An Evaluation of Attitude-Independent Magnetometer-Bias Determination Methods*

J. Hashmall
Computer Sciences Corporation
Lanham-Seabrook, Maryland, USA 20706

J. Deutschmann
National Aeronautics and Space Administration
Goddard Space Flight Center
Greenbelt, Maryland, USA 20771

Abstract

Although several algorithms now exist for determining three-axis magnetometer (TAM) biases without the use of attitude data, there are few studies of the effectiveness of these methods, especially in comparison to attitude-dependent methods. This paper presents results of a comparison of three attitude-independent methods and an attitude-dependent method for computing TAM biases.

The comparisons are based on in-flight data from the Extreme Ultraviolet Explorer (EUVE), the Upper Atmosphere Research Satellite (UARS,) and the Compton Gamma-Ray Observatory (GRO). The effectiveness of an algorithm is measured by the accuracy of attitudes computed using biases determined with that algorithm. The attitude accuracies are determined by comparison with known, extremely accurate, star-tracker-based attitudes.

In addition, the effect of knowledge of calibration parameters other than the biases on the effectiveness of all bias determination methods is examined.

1. Introduction

The recent emphasis on cost reduction in space missions has led to renewed interest in the use of magnetometers as attitude sensors. Because of their role in momentum management systems, three-axis magnetometers (TAMs) are nearly ubiquitous on near Earth-orbiting, three-axis-stabilized spacecraft. The cost of using TAMs for attitude determination is minimal, not only because of their use in momentum control, but also because they are inexpensive, reliable sensors. For these reasons, methods have been proposed to not only compute spacecraft attitudes, but also rates and even ephemerides (References 1–3).

Magnetometers contribute to attitude knowledge because the observed magnetic field best approximates a reference magnetic field when they are expressed in the same coordinate system and the transformation between the local spacecraft coordinate system and an Earth-reference system defines the spacecraft attitude. To best use TAMs for attitude determination, calibration parameters that are used to adjust observed magnetic fields must be determined. Chief among the calibration parameters that can be determined are magnetometer biases.

Magnetometer biases can be determined using a number of different algorithms (References 4–6). Although magnetometer bias determination methods all must use the observed magnetic field and the reference magnetic field (computed using the spacecraft ephemeris), some methods make use of additional information such as attitude rates (which can be obtained from gyroscope measurements) or spacecraft attitudes (obtained using more accurate sensors). Prior to bias determination, the magnetometer observations may be corrected for other calibration parameters, including magnetometer misalignments, nonunity scale factors, and the time-dependent effects of other instruments onboard the spacecraft that produce magnetic fields.

The purpose of this paper is to compare the utility of magnetometer biases computed using different attitude-independent methods for computing magnetometer-based spacecraft attitudes. Their utility is determined by comparing the accuracy of attitudes computed using identical magnetic field observations adjusted using the various computed biases. The data for computing the biases and attitude accuracies are obtained from flight data on three missions supported by the National Aeronautics and Space Administration (NASA) Goddard Space Flight Center (GSFC). These comparisons are made using magnetometer data adjusted with various combinations of other calibration parameters applied. Biases determined using an accurate attitude-dependent method are included for comparison.

2. Background

The three methods that were compared include two that require no rate data and one that does. They consist of the TWOSTEP, TAMCAL, and CFADS algorithms. In addition to these three attitude-independent methods, an attitude-dependent method was used to compute biases for comparison. This attitude-dependent method is referred to as the CFADS attitude-dependent algorithm.

TWOSTEP Algorithm—Attitude Independent (Reference 6)

The TWOSTEP method determines the bias by first computing a centered bias estimate based on a cost function derived from centering the measurements (about weighted averages of the measurements), where the centered measurements depend only linearly on the magnetometer bias. The centered bias estimate is then used as an initial estimate to compute a corrected bias, where the correction is due to the center term. This method was developed to correct the incorrect treatment of correlations in the Gambhir algorithm and to include information from data that that algorithm discards. The TWOSTEP method is attitude independent. Although it has been extended to compute calibration parameters other than the bias, only bias estimates are considered in this work.

