

CHIRAL BOSONIZATION OF SUPERCONFORMAL GHOSTS

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Abstract

We explain the difference of the Hilbert space of the superconformal ghosts (β, γ) system from that of its bosonized fields φ and χ . We calculate the chiral correlation functions of φ , χ fields by inserting appropriate projectors.

Recently, many authors have investigated the bosonization of superconformal ghosts β and $\gamma^{1,2}$. Unlike the fermionic ghosts b and c , the bosonization of (β, γ) system have some problems.

Locally, (β, γ) system is equivalent to two scalar fields φ and χ^3 . Although the chiral correlation functions of β, γ fields have been calculated^{1,2}, the calculation of the chiral correlation functions of φ, χ fields will be troublesome. Besides the redundant zero-modes of the bosonized fields, the main reason is that φ, χ fields have a large Hilbert space than (β, γ) system. In ref. (4), this enlargement was explained as caused by the freedom of choosing the background ghost charge, the so-called picture, and by introducing projectors which specify the picture of each loop, the Hilbert space of φ, χ fields are restricted to the degrees of freedom of the (β, γ) system. In this paper, we explain this problem from an elementary point of view, and then apply new projector for the calculation of the chiral correlation functions of φ, χ fields.

We consider the (β, γ) system corresponding to superstring theory, i. e. with conformal dimensions $\frac{3}{2}$, $-\frac{1}{2}$ respectively. Locally, (β, γ) system is identified with a scalar field φ and a pair of fermions ζ, η with conformal weights 0, 1 respectively.

$$\beta = \partial\zeta e^{-i\varphi} \quad \gamma = \eta e^{i\varphi} \quad (1)$$

The φ field is coupled to background charge $Q=2$, and is described by the action

$$S[\varphi] = \frac{1}{2\pi} \int d^2z \left(-\partial_z \varphi \partial_{\bar{z}} \varphi - \frac{i}{2} \sqrt{g} R \varphi \right) \quad (2)$$

where, $g_{z\bar{z}}$ is a Riemann metric and R is the corresponding scalar curvature. We can again bosonizing (ζ, η) system via another scalar field χ

$$\zeta = e^{i\chi} \quad \eta = e^{-i\chi} \quad (3)$$

The χ field is coupled to background charge $Q = -1$ and is described by the action

$$S[\chi] = \frac{1}{2\pi} \int d^2z \left(\partial_z \chi \partial_{\bar{z}} \chi + \frac{i}{4} \sqrt{g} R \chi \right) \quad (4)$$

φ and χ fields are both restricted to taking values on a unit circle $\mathbb{R}/2\pi\mathbb{Z}$; this compactification results soliton configurations on Riemann surface Σ_g with genus $g > 0$, and insures the necessary holomorphic factorization⁵.

The classical soliton sectors can be labeled by the winding numbers for the canonical homology basis (a_i, b_j) . The soliton solutions of φ, χ fields with winding numbers (n_i, b_j) are given by

$$\begin{aligned} \varphi_{nm}(z) &= \pi(m + \bar{\tau}n)(\text{Im}\tau)^{-1} z + c.c. \\ \chi_{nm}(z) &= i\pi(m + \bar{\tau}n)(\text{Im}\tau)^{-1} z + c.c. \end{aligned} \quad (5)$$

where τ is the period matrix of Σ_g . For simplicity, we have denoted the Jacobi map $\int_{p_0}^z \omega$ as z . The corresponding action

$$\begin{aligned} S[\varphi_{nm}] &= \frac{\pi}{2}(m + \bar{\tau}n)(\text{Im}\tau)^{-1}(m + \tau n) + 2S_b \\ S[\chi_{nm}] &= \frac{\pi}{2}(m + \bar{\tau}n)(\text{Im}\tau)^{-1} - S_b \\ S_b &= \pi(m + \bar{\tau}n)(\text{Im}\tau)^{-1} \Delta - c.c. \end{aligned} \quad (6)$$

where, Δ is Riemann class.

We consider the following correlation functions

$$A_\delta = \int [d\varphi d\zeta d\eta]_\delta e^{-s[\varphi, \zeta, \eta]} \prod_{a=1}^{n+1} \zeta(\chi_a) \prod_{b=1}^n \eta(y_b) \prod_{c=1}^n e^{iq_c \varphi(z_c)} \quad (7)$$

where q_c are integer satisfying $\sum q_c = 2(g-1)$, and δ is a specific spin structure.

If φ and χ are treated independently, the result will be different from that of the corresponding β, γ fields. We notice that

- a) the bosonized fields have redundant zero-modes of ζ and η fields.
- and b) the (φ, χ) system has a larger Hilbert space than that of the (β, γ) system, since φ and χ are not independent globally. Thus we must have appropriate constraints, otherwise, some global configurations will be computed repeatedly.

The first aspect can be resolved by inserting operators $\delta(\zeta(\chi))$, $\prod_{i=1}^g \delta(\eta(\tau_i))$ to remove zero-modes of ζ, η fields. η has zero-modes at $i=1, \dots, g$, and ζ has a constant zero-mode, thus χ is an arbitrary point on Σ_g . In order to avoid to compute the similar part of global configurations of φ, χ fields, we introduce projector

$$\delta(m_\chi - m_\varphi) \delta(n_\chi - n_\varphi) \quad (8)$$

to restrict φ, χ on the same soliton sector at the same time.

