ATOMIC DIPOLE SQUEEZING IN THE CORRELATED TWO—MODE TWO—PHOTON JAYNES —CUMMINGS MODEL

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Abstract

In this paper, We study the atomic dipole squeezing in the correlated two—mode two—photon JC model with the field initially in the correlated two—mode SU(1,1) coherent state. The effects of detuning, field intensity and number difference between the two field modes are investigated through numerical calculation.

1 Introduction

The production and nonclassical properties of quantized electromagnetic fields and their interaction with matters have been the topics of fundamental importance in quantum optics, and both saw remarkable development in the past decade. In the first aspect, the correlated two—mode states of radiation fields, such as the two—mode squeezed state[1], the pair coherent state[2] and correlated two—mode SU(1,1) coherent state[3], have received a great deal of attentions among researchers. These states usually display nonclassical properties including field squeezing, antibunching and sub —poissonian photon statistics. In the other aspect, the theoretical model for the interaction of correlated two—mode fields and a two—level atom, better known as the generalized Jaynes—Cummings model[4], was investigated for field squeezing and atomic dynamics[5]. However, little attention has been paid to the atomic dipole squeezing in these systems.

It is well known that the atomic dipole squeezing, in much the same way as the field squeezing, is the reduction of fluctuation of one component of the dipole moment while keeping the uncertainty relations with the other component at the same time. As it is shown[6] that squeezed atom radiates squeezed lights, it is of importance to study the squeezing of the atomic variables. In the present paper, we devote a study to the squeezing of the atomic dipole moment in the correlated two—mode two—photon JC model.

2 The Hamiltonian and State Vector

We consider a system comprising of a two—level atom interacting with the correlated two

—mode, two—photon field. The Hamiltonian for the system in the dipole and the rotating—wave approximation is given by the following expression:

$$H=\omega_0\sigma_3/2+\omega_1(a_1^+a_1+1/2)+\omega_2(a_2^+a_2+1/2)+\lambda(a_1^+a_2^+\sigma_-+\sigma_+a_1a_2), \quad (h=1) \tag{1}$$

where ω_0 is the atomic transition frequency, ω_i is the frequency of the field of mode i; $a_1(a_1^+)$ and $a_2(a_2^+)$ are the field annihilation (creation) operators of the two modes, respectively; σ_3 is the atomic inversion operator and $\sigma_\pm$ are the atomic transition operators; λ is the atom—field coupling constant.

We assume that the atom is initially in the coherent superposition state of the excited state $|e>$ and the ground state $|g>$, and the field in any correlated two—mode state. The initial atom—field state is given by

$$|\Psi(0)>=|A(0)>\otimes|F(0)> \tag{2}$$

where $|A>=\cos\frac{\beta}{2}|e>+e^{i\varphi}\sin\frac{\beta}{2}|g>=A_1|e>+A_2|g>$

and $|F(0)>=\sum_n C_{n,q}|n+q,n>$

At any time $t>0$, the state vector of the system is found from the Hamiltonian (1) to be

$$|\Psi(t)>=A_1\sum_{n=0}^{\infty}\exp\{-i[\omega_1(n+q+1)+\omega_2(n+1)]t\}C_{n,q}G_1|+;n+q,n>-iA_2\sum_{n=0}^{\infty}\exp\{-i[\omega_1(n+q)+\omega_2 n]t\}C_{n,q}H_1|+;n+q-1,n-1>$$

$$+A_2\sum_{n=0}^{\infty}C_{n,q}\exp\{-i[\omega_1(n+q)+\omega_2 n]\}G_2|-;n+q,n>$$

$$-iA_2\sum_{n=0}^{\infty}C_{n,q}\exp\{-i[\omega_1(n+q+1)+\omega_2(n+1)]t\}.H_2|-;n+q+1,n+1> \tag{3}$$

with

$$G_1=\cos At-\frac{i\delta}{2}\frac{\sin At}{A},\quad G_2=\cos Bt-\frac{i\delta}{2}\frac{\sin Bt}{B}$$

$$H_1=\alpha_{n-1,q}\frac{\sin Bt}{B},\quad H_2=\alpha_{n,q}\frac{\sin At}{A}$$

$$A=[\delta^2/4+\alpha_{n,q}^2]^{1/2},\quad B=[\delta^2/4+\alpha_{n-1,q}^2]^{1/2}$$

$$\alpha_{n,q}^2=\lambda^2(n+q+1)(n+q),$$

3 Atomic Dipole Squeezing

We define the slowly varying atomic dipole operators as

$$\sigma_1=\frac{1}{2}(\sigma_+\exp(-i\omega_0 t)+\sigma_-\exp(i\omega_0 t)) \qquad (4)$$

$$\sigma_2=\frac{1}{2i}(\sigma_+\exp(-i\omega_0 t)-\sigma_-\exp(i\omega_0 t)) \qquad (5)$$

which correspond to the dispersive and absorptive parts of the dipole moment, respectively.

