MULTI-PARTICLE INTERFEROMETRY BASED ON DOUBLE ENTANGLED STATES

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Abstract

A method for producing a 4-photon entangled state based on the use of two independent pair sources is discussed. Of particular interest is that each of the pair sources produces a two-photon state which is simultaneously entangled in both polarization and space-time variables. Performing certain measurements which exploit this double entanglement provides an opportunity for verifying the recent demonstration of nonlocality by Greenberger, Horne, and Zeilinger.

1 Introduction

The incompatibility of quantum mechanics with some of the intuitive concepts found in the premises of the Einstein-Podolsky-Rosen argument (EPR) [1] has recently been shown through the remarkable demonstration of nonlocality by Greenberger, Horne, and Zeilinger (GHZ) [2]. Whereas traditional Bell-type [3] arguments have been based on the statistical correlations of two entangled particles, the GHZ theorem relies only on the perfect correlations of three or more particles to exhibit a blatant contradiction in the premises of EPR. The beauty and simplicity of this theorem has provided strong motivation for the experimental construction of a multi-particle entangled state.

In this paper we present a method for constructing a 4-photon state entangled in space-time variables. As has been shown in the pioneering work of Yurke and Stoler [4] [5], EPR effects can arise even when the particles do not originate from one central decaying source. Yet rather than basing our system on four independent single particle sources, we will use two independent pair sources.

The use of two independent pair sources in two-photon correlation experiments has been discussed by Zukowski, Zeilinger, Horne, and Ekert [6], and by Pavicic and Summhammer [7]. Here, however, the interesting feature of the pair sources is that they each produce two-photon states which are simultaneously entangled in both polarization *and* space-time variables. This double entanglement is used to overcome a basic problem in 4-photon experiments which is introduced through the simple example of the double Franson-interferometer [8]. Throughout the paper simplified models in which the states evolve along the optical paths are used to highlight the importance of the double entanglement.

2 The Double Franson-Interferometer Example

Perhaps the easiest way to envision constructing a 4-photon space-time entangled state from two pair sources would be to use two standard Franson-interferometer setups, and a 4-fold coincidence counting scheme such as that shown in Figure 1. Here, two type-I down-conversion crystals, X and X', are coherently pumped by the same continuous wave (CW) laser source. The signal and

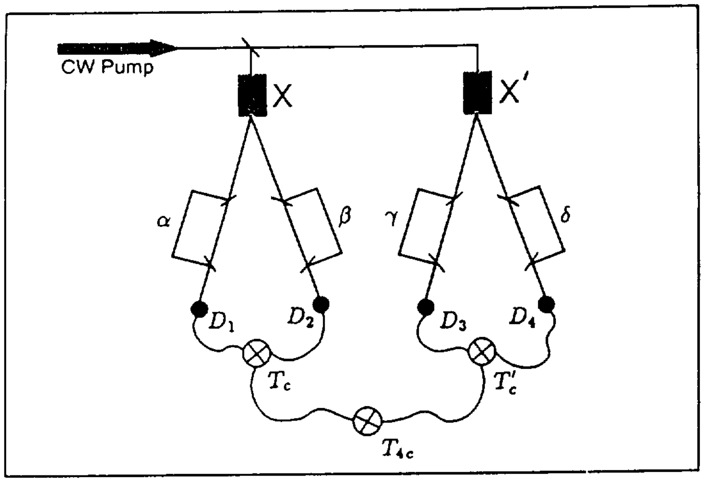

FIG.1. The double Franson-interferometer simply consists of two standard Franson-Interferometer setups coherently pumped by a single laser.

idler photons of each down-converted pair can follow either a long or short path to their respective detectors. For simplicity α, β, γ, and δ, which are the phase delays between the long and short paths in the four arms, can be taken to be equal.

Considering the single Franson-interferometer setup following crystal X, one would first perform the standard procedure of making the two-photon coincidence circuit time window, T_c, much shorter than the time delays associated with α and β. By doing this the two-photon probability amplitudes corresponding to one photon of a down-converted pair taking the short path while the other takes the long path can not contribute to the two-photon coincidence counting rate, and the two-photon space-time entangled state originating from crystal X is: $|\psi_2\rangle \sim |S\rangle_1|S\rangle_2 + e^{i(\alpha+\beta)}|L\rangle_1|L\rangle_2$, where $|S\rangle_1$ denotes the photon taking the short path to detector 1, and so on. By reducing T_c' a similar state is realized in the other two-photon setup : $|\psi_{2'}\rangle \sim |S'\rangle_3|S'\rangle_4 + e^{i(\gamma+\delta)}|L'\rangle_3|L'\rangle_4$, where the primes always indicate origination in crystal X'.

