QUANTUM MECHANICS OF A TWO PHOTON STATE

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Abstract

We review the formalism for describing the two photon state produced in spontaneous parametric down conversion.

1 Introduction

In this discussion we will first outline the general theory of optical spontaneous parametric down-conversion (OPDC). We will then discuss the phase matching conditions. After this we will discuss the classification of OPDC into type-I and type-II. Finally we will present our picture of the two photon state generated by the thory.

The work discussed in this paper is the result of the efforts of the members of the UMBC Quantum Optics Group:

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2 Optical Parametric Downconversion

Optical Parametric down conversion is modeled (in the interaction picture) by the interaction Hamiltonian [1][2]

$$H_1 = \frac{\epsilon_0}{2} \int d^3r \chi^{(2)}_{pbc} E_p(\mathbf{r},t) E_b(\mathbf{r},t) E_c(\mathbf{r},t) \tag{1}$$

where E_p is the pump electric field and $\chi^{(2)}_{abc}$ is the second order susceptibility, ($\chi = \chi^{(1)} + \chi^{(2)} + \chi^{(3)} + \ldots$). The integral is over the intersection of the birefringent crystal and the pump beam. In writing this it is assumed that the crystal does not have a center of symmetry so $\chi^{(2)}_{abc} \neq 0$ and that wave length of the light is much greater than atomic dimensions so the crystal can be treated in the continuum limit. The pump is be treated classically. For spontaneous optical parametric down conversion the wave function incident on the crystal is assumed to be the vacuum, $|\Psi> = |0>$.

Using first order perturbation theory, we can compute the wave function produced at the output face of the crystal. It is a superposition of the vacuum and a two-photon state. The two photon beams are often referred to as the signal and idler beams. In our case, we choose the orientation of the optic axis of the crystal and the polarization of the pump beam so that the produced photons have orthogonal polarizations corresponding to ordinary (o-rays) and extraordinary (e-rays) rays in the crystal.

$$\begin{aligned} |\Psi> &= |0> - \frac{i}{\hbar}\int_{-\infty}^{\infty} dt H_1 |0> \\ &= |0> + \sum_{\mathbf{k},\mathbf{k}'} F_{\mathbf{k},\mathbf{k}'} a^{\dagger}_{o\mathbf{k}'} a^{\dagger}_{e\mathbf{k}'} |0>, \end{aligned} \tag{2}$$

$$F_{\mathbf{k},\mathbf{k}'} = \Gamma_{\mathbf{k},\mathbf{k}'} \delta(\omega_{o\mathbf{k}} + \omega_{e\mathbf{k}} - \omega_p) L h(L\Delta_{\mathbf{k},\mathbf{k}'}) h_{tr}(\mathbf{k},\mathbf{k}'). \tag{3}$$

In Eq. (3) $Lh(L\Delta_{\mathbf{kk}'})$ comes from the integral over the length L of the crystal,

$$h(x) = \frac{1 - e^{-ix}}{ix} \tag{4}$$

$$\Delta_{\mathbf{kk}'} = k_p - k_z - k_z' \tag{5}$$

The integral over the area A of the intersection of the beam cross-section and the crystal gives

$$h_{tr}(\mathbf{k},\mathbf{k}') = \int_A d^2\rho\, e^{i(\mathbf{k}+\mathbf{k}')\cdot\rho}. \tag{6}$$

The time integral gives the 2π times the Dirac delta function which is the *steady-state or frequency phase matching condition.* If we assume that the crystal is very large and the pump beam has a large cross section, then the integrals can be taken to extend over an infinite volume. This leads to the *wave number phase matching condition*

$$\mathbf{k} + \mathbf{k}' = k_p \hat{\mathbf{e}}_z. \tag{7}$$

The assumption of a monochromatic pump beam gives

$$\omega_p = \omega_{o\mathbf{k}} + \omega_{e\mathbf{k}'}. \tag{8}$$

3 The properties of the two photon state

3.1 Terminology

We introduce some terminology for OPDC.

