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Final Report

of

Investigation Of Source Location Determination From Magsat Magnetic Anomalies: The Euler Method Approach
(NCC 5-70)

prepared by

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INVESTIGATION OF SOURCE LOCATION DETERMINATION FROM MAGSAT MAGNETIC ANOMALIES: THE EULER METHOD APPROACH

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SUMMARY

During the auspices of this award, the applicability of the Euler method of source location determination was investigated on several model situations pertinent to satellite-data scale situations as well as Magsat data of Europe. Our investigations enabled us to understand the end-member cases for which the Euler method will work with the present satellite magnetic data and also the cases for which the assumptions implicit in the Euler method will not be met by the present satellite magnetic data. These results have been presented in one invited lecture at the Indo-US workshop on Geomagnetism in Studies of the Earth's Interior in August 1994 in Pune, India, and at one presentation at the XXI General Assembly of the IUGG in July 1995 in Boulder, CO.

A new method, called Anomaly Attenuation Rate (AAR) Method (based on the Euler method), was developed during this study. This method is scale-independent and is appropriate to locate centroids of semi-compact three dimensional sources of gravity and magnetic anomalies. The method was presented during 1996 Spring AGU meeting and a manuscript describing this method is being prepared for its submission to a high-ranking journal (the present version is attached as Appendix A).

The grant has resulted in 3 papers and presentations at national and international meetings and 1 manuscript of a paper (to be submitted shortly to a reputed journal).

Following is the summary of substantive findings of the study for the present satellite magnetic data:

1) Capability of the method in being able to outline the edges of thin and wide magnetic sources: From the perspective of the present satellite magnetic data (~400 km altitude), the lithospheric sources appear as thin plates (~40 km). Contrary to the published literature on this aspect of the method (e.g., Reid et al., 1990), our study upto this point has indicated that for similarly thin and wide sources (thickness to observation altitude ratio of ~1/10), the Euler method is not able to map the edges of the sources sufficiently accurately for non-vertical magnetic inclinations. However, it is possible to map the edges sufficiently precisely as one gets closer to the magnetic poles (i.e., where anomalies symmetrically overlie their sources) and when the remanent magnetization contribution is very small. The model studies in both 1-d and 2-d implementation of the method are in agreement. (This observation suggests that the method may be useful for satellite gravity data because gravity data does not suffer from the same limitations for the purposes of this method.) (See interim progress reports for illustrative figures.)

2) Capability of the method in being able to map the location and the depth of the concentrated magnetic sources:

This aspect of the study was more successful as it was origionally envisaged in the proposal. With the 2-d Euler inversion, it is possible to obtain a large number of (a concentration of) useful solutions for the Kursk magnetic anomaly, Ukraine, and the Kiruna magnetic anomaly, Sweden, from the European Magsat anomaly map. The anomaly attenuation rate (N) of about 2.5, for both these highly magnetic concentrated sources, was based on theoretical consideration that these sources are neither point or spherical sources (with $N = 3$) nor two dimensional (with $N = 2$) from the perspective of Magsat data. With the application of $N = 2.5$, the depths obtained ranged from the Earth's surface to the depth of 60 km. The concentrations of these solutions appear to form a triadic shape in the plan view. It is observed that the range of the depths obtained do not pinpoint the geologic source of the magnetic anomalies investigated (perhaps due to the errors in the data itself, but it is also possible that the geologic sources of these anomalies themselves are scattered throughout the upper lithosphere). However, it is important to realize that this is the first direct magnetic source location determination method that has yielded the source locations of Magsat anomalies within the Earth's upper lithosphere. This is particularly encouraging because most other direct techniques tried by earlier investigators (and also by us during this study) have yielded source locations of geologic magnetic anomalies in the ionosphere. It is also important to recognize the promise of this method for the future, lower altitude satellite magnetic data where the signal-to-noise ratio of the geologic component will be much higher than Magsat. In those cases, it will be possible to reduce the error envelope of the solutions and obtain more precise locations of the large-scale concentrated geologic/magnetic sources. (See interim progress reports for illustrative figures.)

These results and limitations of the method were presented in an invited lecture, cited as:

> Ravat, D., P.T. Taylor, and J.J. Frawley, Interpretation of Satellite Magnetic Anomalies, the Indo-US Workshop on Geomagnetism in Studies of the Earth's Interior, August 1994, Pune, India:

and at the XXI General Assembly of the IUGG, cited as:

> Taylor, P.T., J.J. Frawley, and D. Ravat, Applying Euler's Depth Method to Magsat Data, Boulder, CO, 2-14 July, 1995, IUGG Abstracts Week B, p.B79.

3) Development of a new method for finding centroids of semi-compact sources of gravity and magnetic anomalies:

Recognising the limitations of the Euler method in locating sources of potential-field anomalies from far-field range without the *a priori* knowledge of the anomaly attenuation rate, this new method, called the Anomaly Attenuation Rate (AAR) method, was developed. Based on extensive model studies and real data applications, this method is scale-independent and is able to detect centroids of sources with accuracy better than 10% of the centroid to observation elevation distance (depending on realistic noise level) regardless of the application. The method was used to find centroids of Kentucky, Kiruna, and Kursk Magsat anomalies (they all appear to be within lower crust). These are the first direct and objective estimates of the sources of satellite magnetic anomalies. The detailed discussion of this new method is written as a manuscript (to be submitted shortly to a reputed journal) and is attached as Appendix A.

The method and the results were also presented during the 1996 Spring AGU meeting, and are cited as:

> Ravat, D., and P.T. Taylor, 1996, Source Depths of Kentucky, Kiruna, and Kursk Magsat Magnetic Anomalies Derived from the Anomaly Attenuation Rate Method, Eos Trans. AGU., 77(17), Spring Meeting Suppl., S85.

REFERENCES LISTED

Reid, A.B., Allsop, J.M., Grasner, H., Millet, A.J., and Somerton, I.W., 1990, Magnetic interpretation in three dimensions using Euler deconvolution: Geophysics, v. 55, p. 80-91.

Appendix A

A new method of determining the depths to centroids of three-dimensional sources of magnetic anomalies with examples from environmental and satellite magnetic anomalies

Abstract. A new method, called the Anomaly Attenuation Rate (AAR) method, is developed for determining source-depth to the centroid (the geometric center) of "semi-compact" sources. The method involves computations of radial averages of AARs with increasing distances from a range of estimated source centers (estimated usually from the Euler method). For well-isolated magnetic anomalies from "semi-compact" sources, the theoretical AARs can range from ~2 (close to the sources) to ~3 (in the far-field region). When the estimated source centroid is incorrect, the far-field AARs either exceed far beyond or fall short of the theoretical value of 3. Thus, a graphical leveling-off of the far-field AARs around the value of 3 (considering the error-bars) indicates the upper (deeper) bound of the centroid location. Similarly, in the case of magnetics, the near-field AARs lower than their theoretical value of 2 indicate the lower (shallower) bound of the centroid location (although this bound can be used effectively only in a few cases). For satellite magnetic anomalies, the method is appropriate only for high amplitude, semi-circular anomalies due to the inherent low signal-to-noise ratio of satellite magnetic anomalies. Also, because satellite anomalies involve contribution from multiple sources, the centroid must be interpreted as a weighted average of geometric centers of multiple sources of

magnetization contrasts. Model studies indicate that the AAR method is able to place depths within ±20-30 km of actual center locations from 400 km observation altitude. Thus, the method may be able to discriminate between upper crustal, lower crustal, and mantle magnetic sources. The results from the prominent Kentucky anomaly are well-resolved (centroid depth ~30 km below the Earth's surface). For the Kiruna Magsat anomaly, despite the deleterious effects from neighboring anomalies, reasonable centroid depths could be obtained (between 20 and 30 km). The centroid depths are more scattered for the Kursk anomaly, ranging from 20 to 50 km depth. This spread of depths may indicate that magnetic anomalies due to the Kursk iron-formations and the lower crustal sources combine to form the Kursk Magsat anomaly.

Introduction

A new method of determining the depths to the centroids (geometric centers) of 3-dimensional, semi-compact sources of well-isolated magnetic (and gravity) anomalies is presented in this paper. The motivation of developing the method came from the inability of some of the well-known methods of magnetic depth determination in directly extracting the depth-to-source information for the presently available satellite magnetic anomalies of geologic origin. The problem of direct source-depth determination is most challenging for satellite magnetic anomalies for the following reasons: (1) even for the best of the satellite magnetic anomaly maps that can be prepared with the present state-of-the-art processing technology [see *Ravat et al.*, 1995], the anomaly signal-to-noise ratio is far lower than the anomalies from either ground or aeromagnetic survey data (the important source of "noise" for this problem mainly comes from imprecise external field removal); (2) because of the very high observation altitude (average about 400 km for Magsat), even distinct anomalies from satellite magnetic data represent the anomaly coalescing effect of multiple geologic sources or layers located at various depths in the Earth's upper lithosphere and spread regionally over the distances of hundreds of kilometers; and (3) also because of the high observation altitude, the problem of distinguishing the tops of the (bulk or ensemble) sources from their bottoms is ill-posed, especially for magnetic data (in the retrospect, the problem is nearly impossible). Despite the low odds of obtaining precise source-depths from the satellite data, obtaining the direct depth estimates within ±20-30 km of the actual source region (estimates which are not compromised by subjective choice of parameters) is still a useful endeavor in this particular case for various scientific reasons. First, the geologic origin of satellite magnetic anomalies has never been directly proven. The anomalies are the ultimate residual of a processing sequence that elimates modeled magnetic fields from the geodynamo and the external fields and suppresses non-replicatable parts that cannot be geologic in origin. While the satellite magnetic field we now believe to be geologic in origin [similar to the results of *Ravat et al.*, 1995] cannot certainly come from any known aspects any other source of the magnetic field but the geologic sources and it agrees many times with what we think the geologic magnetic field should represent, the origin ascribed to the satellite magnetic anomalies is ultimately a logical deduction. For this reason, it is a worthwhile task just to be able to prove that the source-depths of satellite magnetic anomalies lie indeed in the Earth's upper lithosphere (especially, when several direct techniques have failed). Second, on rock magnetic grounds, the source regions (i.e., where in the upper lithosphere) of satellite magnetic anomalies have been debated for the last twenty years. *Wasilewski et al.* [1979] and *Wasilewski and Mayhew* [1992] have argued that, in the continental setting, lower crustal mafic granulites represent the most prominent magnetic sources that can give rise to the observed satellite magnetic anomalies. On the other hand, *Toft and Haggerty* [1988] envisaged a possible contribution to the anomalies from mantle regions due to distributions of metallic iron in the mantle. This view was based on the native iron (which has higher Curie temperature than the magnetic iron-oxides commonly encountered in the Earth's crust) found in some of their West

African xenoliths, but could be extended to ancient cratonic regions that have been stable and in a low oxidation (or reducing) state for a long period of time [*Toft,* 1989; personal communication]. Bulk magnetization of such a mantle down to the Curie depth of metallic iron will be far lower than the lower crustal granulites and, thus, no magnetic depth determination technique is likely to resolve it separately from the lower crustal granulite sources, especially from satellite altitudes. Despite this difficulty, an independent evaluation is still valuable of the principal source depths of geologic magnetic sources whether in the Earth's upper crust, lower crust, or mantle.

Previous Source-depth Determination Attempts From Satellite Magnetic Data

The main purpose behind discussing these previous source-depth determination attempts is to focus on the inadequacies of the methods for satellite altitude magnetic data. We believe that benefits of this hindsight will be appreciated by unwary researchers venturing into this task.

