There aren't Non-standard Solutions for the Braid Group Representations of the QYBE Associated with 10-D Representations of SU(4).

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Abstract

In this paper by employing the weight conservation and the diagrammatic techniques we show that the solutions associated with the 10-D representations of SU (4) are standard alone.

1 Introduction

It is well known that the quantum Yang-Baxter equations (QYBE) play an important role in various theoretical and mathematical physics, such as completely integrable system in (1+1)-dimensions, exactly solvable models in statistical mechanics, the quantum inverse scattering method and the conformal field theories in 2-dimensions. Recently, much remarkable progress has been made in constructing the solutions of the QYBE associated with the representations of lie algebras. It is shown that for some cases except the standard solutions, there also exist new solutions, but the others have not non-standard solutions. In reference 11, we derived the braid group representations associated with the 10-dimensional representation of SU (4) and corresponding trigonometric and rational solutions. In this paper, the classical limit of the braid group representations is checked. Then it is shown that the solutions associated with the 10-dimentional representations are standard alone.

2 Classical Limit

It is well known that in the classical limits as $q \to 1$ the standard solution of QYBE require that[10]

$$S\Big|_{q\to 1} = P\,[\,I + (q-1)\,r\,] + o[\,(q-1)^2\,] \qquad (2.1)$$

and

$$\Phi_v^T r \Phi_v = C_V = 2C_R - C_{Ev} \qquad (2.2)$$

where P is the permutation operator and Φ_V stands for the normalized classical eigenvectors, r is the classical r-matrix, C_R and C_{Ev} are the Casimirs. The eigenvalues are given by

$$\lambda_v = (\pm)\, q^{C_v} \qquad (2.3)$$

In Ref. (11), We have derived the braid group representations associated with the 10-D

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representations of SU (4). The Casimir eigenvalues of S-matrix were given by

$$\lambda_1=q^{-3},\quad \lambda_2=-q,\quad \lambda_3=q^3 \tag{2.4}$$

From the result of Ref. (11) we know that there are some fundamental submatrices, A_1, $A_2^{(1)}$ $A_3^{(1)}$, $A_4^{(2)}$, $A_4^{(3)}$ and $A_6^{(2)}$, and others can be expressed by direct sum of the fumdamental submatrices. So we discuss only the classical limits of this submatrices.

For example, we discuss only $A_3^{(1)}$:

$$A_3^{(1)}=\begin{bmatrix} 0 & 0 & q \\ 0 & q^{-1} & qw \\ q & qw & (1-q^2)\,w \end{bmatrix} \tag{2.5}$$

$$A_3^{(1)}\Big|_{q\to 1}=\begin{bmatrix} 0 & 0 & q \\ 0 & 2-q & 4\,(1-q) \\ q & 4\,(1-q) & 0 \end{bmatrix} \tag{2.6}$$

$$r_3^{(1)}=\begin{bmatrix} 1 & -4 & 0 \\ 0 & -1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \tag{2.7}$$

$$\Phi_1^T=\frac{1}{\sqrt{6}}(1\ \ 2\ \ 1),\quad \Phi_2^T=\frac{1}{\sqrt{2}}(1\ \ 0\ \ -1),\quad \Phi_3^T=\frac{1}{\sqrt{3}}(1\ \ -1\ \ 1) \tag{2.8}$$

and

$$\Phi_v^T r_3^{(1)}\Phi_v=C_v=\begin{cases} 3 & v=1 \\ 1 & v=2 \\ 3 & v=3 \end{cases} \tag{2.9}$$

Therefore the solutions of QYBE are standard.

3 About absence of the nonstandard solution

From Ref. (11) we have known that so long as $u_6=u_2=u_{-4}=u_{-6}$, there exists the alone solution. In Ref. (11), we have

$$\begin{cases} u_4 q_8^{2\,(2.4)}+w_{10}^{(4.6)}\,p_8^{2\,(2.6)}=w_{10}^{(4.6)}\,p_6^{2\,(2.4)} \\ u_6 q_8^{2\,(2.4)}+w_{10}^{(4.6)}\,u_4^2=u_4 w_{10}^{2\,(2.6)}+w_{10}^{(4.6)}\,p_{10}^{2\,(4.6)} \\ u_4^2=P_{10}^{(4.6)}\,p_6^{2\,(4.6)} \end{cases} \tag{3.1}$$

$$\begin{cases} u_6 p_8^{(2.6)} = p_{10}^{2\,(4.6)} \\ u_2 p_8^{(2.6)} = p_6^{2\,(2.4)} \end{cases} \quad (3.2)$$

$$\begin{cases} u_{-5}\, q_{-10}^{2\,(-6.-5)} + w_{-9}^{(-5.-4)} \quad p_{-10}^{2\,(-6.-4)} = w_{-9}^{(-5.-4)} \quad p_{-11}^{2\,(-6.-5)} \\ u_{-4} q_{-10}^{2\,(-6.-5)} + w_{-9}^{(-5.-4)} \quad u_{-5}^{2} = w_{-9}^{2\,(-5.-4)} \quad u_{-5} + w_{-9}^{(-5.-4)} . p_{-q}^{2\,(-5.-4)} \\ u_{-5}^{2} = p_{-11}^{(-6.-5)} \quad p_{-9}^{(-5.-4)} \end{cases} \quad (3.3)$$

$$\begin{cases} u_{-6}\, p_{-10}^{(-6.-4)} = p_{-11}^{2\,(-6.-5)} \\ u_{-4} p_{-10}^{(-6.-4)} = p_{-9}^{2\,(-5.-4)} \end{cases} \quad (3.4)$$

$$\begin{cases} u_0 q_0^{2\,(-6.0)} + w_{-6}^{(-6.0)} \quad p_0^{2\,(-6.6)} = w_{-6}^{(-6.0)} \quad p_6^{2\,(0.6)} \\ u_5 q_0^{2\,(-6.0)} + u_0^2 w_6^{(0.6)} = u_0 w_6^{2\,(0.6)} + w_6^{(0.6)} \quad p_6^{2\,(0.6)} \\ u_0^2 = p_6^{(0.6)} \quad p_{-6}^{(-6.0)} \end{cases} \quad (3.5)$$

$$\begin{cases} u_{-6} p_0^{(-6.6)} = p_{-6}^{2\,(-6.0)} \\ u_6 p_0^{(-6.6)} = p_6^{2\,(0.6)} \end{cases} \quad (3.6)$$

From eq . (3 . 1), we have

$$u_4 = p_{10}^{(4.6)} \quad ; \quad p_6^{(2.4)} = p_{10} \quad (3.7)$$

From eq . (3 . 2) + eq . (3 . 6), we have

$$u_2 = u_6 \quad (3.8)$$

$$u_{-5}=p_{-9}^{(-5,-4)}, \quad p_{-9}^{(-5,-4)}=p_{-11}^{(-6,-5)} \tag{3.9}$$

$$u_{-4}=u_{-6} \tag{3.10}$$

$$u_0=p_6^{(0,6)}, \quad p_6^{(0,6)}=p_{-6}^{(-6,0)} \tag{3.11}$$

$$u_{-6}=u_6 \tag{3.12}$$

From eq. (3.8), (3.10) and (3.12) we have

$$u_6=u_2=u_{-4}=u_{-6} \tag{3.13}$$

Therefore the solutions of the QYBE are standard alone.

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