TAMCAL Algorithm—Attitude Independent (Reference 5, 7)

The TAMCAL algorithm minimizes a loss function, L, defined by

$$L = \sum_t w_t \left| \left[\left| \vec{B}_R(t) \right|^2 - \left| \vec{B}_O(t) - \vec{b} \right|^2 \right] \right| \tag{1}$$

where w_t is a weight and $\vec{B}_R(t)$ and $\vec{B}_O(t)$ are the reference and observed magnetic field vectors at time t and $\vec{b}$ is the bias vector. The bias is solved by an iterative refinement of an initial bias estimate. This method is truly attitude independent, because the magnitudes of the reference and estimated observed (observed-minus-estimated bias) magnetic field vectors are used, and these magnitudes are attitude independent. This algorithm is embodied in the Multimission Three-Axis Stabilized Attitude Spacecraft (MTASS) Flight Dynamics Support System (FDSS) magnetometer calibration utility used by the Flight Dynamics Division (FDD) at GSFC.

CFADS Algorithm—Attitude Independent (Reference 7)

The CFADS algorithm for bias determination consists of a particular use of the MTASS FDSS Coarse/Fine Attitude Determination System (CFADS). In this use of CFADS, a batch least-squares method is used to solve for a state vector containing an epoch attitude, rate biases, and TAM biases. The solution uses only TAM observations and attitude rate data (usually obtained from gyroscopes). Because neither attitude information nor data from more accurate attitude sensors are used in it, this method is also considered attitude independent.

CFADS Attitude-Dependent Algorithm (Reference 7)

The CFADS attitude-dependent algorithm uses MTASS FDSS CFADS and solves for the same state vector as does the CFADS attitude-independent algorithm. The difference arises from the additional use of pseudo-observations at times throughout the batch and from the choice of observation weights. Pseudo-observations consist of estimates of the spacecraft attitude previously obtained using more accurate sensors (in this case, star trackers). The CFADS loss function includes differences between a representation of each attitude (from a pseudo-observation), propagated to the epoch time using rate data, and the attitude portion of the state vector. The

pseudo-observation weights are set to values, sufficiently high compared with the magnetic field observations, so that the attitude solution is negligibly affected by the magnetic field observations. This procedure is equivalent to including highly weighted, more accurate sensor observations directly as input to CFADS.

The three spacecraft included in this study were the Extreme Ultraviolet Explorer (EUVE) (Reference 8), the Compton Gamma-Ray Observatory (GRO) (Reference 9), and the Upper Atmosphere Research Satellite (UARS) (Reference 10). Each of the missions contain unique features that must be considered in order to understand the results. The three spacecraft all use fixed-head star trackers (FHSTs) and accurate gyroscopes as primary sensors. Their onboard computer (OBC) attitude accuracies have stringent requirements (1σ attitude accuracy requirements: EUVE = 6 arcseconds, UARS = 20 arcseconds, and GRO = 27 arcseconds); therefore, compared with the TAM-based attitudes described later, the OBC attitudes can be considered truth.

The three spacecraft contain similar magnetometers, and all are in nearly circular orbits. While EUVE and GRO have 28.5-degree (deg) orbital inclinations, the UARS orbit has an inclination of about 57 deg. During the periods when data for this study were obtained, the spacecraft mean altitudes were about 513 kilometers (km) for EUVE, 582 km for UARS, and 379 km for GRO.

EUVE and GRO have magnetic torquer bars (MTBs) situated relatively near the TAMs, while the UARS MTBs were designed to be at a sufficient distance from the TAMs to have only a small effect on the measured magnetic field. As a result, MTB data were not available to FDD in the UARS telemetry. None the less, FDD personnel have seen data that indicate the possibility of an uncalibrated MTB effect of 1 to 3 milligauss (mG) on UARS. For EUVE and GRO, the time-dependent effects of the MTBs can be minimized through application of a coupling matrix.

Magnetometer measurements on UARS and EUVE are received with a precision of about 7.8 mG, while on GRO the least significant telemetry bit represents about 0.3 mG.

During the periods when the data for this study were obtained, EUVE and GRO were inertially pointing while UARS rotated at 1 revolution per orbit (RPO) to maintain an Earth-pointing attitude. Continuous telemetry was not available for GRO, and the data gaps of up to 10 minutes were filled with gyro data, created assuming constant attitude rates between the attitudes at either end of each gap. While EUVE and UARS TAM data were received in an unaltered state, the GRO TAM data used had been previously adjusted using prelaunch calibration parameters.