Horizontal Gradient-based Techniques

Attempts of source-depth determination using the usual empirical techniques (e.g., techniques discussed in *Dobrin* [1952] and *Vacquier et al.* [1963]) from satellite magnetic data and also from modeled magnetic anomalies at satellite altitudes with sources placed in the crust yield depths of sources in the "atmosphere" at altitudes between 200-300 km. Some of these attempts are discussed by *Regan and Marsh* [1982]; similar results were also obtained during this study with the refined Magsat data set and with the modeled anomalies with the "straight-slope" technique (attempted during an extremely trying phase of failed attempts of objectively using the Euler method in this study). *Skilbrei* [1993] considered effects of the width and the depth extent of modeled magnetic sources (suitable for ground-magnetic and aeromagnetic elevations) on "straight-slope" depth-determination. He showed that as these parameters (width, depth-to-top, and depth extent) of the source vary, the multiplication factor used in the straight-slope estimate changes. The results of *Skilbrei* [1993] can be stated even more emphatically for satellite altitude data: no objectively usable multiplication indices work for the purpose of depth-determination (with sufficiently small error bounds) from the techniques that use solely the horizontal anomaly gradient. The problem of inconsistent indices caused at high altitudes demonstrates scale dependence of the empirical methods. In general, a scale-independent method should have a better chance of success in objectively determining depths.

Spectral Techniques

One of the widely used scale-independent methods based on the spectral information content of magnetic anomalies from ensemble of sources was devised by *Spector and Grant* [1970]. A similar method based on global harmonic expansions of the field tends to yield an average depth to the top of all magnetization contrasts near about 60 km into the Earth's lithosphere (*C. Voorhies*, 1996; personal communication). As an average, this estimate to depth to the top of an 'average' magnetic layer can be considered too deep because even in areas of exceptionally low geotherms the Curie temperature of magnetite will be reached at this depth. In order to sense the depth to the bottom from the average Magsat altitude from the spectral techniques, the physical dimension of a magnetic profile (or the smallest side of a rectangular area considered) would need to be at least 5300 km long [based on *Connard et al.*, 1983; *Blakely*, 1995]. Obtaining signals at such long wavelengths comprised purely of geologic origin is nearly an impossible task because of the interference from the truncated main field (13th degree and order) and the imperfections in ionospheric field corrections. Referencing the above spectral techniques, *Agarwal et al.* [1992] have, perhaps mistakenly, used incorrect formulas and methodology to arrive at depths to the bottom of a magnetic layer under the Deccan Plateau in central India from Magsat data and, therefore, their depths cannot be taken as valid depth estimates from satellite data. Their figures clearly illustrate that the information regarding the depth to the magnetic bottom is not present in their plots (due partly to their use of inadequate profile and map dimensions ranging between 1200-1800 km) and if one uses their figures to compute the depth to the top of a magnetic ensemble of sources (after correcting for inaccurate labeling), these depths lie at about 400 km altitude in the atmosphere.*

The Method of Anomaly Moments

Bhattachayya and Leu [1975] developed a wavenumber-domain method of determining centroids of sources of aeromagnetic anomalies from the moments of anomalies. This method was extended for ensemble of sources by *Okubo et al.* [1985, 1989]. We have not attempted these methods on satellite anomaly data sets.

The Euler Method

The readers are referred to *Ravat* [1997] and references therein for a more complete analysis of the Euler method. Here only aspects pertinent to this paper are summarized and, when not referred specifically to other references, all of the following statements in this subsection are based directly on the results of *Ravat* [1997] and references therein.

The method, based on Euler's theorem of homogeneous functions, is scale-independent and is extremely valuable for two classes of problems. The first class of problems is for sources of potential-field anomalies that can be considered dipoles or magnetized spheres (or monopoles or spherical masses in the case of gravity anomalies),

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* [Note to Reviewers: The intent here is not to attack someone. We completely understand that genuine mistakes are made by everybody; we are no exception. But if one reads this paper and the references therein carefully, one gets the impression of gross carelessness and disregard for the very minimum of scientific standards. None of the references they quote for their formula give the formula given in the above paper. Moreover, the formula they use cannot be correct because the depth in the formula is directly dependent on the "gate length" of the profile or the map; consequently, the longer the gate length, the deeper the derived depth. We will remove this "note" after the review.]

lines of dipoles or infinitely long magnetized cylinder (or similar cylindrical masses), and boundaries of magnetization contrasts (or mass contrasts) [e.g., *Thompson*, 1982; *Reid et al.*, 1990; *Blakely*, 1995]. When the source of an anomaly can be ascribed (without violating the shape assumptions) as one of the above, the source locations from this method are fairly precise. This is because one of the key parameters needed in the use of this method is the rate at which an anomaly attenuates (or the anomaly attenuation rate (AAR)) with increasing distance from a source and the value of the AAR is dependent on the shape of the source (for magnetics: sphere = 3, cylinder = 2, step = 1; for gravity: sphere = 2, cylinder = 1, step = ~0). The importance of meeting these assumptions is that as long as the shape assumption remains valid (at all source-to-observation distances encountered in the analysis of a given anomaly), the prescribed AARs do not change with changing source-to-observation distance. The shape assumptions of the method for the latter two cases are met realistically only for low altitude surveys. Many geologic sources of interest are arbitrarily three-dimensional (meaning, somewhere in between an infinite two-dimensional shape and a sphere) and, hence, the AARs are somewhere in between 2 & 3 for magnetic and 1 & 2 for gravity applications. The problem in these cases is compounded substantially because, for such non-Euclidean AARs, the AARs themselves change with changing source-to-observation distances. Thus, as a general case, it is not possible to determine the source-depths of such sources with the Euler method. As mentioned earlier, analysis of modeled and actual satellite anomalies showed that the large magnitude of objectively determined error-bars from these examples undermined the utility of the derived depth estimates. However, the horizontal location of the centroid of a three-dimensional source can be reasonably well-determined regardless of the precision of the AAR.

The second class of depth-determination problems for which the use of the Euler method can almost always yield acceptably small errors is when source-to-observation distances are very small. This is a result of the scale-independent nature of the method; in other words, keeping all other factors the same, the percent error in depth (from the observation elevation) is same regardless of the source-to-observation distance and, consequently, the depth errors in physical units are larger for larger source-to-observation distances and vice versa. A retrospective calculation of a blind worst-case scenario for an arbitrarily three-dimensional source suggests a 25% depth error, which from satellite atlitudes translates into ± 100 km depth error - a hopelessly useless depth-determination at best.

Werner Deconvolution

The Werner deconvolution method [*Hartman et al.*, 1971; *Jain*, 1976] is also scale-independent, but is appropriate for the analysis of magnetic anomalies whose sources can be regarded as either thin dikes or wide sills from the observation elevation. Examination of satellite magnetic anomaly maps shows that the assumptions regarding source shapes are not satisfied by satellite magnetic anomalies and, therefore, the method is not suitable for the analysis of satellite magnetic anomalies.

A New Method For Locating Centroids Of Compact, Three-dimensional Sources From Well-isolated Anomalies: The Anomaly Attenuation Rate (AAR) Method

Having established that most present depth-determination methods are not capable of objectively finding source depths from specific satellite magnetic anomalies, a restricted class of such problems was examined, particularly with the Euler method, for a methodology that might work in a given set of restrictive circumstances. This resulted in the development of a new method, named here the anomaly attenuation rate (AAR) method.

In the AAR method, it is assumed that reasonably well-isolated anomalies are available from "semi-compact", three-dimensional (but not necessarily idealized) sources of potential-field anomalies. The premise of the method is that as one goes farther away from sources of three-dimensional geometry, the anomaly attenuation rate should approach the value of 3 in magnetic and 2 in gravity applications. This premise also applies to an ensemble of three-dimensional sources whose individual anomalies coalesce into an anomaly that appears indistinguishable from the anomaly from a single arbitrary three-dimensional source. The word "semi-compact" is not intended to mean spherical, but sphericity is one extreme of its range. It is particularly used here to define anomalies of sources whose anomaly attenuation rates vary between those of two-dimensional sources and spheres, <u>at the given source-to-observation distance</u>. As a qualitative guide, an average steel drum can be regarded as semi-compact by a magnetic anomaly sensed 1 or 2 m away from the drum; a large igneous pluton is semi-compact to a magnetic anomaly sensed at 30 or 40 km elevation above the pluton; and a fairly large block of crust, say a few hundred kilometers on its sides, is semi-compact to a magnetic anomaly sensed at near-Earth satellite altitudes (in all these magnetic cases; the anomaly attenuation rates will be between 2 and 3).

The concept is illustrated in Figure 1. In explaining the method however, we take liberty to postpone the discussion of model studies and discuss examples from simple, but measured anomalies from steel-drums (Figure 2; see *Ravat* [1996] for the examples). Referring to Figure 1, the method can be divided into three steps: (1) First, the horizontal geometric center of magnetization (or mass, for gravity) of the three-dimensional source is found through the Euler method using N (the structural index) in the neighborhood of 2.5 (as shown in *Ravat* [1997], the horizontal location of the source is not compromised by the imprecise knowledge of the N); (2) Next, with a small moving window (small with respect to the width of the anomaly) and using the horizontal source-center locations determined in the first step, the local anomaly attenuation rates are computed over the entire anomaly, <u>for</u>

a sequence of (assumed) depths to the centers of the three-dimensional source. To obtain more representative values of the anomaly attenuation rates, the calculated AARs are radially averaged (i.e., symmetrically with respect to the (assumed) source-centers) according to the distance from the (assumed) centroid locations; and (3) These AARs and the average of their standard error [e.g., *Press et al.*, 1986] are then plotted with respect to the source-to-window distance, for each assumed depth to the center location used in the second step. Figures 2a and 2b show the examples of these plots from the measured anomalies of a single 55 gallon drum (placed vertically on the ground with sensor elevation 8') and a set of four such drums (placed adjacent to each other in a square formation).

Because the upper limit of the theoretical anomaly attenuation rate is 3 for compact magnetic sources (2 for compact mass variations), one can exclude the centroid-depths of the second step that result in AARs much higher than the theoretical maximum value (considering the errors in AARs, Figure 2). This is an upper bound for the deepest possible centroid location. On the other hand, if a window directly above the horizontal center of the source yields anomaly attenuation rates much lower than the theoretical minimum of the three-dimensional sources (i.e., 2 for magnetics) (considering once again the errors in AARs), then those source depths can be discarded (this does not happen in the examples shown in Figure 2, but will be illustrated later). The acceptable AARs on the theoretical minimum end of the spectrum constitute a bound for the shallowest possible centroid location. Because, theoretically, AAR is expected to change gradually with the distance to the sources, the AAR curves should be smooth. The degree of their raggedness generally reflects data quality and the validity of the estimate of the average of the derived AARs (the validity depends on the quantity of AARs averaged) and it may be used qualitatively along with the above error-bars in the decision-making process. As seen in the magnetic anomaly examples in Figure 2, this method works quite effectively in determining the centroids of semi-compact, three-dimensional sources (with error level in the determination better than 4% of the centroid-to-observation distance).

The flatter portions of the AAR curves in the far-field region (near the AAR value of 3 in the cases in Figure 2) are quite important in making the determination that anomalies being considered are adequately isolated and interference from neighboring anomalies is not a compromising factor. When the far-field flatter portions are not observed (i.e., when the curves continue to climb without even a slightest indication of leveling off near the AAR value of 3), no valid determination of the maximum depth to the centroid can be made. Such cases can arise from the interference from neighboring anomalies. Also, even in the cases of well-isolated anomalies, in the far-field regions the signal-to-noise ratio is generally lower than over the central parts of the anomaly (because the signal is often close to zero). In such cases, the flatter portions of the AAR curves may oscillate depending on the signal-to-noise ratio in the given situation. As an example, in Figure 2b, where the signal is roughly four times that of the case of a single drum (Figure 2a), the far-field portions of the AAR curves are a lot smoother (and in fact, the error bars of AAR values are also smaller than the plotting symbols) than the similar far-field portions of the curves from the single drum example. These additional criteria can be useful in understanding the quality of the derived result from this method.