Therefore, the 4-photon quantum state realized by combining the outputs of these two-photon coincidence circuits into a 4-photon coincidence circuit would be the product:

$$\begin{aligned}
|\psi_4\rangle &\sim \left(|S\rangle_1|S\rangle_2 + e^{i(\alpha+\beta)}|L\rangle_1|L\rangle_2\right) \otimes \left(|S'\rangle_3|S'\rangle_4 + e^{i(\gamma+\delta)}|L'\rangle_3|L'\rangle_4\right) \\
&= |S\rangle_1|S\rangle_2|S'\rangle_3|S'\rangle_4 + e^{i(\gamma+\delta)}|S\rangle_1|S\rangle_2|L'\rangle_3|L'\rangle_4 + \\
&\quad e^{i(\alpha+\beta)}|L\rangle_1|L\rangle_2|S'\rangle_3|S'\rangle_4 + e^{i(\alpha+\beta+\gamma+\delta)}|L\rangle_1|L\rangle_2|L'\rangle_3|L'\rangle_4. \qquad (1)
\end{aligned}$$

This, however, is not a 4-photon space-time entangled state because of the two middle terms, describing the amplitudes where some photons followed the short paths while others followed the long paths. What is desired is the ellimination of these two terms so that the 4-fold coincidence counting rate shows the signature interference due to the two indistinguishable processes in which all 4 photons take their short paths, or all 4 take their long paths.

At first glance, one might be tempted to try and achieve this ellimination by reducing the 4-fold coincidence time window, T_{4c}, (as had been done in each of the two-photon coincidence circuits) to "cut-off" these unwanted terms. However, this will not work, as can be seen through the following simplistic indistinguishability argument: Consider the case when a down conversion pair is "born" in crystal X at some time t and each of the photons follows its long path en route to detectors 1 and 2. Then at some later time $t+\tau$ a pair is born in cyrstal X' but these photons follow their short paths to detectors 3 and 4. Well, in the extremely unfortunate circumstance that τ is exactly equal to the time delay between the short and long paths, all 4 detectors will fire simultaneously. Thus, even though in principle T_{4c} can be made extremely small, the 4-fold coincidence count resulting from this type of $|L\rangle_1|L\rangle_2|S'\rangle_3|S'\rangle_4$ amplitude is indistinguishable from a $|L\rangle_1|L\rangle_2|L'\rangle_3|L'\rangle_4$ or $|S\rangle_1|S\rangle_2|S'\rangle_3|S'\rangle_4$ amplitude when the two pairs were born at exactly the same time. In other words, simple attempts at space-time based projective measurements will not result in a space-time entangled state.

However, these two unwanted middle terms can be elliminated in a similar setup where each of the two-photon states is entangled in both polarization and space-time variables. As will be seen, the final projective measurements can be based on polarization, leaving a 4-photon state entangled in space-time varibles.

3 The Two-Photon Double Entangled State

Since we will solve the above problem by constructing our 4-photon entangled state from two double entangled two-photon states, we will briefly review their interesting features. The two-photon state which is simultaneously entangled with respect to both polarization and space-time variables has been observed [9], and even used to demonstrate two different types of violations of Bell's inequalities in a single experimental setup [10].

One way to construct such a state is shown in the cartoon schematic of Figure 2a. Consider a down-conversion crystal, X, cut at a type-II phase matching angle [11] which produces pairs of orthogonally polarized signal (parallel to the e-ray plane of crystal X) and idler (parallel to the o-ray plane of crystal X) photons that travel collinearly in the same direction as the pump. Ideally, the crystal should be thin enough so that its birefringence does not impart any significant temporal phase lag between the two down-converted photons, although in practice a thick crystal may be used followed by a compensation device [12] [13].

At this point the state can be roughly described by polarization kets: $|\psi\rangle \sim |\mathbf{o}\rangle \otimes |\mathbf{e}\rangle$. After filtering out the pump beam, the down-converted photons pass through a thin birefringent crystal, BC, whose fast and slow axes are aligned at $\pm 45^o$ to the signal and idler polarizations (see Figure 2b). Thus, upon encountering BC, the state emerging from the crystal evolves as:

$$|\mathbf{o}\rangle \otimes |\mathbf{e}\rangle \longrightarrow \frac{1}{2}\left(|\mathbf{F}\rangle + |\mathbf{S}\rangle\right) \otimes \left(-|\mathbf{F}\rangle + |\mathbf{S}\rangle\right) = -\frac{1}{2}\left(|\mathbf{F}\rangle|\mathbf{F}\rangle - |\mathbf{S}\rangle|\mathbf{S}\rangle\right) \tag{2}$$

where $|\mathbf{F}\rangle$ and $|\mathbf{S}\rangle$ describe photons polarized along the fast and slow axes of BC.