3. Method and Results

Prior to bias determination, the magnetometers on each spacecraft were calibrated using a complete, attitude-dependent calibration method applied to a long span (8 to 14 orbits) of data. This method is different from the attitude-dependent bias determination method. The calibrations performed on EUVE, UARS, and GRO have been verified by computing TAM/gyro attitudes using all of the calibration parameters and achieving attitude accuracies (compared with star-tracker-based attitudes) of about 0.06 deg for UARS and EUVE (Reference 11) and 0.02 deg for GRO.

These calibrations determined parameters in the magnetometer adjustment equation:

$$\vec{B}_o(t) = A_{bs}\left\{\vec{B}_s(t) + C_{st}\vec{D}_t(t)\right\} - \vec{b}_b \tag{2}$$

where

$\vec{b}_b$ = bias vector in the body frame

$\vec{D}_{tb}$ = commanded torquer dipole at time t

C_{st} = torquer coupling matrix that converts from the torquer dipole in the torquer frame to the magnetic field in the magnetometer frame

171

$\vec{B}_x$ = measured magnetic field in the sensor frame

A_{bs} = alignment/scale factor matrix that both converts magnetic field estimates in the sensor frame to the body frame and corrects them for scale factor deviations in the magnetometer

The bias vector compensates for any time-independent magnetic fields in the spacecraft, the torquer dipole vector and coupling matrix compensate for the major source in the time-varying spacecraft magnetic field, and the alignment/scale factor matrix puts the results into the body frame while compensating for linear errors in the magnetometer response. This matrix, which will be henceforth referred to as the A-matrix, is not constrained to be either orthogonal or normal and can be viewed as being composed of columns representing the alignment of the three independent one-axis magnetometers, each multiplied by its separate scale factor.

For each spacecraft, a 193-minute (approximately 2 orbits) span of data was selected for bias determination. This span was selected so that magnetometer, magnetic torquer (except for UARS), gyroscope, and OBC reference attitudes were available. The data were independent of those used for the full calibration of the magnetometers. To simulate conditions in which some of the calibration parameters might be unknown or inaccurate, the magnetometer data for EUVE and GRO were adjusted in four ways to produce four separate sets of corrected magnetometer data. UARS data, because magnetic torquer data were not available, were adjusted in only the two ways that do not require torquer data.

The four types of adjustments made were the following:

1. No adjustment

2. Coupling only: $\vec{B}_o(t) = \vec{B}_x(t) + C_{st}\vec{D}_t(t)$

3. A-matrix only: $\vec{B}_o(t) = A_{bs}\vec{B}_x(t)$

4. Both: $\vec{B}_o(t) = A_{bs}\left[\vec{B}_x(t) + C_{st}\vec{D}_t(t)\right]$

Note that no bias adjustments were performed.

For each of the sets of adjusted data, separate bases were determined using each of the three attitude-independent bias determination methods and, for comparison, using the CFADS attitude-dependent method. The computed biases are shown in Table 1.

Separate sets of data spanning 193 minutes for each spacecraft were selected and adjusted in the four ways detailed above. In addition, the corresponding biases were subtracted from the data (biases determined using the no-adjustment case of the first set of data, subtracted from the no-adjustment case of the second set of data, etc.). Each of these sets of adjusted data with bias correction were used with gyroscope data to determine spacecraft attitudes at 5-second intervals over the 193 minutes. The attitude solutions were computed using a batch least-squares method with gyroscope biases and attitude as the only elements of the state vector. Similar attitude solutions were obtained with each set of adjusted data from which no magnetometer biases had been subtracted. In all attitude computations, the reference fields were obtained using the International Geomagnetic Reference Field (IGRF) model, evaluated to 10th order, and using the 1990 definitive coefficients including the secular terms.

A total of 20 sets of attitudes each were computed for EUVE and GRO and an additional 10 for UARS. The attitude rotations between the computed attitude and the reference (OBC—star-tracker-based) attitude were expressed as an Euler sequence at 5-second intervals, and the root-mean-square (RMS) of these error rotations was determined. Table 2 contains these RMS errors over all axes* throughout the period. These values represent the accuracy of the attitudes obtained with the different biases and are labeled as attitude errors.

