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LARGE DEFLECTION ANALYSIS OF A TENSION-FOIL BEARING

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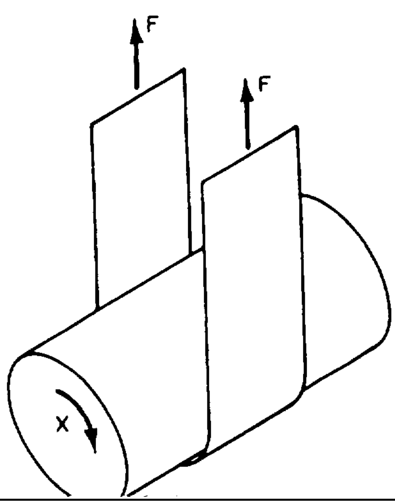
INTRODUCTION

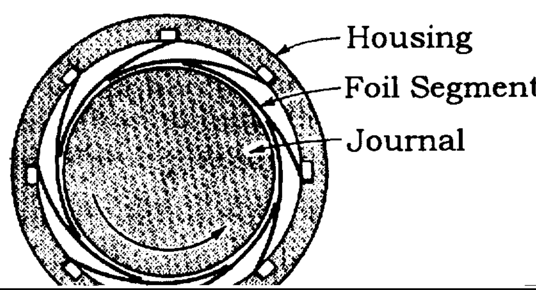
The rolling element bearings (REB's) which support many turbomachinery rotors offer high load capacity, low power requirements, and durability. Two disadvantages of REB's are:

- rolling or sliding contact within the bearing has life-limiting consequences; and
- REB's provide essentially no damping.

The REB's in the Space Shuttle Main Engine (SSME) turbopumps must sustain high static and dynamic loads, at high speeds, with a cryogenic fluid as lubricant and coolant. The pump end ball bearings limit the life of the SSME high pressure oxygen turbopump (HPOTP). Compliant foil bearing (CFB) manufacturers have proposed replacing turbopump REB's with CFB's. CFB's work well in aircraft air cycle machines, auxiliary power units, and refrigeration compressors. In a CFB, the rotor only contacts the foil support structure during start up and shut down. CFB damping is higher than REB damping. However, the load capacity of the CFB is low, compared to a REB. Furthermore, little stiffness and damping data exist for the CFB. A rotordynamic analysis for turbomachinery critical speeds and stability requires the input of bearing stiffness and damping coefficients.

The two basic types of CFB are the tension-dominated bearing (Figure 1) and the bending-dominated bearing (Figure 2). Many investigators have analyzed and measured characteristics of tension-dominated foil bearings, which are applied principally in magnetic tape recording. The bending-dominated CFB is used more in rotating machinery. Recently, a new tension-foil bearing configuration has been proposed for turbomachinery applications.





Previous foil bearing models have included a plate analysis for the foil in which small deflections are assumed. If the deflection of a plate is greater than about 20% of the plate thickness, a large deflection analysis should be used [1]. In a tension foil bearing, the foil deflection may be several times the foil thickness. The goal of the present work is to develop bench-mark ANSYS finite element analyses of a uniformly loaded plate, and a FORTRAN code for the large deflection analysis of a tension-foil bearing. In this report, results of ANSYS finite element analyses of a thin plate under uniform load are presented. Progress on the development of the FORTRAN code are also presented.

THEORY

Von Karman's equations for the transverse deflection w of a plate under a transverse pressure load are [1]

$$\frac{\partial^4 F}{\partial x^4}+2\frac{\partial^4 F}{\partial x^2 \partial y^2}+\frac{\partial^4 F}{\partial y^4}=E\left[\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2-\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}\right] \tag{1}$$

$$\frac{Eh^3}{12(1-\nu^2)}\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2 \partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)=q+h\left(\frac{\partial^2 F}{\partial y^2}\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 F}{\partial x^2}\frac{\partial^2 w}{\partial y^2}-2\frac{\partial^2 F}{\partial x \partial y}\frac{\partial^2 w}{\partial x \partial y}\right) \tag{2}$$