It has been erroneously reflected by some people at scientific conferences that this method is suitable only for point sources or spheres or similar objects. This is not so. In the case of such simpler shapes, the AARs at all source-to-observation distances yield the theoretical maximum value of AAR (e.g., showing a flat line at the AAR value of 3 in the graphs similar to the above for the curve for the correct depth).

Model Studies

Generalized Model Studies

Generalized scale-independent model studies were performed to explore the advantages and limitations of the method at different magnetic latitudes. The model of a cylinder was used to compute anomalies (Talwani, 1965) with the source magnetized by induction in inducing field inclinations ranging from 0° to 90° (source characteristics: radius = 20 units; depth to the top = 0 units; depth to the bottom = 20 units; observation elevation = 30 units above the depth to the top). Graphs of AAR vs. Centroid-to-observation window distance for the cases with inducing inclinations 30°, 45°, and 75° are presented first because they are similar to the steel drum examples shown earlier (Figures 3, 4, and 5, respectively). One presentation difference from the previously shown examples is that in labeling the AAR curves, instead of the depth error, the various assumed depths to centroids used in the calculation of the AAR curves are shown (because in realistic cases, the true depth errors are not known). Parts (a) of the figures show many AAR curves; for clarity, only the acceptable curves are shown in parts (b) (i.e., acceptable using the criteria discussed earlier in the steel drum examples). All these cases show that, based on objective evaluation criteria, the inferred source centers are slightly deeper than the actual centroids of the modeled source (error level in the determination better than 10% of the centroid-to-observation distance). Based on a small number of tests, the most likely reason for the deeper estimate in these cases is the large diameter of the source in comparison to the centroid-to-observation elevation distance .

The cases between 0° and ~15° inducing inclination are problematic (but for a different reason than usual problems at low magnetic inclinations) due to the peculiarity of the derived AARs when the observation window crosses the anomaly zero values (examples of these cases are not shown here). When the window crosses the anomaly zeroes, the derived AARs go through a crossover effect (similar in appearance to gravity/magnetic edge-effects at the boundaries of sources), the examples of which are discussed at length in *Ravat* [1997]. One example of this crossover effect is presented for the 90° inducing inclination case below. When these AAR crossovers are radially averaged, they show a number of oscillations in the AAR curves which make difficult the determination of acceptable AAR curves. The process of radial averaging makes it possible to average out the crossover effect in the mid-latitudes (30° to over 75°).

The cases close to 90° inducing inclination can also be problematic, due to the same AAR crossover effect, particularly when the anomalies are nearly symmetric with respect to the centroids of their sources. For circularly symmetric anomalies, there is no advantage in radial averaging the AARs - and thus, the zero crossover effect once again makes difficult the determination of acceptable curves (see Figure 6). The explanation of the "inflections" of the AAR curves in Figure 6 is cumbersome and, therefore, the interested readers are referred to *Ravat* [1997]. Despite the difficulty, if one ignores the near-field parts of the AAR curves and focusses only on the far-field AARs, using the curve selection criteria for the far-field region (discussed earlier), one can obtain reasonable estimates (or at least as robust as the mid-latitude estimates) of depths to the centroids of sources.

The results of the model study confirm that the AAR method is useful in determining reasonable estimates of centroids of sources of magnetic anomalies (and by inductive reasoning, also gravity anomalies). For wide source bodies in comparison to the observation altitude, the derived depths will be slightly deeper than the true centroids (but further studies are required in establishing the exact relationships).

Application Of The Method On Satellite Anomalies

Spherical coordinate implementation of the rectangular coordinate Euler equation would have been necessary to study satellite anomalies for the purpose of determining the

AARs in the most general manner (an early version of such a program is available). However, recognizing that the AAR method is applicable only for anomalies of limited spatial extent and originating from "semi-compact" sources, the spherical coordinate implementation is deemed unnecessary. It is also easier to calculate vertical derivatives of anomalies from rectangularly-gridded data sets due to the availability of well-known transformations [see, for example, *Blakely*, 1995]. Thus, it is most optimum to transform spherically-registered (latitude, longitude) data to a rectangular grid using appropriate map projections and then gridding techniques [e.g., minimum curvature, *Briggs*, 1974]. These data and the derivatives can be filtered with a high-cut filter to reduce noise due to gridding (when needed); the high-cut filtering (with a cutoff wavelength of about 200/300 km, which is smaller than the wavelength of the signal content in the satellite data) is required for the satellite altitude anomalies due to low signal-to-noise ratio of the satellite data, but it was not required for the environmental cases discussed earlier.

Satellite Magnetic Anomaly Model Study

A model study, with observation and grid parameters similar to the real satellite data, was also performed using the methods outlined above. Spherically-registered modeled anomalies were generated using a few different spherically-registered magnetic source models [using Program SPHERE II, *von Frese et al.*, 1981; modified by *Ravat*, 1989]. Figures 7a and 7b show the examples of the anomaly and the AAR curves from a source about 6° in latitude and longitude centered at 52.5° latitude, extending from 10 km to 30 km in the Earth's crust (anomaly calculated at 400 km elevation). The AAR curves in this and other satellite examples tend to be clustered together because of high observation altitude of satellite data in comparison to dimensions of the source regions. In other words, because of the high observation altitude, the source depths are not as well-resolved as in the other examples shown earlier (this is obviously no surprise) and, as a result, a number of AAR curves distributed over a large depth extent can be deemed acceptable using the selection criteria presented earlier. Despite the limited depth resolution, the acceptable curves/depths are not very far from (certainly within ±30 km) the true depth of the centroid (20 km) (error level in the determination better than 8% of the centroid-to-observation distance). Eventhough this amount of error cannot be useful for solving many geologic problems, the resolution of the method is adequate in addressing various issues related to interpretation of satellite magnetic data (discussed in the introduction).

Approximate Depths of Kentucky, Kursk, and Kiruna Magsat Anomalies

There are only a few Magsat anomalies that meet the criteria/assumptions of the AAR method; namely, the criteria of adequate signal-to-noise ratio, appropriate areal extent of the anomalies, and the form of the anomalies resulting from the net effect of direction of magnetization and the direction of the Earth's field (i.e., the adequate form of the anomalies is what is normally observed from induced magnetization for magnetic field inclinations between ~30° and ~90°). It is important to note that the above description and the criteria do not preclude anomalies that have significant contribution from remanent magnetization. In fact, anomalies of both the steel drum examples discussed earlier contain a large contribution from remanent magnetization [see *Ravat*, 1996]. Results from three anomalies that appear to fit the above assumptions/criteria are presented here. These are the well-known Kentucky, Kursk, and Kiruna Magsat anomalies. There are a number of other high amplitude anomalies in Magsat data in the polar regions; however, in this initial study, the anomalies with questionable amount of ionospheric noise were not dealt with. The Bangui Magsat anomaly, on the other hand, is too close to magnetic equator and also has a considerable east-west extent and thus is not deemed adequate for this study.

In the analysis of individual anomalies, it is advantageous to apply the AAR method on residual magnetic fields of the sources. This is because the method depends on the observations of flat portions of the AAR curves in the far-field region; if, in the far-field region, the effect of neighboring sources is prominent, then the flattening of the AAR curves cannot be observed. In the far-field region, the amplitude of signal due to the sources/anomaly of interest is also small in comparison to regions closer to the sources. Unfortunately, attempts of obtaining residuals from the above three anomalies using a number of different state of the art techniques did not yield satisfactory results (to some degree not precisely knowing the amount of remanence puts further limitations on knowing what the residual should look like. As discussed later, remanence is undoubtedly present in the cases of Kentucky and Kursk anomalies). Interference from "neighboring sources" can be minimized by limiting the area of the map for computation of the AARs and for the radial averaging to only parts that unambiguously appear related to the anomaly of interest. This approach of minimizing the effect of neighboring sources can limit the identification of acceptable AAR curves because the approach truncates the far-field, flatter parts of the AAR curves. When the effect of neighboring sources is prominent, the AAR curves continue to climb, without leveling off in the far-field region, but sometimes giving a hint of a possibly acceptable solution through a sudden change in the curvature of the AAR curves (as seen in the example of AAR curves from Kiruna Magsat anomaly below).

Figures 8, 9, and 10 show the anomalies and the derived AAR curves from Kentucky, Kursk, and Kiruna Magsat anomalies, respectively. The anomaly parts of the figures also show the locations of clusters of Euler solutions computed for the purpose of identifying the horizontal geometric center of magnetization of the collection of the sources of these anomalies (the first step of the method). The respective AAR curves were derived with respect to the horizontal center location and assuming a different depth (for each of the AAR curve) (the second step). The AAR curves for the Kentucky Magsat anomaly (Figure 8b) have the best resolving power of the three anomalies presented; only one curve, representing the depth of 30 km, shows some flattening near the AAR value of 3. For the Kursk Magsat anomaly, the AAR curve representing the depth of 50 km shows the most far-field flattening and the acceptable near-field values (Figure 9b). However, it is possible that the curves for the depths 20, 30, and 40 km could also meet the criteria considering the errors in the derived AARs. Had it been possible to extend the area of compilation of AARs without including the effects of neighboring anomalies, a more robust determination of the depth to the centroid of the Kursk Magsat anomaly would have been possible. The effect of the neighboring sources is most prominently observed in the AAR curves for the Kiruna Magsat anomaly (Figure 9b). In this case, the "knee" or abrupt change in the curvature of the AAR curves near the AAR value of 3 is the only indication that the two of the AAR curves (the depths of 20 and 30 km) could have leveled off, had there been no interference from neighboring anomalies. The inferred depths, however, are not unreasonable and as suggested later in the discussion section may very well represent realistic geometric centers of magnetization of the sources of Kiruna Magsat anomaly.

Discussion

Source Depths

Previous interpretations of depths to sources of Kentucky, Kursk, and Kiruna Magsat anomalies are consistent with the depth to the centroids obtained in this study. The Kentucky Magsat anomaly overlies one of the most intense areas of aeromagnetic anomalies in the U.S. (~ 1000 nT in amplitude) and occurs at the intersection of Grenville Front in Kentucky/Tennessee and the line of positive magnetic anomalies associated with the Tennessee-Illinois-Kentucky Lineament [TIKL, *Ravat*, 1984; also called South-Central Magnetic Lineament or SCML by *Hildenbrand*, 1985]. *Mayhew et al.* [1982] have forward-modeled the magnetic sources of part of the Kentucky aeromagnetic anomaly with realistic, but high magnetization contrasts; the vertical extent of sources in their model cover almost the entire crust. Thus, the derived 30 km depth to the centroid of Kentucky Magsat anomaly from the AAR method is very realistic in this part of the midcontinent of the U.S.A.

Both Kursk and Kiruna Magsat anomalies are known worldwide for their rich near-surface iron deposits. However, in modeling these anomalies, *Ravat et al.* [1993] found that realistic estimates of the dimensions and magnetic properties of the iron-ore and other nearby iron-rich formations were insufficient in producing the satellite magnetic anomalies associated with these iron deposits. Therefore, in addition to the iron deposits, they hypothesized deep-seated crustal sources for these Magsat anomalies. In addition, *Taylor and Frawley* [1996] have recently estimated source depths of aeromagnetic anomalies over Kiruna, Sweden, using spectral techniques. Their results suggest multiple ensemble of sources, with the top of the deepest determinable ensemble in the neighborhood of 9-10 km (bottom not sensed). In the context of the results of these studies, the depths of centroid derived from the AAR method are very reasonable for both Kursk and Kiruna Magsat anomalies.