- Collinear **k** and **k**' are parallel to $\mathbf{k}_p$
- Degenerate $\omega_1 = \omega_2$
- Type-I Signal and idler have same polarization
- Type-II Signal and idler have orthogonal polarization

We next remind the reader of the definition of an *entangled state* [3]. For two degrees of freedom, we say a state $\Psi(1,2)$ is entangled if it is not a product state, i.e. $\Psi(1,2) \neq \phi(1)\phi(2)$. A simple example of an entangled state is the singlet state of two spin-1/2 particles.

3.2 OPDC two photon state

[4] The state in Eq.(2) is entangled in wave number and frequency or, equivalently, in space and time because $F_{\mathbf{kk}'}$ does not factor into a function of **k** and **k**'. In general, it is not entangled in polarization.

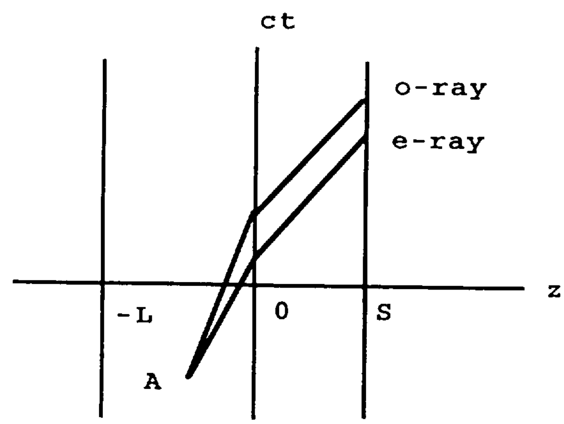

FIG. 1. A Feynman-like diagram showing a pair created at point A inside the crystal. For the case shown the speed of the e-ray is greater than that of the o-ray in the crystal. Since the first photon is always the e-ray, the state is not entangled in polarization.

The simplest experiment to study the two photon state is illustrated below.

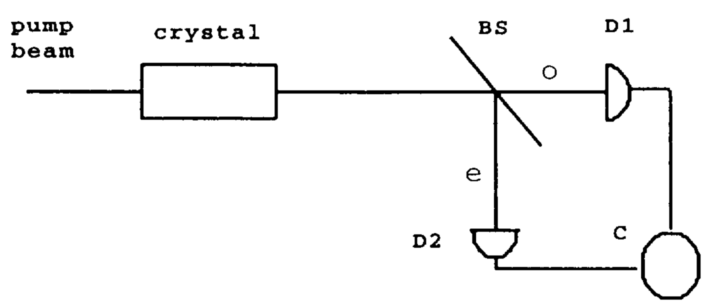

FIG. 2. A collinear, type-II experiment. The beam splitter seperates the polarizations and sends them to the two detectors D1 and D2. A coincident counter, C, detects coincidences.

For this is a collinear, type-II experiment the output is given by the coincident counting rate

$$R_c = \lim_{T\to\infty} \frac{1}{T} \int_0^T dT_1 \int_0^T dT_2 < \Psi|E_1^{(-)}E_2^{(-)}E_2^{(+)}E_1^{(+)}|\Psi > S(T_1 - T_2) \tag{9}$$

where S(t) is a coincidence time window. The probability of a coincidence is

$$< \Psi|E_1^{(-)}E_2^{(-)}E_2^{(+)}E_1^{(+)}|\Psi >= |A(t_1,t_2)|^2 \tag{10}$$

$$A(t_1,t_2) =< 0|E_2^{(+)}E_1^{(+)}|\Psi > \tag{11}$$

$$t_j = T_j - z_j/c \tag{12}$$

where T_j is the time at which a photon is detected at detector j which is a distance z_j from the output surface of the crystal. A is called the *two-photon amplitude* or *biphoton*. The two-photon amplitude is of the form

$$A(t_1,t_2) = v(t_1+t_2)u(t_1-t_2) \tag{13}$$

$$u(t) = e^{i\omega_d \frac{t}{2}}\Pi(t), \tag{14}$$

$$v(t) = v_0 e^{i\omega_p \frac{t}{2}} \tag{15}$$

$$\omega_d = \Omega_o - \Omega_e. \tag{16}$$

The quantities Ω_o and $\Omega_e, (\Omega_o + \Omega_e = \omega_p)$ are chosen for convenience. For details see [4].