In equations (1) and (2), the xy directions are in the plane of the plate, h is the plate thickness, E and ν are Young's modulus and Poisson's ratio, and q is the transverse pressure. F is called the "Airy stress function", and is related to the normal and shear loads per unit length, Q_x, Q_y, and Q_{xy} by:

$$Q_x=h\frac{\partial^2 F}{\partial y^2} \qquad Q_y=h\frac{\partial^2 F}{\partial x^2} \qquad Q_{xy}=-h\frac{\partial^2 F}{\partial x \partial y} \tag{3}$$

The von Karman equations account for the in-plane loading and deflections which occur when a plate is loaded by an external transverse pressure. If the transverse deflection is less than 20% of the plate thickness, the in-plane loads and deflections are neglected, and equations (1) and (2) are replaced by the "small deflection" equation:

$$\frac{Eh^3}{12(1-\nu^2)}\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2 \partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)=q \tag{4}$$

Previously, the finite difference approach was used to program equation (4) for small deflection analyses of a foil. A finite difference template set up to represent the equation was modified at the edges to model clamped conditions on two ends of the foil, a free edge along the length of the foil, and a line of symmetry at the mid-width of the foil. An iterative solution technique was used.

In order to apply the finite difference technique to solve the von Karman equations, one would solve for the plate deflections iteratively assuming the Airy stress function is zero, use the deflection solution to solve iteratively for the Airy stress function, and continue to iterate until the solutions for deflections and Airy stress function converge. In addition, modeling the boundary conditions is more difficult for the solution of von Karman's equations than for the small deflection equation. As a result, the finite element approach was chosen as the numerical method for the large deflection analysis.

In a finite element analysis of a continuous structure, the structure is subdivided into smaller parts called "elements", which are connected at points called "nodes". Each element is represented by a stiffness matrix, which relates forces and displacements at the nodes. The stiffness matrix is a function of the element stress/strain state. For small deflections, a first order approximation for strain is adequate. For large deflections, the solution is iterative and the stiffness matrix must be modified to include additional terms in the strain approximation, and to include "stress stiffening": the in-plane stresses carry some of the transverse load.

ANSYS MODELS

Before beginning the FORTRAN code development, the ANSYS finite element program was used to model a foil under a uniform pressure load. The elastic quadrilateral shell element, Stif63, was used to model the plate. This element can be used to model stress stiffening and large deflection effects. A 2 inch long, 1 inch wide, 0.01 inch thick Inconel foil was modeled. Since a uniform load is assumed, two lines of symmetry exist. The ANSYS models are for one fourth of the foil, with 50 square elements, and clamped and free boundary conditions.

Figure 3 is a plot of maximum displacement predictions for the uniformly loaded Inconel foil. The three lines plotted are the results of ANSYS large deflection analyses, ANSYS small deflection analyses, and small deflection finite difference analyses. The small deflection analysis results agreed within 5% at all nodes in the models. The small deflection and large deflection results begin to differ at less than 0.5 psi uniform load. At 10 psi, the maximum deflection predicted by the large deflection analysis is less than 20% of that predicted by the small deflection analyses. The predictions of the finite element program which is described in the next section will be compared to the ANSYS results in figure 3.

FINITE ELEMENT MODEL

The finite element program described in this section is similar in structure to the PLASTOSHELL program in reference [2]. Many elements used to model the deflection of a plate under a transverse load have proven too stiff when the plate modeled is very thin and the deflection is large. As a result, Hughes and Cohen [3] developed the "heterosis" element, which is used in the present analysis. The element has nine nodes: four corner nodes, four mid-side nodes, and a center node.

