we assume that the detuning parameter $\Delta = \omega_L - (\omega_S + \omega_A)/2$ is equal to zero and define

$$\xi_S(t) = u_S(t)\xi_S + v_S(t)\xi_A^*, \qquad \xi_A(t) = u_A(t)\xi_A + v_A(t)\xi_S^*.$$

The angular brackets denotes the average over the initial states of the Stokes and the anti–Stokes fields. The coefficients in the above expressions are obtained from solution of the Heisenberg equations of motion and the subsequent reduction of the Boson modes in the normal characteristic function using disentangling theorems. Letting $\Gamma = (\gamma_S - \gamma_A)|E_L|^2/2$, where the laser field is $E_L = e_L \exp(i\omega_L t)$ and the parameters

$$\gamma_S = 2\pi|g(\omega_B)|^2 \rho(\omega_B), \qquad \gamma_A = 2\pi|\kappa(\omega_B)|^2 \rho(\omega_B),$$

introduced from the Markoff approximation with the Boson excitation frequency $\omega_B = \omega_L - \omega_S$, the results are

$$
\begin{aligned}
u_S(t) &= \frac{1}{\gamma_S - \gamma_A}(\gamma_S e^{\Gamma t} - \gamma_A); \qquad u_A(t) = \frac{1}{\gamma_S - \gamma_A}(\gamma_S - \gamma_A e^{\Gamma t}); \\
v_S(t) &= -v_A(t) = \frac{\sqrt{\gamma_S \gamma_A}}{\gamma_S - \gamma_A}(e^{\Gamma t} - 1)e^{i(2\phi_L + \psi_S - \psi_A)}; \\
B_S(t) &= \frac{1}{(\gamma_S - \gamma_A)^2}\left(\gamma_S^2(e^{2\Gamma t} - 1) + 2\gamma_S\gamma_A(1 - e^{\Gamma t})\right) + \frac{\gamma_A \bar{n}_V}{\gamma_S - \gamma_A}(e^{2\Gamma t} - 1); \\
B_A(t) &= \frac{\gamma_S\gamma_A}{(\gamma_S - \gamma_A)^2}(e^{\Gamma t} - 1)^2 + \frac{\gamma_A \bar{n}_V}{\gamma_S - \gamma_A}(1 - e^{2\Gamma t}); \\
D_{SA}(t) &= \frac{\sqrt{\gamma_S\gamma_A}}{\gamma_S - \gamma_A}\left(\frac{1}{\gamma_S - \gamma_A}(e^{\Gamma t} - 1)(\gamma_A - \gamma_S e^{\Gamma t}) + \bar{n}_V(e^{2\Gamma t} - 1)\right)e^{i(2\phi_L + \psi_S - \psi_A)}.
\end{aligned}
\tag{3}
$$

The phases are defined by $E_L = |E_L|\exp(i\phi_L), g = |g|\exp(i\psi_S)$ and $\kappa = |\kappa|\exp(i\psi_A)$.

The usual definition of the two–mode operators is a linear combination of the creation and annihilation operators. However, we find that the model discussed here does not yield the usual squeezed state correlations between the Stokes and anti–Stokes fields. The coupling through the reservoir is also expected to degrade the coherence developed between the Stokes and the anti–Stokes fields during evolution. It is, therefore, surprizing that the fields do display quantum coherences in the higher–order correlations between the fields. To show this we adopt of the definitions of sum squeezing and difference squeezing used by Hillery [3].

2.1 Sum Squeezing

For sum squeezing we define the operators

$$V_1 = \frac{1}{2}(A_S^\dagger A_A^\dagger + A_S A_A), \qquad V_2 = \frac{i}{2}(A_S^\dagger A_A^\dagger - A_S A_A). \tag{4}$$

The product of their standard deviations, ΔV_i, satisfies the Heisenberg inequality

$$\Delta V_1 \Delta V_2 \geq \frac{1}{4}\langle N_A + N_S + 1\rangle. \tag{5}$$

The operators are in a quantum state, said to be sum squeezed in the V_1 direction when the variance of V_1 satisfies the inequality

$$(\Delta V_1)^2 < \frac{1}{4}\langle N_A + N_S + 1\rangle. \tag{6}$$

To determine whether the dynamics produces a higher–order squeezed state, we define the shifted variance

$$\delta V_1^2 = (\Delta V_1)^2 - \frac{1}{4}\langle N_A + N_S + 1\rangle; \tag{7}$$

which is negative in the region of the quantum state.