For all three Magsat anomalies studied, independent estimates/models discussed above infer the vertical extent of the magnetic sources from near-surface to deep crustal levels (considering that these are some of the highest amplitude and wide wavenumber range aeromagnetic anomalies in world, it is not unreasonable to expect source regions of large horizontal and vertical dimensions). While the results of the AAR method indicate centroids of the ensemble of sources in the lower continental crust and are consistent with magnetization models that suggest origin of bulk of the satellite magnetic anomalies from highly magnetic lower crustal granulites [*Wasilweski et al.*, 1979; *Wasilewski and Mayhew*, 1992], these results cannot refute some contribution from the uppermost mantle, at least in these specific regions. Because the method determines approximate depth of the centroid of sources (and in all the above cases, ensemble of sources), bottom of the source cannot be readily inferred without the knowledge of the top of the source. Further reduction in the resolution of the method is caused by large source-to-observation distances involved in the cases of satellite anomalies. Despite these limitations, the results have proven by direct means that the sources of Magsat anomalies are indeed in the upper lithosphere (previously known from logical deduction).

Remanence

Another useful deduction of the study relates to the presence of remanence for the sources of satellite magnetic anomalies. Horizontal locations of the centers of sources inferred from the Euler method based on the clusters of maximum number of Euler solutions for Kentucky (southern part) and Kursk (southeastern part) Magsat anomalies (Figures 8a and 9a) lie significantly away from the likely locations of the horizontal centers of the sources, had the sources of these anomalies were magnetized by induction (considering the geomagnetic inclinations and declinations, the horizontal centers for these anomalies would have been in the northern parts of the anomalies). Because the determination of the horizontal centers of semi-compact sources from the Euler method is not affected by the presence of remanence, the derived center locations of the bulk sources of these anomalies indicate that remanent magnetization is an important contributor toward the cause of these anomalies. In the case of Kiruna Magsat anomaly, magnetic latitude is sufficiently high and therefore it is not possible to conclude the presence of remanence for this anomaly by direct means (although remanence may be present in the direction not far away from the induced component of magnetization).

A Possible Alternative To The AAR Method In Certain Situations

As an alternative to the AAR method, and only to explore another possible approach for the semi-compact sources, it is possible to upward continue the anomaly, by trial-and-error, to an elevation where the anomaly attenuation rate attains its maximum value. The centroid location can then be determined in one step by using the theoretical maximum value of AAR in the Euler method. However, for some sources the magnitude of the upward continued anomaly will decrease to a level that *a posteriori* error estimates in the location could be unacceptable (especially for sources where the near-field AARs are closer to the theoretical minimum value of the AAR). In realistic applications, with this alternative approach, problems related to the isolation of the anomaly (in other words, improper removal of the regional anomaly) may also be overemphasized in the upward continued data (because upward continuation is a smoothly-varying lowpass filter), leading to inaccurate results.

Conclusion

A new method, called the Anomaly Attenuation Rate (AAR) method, has been developed to determine centroids of semi-compact sources of magnetic anomalies. The method is shown to be scale-independent with examples of a wide spectrum of anomalies, from environmental to satellite magnetic data. The method leads to useful centroid locations when signal-to-noise ratio of the anomalies is high (accuracy better than 10% of the centroid-to-observation distance and significantly better when adequate residual magnetic anomalies are available).

Application of the method on Kentucky, Kursk, and Kiruna Magsat anomalies have shown that the centroids of these anomalies lie in the lower crust, supporting high magnetization of lower crustal granulites as the principal cause of the bulk of these anomalies. However, the results cannot entirely refute possible contributions toward the anomalies from the uppermost mantle of these specific regions.

The locations of horizontal centers of the sources with respect to the anomalies of Kentucky and Kursk conclusively show, by direct means, that remanent magnetization is a significant contributor toward these sources of satellite magnetic anomalies.

And finally, the results have proven by direct means that the sources of Magsat anomalies are indeed in the upper lithosphere (previously known from logical deduction) -

a conclusion that is very gratifying after working on the geologic interpretation of satellite magnetic data for over a decade.

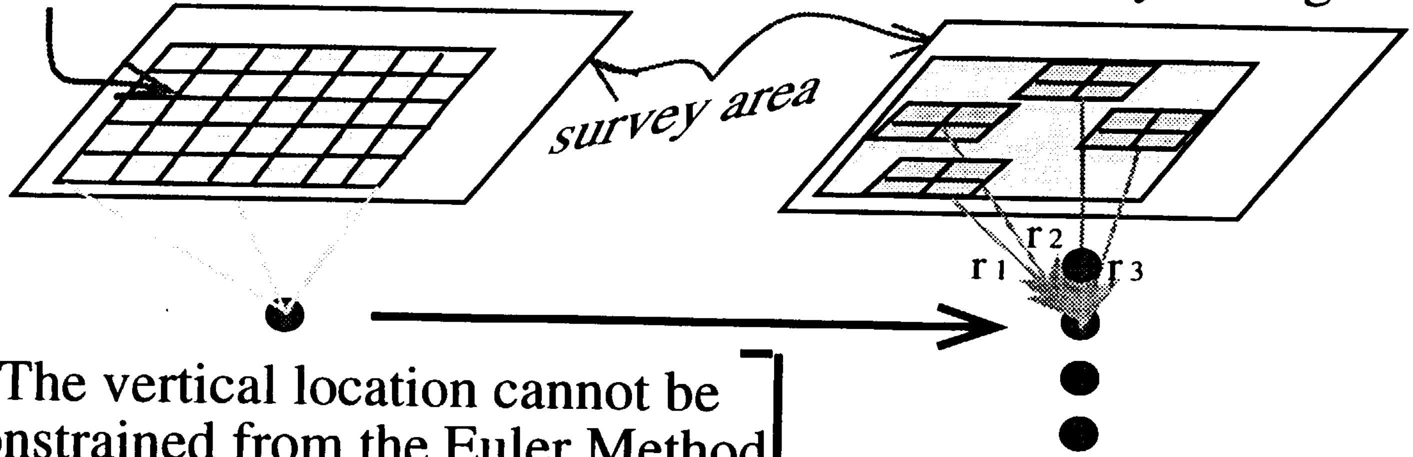
The Anomaly Attenuation Rate Method
(for "concentrated" sources)

Step 1

A cluster of Euler solutions
is used to determine
the horizontal location

Step 2

Assuming different depths
to center, AARs are calculated
and radially averaged



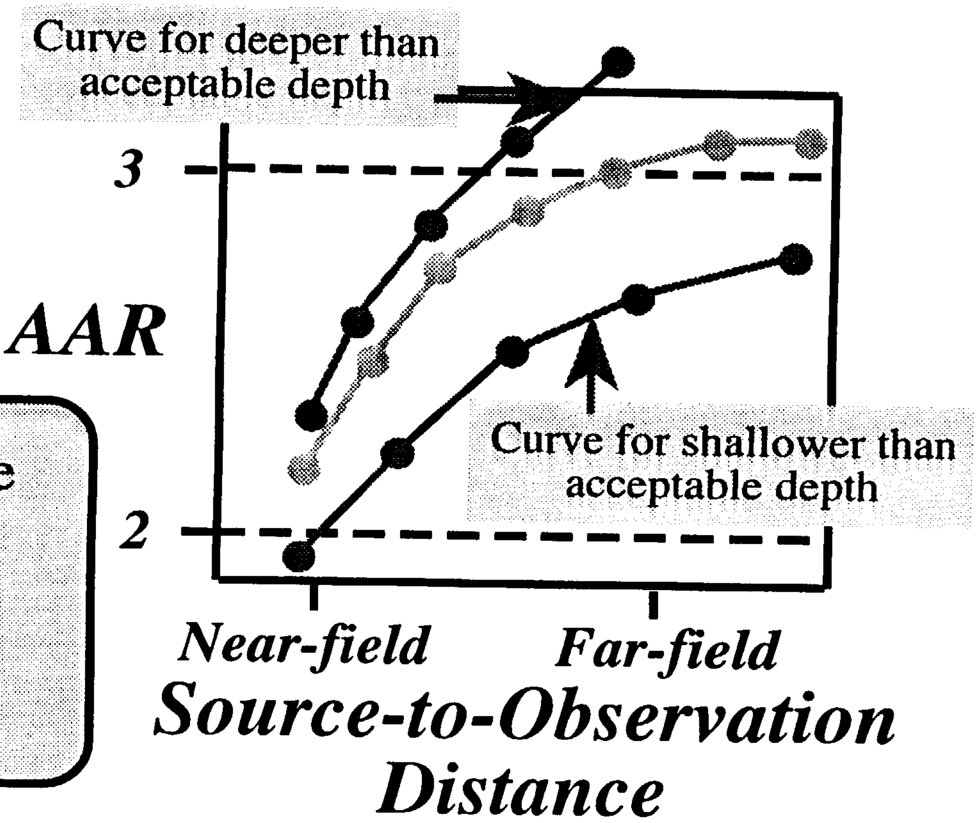
[The vertical location cannot be
constrained from the Euler Method
without the knowledge of *N*]

Figue 1

Step 3

For each depth,
the AARs are
graphically plotted
w.r.t. source-to-
observation distance

When Far-field AARs are
close to ~ 3 and when
Near-field AARs are
above ~ 2, the depth is
deemed acceptable



Figue 1 (continued)

