Photon number-phase uncertainty relation in the evolution of the field in a Kerr-like medium

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Abstract

A model of a single-mode field, initially prepared in a coherent state, coupled to a two-level atom surrounded by a nonlinear Kerr-like medium contained inside a very good quality cavity is considered. We derive the photon number-phase uncertainty relation in the evolution of the field for a weak and strong nonlinear coupling respectively, within the Hermitian phase operator formalism of Pegg and Barnett, and discuss the effects of nonlinear coupling of thr Kerr-like medium on photon number-phase uncertainty relation of the field.

1 Introduction

Recently, Agarwal et al[1] have considered the propagation of a single-mode resonant field through a nonlinear Kerr-medium. Bužek et al[2] have dealed with a combination of two models: the Jaynes–Cummings model (JCM) describing the interaction of a single-mode cavity field with a single two-level atom, and a nonlinear Kerr-like medium inside a cavity which may be modelled by an anharmonic oscillator[1,3]. Particularly, they have showed that with increasing nonlinear coupling the period between the revivals of the atomic inversion is shortened and its time evolution becomes more regular. Besides, they also described the squeezing of the cavity mode and the time evolution of the photon-number distribution.

As is well-known, the phase properties of light field is very important in quantum optics. Lately, Pegg and Barnett[4-6] have shown that an Hermitian phase operator of radiation field exists. It can be constructed from the phase states. This new phase operator formalism makes it possible to describe the quantum properties of optical phase in a fully quantum mechanics. Gerry[7] has studied the phase fluctuations of coherent light interacting with the anharmonic oscillator using the Hermitian phase operator. Gantsog et al[8,9] have studied the phase properties of self-squeezed states generated by the anharmonic oscillator, elliptically polarized light propagating through a Kerr medium and a damped anharmonic oscillator using the Hermitian phase operator.

In this paper we consider a generalized JCM with an additional Kerr-like medium, namely, a combined model that comprises the JCM and the anharmonic oscillator model (AOM) used to describe a Kerr medium. We deal with not only the field-Kerr medium interaction, but also the field-atom interaction. We derive the photon number-phase uncertainty relation in the evolution of the field for a weak and strong nonlinear coupling respectively, within the Hermitian phase operator formalism of Pegg and Barnett, and discuss the effects of nonlinear coupling of the Kerr medium on the number-phase uncertainty relation of the field.

2 The model

We consider a model which consists of a single two-level atom surrounded by a nonlinear Kerr-like medium contained in a high-Q single-mode cavity. The cavity mode is coupled to the Kerr-like medium as well as to the two-level atom. The Kerr-like medium can be modelled as an anharmonic oscillator [1,3]. In the adiabatic limit, the effective Hamiltonian of the system involving only the photon and atomic operators in rotating-wave approximation, is[1,2]

$$H_{eff} = \hbar\omega(a^+a + \frac{1}{2}) + \hbar\omega_0 S^z + \hbar\chi a^{+2}a^2 + \hbar g(S^+a + a^+S^-), \tag{2.1}$$

where a and a^+ are the annihilation and creation opreators of the field mode, $S^\pm$ and S^z are the spin-flip and inversion operators of the atom respectively, g is the field-atom coupling constant and χ describes the strength of the quadratic nonlinearity modelling the Kerr medium, ω_0 is the frequency of the atomic transition, the frequency ω is

$$\omega = \omega_f - \lambda^2/(\omega_k - \omega_f), \tag{2.2}$$

Where ω_f and ω_k are the frequency of the field mode and the anharmonic oscillator modelling the Kerr medium respectively, and λ is the field-Kerr medium coupling constant.