The moments of these operators are calculated by using the characteristic function and the result for the sum squeezing shifted variance of V_1 is

$$
\begin{aligned}
\delta V_1^2 &= \frac{1}{4}\Big\langle \big[(\xi_S(t)\xi_A(t))^2 + 4D_{SA}(t)\xi_S(t)\xi_A(t) + 2(D_{SA})^2 + 2D_{SA}^*\xi_S(t)\xi_A(t) + \text{c.c.}\big] \\
&\quad + 2\big[|\xi_S(t)\xi_A(t)|^2 + B_S(t)|\xi_A(t)|^2 + B_A(t)|\xi_S(t)|^2 + |D_{SA}(t)|^2 + B_S(t)B_A(t)\big]\Big\rangle \\
&\quad - \frac{1}{4}\langle \xi_S(t)\xi_A(t) + D_{SA}(t) + \text{c.c.}\rangle^2.
\end{aligned}
\tag{8}
$$

2.2 Difference Squeezing

For the definition of difference squeezing, define

$$W_1 = \frac{1}{2}(A_S A_A^\dagger + A_S^\dagger A_A), \qquad W_2 = \frac{i}{2}(A_S A_A^\dagger - A_S^\dagger A_A). \tag{9}$$

The state is difference squeezed in the W_1 operator when the variance of the operator satisfies the inequality ($\langle N_S\rangle > \langle N_A\rangle$)

$$(\Delta W_1)^2 < \frac{1}{4}\langle N_S - N_A\rangle. \tag{10}$$

The moments are calculated from the characteristic function, as discussed already in the previous subsection. We also define a shifted variance of W_1 in analogy with Eq. (7)

$$\delta W_1^2 = (\Delta W_1)^2 - \frac{1}{4} \langle N_A - N_S \rangle \tag{11}$$

which is negative when the state is squeezed along the W_1 direction. For the difference squeezing variable W_1 we have the following expression

$$\begin{aligned}
\delta W_1^2 &= \frac{1}{4} \Big\langle \Big[(\xi_S(t)\xi_A^*(t))^2 + 2D_{SA}^*(t)\xi_S(t)\xi_A(t) + \text{c.c.} \Big] + 2 \Big[|\xi_S(t)\xi_A(t)|^2 + B_S(t)|\xi_A(t)|^2 \\
&\quad + B_A(t)|\xi_S(t)|^2 + |D_{SA}(t)|^2 + B_S(t)B_A(t) + |\xi_A(t)|^2 + B_A(t) \Big] \Big\rangle - \frac{1}{4} \langle \xi_S(t)\xi_A^*(t) + \text{c.c.} \rangle^2 .
\end{aligned} \tag{12}$$

3 Results

There are several parameters occuring in the model and appearing in Section 2. The dynamical parameters, i.e. those appearing in the evolution equations have been previously defined. We note that the detuning is assumed to be small in our model and this parameter is set to zero. The initial states of the fields represent another set of important parameters. The choice of an initial state for the Stokes and anti–Stokes fields is dictated by experimental conditions. We restrict our discussion to combinations of two experimentally useful initial states: the coherent state and the chaotic state. Using one of the choices, we examine the quantum correlations developed betwwen the electromagnetic fields; of course, other situations, such as, a Fock state or a squeezed vacuum state could also be identified. The Boson field is considered to be in a chaotic state with an average number of excitations $\bar{n}_B$; when the Stokes and/or anti–Stokes fields are in a chaotic state, then their phases are randomized and their statistical properties are also represented by their average photon number $\bar{n}_S$ and $\bar{n}_A$, resp. When the Stokes and anti–Stokes fields are in coherent states, in addition to the average photon number, the phase of the fields, ϕ_S and ϕ_A, is also needed.

The plot of Figure 1 is a display of the shifted variance of the operator V_1 versus the interaction time t for the three different values of the phase $\phi = 2\phi_L - \psi_S - \psi_A$. The Stokes and anti–Stokes fields are both initially in a coherent state, $n_S = n_A = 2$, and the reservoir is in the vacuum state $\bar{n}_V = 0$. The time has been scaled to the product, $\gamma|E_L|^2$, where E_L is the laser field amplitude and in the results presented here we set $\gamma = \gamma_S = \gamma_A$, i.e. the damping constants are equal. The region of the curves with negative ordinate values corresponds to the case when light is V_1–sum squeezed. The phase value of $\phi = \pi/2$ continues to decrease as the interaction time increases which means that for large times squeezing occurs near the point $\phi = \pi/2$. As the average number of excitations is increased in the Boson reservoir, the region for squeezing deteriorates.