AFOIL, the FORTRAN code for the large displacement analysis of a tension-foil bearing, is an 800 line, double precision program. Presently, AFOIL includes the following subroutines:

- INPUT - reads the problem description from a file;

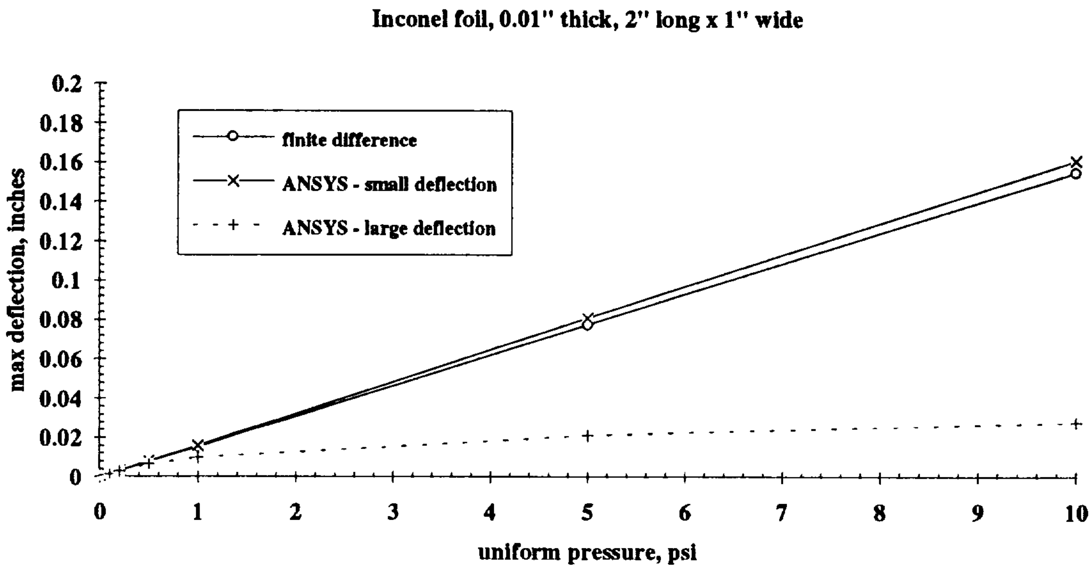


Figure 3. Finite difference/ANSYS comparisons

- ZERO - intializes vectors and matrices;
- ELAS - generates the elasticity matrix for the foil;
- MESH - identifies the location of element nodes;
- BGL - reads the pressure input from a file, sets up the linear and nonlinear parts of the strain/displacement matrix [*B*], and computes the nodal loads due to pressure;
- SHAPE - evaluates the shape functions used to compute [*B*] and the nodal loads;
- BFILL - fills [*B*];
- BOUNDS - fixes the boundary conditions, two clamped ends, one free edge, and one line of symmetry;
- STIFF - computes the stiffness matrix for each element;
- FRONT - solves for the displacements using the frontal technique [4];
- RESID - computes reaction forces and residual forces for a convergence check;
- CONV - checks for convergence of the iterative process; and
- OUT - writes the results to an output file.

Presently, AFOIL runs, but computes incorrect deflections. It appears that the problem is in the FRONT subroutine. If the problem continues despite debugging efforts, a slower Gaussian elimination method will be substituted for the frontal technique.

CONCLUSIONS

ANSYS models of a thin foil under uniform transverse pressure indicate the importance of using a nonlinear large deflection analysis for computing the foil deflections. An 800 line, double precision FORTRAN computer program, AFOIL, has been developed for the analysis of the foil in a tension-foil bearing. However, debugging of AFOIL is not complete. The solution technique may be changed if debugging of the frontal subroutine does not correct the current problem.

REFERENCES

[1] Szilard, Rudolph, 1974, *Theory and Analysis of Plates: Classical and Numerical Methods*, Prentice Hall, Englewood Cliffs, NJ.

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[3] Hughes, T. J. R., and Cohen, Martin, 1978, "The 'Heterosis' Finite Element for Plate Bending," *Computers and Structures*, Vol. 9, pp 445-450.

[4] Hinton, E. and Owen, D. R. J., 1977, *Finite Element Programming*, Academic Press, London.