---

* Except for UARS where a timing error introduced a systematic offset in the pitch component. In the case of UARS, the mean of the other two axes is given.

Table 1. Comparison of Magnetometer Bias Computed Using Different Methods

Spacecraft	Adjustment	Axis	Bias Determination Method			
			TWOSTEP	TAMCAL	CFADS	Attitude Dependent
EUVE	None	X	5.53	7.06	3.41	6.00
		Y	1.43	2.57	0.32	−3.26
		Z	−10.74	−11.65	−10.59	−10.08
	Coupling Matrix	X	−3.27	−1.70	−2.94	−1.94
		Y	3.76	2.60	3.02	0.64
		Z	−2.19	−3.15	−1.74	−0.54
	A-Matrix	X	9.81	15.38	7.78	8.57
		Y	−5.41	−13.51	−5.69	−5.83
		Z	−6.80	−4.04	−7.21	−8.04
	Both	X	0.89	6.33	1.19	0.40
		Y	−2.99	−11.00	−2.76	−1.70
		Z	1.81	4.67	1.62	1.51
GRO	None	X	0.41	2.31	0.47	1.39
		Y	−0.96	−1.94	−1.13	−1.99
		Z	−6.10	−6.65	−2.86	−2.82
	Coupling Matrix	X	0.46	2.35	0.35	1.24
		Y	−1.11	−2.09	−1.38	−2.13
		Z	−6.12	−6.67	−2.82	−2.74
	A-Matrix	X	−2.42	−3.36	−2.43	−2.39
		Y	−2.50	−4.96	−2.54	−2.38
		Z	−2.24	1.20	−2.07	−2.06
	Both	X	−2.37	−3.32	−2.55	−2.51
		Y	−2.63	−5.10	−2.75	−2.51
		Z	−2.26	1.18	−2.00	−1.99
UARS	None	X	−4.82	−4.77	−3.54	−3.07
		Y	−7.58	−7.36	−7.14	−8.34
		Z	−4.96	−4.91	−4.71	−4.49
	A-Matrix	X	−2.81	−0.70	−1.58	−2.25
		Y	−6.58	−5.21	−6.37	−6.27
		Z	−2.60	−0.07	−2.05	−2.16

For each spacecraft and each bias source, the mean values of the attitude errors were computed and are presented as rows labeled "mean" under each spacecraft in Table 2. Similarly, for each bias source, the mean of the attitude errors for all spacecraft and each adjustment method, as well as the overall mean for all spacecraft and all adjustments, are presented as rows labeled by the adjustment method under the category "mean" in this table. These data are also presented for the three spacecraft as Figures 1, 2 and 3. Note that in these figures, the order of the series has been changed to improve visibility.

To more clearly show the relative accuracies, the attitude accuracy results were converted to ratios. Table 3 presents the ratios of the attitude accuracies given in Table 2. For each row of attitude errors in Table 2, the value in each column (representing a bias determination method) was divided by the attitude error from biases obtained using the attitude-dependent method (representing the "best" biases with this adjustment). The resulting ratios show how close to the "best" adjustment a given set of biases comes. An additional column shows the attitude errors for each adjustment divided by the attitude errors obtained using all available adjustments. The biases used to obtain both the numerator and denominator of these ratios were found using the attitude-dependent method. This column shows how important each type of adjustment is for each spacecraft.

Table 2. Comparison of Magnetometer Based Attitude Accuracies Computed Using Different TAM Biases