To isolate the effects of the nonlinear coupling of the Kerr medium from that of the finite detuning, we restrict in the case of the resonance ($i.e., \omega_0 = \omega$). Let us assume that the atom is initially in the excited state $|e>$ and the field mode is prepared in a coherent state $|\alpha>$. The initial state vector $|\psi(0)>$ of the system is

$$|\psi(0)>= |\alpha> \otimes |e>= \sum_{n=0}^{\infty} b_n e^{in\beta}|n, e>, \tag{2.3}$$

where

$$b_n = \exp(-\bar{n}/2)(\bar{n}^n/n!)^{1/2}. \tag{2.4}$$

In the interaction picture, the state vector of the system at a later time t is found from the Hamiltonian (2.1) to be

$$|\psi(t)>= \sum_{n=0}^{\infty} b_n e^{in\beta} e^{-i\chi n^2 t}.\{[\cos(\frac{1}{2}\Omega_{n\chi}t) + i\frac{2\chi n}{\Omega_{n\chi}}\sin(\frac{1}{2}\Omega_{n\chi}t)]|n, e>$$

$$-i\frac{2g\sqrt{n+1}}{\Omega_{n\chi}}\sin(\frac{1}{2}\Omega_{n\chi}t)|n+1, g>\}, \tag{2.5}$$

where $|g>$ is the ground state of the atom, $\Omega_{n\chi}$ is the generalized Rabi frequency defined by

$$\Omega_{n\chi} = [4g^2(n+1) + 4\chi^2 n^2]^{1/2}, \tag{2.6}$$

It is obvious that, for $\chi = 0$, the state vector $|\psi(t)>$ given by Eq.(2.5) describes the dynamics of the ordinary JCM.

3 The phase variance of the cavity field

Based on the Hermitian phase formalism of Pegg and Barnett[4-6], The complete set of $s+1$ orthonormal phase state is defined by

$$|\theta_m> = \frac{1}{\sqrt{s+1}} \sum_{n=0}^{s} \exp(in\theta_m)|n>, \tag{3.1}$$

where $\theta_m = \theta_0 + 2\pi m/(s+1), m = 0, 1, 2, \cdots, s$, and θ_m is an arbitrary real number. The Hermitian phase operator is given by

$$\hat{\Phi}_\theta = \sum_{m=0}^{s} \theta_m|\theta_m><\theta_m|, \tag{3.2}$$

Clearly, phase state $|\theta_m>$ are eigenstates of $\hat{\Phi}_\theta$ with the eigenvalues θ_m. The eigenvalues θ_m are restricted to lie within a phase window between θ_0 and $(\theta_0 + 2\pi)$. It has to be noted That, after all expectation values of the phase variables associated with the phase properties of the field have been calculated in the finite $(s+1)-$dimensional space, s is allowed to tend to infinity. The phase distribution of the state given by Eq.(2.5) is

$$p(\theta_m, t) = |<\theta_m|\psi(t)>|^2, \tag{3.3}$$

with the expectation value and the variance

$$<\hat{\Phi}_\theta> = \sum_m \theta_m P(\theta_m, t), \tag{3.4}$$

$$<\Delta\hat{\Phi}_\theta^2> = \sum_m (\theta_m - <\hat{\Phi}_\theta>)^2 P(\theta_m, t). \tag{3.5}$$

We choose the reference phase $\theta_0 = \beta - \pi s/(s+1)$, and introduce a new phase label $\mu = m - s/2$, which goes in integer steps from $(-s/2)$ to $(s/2)$. Then the phase distribution becomes symmetric in μ. In the limit as s tends to infinity, the continuous phase variable can be introduced replacing $\mu 2\pi/(s+1)$ by θ and $2\pi/(s+1)$ by $d\theta$. Then we can find a continuous phase distribution

$$P(\theta, t) = \frac{1}{2\pi}\{1 + 2\sum_{n>n'} b_n b_{n'}[A_{nn'}\cos[(n-n')\theta + (n^2 - n'^2)\chi t]$$