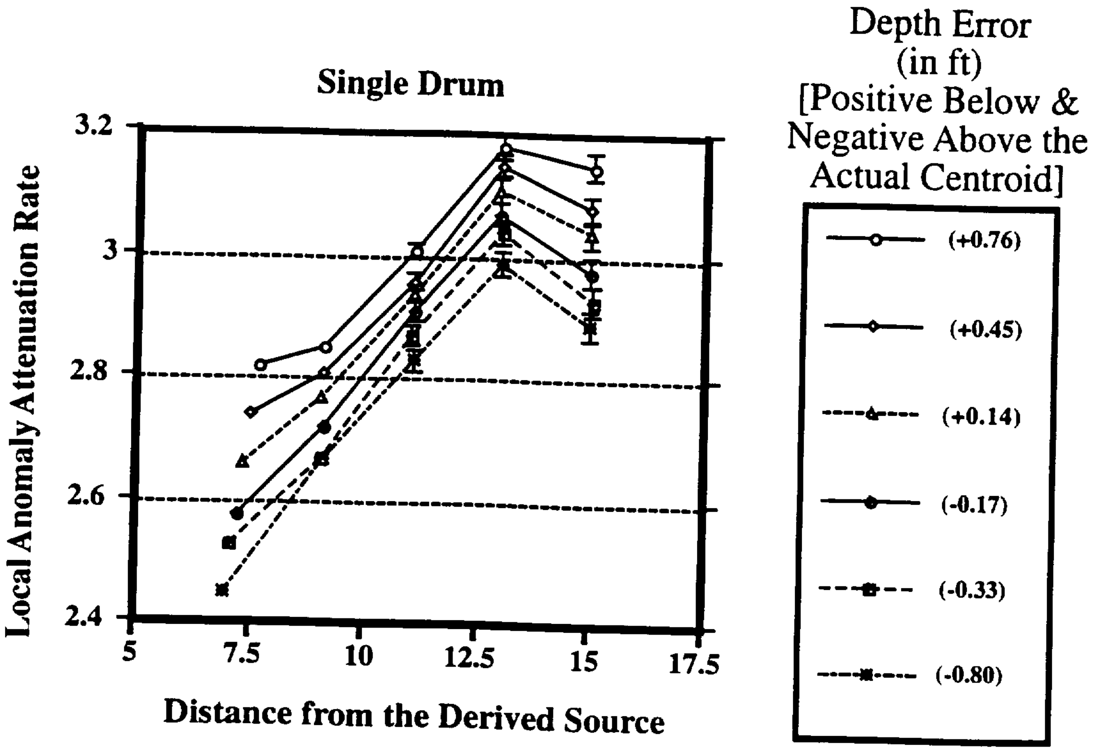


Figure 2a

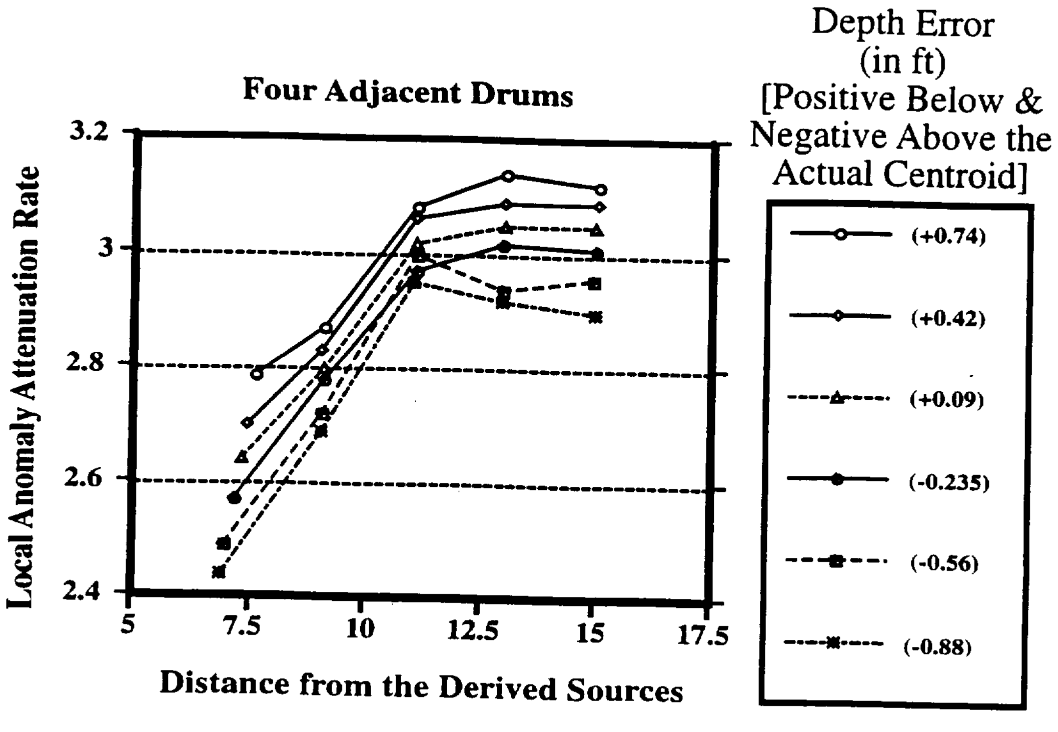


Figure 2b

Inclination = 30°

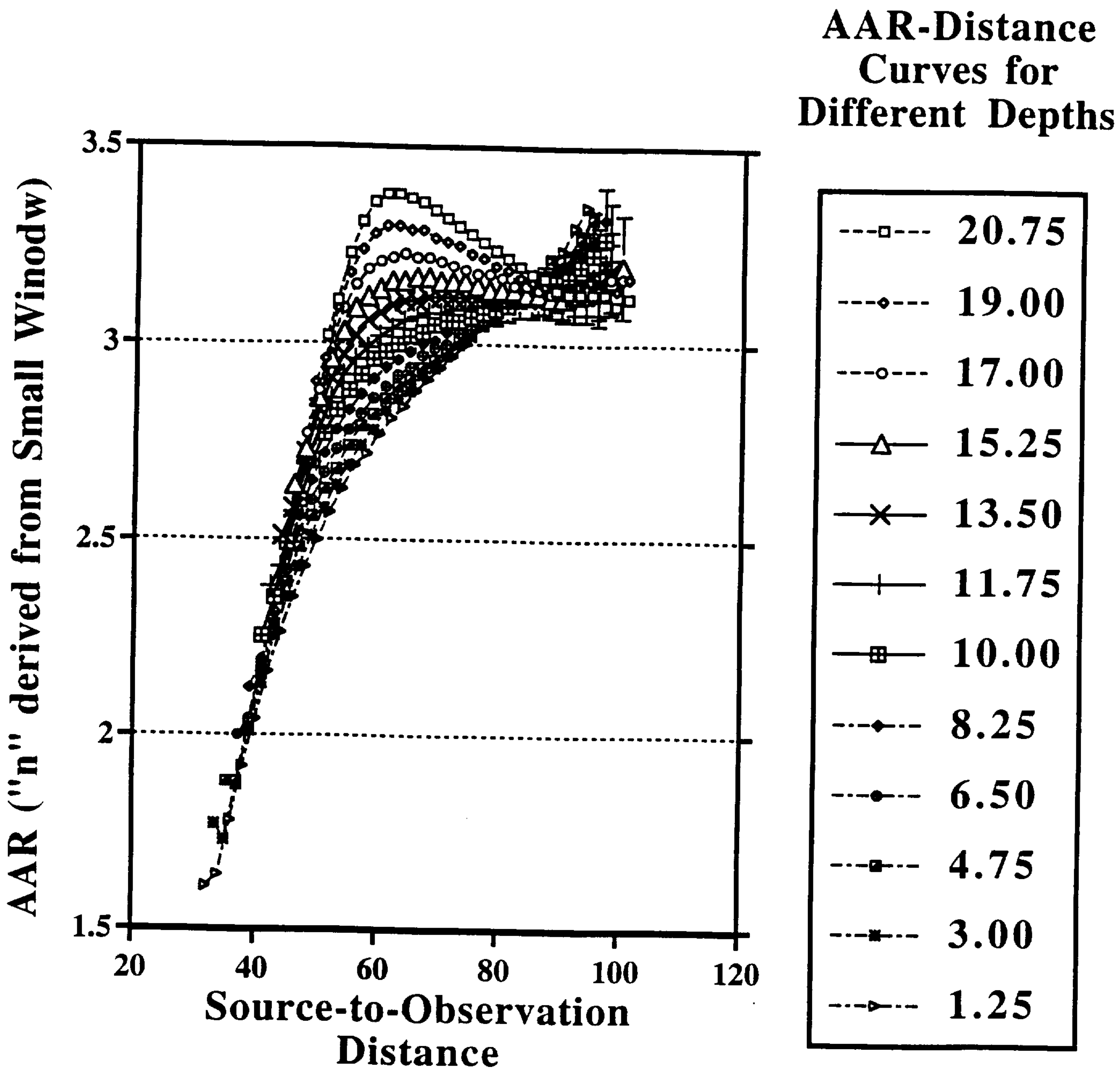


Fig 3a

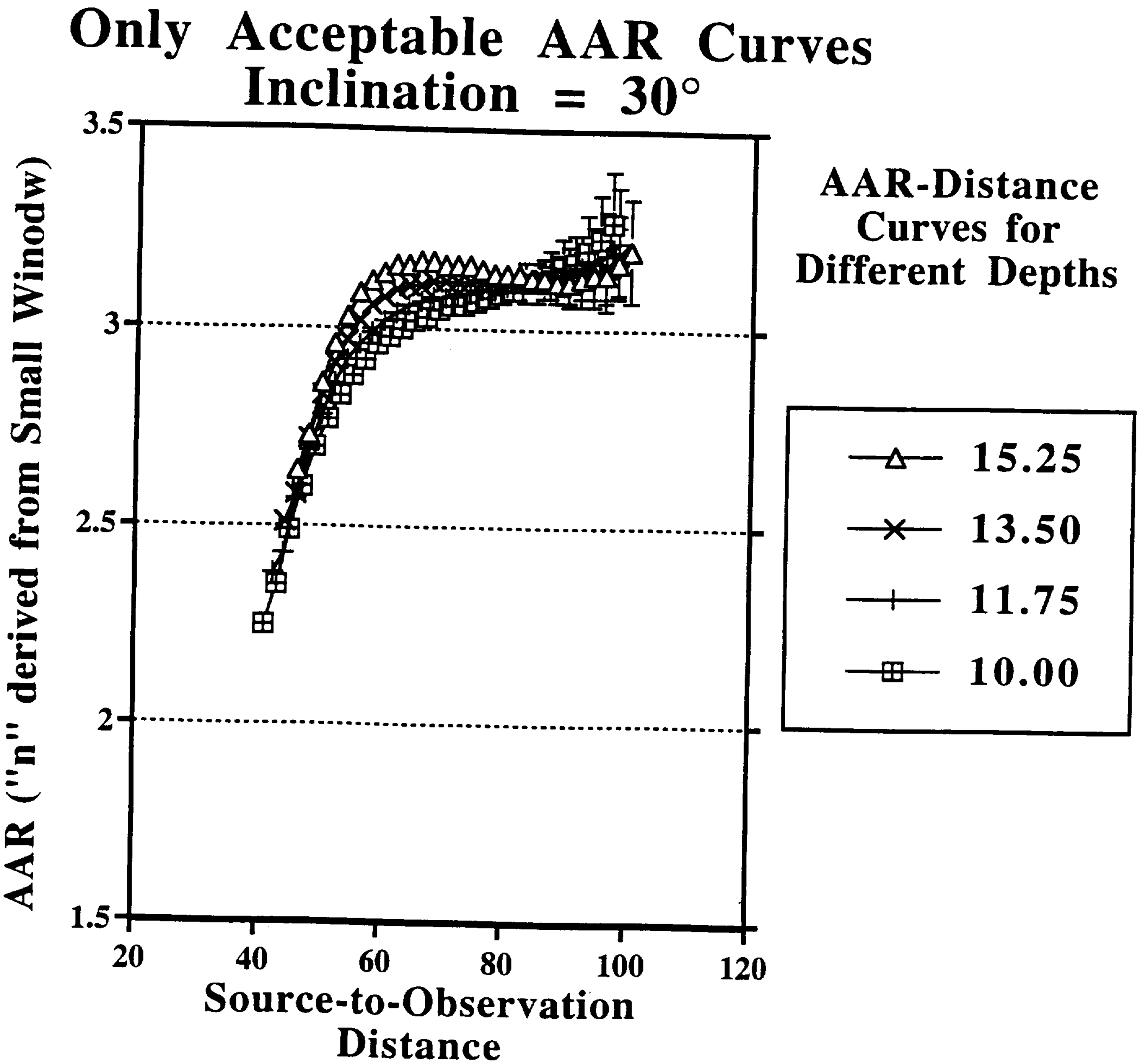

Only Acceptable AAR Curves
Inclination = 30°
AAR ("n" derived from Small Winodw)
3.5
3
2.5
2
1.5
20
40
60
80
100
120
Source-to-Observation Distance
AAR-Distance Curves for Different Depths
15.25
13.50
11.75
10.00