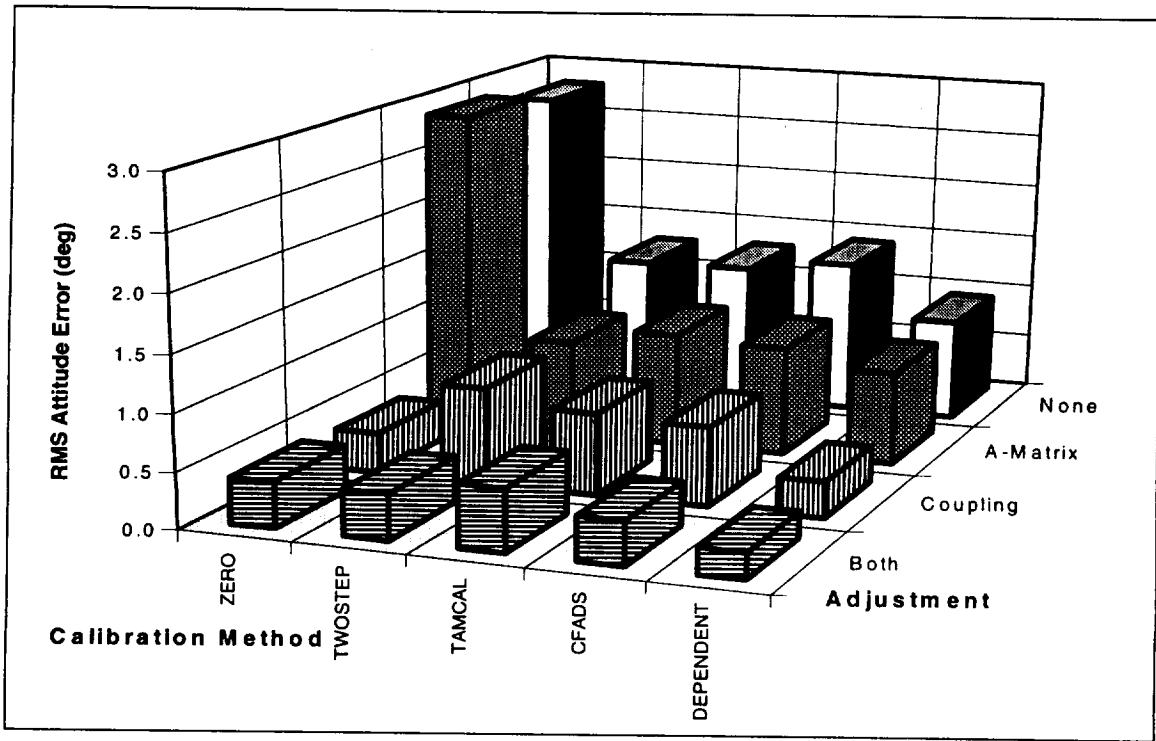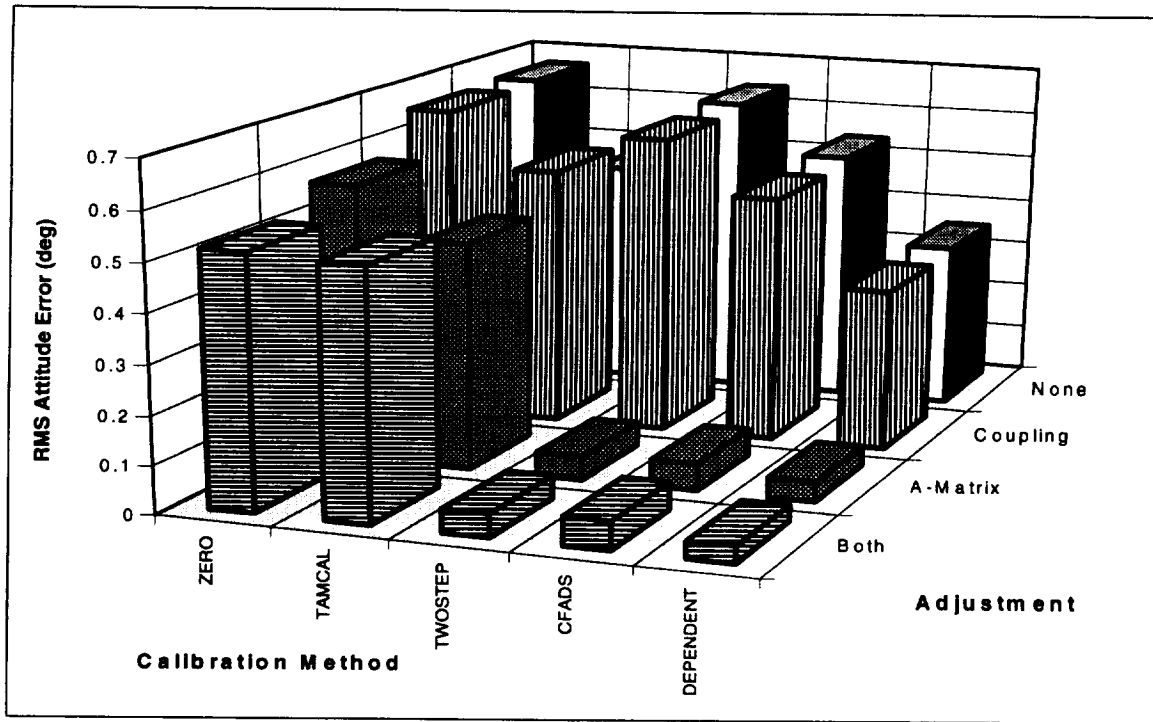| Spacecraft	Adjustment	Attitude Error (Degrees)				
		With Bias Determination Method:				
		No Bias	TWOSTEP	TAMCAL	CFADS	Attitude Dependent
EUVE	None	2.795	1.258	1.276	1.388	0.916
	Coupling Matrix	0.364	0.864	0.734	0.704	0.323
	A-Matrix	2.895	0.873	1.021	0.985	0.847
	Both	0.408	0.407	0.553	0.388	0.230
	Mean	1.616	0.851	0.896	0.866	0.579
GRO	None	0.665	0.639	0.478	0.534	0.347
	Coupling Matrix	0.658	0.626	0.544	0.518	0.337
	A-Matrix	0.573	0.050	0.470	0.057	0.049
	Both	0.519	0.048	0.429	0.058	0.034
	Mean	0.604	0.341	0.479	0.292	0.192
UARS	None	1.694	0.823	0.837	0.790	0.693
	A-Matrix	0.753	0.143	0.407	0.131	0.049
	Mean	1.224	0.483	0.622	0.461	0.371
Mean	None	1.718	0.907	0.864	0.904	0.652
	Coupling Matrix	0.511	0.745	0.639	0.611	0.330
	A-Matrix	1.530	0.469	1.162	0.501	0.422
	Both	0.464	0.227	0.489	0.223	0.132
	Mean	1.148	0.558	0.666	0.540	0.381