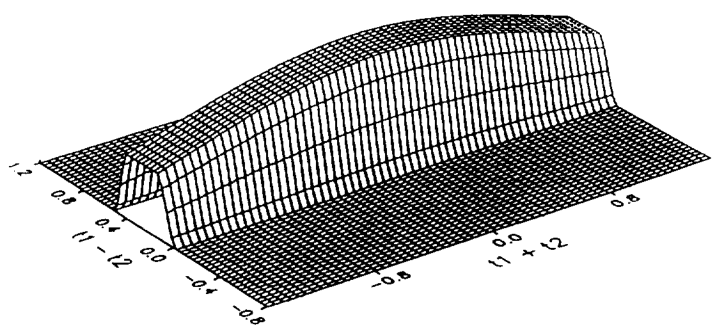

FIG. 3. An illustration of the two-photon amplitude. In most experiments the width in $t_1 - t_2$ is much smaller than the length in $t_1 + t_2$. The latter is determined by the coherence length of the pump.

3.3 Double entanglement

It is possible to entangle the polarization as well as the energy and momentum. The simplest experiment for seeing this is shown in figure 4 below.

A birefringent crystal is placed in the path of the rays to compensate for the different group velocities of the o- and e-rays. If the e-ray emerges from the crystal first, the compensator is arranged so the e-ray passes along its slow axis. The length of the compensator may be varied so that it introduces a delay τ in the e-ray relative to the o-ray.

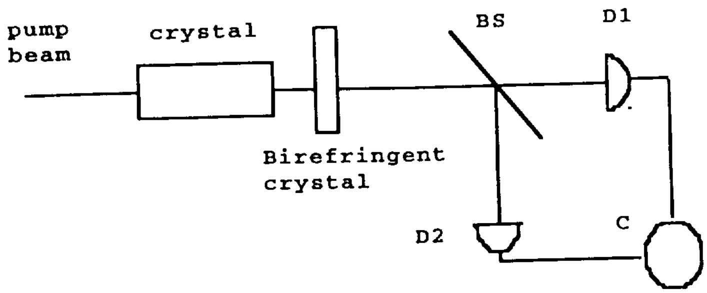

FIG. 4. The use of a birefringent crystal as a compensator.

If the beam splitter is a 50-50 beam splitter, then the two photon amplitude becomes

$$A(t_1, t_2) = \frac{1}{\sqrt{2}} v(t_1 + t_2 - \tau)[u(t_1 - t_2 + \tau) - u(-t_1 + t_2 + \tau)]. \quad (17)$$

The minus sign comes from the reflection off the mirror. The figure below illustrates the form of the bracketed term in Eq.(17). The probability amplitude will show interference between these two terms if τ is chosen so the two terms overlap. The xounting rate is then vee shaped, going to zero for complete overlap. We refer you to Dr. Sergienko's talk for details of the experimental