Now, according to Riemann-Roch theorem, (7) must be modified as follow

$$A_\delta = \int [d\varphi d\zeta d\eta]_\delta e^{-s[\varphi, \zeta, \eta]} \prod_{a=1}^n \zeta(x_a) \prod_{b=1}^n \eta(y_b) \prod_{c=1}^n e^{iq_c \varphi(z_c)} \quad (9)$$

Inserting our projector

$$\delta(\zeta(x)) \prod_{i=1}^g \delta(\eta(r_i)) \delta(m_\varphi - m_\chi) \delta(n_\chi - n_\varphi)$$

we have

$$A_\delta = \left\langle \prod_{a=1}^n \zeta(x_a) \prod_{b=1}^n \eta(y_b) \prod_{c=1}^n e^{iq_c \varphi(z_c)} \cdot \delta(\zeta(x)) \prod_{i=1}^g \delta(\eta(r_i)) \delta(m_\chi - m_\varphi) \delta(n_\chi - n_\varphi) \right\rangle_\delta$$

For the δ -functions with fermion arguments, $\delta(\zeta) = \zeta$, $\delta(\eta) = \eta$, and labelling the arbitrary X as X_{n+1} , we get

$$A_\delta = \left\langle \prod_{a=1}^{n+1} e^{ix(x_a)} \prod_{b=1}^n e^{-iy(y_b)} \prod_{c=1}^n e^{iq_c \varphi(z_c)} \prod_{i=1}^g e^{-ix(r_i)} \delta(m_\chi - m_\varphi) \delta(n_\chi - n_\varphi) \right\rangle_\delta$$

This result can be written as a soliton sum $\Lambda_{\text{sol}, \delta}$ multiplied the amplitude of zero soliton sector A_{00}

$$A_\delta = \Lambda_{\text{sol}, \delta} \cdot A_{00} \quad (10)$$

A_{00} is the result of the single-valued part of φ, χ fields. It is trivial that

$$A_{00} = \exp \left\{ 2\pi i \text{Im} \left(\sum x_a - \sum y_b - \sum r_i + \sum q_c z_c - \Delta \right) (\text{Im} \tau)^{-1} \text{Im} \left(\sum x_a - \sum y_b \right. \right.$$

$$\left. \left. - \sum r_i + \sum q_c z_c - \Delta \right) \right\} \times \left| \frac{\prod_c \sigma(z_c)^{-2q_c}}{\prod_{c_1 < c_2} E(z_{c_1}, z_{c_2})^{q_{c_1} q_{c_2}}} \cdot \frac{\prod_b \sigma(y_b) \prod_{r_i} \sigma(r_i)}{\prod_a \sigma(x_a)} \right.$$

$$\left. \times \frac{\prod_{a_1 < a_2} E(x_{a_1}, x_{a_2}) \prod_{b_1 < b_2} E(y_{b_1}, y_{b_2}) \prod_{r_1 < r_j} E(r_1, r_j) \prod_{b < r_i} E(y_b, r_i)}{\prod_{a, b} E(x_a, y_b) \prod_{a, r_i} E(x_a, r_i)} \right|^2 \quad (11)$$

Using Poisson summation formula, we get

$$\Lambda_{\text{sol}, \delta} = (\det \text{Im} \tau) \exp \left\{ -2\pi \text{Im} \left(\sum x_a - \sum y_b + \sum q_c z_c - \sum r_i - \Delta \right) (\text{Im} \tau)^{-1} \cdot \text{Im} \left(\sum x_a - \sum y_b + \sum q_c z_c - \sum r_i - \Delta \right) \right\} \cdot \left| \sum_{\mathbf{p}_x} \exp \left\{ \pi i \mathbf{p}_x \tau \mathbf{p}_x + 2\pi i \mathbf{p}_x \left(\sum x_a - \sum y_b - \sum r_i + \Delta \right) \right\} \right. \\ \left. \cdot \left| \sum_{\mathbf{p}_\varphi} \exp \left\{ -\pi i (\mathbf{p}_\varphi + \delta') \tau (\mathbf{p}_\varphi + \delta) + 2\pi i (\mathbf{p}_\varphi + \delta) \left(\sum q_c z_c - 2\Delta + \delta' \right) \right\} \right|^2 \right. \quad (12)$$

Here, $\delta, \delta' \in (\frac{1}{2} \mathbf{z}/\mathbf{z})^s$, and $\delta = [\frac{\delta'}{\delta}]$

From (11) and (12), holomorphic anomaly factors of A_{sol} and $\Lambda_{\text{sol}, \delta}$ can cancel each other. Thus we can have chiral correlation functions

$$A_{\delta}^{\text{chiral}} = (\det \text{Im} \tau)^{\frac{1}{2}} \frac{\prod_{b_0=1}^n \theta[\delta] (-y_{b_0} + \sum x_a - \sum y_b + \sum q_c z_c - 2\Delta)}{\prod_{a_0=1}^{n+1} \theta[\delta] (-x_{a_0} + \sum x_a - \sum y_b + \sum q_c z_c - 2\Delta)} \\ \cdot \frac{\prod_{a_1 < a_2} E(x_{a_1}, x_{a_2}) \prod_{b_1 < b_2} E(y_{b_1}, y_{b_2})}{\prod_{a, b} E(x_a, y_b) \prod_{c_1, c_2} E(z_{c_1}, z_{c_2})^{q_{a_1} q_{a_2}} \prod_c \sigma(z_c)^{2q_c}} \quad (13)$$

Thus, by inserting appropriate projector to remove the zero-modes of ζ, η fields and restrict φ, χ on the same soliton sector, we get the correct chiral correlation functions of φ, χ fields. As compared with ref. (4), our approach is more comprehensive.

References

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