In order for a coincidence detection to occur, the photon pair must be split by 50/50 beam splitter BS. In each of the output ports of BS are delay units, which could be variable thickness birefringent material or even Pockel's cells, that are oriented with their fast and slow axes parallel

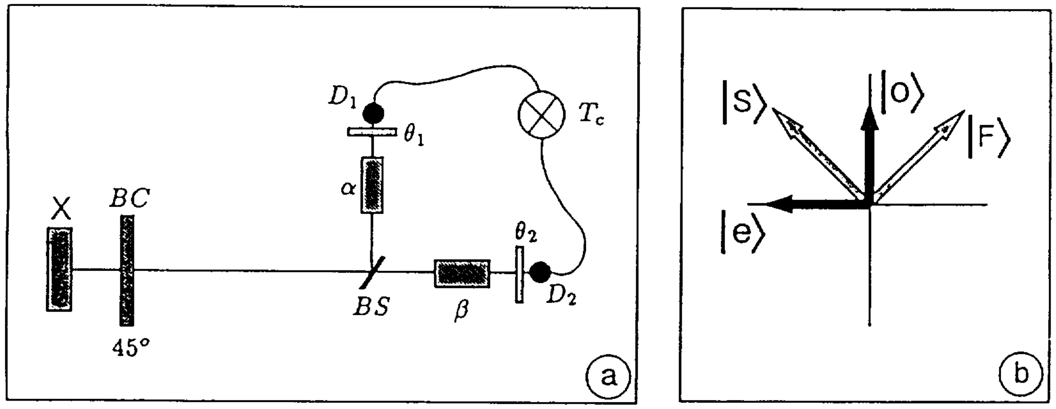

FIG.2. a) A cartoon schematic of the experiment which can realize the two-photon double entangled state. b) The polarization orientations of the signal (e-ray) and idler (o-ray) photons, and the fast and slow axes of BC.

to those of BC. In this way we can impart variable space-time phase delays, α and β, between the fast and slow "paths" leading to each detector. Behind each delay unit is a polarization analyzer (θ_1 and θ_2) and a detector.

It is easy to see that after BS and the delay units,

$$|\psi\rangle \longrightarrow \left(|\mathbf{F}\rangle_1|\mathbf{F}\rangle_2 - e^{i(\alpha+\beta)}|\mathbf{S}\rangle_1|\mathbf{S}\rangle_2\right) \tag{3}$$

State 3 is the double entangled state. Note that as we vary the phase delay $\alpha + \beta$ we see a space-time interference between the two indistinguishable amplitudes in which both photons followed their fast paths and both photons followed their slow paths, in exact analogy to the standard Franson-interferometer. Furthermore, if we go to a space-time coincidence counting rate minimum or maximum (e.g. $\alpha + \beta = 0, \pi$) we may rotate the analyzers θ_1 and θ_2 and see a polarization interference in analogy to that seen in some of the earlier tests of Bell's inequalties. It should be emphasized that there was no need of a short coincidence time window to see this effects.

4 The 4-Photon Space-Time Entangled State

We now proceed to employ two of these double entangled two-photon setups in a manner analogous to the use of two Franson-interferometers in Figure 1. Additionally, we insert an extra 50/50 beam splitter, EBS, so that photons transmitted by BS and BS' can reach either detector 2 or detector 4, as is shown in Figure 3a. Furthermore, we align the fast and slow axes of the elements in the primed system orthogonal to those of the unprimed system (see Figure 3b). As in equation 1, the

4-photon state here is the product of two two-photon entangled states (state 3 and its analog in the primed system), so that taking into account the action of EBS and ignoring the terms which will not contribute to the 4-fold coincidence counting rate, it is not difficult to see that [14]:

$$\begin{aligned} |\psi_4\rangle \sim\ & |\mathbf{F}\rangle_1|\mathbf{F}'\rangle_3\{|\mathbf{F}\rangle_2|\mathbf{F}'\rangle_4 - |\mathbf{F}'\rangle_2|\mathbf{F}\rangle_4\} \\ & -e^{i(\gamma+\delta)}|\mathbf{F}\rangle_1|\mathbf{S}'\rangle_3\{|\mathbf{F}\rangle_2|\mathbf{S}'\rangle_4 - |\mathbf{S}'\rangle_2|\mathbf{F}\rangle_4\} \\ & -e^{i(\alpha+\beta)}|\mathbf{S}\rangle_1|\mathbf{F}'\rangle_3\{|\mathbf{S}\rangle_2|\mathbf{F}'\rangle_4 - |\mathbf{F}'\rangle_2|\mathbf{S}\rangle_4\} \\ & +e^{i(\alpha+\beta+\gamma+\delta)}|\mathbf{S}\rangle_1|\mathbf{S}'\rangle_3\{|\mathbf{S}\rangle_2|\mathbf{S}'\rangle_4 - |\mathbf{S}'\rangle_2|\mathbf{S}\rangle_4\} \end{aligned} \tag{4}$$