$$+ B_{nn'}\sin[(n-n')\theta + (n^2 - n^2)\chi t]]\}, \tag{3.6}$$

where

$$A_{nn'} = \cos(\frac{1}{2}\Omega_{n\chi}t)\cos(\frac{1}{2}\Omega_{n'\chi}t) + \frac{4g^2\sqrt{(n+1)(n'+1)} + 4\chi^2 nn'}{\Omega_{n\chi}\Omega_{n'\chi}}\sin(\frac{1}{2}\Omega_{n\chi}t)\sin(\frac{1}{2}\Omega_{n'\chi}t)$$

$$B_{nn'} = (2\chi n/\Omega_{n\chi}) \sin(\frac{1}{2}\Omega_{n\chi}t) \cos(\frac{1}{2}\Omega_{n'\chi}t) - (2\chi n'/\Omega_{n'\chi}) \cos(\frac{1}{2}\Omega_{n\chi}t) \sin(\frac{1}{2}\Omega_{n'\chi}t). \tag{3.7}$$

The function $P(\theta, t)$ is normalized so that

$$\int_{-\pi}^{\pi} P(\theta, t) d\theta = 1. \tag{3.8}$$

If the mean photon number in the field is large, $\bar{n} >> 1$, the coefficient b_n in Eq.(2.4) can be well approximated by a continuous Gaussian distribution

$$b_n = (2\pi\bar{n})^{-1/4} \exp[-(\bar{n} - n)^2/4\bar{n}]. \tag{3.9}$$

If Eq.(3,9) is substituted into Eq.(3.6) and the summation in Eq.(3.6) is replaced by an appropriate integral over the variable n, one can approximately work the phase distribution.

We will consider two limit cases:

(1)The weak nonlinear coupling which is defined by the condition $g^2\bar{n} >> \chi^2\bar{n}^2$, with $\bar{n} >> 1$. In this case the generalized Rabi frequency can be approximated as

$$\Omega_{n\chi} \approx g[(\bar{n})^{1/2} + n(\bar{n})^{-1/2}](1 + \frac{1}{2}\varepsilon_\chi^2), \tag{3.10}$$

where

$$\varepsilon_\chi = (\chi^2\bar{n}^2/g^2\bar{n})^{1/2}. \tag{3.11}$$

Then using Eqs.(3,9) and (3.10) and replacing the summation in Eq.(3.6) by an appropriate integral over n , we obtain

$$P(\theta, t) = \frac{1}{2\pi}(\frac{2\pi\bar{n}}{1 + 16\bar{n}^2\chi^2t^2})^2 \cdot \{(1 + \varepsilon_\chi) \exp[-\frac{2\bar{n}}{1 + 16\bar{n}^2\chi^2t^2}[2\bar{n}\chi t + (\theta - \frac{gt}{2\sqrt{\bar{n}}})^2]]$$

$$+(1 - \varepsilon_\chi) \exp[-\frac{2\bar{n}}{1 + 16\bar{n}^2\chi^2t^2}[2\bar{n}\chi t + (\theta + \frac{gt}{2\sqrt{\bar{n}}})^2]]\}. \tag{3.12}$$

According to Eqs.(3.4) and (3.5), using Eq.(3.12), and taking into account $\theta_m = \theta + \beta$, we can directly find an expectation value of the phase operator and its variance

$$< \hat{\Phi}_\theta >= \beta + \varepsilon_\chi \frac{gt}{2\sqrt{\bar{n}}}(1 - 4\bar{n}\varepsilon_\chi), \tag{3.13}$$

$$< \Delta\hat{\Phi}_\theta^2 >= \frac{1}{4\bar{n}} + \frac{(gt)^2}{4\bar{n}}(1 + 16\bar{n}\varepsilon_\chi^2). \tag{3.14}$$