Figure 1. EUVE Attitude Errors
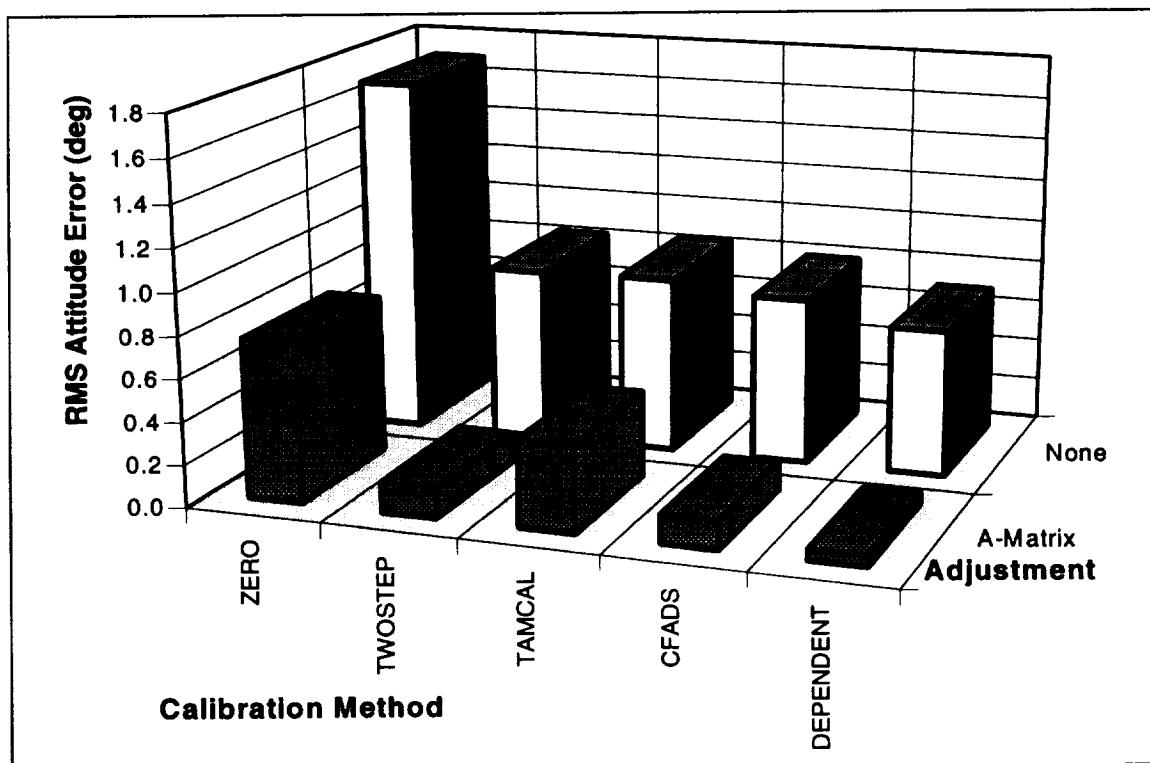

Figure 2. GRO Attitude Errors

175

Figure 3. UARS Attitude Errors

Table 3. Relative Attitude Accuracies Computed Using Different TAM Biases

Spacecraft	Adjustment	Attitude Error Ratio				
		With Bias Determination Method				
		No Bias	TWOSTEP	TAMCAL	CFADS	Overall Ratio
EUVE	None	3.051	1.373	1.393	1.515	3.983
	Coupling Matrix	1.127	2.675	2.272	2.180	1.404
	A-Matrix	3.418	1.031	1.205	1.163	3.683
	Both	1.774	1.770	2.404	1.687	1.000
GRO	None	1.916	1.841	1.378	1.539	10.206
	Coupling Matrix	1.843	1.858	1.524	1.451	9.912
	A-Matrix	11.694	1.020	9.592	1.163	1.441
	Both	15.265	1.770	12.500	1.706	1.000
UARS	None	2.444	1.188	1.208	1.140	14.143
	A-Matrix	15.367	2.918	8.306	2.673	1.000

5. Discussion and Conclusions

For interpretation of the results, EUVE and UARS represent cases where there is a large error in unadjusted calibration parameters (e.g., A-matrix and coupling matrix) as well as larger data noise. The dominant term in the EUVE and UARS magnetometer noise is telemetry digitization, which has a uniform distribution and a range of 7.8 mG; for GRO, the dominant errors arise from systematic errors in the magnetic field model.

Although UARS is in a higher inclination orbit than either GRO or EUVE, resulting in a larger variation in the magnetic field throughout the orbit and a slight decrease in the accuracy of the reference magnetic field model at extreme latitudes, insufficient examples are available to reach reliable conclusions about the effect of orbit inclination.

Two elementary conclusions are clear and not surprising:

- Attitude-dependent biases yield consistently more accurate attitudes than do any of the attitude-independent biases.

- Attitude-independent biases obtained using rate data generally yield more accurate attitudes than do the other attitude-independent biases.

Comparing the results using biases computed with the TWOSTEP method with those computed using TAMCAL shows the following:

- TWOSTEP yields slightly better attitude accuracies than does TAMCAL. This conclusion is most easily seen from the somewhat lower mean values for TWOSTEP than for TAMCAL in Table 2. The GRO TAMCAL values are obviously seriously flawed.

- TWOSTEP is more robust than TAMCAL. TAMCAL sometimes yields biases that produce attitude errors that are out of the range of the attitude errors produced with the other methods. The attitude accuracies may even be worse than if no biases had been applied. These flawed biases were computed for GRO data where most of the significant adjustment had been applied before any of these computations were performed. Even without any further adjustment, the GRO attitudes are quite accurate. It seems possible that the minimization algorithm used in TAMCAL is unstable when it starts very near a minimum; in this case, an initial large jump may lead to poor results.

Comparing the results using different adjustments to the magnetometer data yields an extremely important conclusion:

The algorithm used to compute magnetometer biases affects attitude accuracies significantly less than does knowledge of the additional parameters (A-matrix and torquer coupling) that can be used to adjust the data. This conclusion is most evident in Table 3 where the attitude error ratios vary far less among the calibration methods than does the overall attitude error ratio among the adjustment types.

Comparing the results from the three spacecraft studied indicates the following:

- Comparing the results for EUVE and GRO show that for EUVE the major contributing factor to attitude error is uncertainty in the coupling matrix; whereas for GRO, the uncertainty in the A-matrix dominates. The relative importance of the calibration parameters varies from spacecraft to spacecraft.