Although analagous to equation 1, we see that the inclusion of the extra beam splitter has essentially divided each of the four possible 4-photon amplitudes into two equal phase parts, as indicated by the curly bracketed terms in equation 4. Based on the polarization, it is these equal

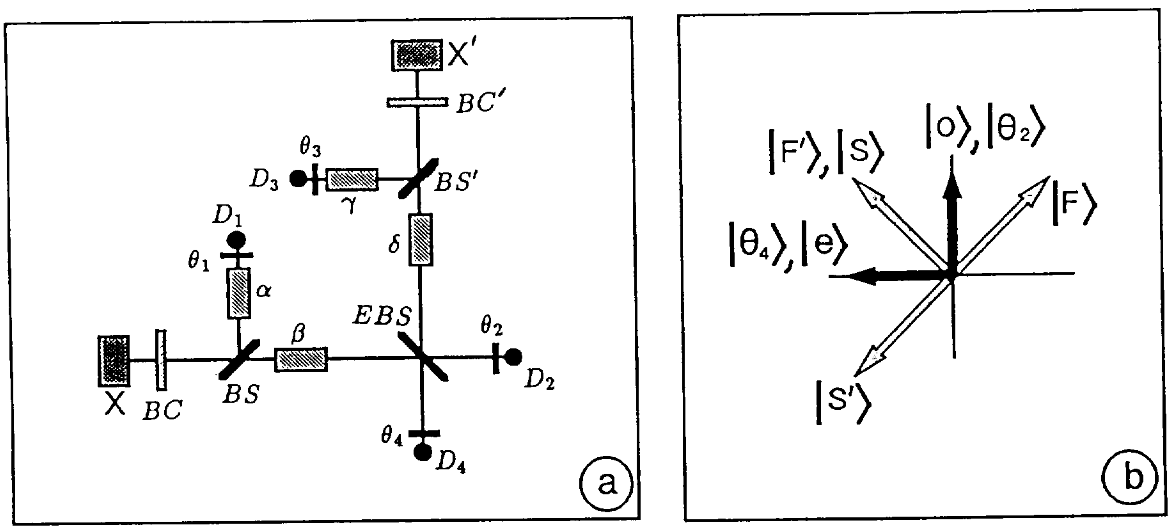

FIG.3. a) The envisioned scheme consists of two coherently pumped "double entangled two-photon state" setups which overlap through the use of an extra beam splitter (EBS). b) The important polarization orientations of the various elements in the scheme. Note that BC' is orthogonal to BC.

phase parts which will constructively or destructively interfere to produce the space-time entangled state.

For example, we consider the projection of the state on to the polarization analyzers and note from Figure 3b that $|\mathbf{F}\rangle$ and $|\mathbf{S}'\rangle$ are antiparallel. Thus, regardless of the settings of analyzers θ_2 and θ_4 the two equal phase parts in the curly brackets of the second term will subtract and this corresponding 4-photon amplitude will vanish. Likewise, since $|\mathbf{S}\rangle$ and $|\mathbf{F}'\rangle$ are parallel, the two equal phase parts in the curly brackets of the unwanted third term will also subtract. Furthermore, we note that if we orient θ_2 and θ_4 as shown in Figure 3b, and define the relevant part of the polarizer projection operator as $\mathcal{P} \equiv |\theta_2\rangle|\theta_4\rangle\langle\theta_4|\langle\theta_2|$, then in the curly brackets of the first 4-photon term:

$$\mathcal{P}|\mathbf{F}\rangle_2|\mathbf{F}'\rangle_4 = -\mathcal{P}|\mathbf{F}'\rangle_2|\mathbf{F}\rangle_4, \tag{5}$$

and these two equal phase parts *add together*. The same is true inside the curly brackets of the last 4-photon term.

In other words, since the polarizations are associated with space-time paths, the amplitudes in which some photons take the fast paths while others take the slow paths are seen to vanish, while those in which all four photons take the fast paths or all four photons take the slow paths remain!

The remaining 4-photon state is entangled in space-time variables:

$$|\psi_4\rangle = |F\rangle_1|F\rangle_2|F\rangle_3|F\rangle_4 + e^{i(\alpha+\beta+\gamma+\delta)}|S\rangle_1|S\rangle_2|S\rangle_3|S\rangle_4. \tag{6}$$

It is interesting to see that with the above choice of θ_2 and θ_4 settings, there is no dependence on the difference in pair birth times (provided it is within the coherence time of the pump), nor any reliance on any type of ultra-short coincidence time windows provided we can assure at most one pair of photons from each crystal is in the system at any given time.

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