(2)The strong nonlinear coupling which is defined by the condition $g^2\bar{n} << \chi^2\bar{n}^2$, with $\bar{n} >> 1$. In this case the generalized Rabi frequency can be approximated as

$$\Omega_{nx} \approx 2\chi\bar{n}(1 + \frac{1}{2}\varepsilon_g^2), \tag{3.15}$$

where

$$\varepsilon_g = (g^2\bar{n}/\chi^2\bar{n}^2)^{1/2}. \tag{3.16}$$

Then, we find

$$P(\theta,t) = \frac{(8\pi\bar{n})^{1/2}}{2\pi}(1+16\bar{n}^2\chi^2t^2)^{-1/2}\exp[-\frac{2\bar{n}}{1+16\bar{n}^2\chi^2t^2}(\theta+2\bar{n}\chi t)^2] \qquad (3.17)$$

$$<\hat{\Phi}_\theta> = \beta - 2\bar{n}\chi t, \qquad (3.18)$$

$$<\Delta\hat{\Phi}_\theta^2> = \frac{1}{4\bar{n}} + 4\bar{n}(\chi t)^2. \qquad (3.19)$$

We see that the average value of the phase is not equal to the initial quantity β, and the phase variance is always enhanced. For the weak nonliner coupling, the enhancement of the phase variance is proportional to $(gt)^2/4\bar{n}$, this is similar to that of the resonance field in coherent state JCM[10]. For the strong nonlinear coupling, the enhancement of the phase variance is proportional to $4\bar{n}(\chi t)^2$. Obviously, the enhancement of the phase variance in the strong nonlinear coupling case is larger than that in the weak nonlinear coupling case.

4 The number-phase uncertainty relation

It is not difficult to calculate the variance of the photon-number for the state given by Eq.(2.5). Using $\hat{n} = a^+a$, for the weak nonlinear coupling we obtain

$$<(\Delta\hat{n})^2> = <\hat{n}^2> - <\hat{n}>^2 \approx \bar{n} + \sqrt{\bar{n}}gt\exp[-(gt)^2/2]\sin(2\sqrt{\bar{n}}gt). \qquad (4.1)$$

¿From Eqs.(3.14) and (4.1), we find that the number-phase uncertainty relation is

$$<(\Delta\hat{n})^2><\Delta\hat{\Phi}_\theta^2> \approx \frac{1}{4} + \frac{1}{4}(gt)^2\{1+16\bar{n}\varepsilon_\chi^2 + 16\sqrt{\bar{n}}gt\varepsilon_\chi^2\exp[-(gt)^2/2]\sin(2\sqrt{\bar{n}}gt)\}. \qquad (4.2)$$

For the strong nonlinear coupling, we find

$$<(\Delta\hat{n})^2> \approx \bar{n} + 2\bar{n}\chi t\varepsilon_g^2\exp[-2\bar{n}(\chi t)^2]\sin(2\bar{n}\chi t), \qquad (4.3)$$

$$<(\Delta\hat{n})^2><\Delta\hat{\Phi}_\theta^2> \approx \frac{1}{4} + 4\bar{n}^2(\chi t)^2\{1+2\chi t\varepsilon_g^2\exp[-2\bar{n}(\chi t)^2]\sin(2\bar{n}\chi t)\}. \qquad (4.4)$$

We see that, the uncertainty product during the evolution is expanded. The expansion of the uncertainty product is fast in strong nonlinear coupling case. This is similar to that of the self-squeezed state generated by the anharmonic oscillator[8]. This occurs because both the field-atom interaction[11] and the field-Kerr medium interaction are nonlinear, moreover the nonlinear interaction strength of the latter is larger than that of the former.

5 Summary

In the present paper we consider a generalized JCM in the presence an additional Kerr-like medium. we have derived the photon number-phase uncertainty relation in the evolution of a resonant field for a weak and strong nonlinear coupling, within the Hermitian phase operator formalism of Pegg and Barnett. We have shown that the nonlinear coupling of the cavity mode to Kerr-like medium leads to the enhancement of the phase variance of the field and the expansion of the uncertainty product. Particularly, the expansion of the uncertainty product is fast in strong nonlinear coupling case. We have indicated that this is similar to that of the self-squeezed state generated by the anharmonic oscillator. We have also indicated that the nonlinear interaction strength of the field-Kerr medium is larger than that of the field-atom.

Acknowledgments

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