Fig 3b

Inclination = 45°

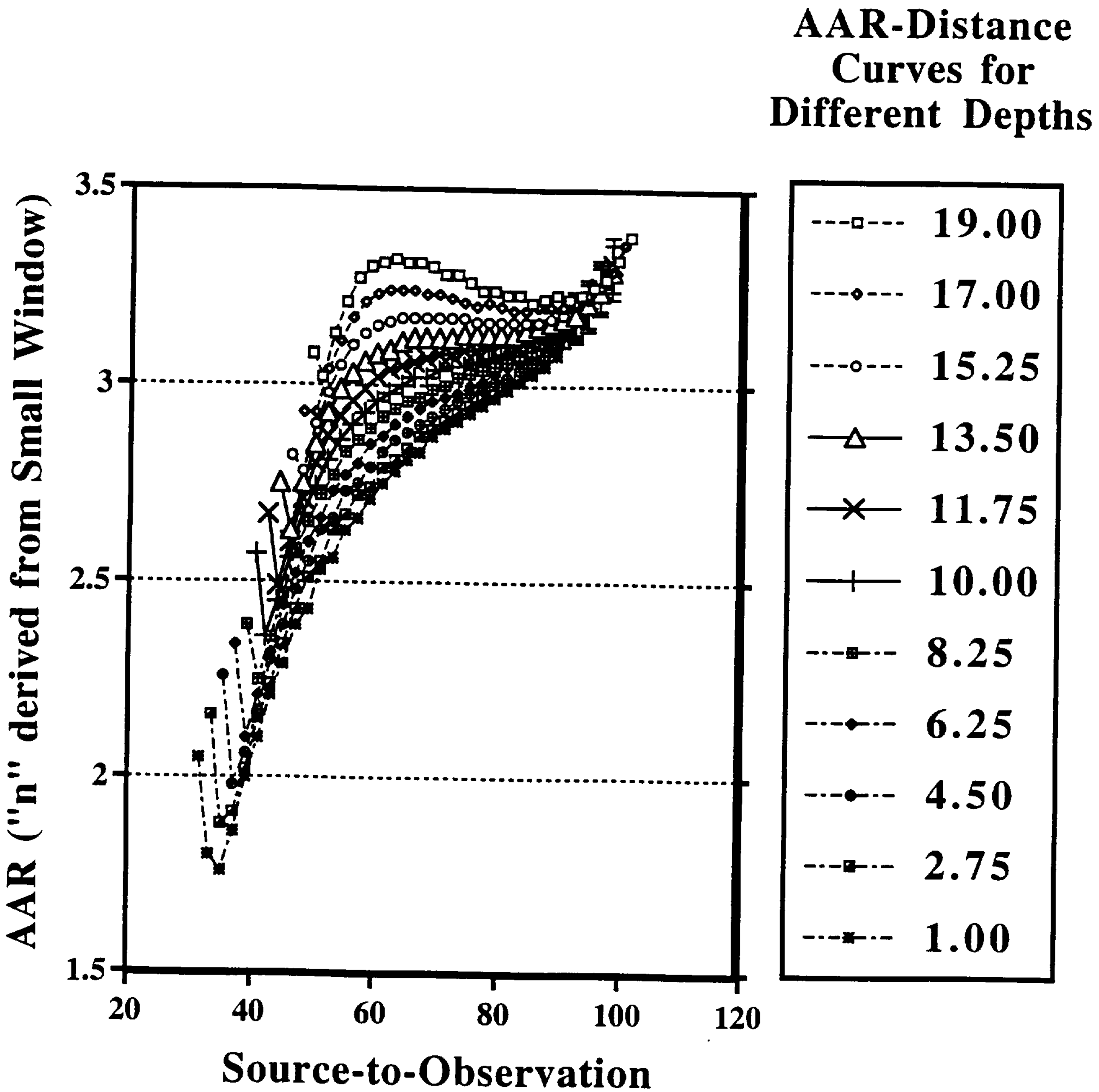


Fig. 4a

Only Acceptable AAR Curves
Inclination = 45°

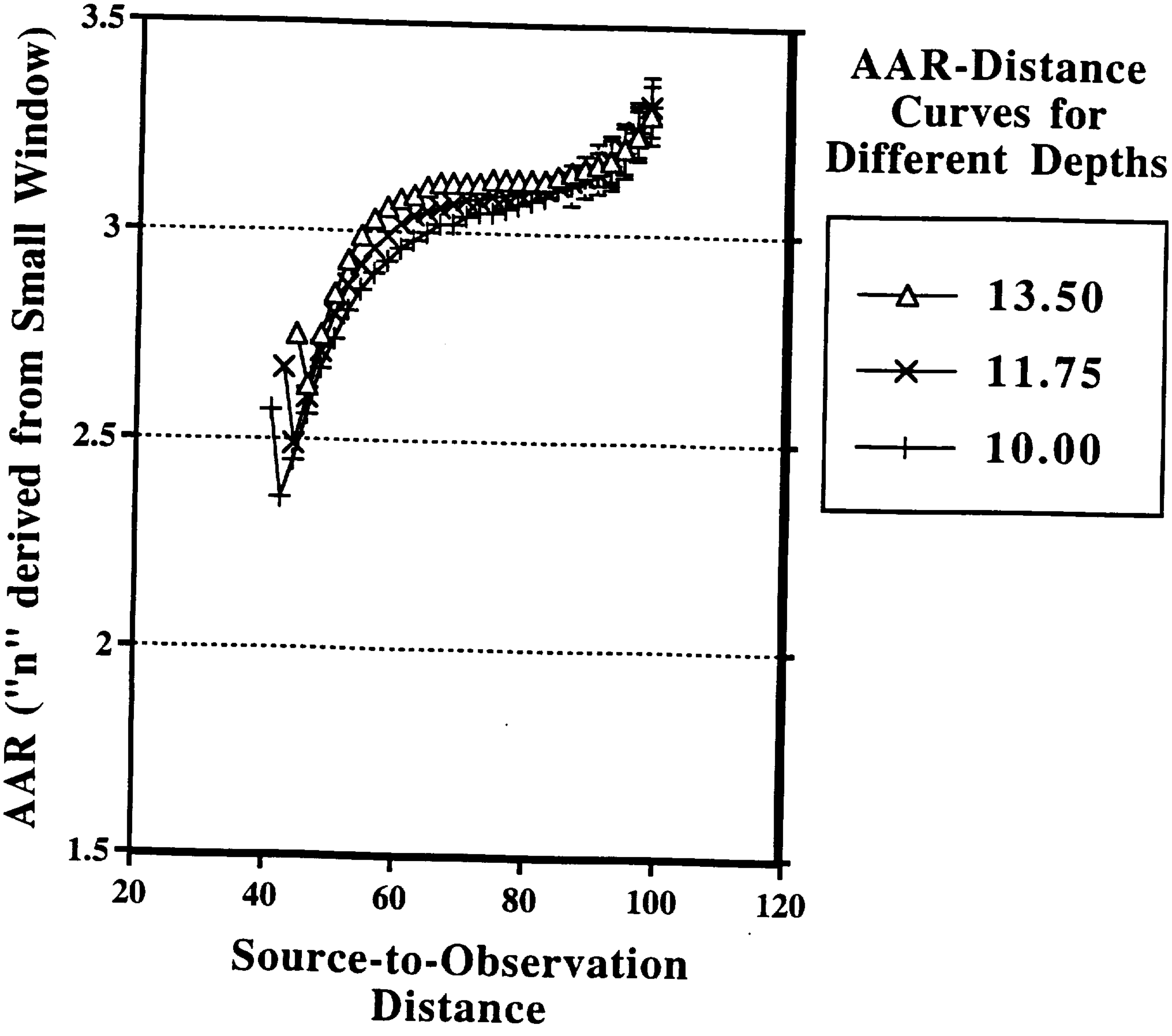


Fig. 4b

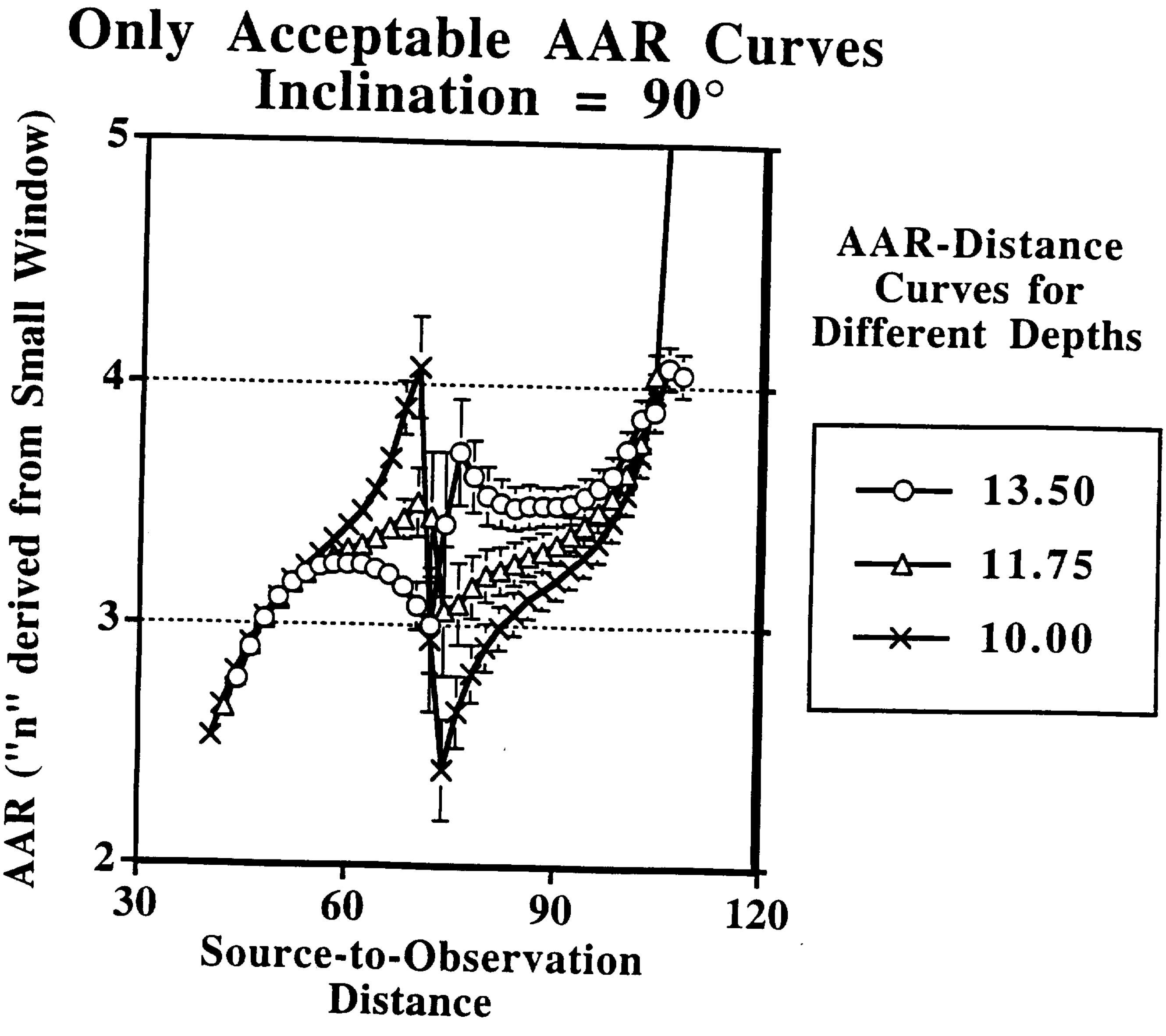

Only Acceptable AAR Curves
Inclination = 90°
AAR ("n" derived from Small Window)
5
4
3
2
30
60
90
120
Source-to-Observation Distance
AAR-Distance Curves for Different Depths
13.50
11.75
10.00