When both the Stokes and the anti–Stokes fields are initially in a chaotic state, the sum squeezing variable V_1 still shows squeezing and the phase $\phi = \pi/2$ is very robust to the values of the initial state (Figure 2). We note that the initial value of the shifted variance has been changed by the initial chaotic state of the variables.

No squeezing was found for the variable W_1, either with coherent or chaotic initial states.
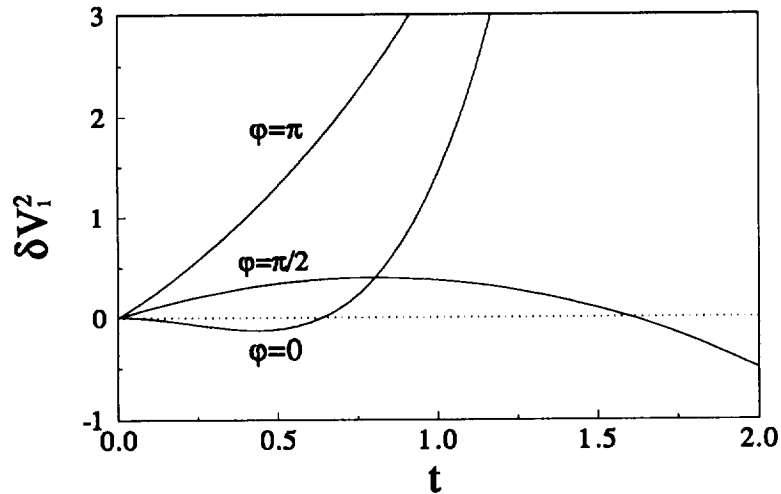
Figure 1: Plot of the sum squeezing shifted variance versus the interaction time for initially coherent Stokes and anti–Stokes fields. The phase $\phi = 2\phi_L + \psi_S - \psi_A$ has the values 0, $\pi/2$ and π.

4 Summary

In this paper we have examined a special model for the interaction between two modes in a cavity mediated by a Boson reservoir field [5, 6]. We find sum squeezing, a form of higher–order squeezing, over a range of interaction times and initial states. There are two salient features of our results; first, the intermediate field has a continuous spectrum of a reservoir, but still the two fields develop quantum mechanical correlations; and second, the quantum nature of the correlations is not manifest through the usual first order or even simple higher–order correlations among the operators, but through special combinations of the field operators.

There are other models where the fields are mediated by either electronic or acoustic fields, eg. a polariton or Brillouin scattering model [5, 6, 11]; these processes are analogous to the present model where the directly coupled fields are not detected in an experiment. In such cases experiments designed to measure higher–order correlations can reveal the underlying quantum correlations induced through the fields.

Acknowledgments

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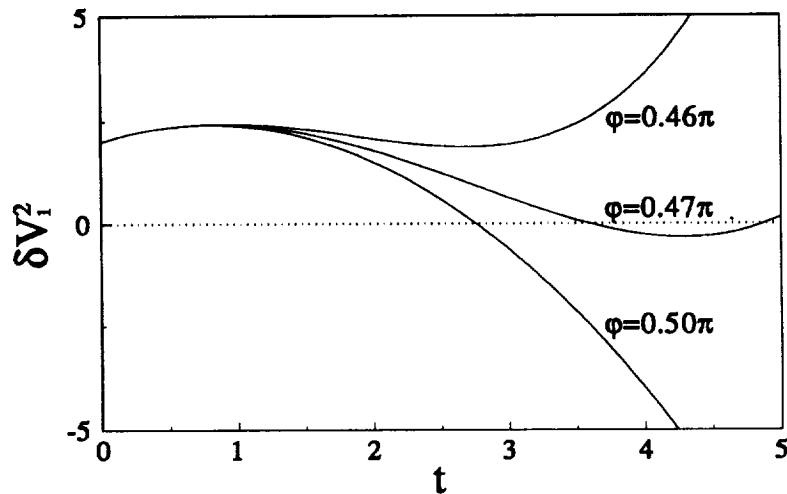[2] K. Zaheer and S. Zubairy, Adv. in Atomic, Molecular and Optical Physics **28**, 143 (1989).

Figure 2: The sum squeezing shifted variance versus time for initially chaotic Stokes and anti–Stokes fields.

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