- Comparing the attitude errors with all adjustments made and with attitude-dependent biases shows that the better sensor digitization on GRO yields significantly improved magnetometer-based attitude accuracies than the coarse sensor digitization on UARS and EUVE.

These results have shown that to maximize the performance of magnetometers for attitude determination, complete on-orbit calibration should be performed. If this is impossible, accurate prelaunch calibration is desirable. Magnetometer data should be adjusted to compensate for misalignment, scale factor, and torquer coupling, as well as for biases.

177

Where possible, attitude-dependent bias calibration is preferred. If accurate attitudes are unavailable, inclusion of attitude rate data in the bias calibration improves results. If neither accurate attitude data nor rate data are available, the TWOSTEP method yields more reliable results than does TAMCAL. In addition, the TWOSTEP algorithm is very easy to apply; produces results quickly; and, even in the absense of other calibration, does significantly improve attitudes.

Further work on this subject is needed to evaluate the usefulness of various algorithms (including TWOSTEP) for determining the full set of magnetometer calibration parameters. Extension of this work to spacecraft in polar orbits could also provide further insight into the relative merits of calibration algorithms in different conditions.

References

1. Challa, M., G. Natanson, J. Deutschmann, and K. Galal, *A PC-Based Magnetometer-Only Attitude and Rate Determination System for Gyroless Spacecraft*, Paper No. 07, presented at the GSFC FDD Flight Mechanics/Estimation Theory Symposium 1995, NASA/Goddard Space Flight Center, Greenbelt, Maryland, May 16–18, 1995

2. Shorsi, G., and Bar-Itzhack, "Satellite Autonomous Navigation Based on Magnetic Field Measurements," *Journal of Guidance, Control, and Dynamics*, Vol. 18, No. 4, July–August 1995, pp. 843-850

3. Ketchum, E., *Autonomous Spacecraft Orbit Determination Using the Magnetic Field and Attitude Information*, Paper No. AAS 96-005, presented at the 19th Annual AAS Guidance and Control Conference, Breckenridge, Colorado, February 1996

4. Computer Sciences Corporation, CSC/TM-75/6248, *Determination of Magnetometer Biases Using Module RESIDG*, B. Gambhir, March 1975

5. Davenport, P., W. Rumpl, and G. Welter, *In-Flight Determination of Spacecraft Magnetic Bias Independent of Attitude*, presented at the GSFC FDD Flight Mechanics/Estimation Theory Symposium 1988, NASA/Goddard Space Flight Center, Greenbelt, Maryland, May 1988

6. Shuster, M., and R. Alonso, *Magnetometer Calibration for the First Argentine Spacecraft*, Paper No. AAS 95-302, presented at the AAS/AIAA Astrodynamics Specialist Conference, Halifax, Nova Scotia, Canada, August 1995

7. Goddard Space Flight Center, Flight Dynamics Division, 553-FDD-93/032R1UD0, *Multimission Three-Axis Stabilized Spacecraft (MTASS) Flight Dynamics Support System (FDSS) Mathematical Background, Revision 1*, M. Lambertson, J. Keat, and E. Scheidker (CSC), prepared by Computer Sciences Corporation, June 1994

8. Goddard Space Flight Center, Flight Dynamics Division, 554-FDD-93/006R0UD0, *Extreme Ultraviolet Explorer (EUVE) Postlaunch Report*, J. Bez (CSC), prepared by Computer Sciences Corporation, March 1993

9. Goddard Space Flight Center, Flight Dynamics Division, 554-FDD-91/114R0UD0, *Gamma Ray Observatory (GRO), Flight Dynamic Analysis Team Postlaunch Report*, L. Kulp and W. Davis (CSC), prepared by Computer Sciences Corporation, February 1992

10. Goddard Space Flight Center, Flight Dynamics Division, 554-FDD-92/029R0UD0, *Upper Atmosphere Research Satellite (UARS) Postlaunch Report*, J. Bez (CSC), prepared by Computer Sciences Corporation, March 1992

11. Hashmall, J., K. Liu, and M. Rokni, *Accurate Spacecraft Attitudes From Magnetometer Data*, Paper No. MS95/007, presented at the CNES International Symposium on Space Dynamics, Toulouse, France, June 19–23, 1995