Fig. 6

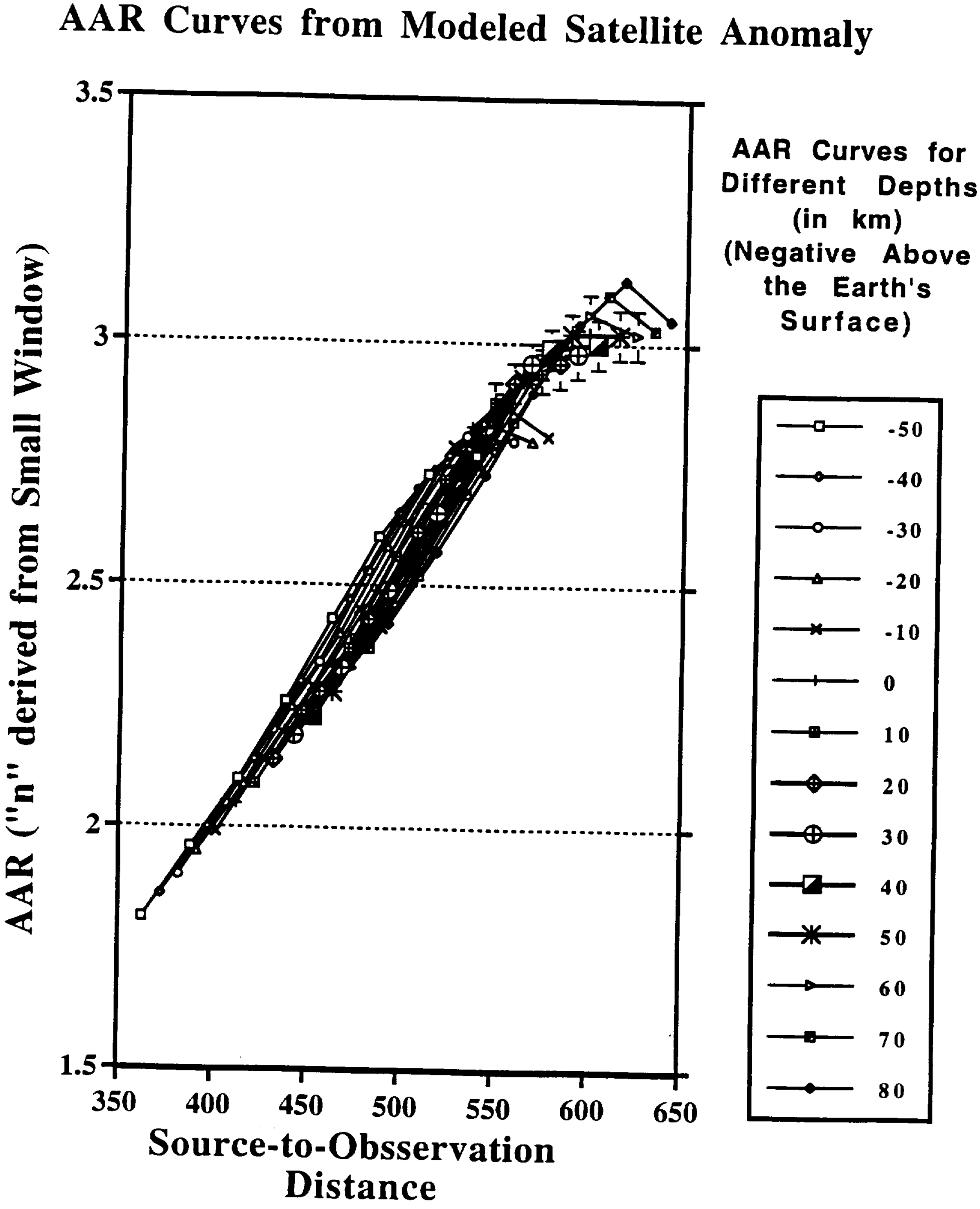

AAR Curves from Modeled Satellite Anomaly
AAR ("n" derived from Small Window)
3.5
3
2.5
2
1.5
350
400
450
500
550
600
650
Source-to-Obsservation Distance
AAR Curves for Different Depths (in km) (Negative Above the Earth's Surface)
-50
-40
-30
-20
-10
0
10
20
30
40
50
60
70
80

Fig 7

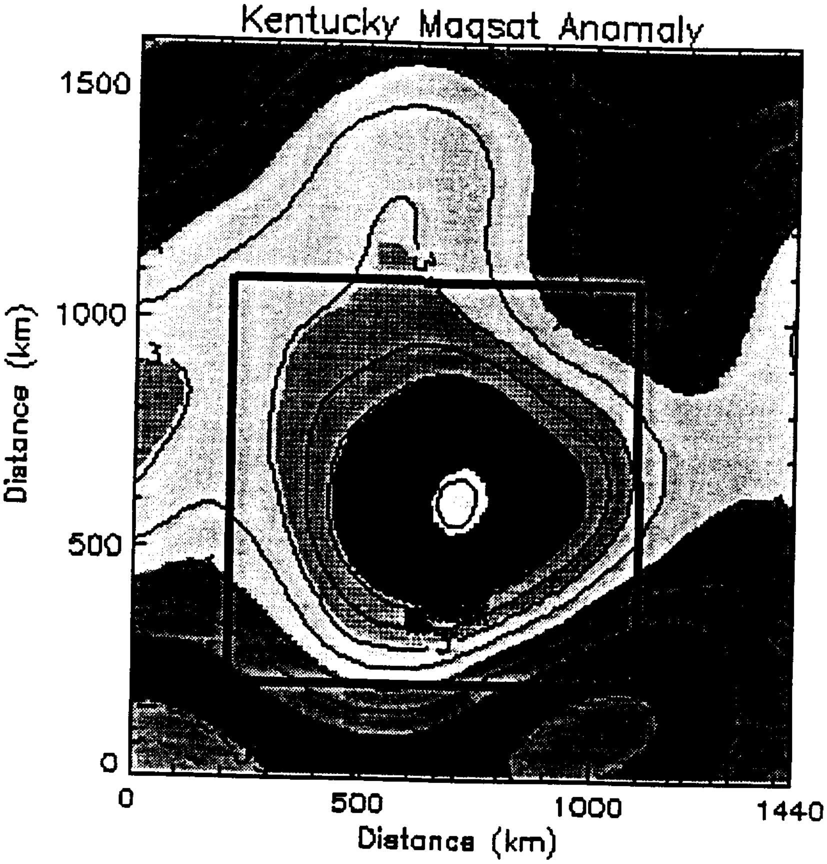

Kentucky Magsat Anomaly
1500
1000
500
0
Distance (km)
0
500
1000
1440
Distance (km)

Fig. 8a.

AAR Curves from Kentucky Magsat Anomaly

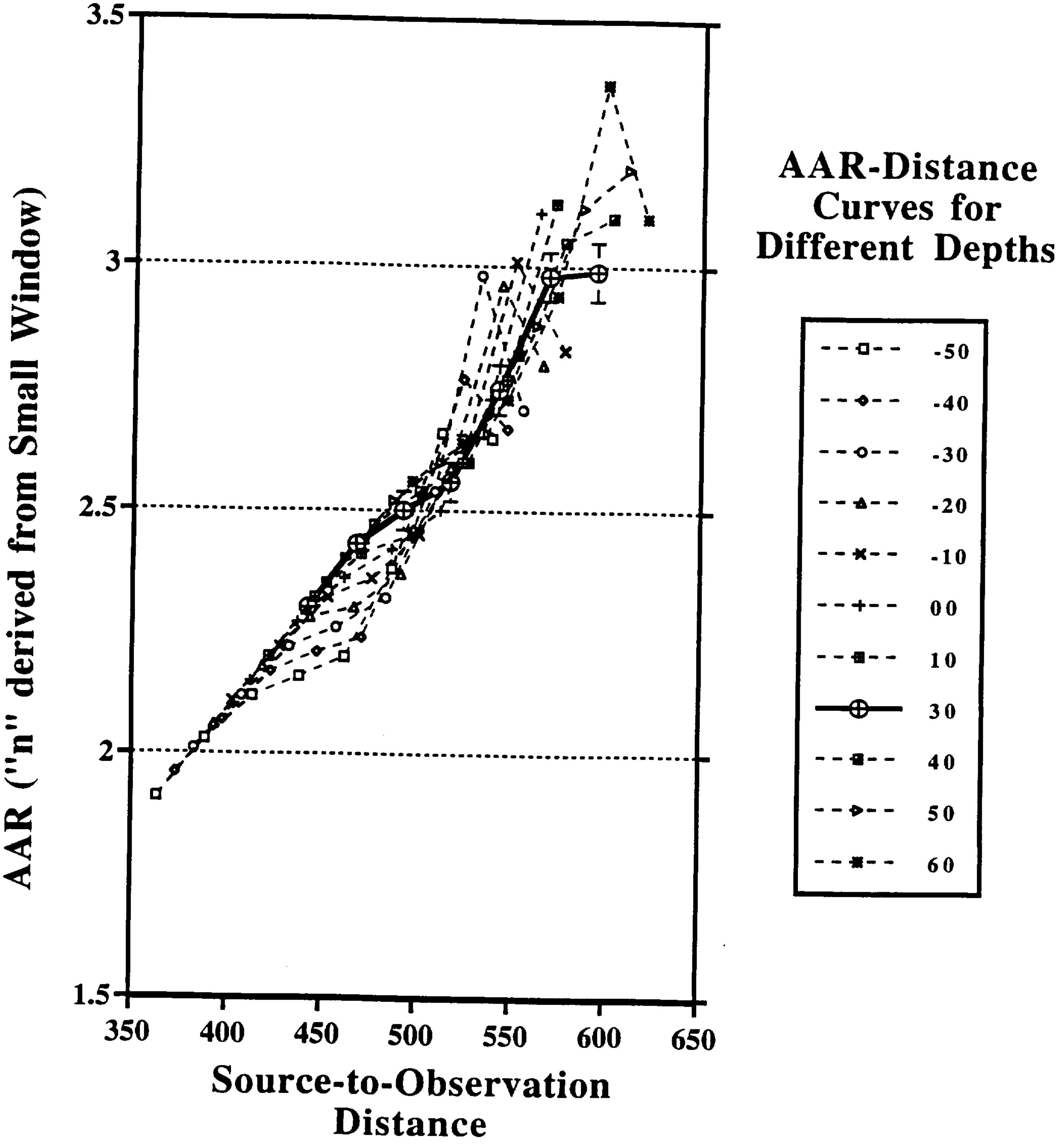


Fig. 8b.

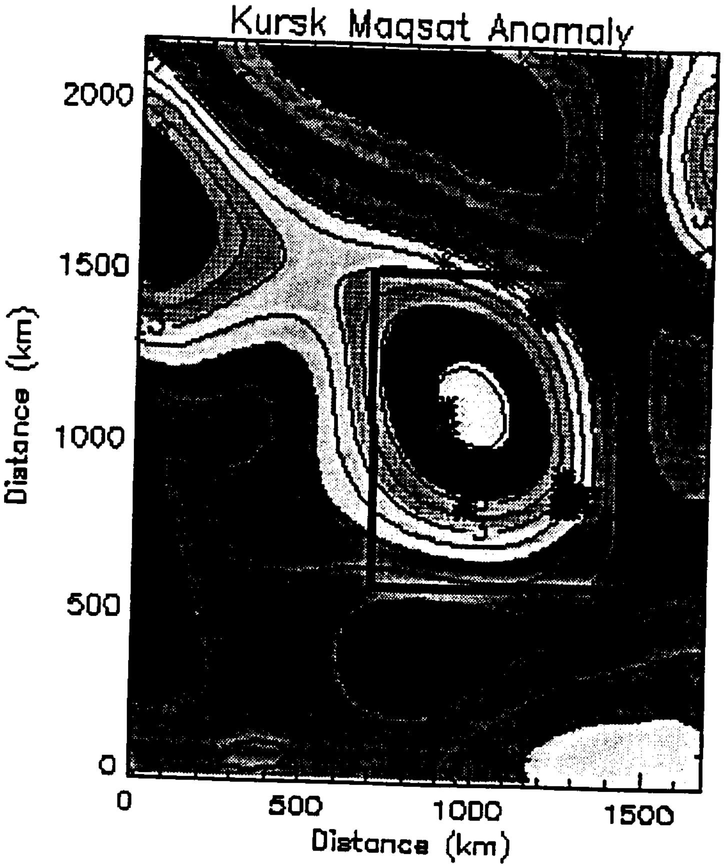

Kursk Magsat Anomaly
2000
1500
1000
500
0
Distance (km)
0
500
1000
1500
Distance (km)

Fig. 9a.