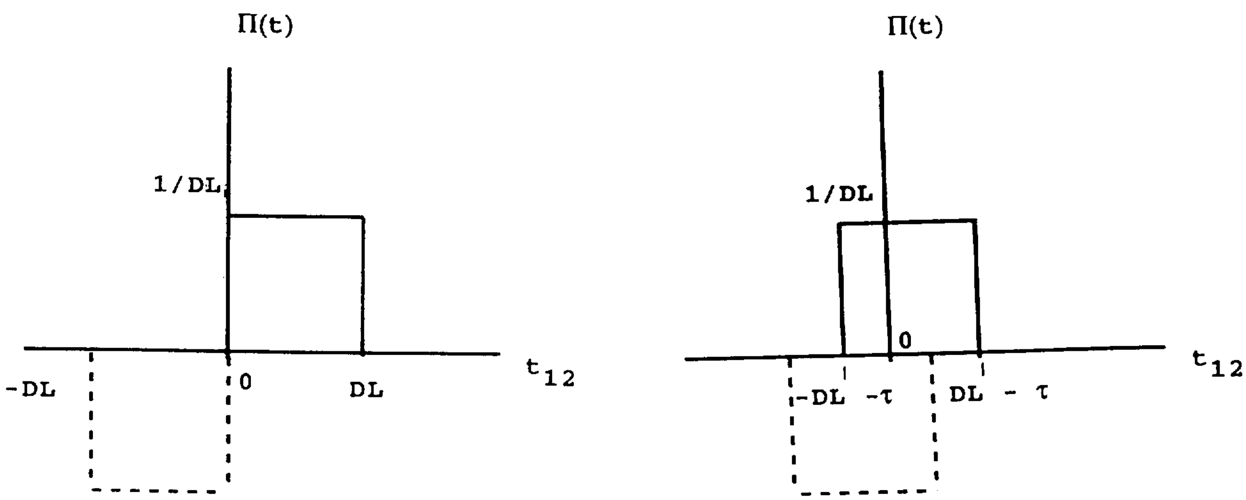

FIG. 5. The form of the amplitude in Eq. (17) is shown for no overlap and partial overlap.

We also illustrate this effect using Feynman-like diagrams for a some typical pairs.

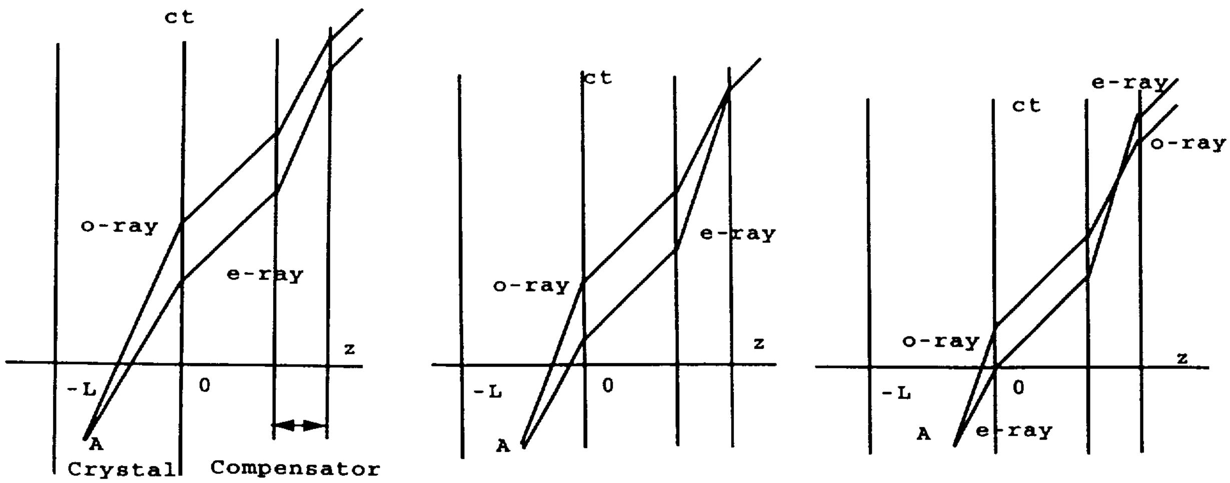

FIG. 6. These diagrams illustrate how the compensator effects pairs that are created at point A near the input, at the center, and near the output of the crystal.

4 Conclusion

We have a good understanding of the structure of the two-photon amplitude both theoretically and experimentally. The experimental results have been reported by other members of our group at this meeting. We have recently completed some work on the transverse correlations of the signal and idler beams.

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