The atomic state is said to be squeezed if the variance satisfies the condition

$$(\triangle\sigma_i)^2<\frac{1}{4}\mid<\sigma_3>\mid \qquad i=1 \text{ or } 2 \qquad (6)$$

This condition can be rewritten as

$$S_i=(\triangle\sigma_i)^2-\frac{1}{4}\mid<\sigma_3>\mid<0 \qquad (7)$$

In carrying out the numerical calculations, we assume that the initial field is in the correlated two-mode SU(1,1) coherent state[3]

$$\mid F(0)>=(1-\mid\zeta\mid^2)^{1+q/2}\sum_{n=0}^{\infty}\left[\frac{(n+q)!}{n!q!}\right]^{1/2}\zeta^n\mid n+q,n> \qquad (8)$$

where $\zeta=-\mathrm{th}(\theta/2)\exp(-i\varphi)$ and where $0<\theta<\infty$ and $0\leqslant\varphi\leqslant 2\pi$. For simplicity, we set $\varphi=0$. Also we focus on the effects of detuning, photon number and the number defference between the two modes on the atomic dipole squeezing.

We assume the atom to be initially in the ground state. In the case of on-resonance excitation, the dispersive part of dipole moment does not squeeze, as is shown by the theoretical expression of the squeezing function S_1. The evolution of S_2 vs reduced time λt for different photon numbers (N_2) and number differences (q) are shown in Figs. 1~3. It is evident from Fig. 1, where $q=0$, that S_2 exhibits exactly periodic fluctuation behavior, with periodic time $\lambda t=\pi$. Good squeezing for σ_2 is found in the case of weak initial field with $N_2=1$, as shown in Fig. 1a, where σ_2 is squeezed almost all the time except when $\lambda t=k\pi$ $(k=0,1,2\cdots\cdots)$. With the initial field becoming more intensive, both the digree and duration of squeezing grow smaller. (Fig. 1)

When there is number difference between the two modes, the time evolution of S_2 no longer shows periodic behavior (Figs2~3). In weak initial field cases, for example $N_2=1$ (Fig. 2a, Fig. 3a), the fluctuations of S_2 are small and squeezings recur. The first squeezing in the case of $q=1$ lasts longer than that of the case $q=5$, but larger and longer squeezings recover in the case of $q=5$. In both cases the degree and duration fo squeezing get smaller as the initial