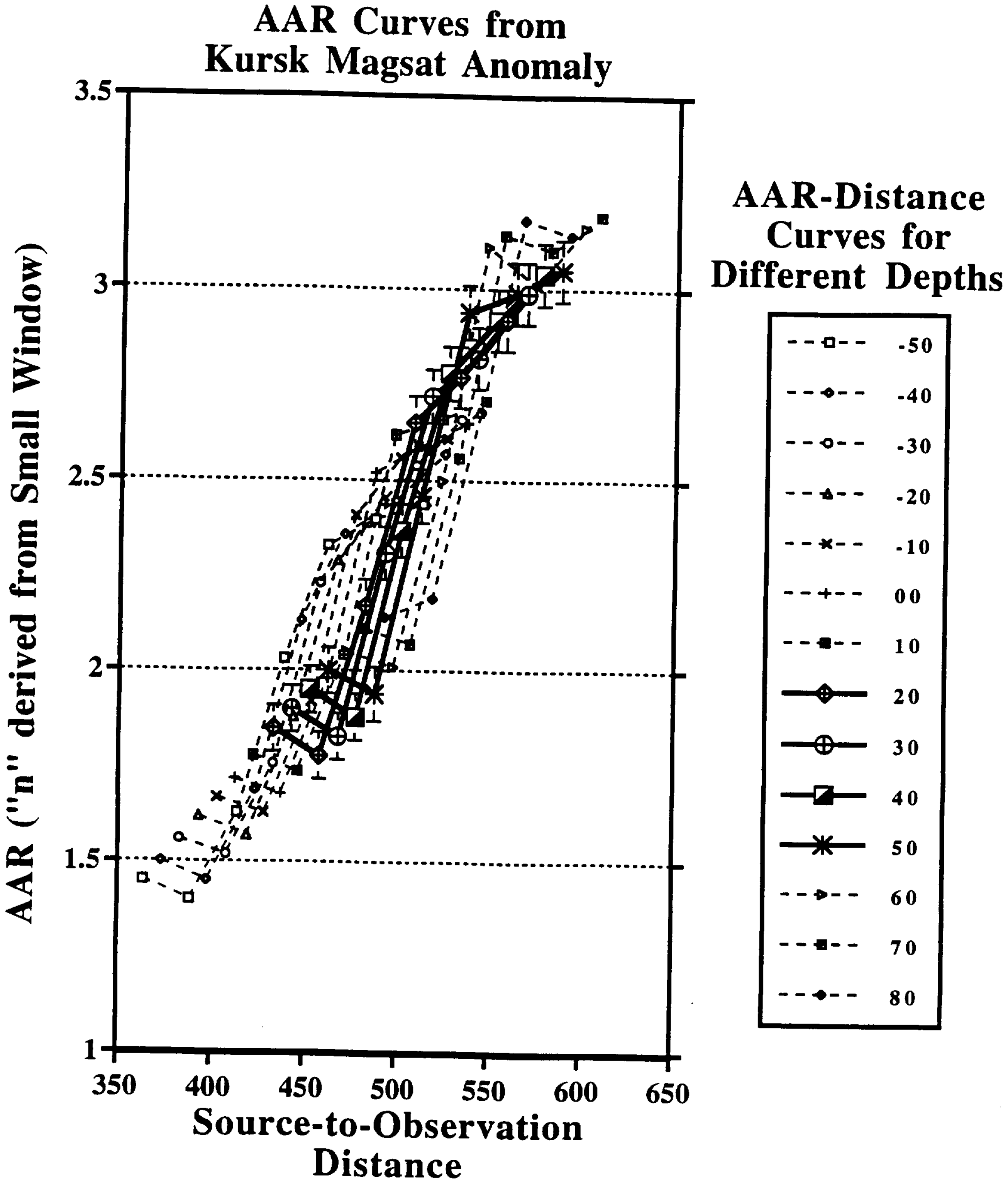

AAR Curves from Kursk Magsat Anomaly
AAR ("n" derived from Small Window)
3.5
3
2.5
2
1.5
1
350
400
450
500
550
600
650
Source-to-Observation Distance
AAR-Distance Curves for Different Depths
-50
-40
-30
-20
-10
00
10
20
30
40
50
60
70
80

Fig. 9b.

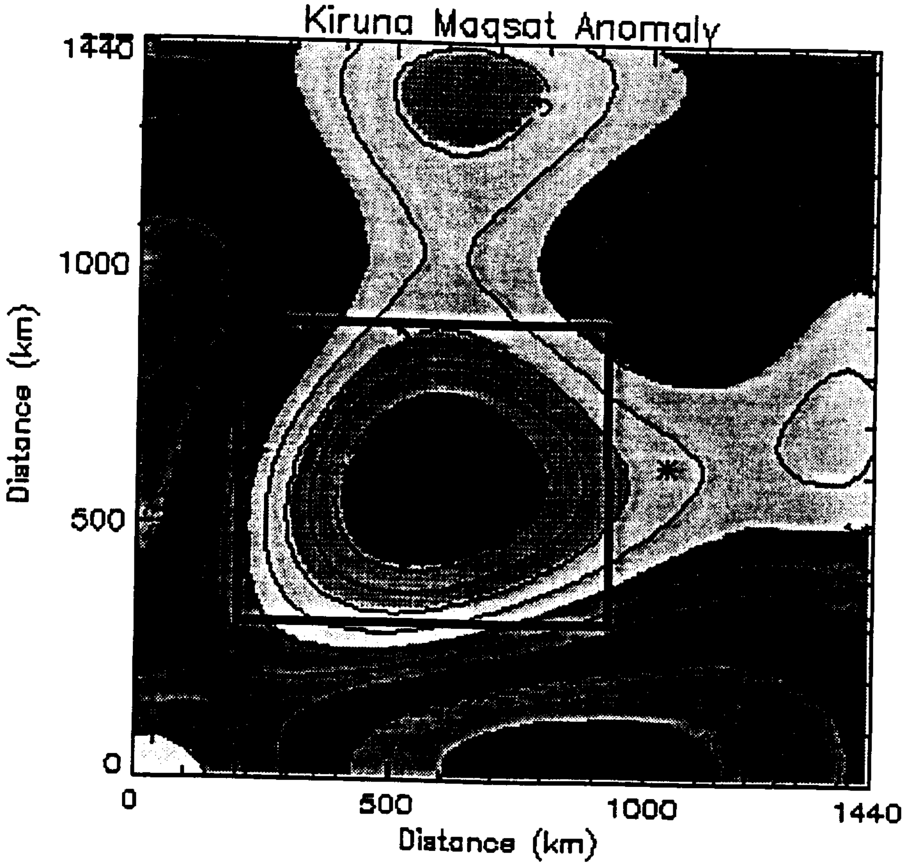
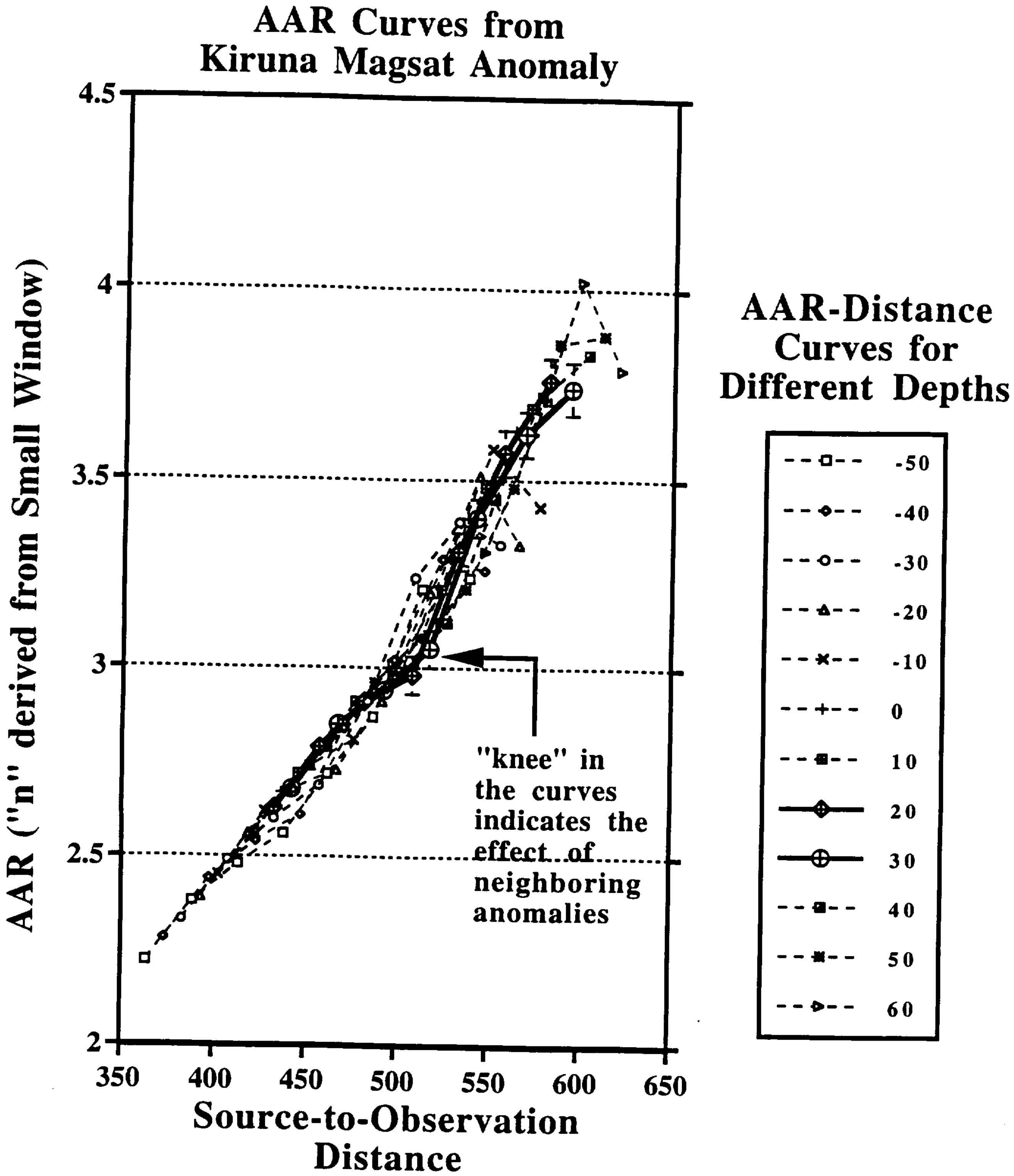

Kiruna Magsat Anomaly
1440
1000
500
0
Distance (km)
0
500
1000
1440
Distance (km)
*

Fig. 10a



AAR Curves from Kiruna Magsat Anomaly

Fig. 10b.