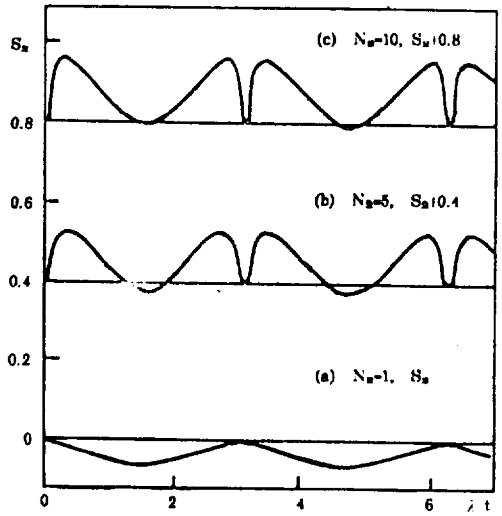

Fig.1 Evolution of S_z for A_1=0, δ=0, q=0

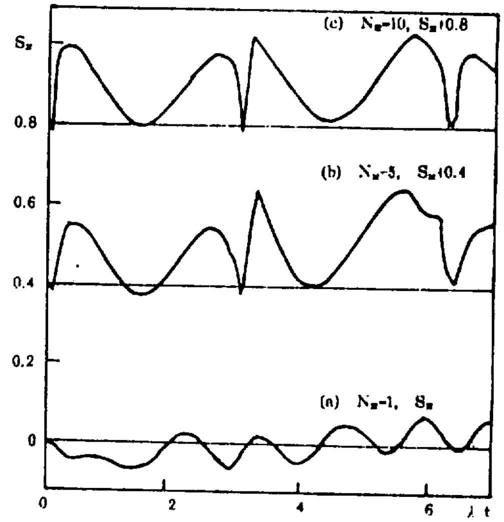

Fig.2 Evolution of S_z for A_1=0, δ=0, q=1

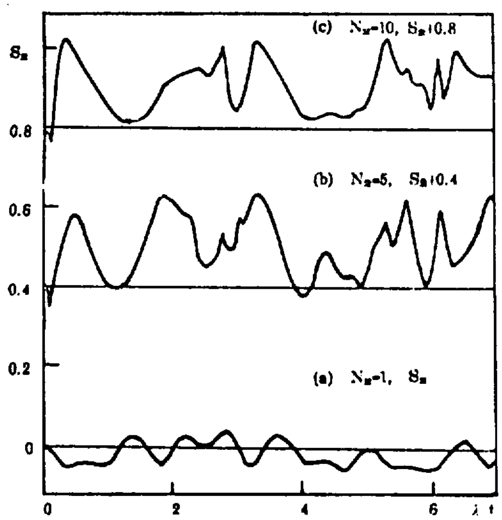

Fig.3 Evolution of S_z for A_1=0, δ=0, q=5

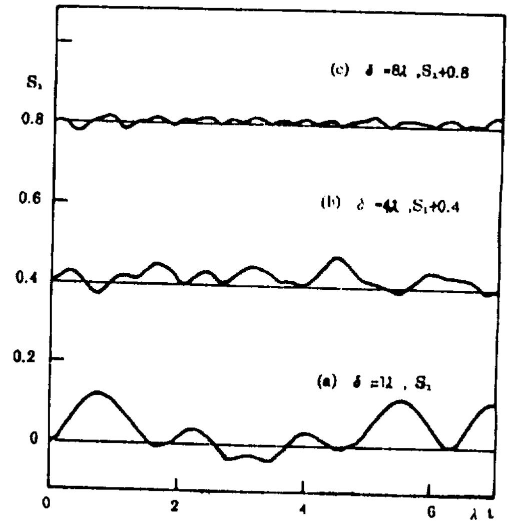

Fig.4 Evolution of S_1 for A_1=0, q=1, N_z=1

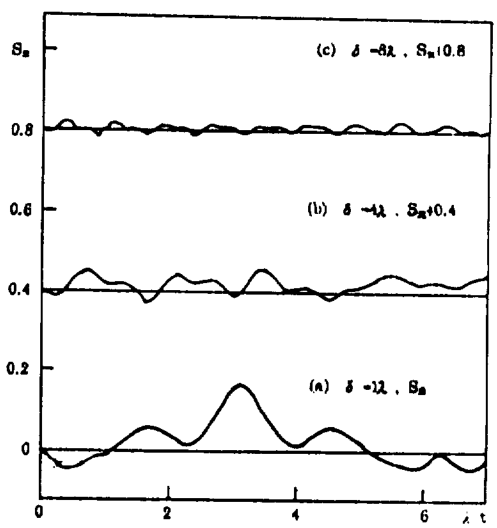

Fig.5 Evolution of S_z for A_1=0, q=1, N_z=1

field becomes more intensive. As it is observed from Fig. 3b and Fig. 3c, only short and small squeezings occur when $N_2=5$ and 10.

The above results reveal that atomic squeezing is both intensity and phase dependent of the initial field. Two identical field modes ($q=0$), which mean that both modes have identical number and phase distributions, result in periodic fluctuation and squeezing. On the other hand, two different field modes ($q=0$) lead to nonperiodic fluctuation and weaker squeezing effects.

The time evolution of S_1 and S_2 for different off—resonance excitations δ and for $q=1$ and $N_2=1$ are shown in Fig. 4 and Fig. 5 respectively. It is seen that both S_1 and S_2 squeeze recurrently and alternatively. When the detuning is larger, the fluctuation and squeezing become smaller due to weaker coupling between field and atom. We have also studied the cases when q is large and found that squeezing exists only for small detuning (not shown).

4 Conclusion

In summary, we have investigated the atomic dipole squeezing for the correlated two —mode two—photon JC model with the field initially in the correlated two—mode SU(1,1) coherent state. It is shown that in the on—resonance excitation and when the numbers of the two modes are equal, periodic squeezing is found for the absorption part of the dipole . Good squeezing is observed when the atom is initialy in the ground state and the initial field is weak . As the number of photon in mode 2 and the number difference grow larger, the degree and duration of squeezing decrease . In off—resonance excitation, both σ_1 and σ_2 exhibit squeezing effects . Detuning generally displays the effects of reducing fluctuation and squeezing even revokes for large detuning .

References

[1]C. M. Caves, B. L. Schumaker, Phys. Rev. A 31, 3068 (1985)

[2]G. S. Agarwal, J. Opt. Soc. Am. B5, 1940 (1988)

[3]C. C. Gerry, J. Opt. Soc. Am. B8, 685 (1991)

[4]E. T. Jaynes, F. W. Cummings, Proc. IEEE, 51, 89 (1963)

[5]C. C. Gerry, R. F. Welch, J. Opt. Soc. Am, B9, 290 (1992)

[6]M. M. Ashraf, the late M. S. K. Ramzi, J. of Mod. Opt. 39, 2245 (1992), and the references therein.