Uncertainty and Intelligence in Computational Stochastic Mechanics

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INTRODUCTION

Classical structural reliability assessment techniques are based on precise and crisp (sharp) definitions of failure and non-failure (survival) of a structure in meeting a set of strength, function and serviceability criteria. These definitions are provided in the form of performance functions and limit state equations. Thus, the criteria provide a dichotomous definition of what real physical situations represent, in the form of abrupt change from structural survival to failure. However, based on observing the failure and survival of real structures according to the serviceability and strength criteria, the transition from a survival state to a failure state and from serviceability criteria to strength criteria are continuous and gradual rather than crisp and abrupt. That is, an entire *spectrum of damage* or failure levels (grades) is observed during the transition to total collapse. In the process, serviceability criteria are gradually violated with monotonically increasing level of violation, and progressively lead into the strength criteria violation. Classical structural reliability methods correctly and adequately include the ambiguity sources of uncertainty (physical randomness, statistical and modeling uncertainty) by varying amounts. However, they are unable to adequately incorporate the presence of a damage spectrum, and do not consider in their mathematical framework any sources of uncertainty of the vagueness type. Vagueness can be attributed to sources of fuzziness, unclearness, indistinctiveness, sharplessness and grayness; whereas ambiguity can be attributed to nonspecificity, one-to-many relations, variety, generality, diversity and divergence. Using the nomenclature of structural reliability, vagueness and ambiguity can be accounted for in the form of realistic delineation of structural damage based on subjective judgment of engineers. For situations that require decisions under uncertainty with cost/benefit objectives, the risk of failure should depend on the underlying level of damage and the uncertainties associated with its definition. A mathematical model for structural reliability assessment that includes both ambiguity and vagueness types of uncertainty was suggested to result in the likelihood of failure over a damage spectrum. The resulting structural reliability estimates properly represent the continuous transition from serviceability to strength limit states over the ultimate time exposure of the structure. In this section, a structural reliability assessment method based on a fuzzy definition of failure is suggested to meet these practical needs. A failure definition can be developed to indicate the relationship between failure level and structural response. In this fuzzy model, a subjective index is introduced to represent all levels of damage (or failure). This index can be interpreted as either a measure of failure level or a measure of a degree of belief in the occurrence of some performance condition (e.g., failure). The index allows expressing the transition state between complete survival and complete failure for some structural response based on subjective evaluation and judgment.

STRUCTURAL RELIABILITY ASSESSMENT

The reliability of an engineering system can be defined as its ability to fulfill its design purpose for some time period. The theory of probability provides the fundamental basis to measure this ability. The reliability of a structure can be viewed as the probability of its satisfactory performance according to some performance functions under specific service and extreme conditions within a stated time period. In estimating this probability, system uncertainties are modeled using random variables with mean values, variances, and probability distribution functions. Many methods have been proposed for structural reliability assessment purposes, such as First-Order Second Moment (FOSM) method, Advanced Second Moment (ASM) method, and computer simulation (Refs. 2 and 4). In this section, two probabilistic methods for reliability assessment are described. They are 1) advanced second moment (ASM) method, and 2) Monte Carlo Simulation (MCS) method with Variance Reduction Techniques (VRT) using Conditional Expectation (CE) and Antithetic Variates (AV).

Advanced Second Moment (ASM) Method

The reliability of a structure can be determined based on a performance function that can be expressed in terms of basic random variables X_i's for relevant loads and structural strength. Mathematically, the performance function Z can be described as

$$Z = Z(X_1, X_2, ..., X_n) = \text{Structural strength - load effect} \tag{1}$$

where Z is called the *performance function* of interest. The failure surface (or the *limit state*) of interest can be defined as $Z = 0$. Accordingly, when $Z < 0$, the structure is in the failure state, and when $Z > 0$ it is in the safe state. If the joint probability density function for the basic random variables X_i's is $f = Z_{x_1, x_2, ..., x_n}(x_1, x_2, ..., x_n)$, then the failure probability P_f of a structure can be given by the integral

$$P_f = \int ... \int f_{x_1, x_2, ..., x_n}(x_1, x_2, ..., x_n)\, dx_1 dx_2 ... dx_n \tag{2}$$

where the integration is performed over the region in which $Z < 0$. In general, the joint probability density function is unknown, and the integral is a formidable task. For practical purposes, alternate methods of evaluating P_f are necessary.

Reliability Index (Safety Index)

Instead of using direct integration as given by Eq. 2, the performance function Z in Eq. 1 can be expanded using a Taylor series about the mean value of X's and then truncated at the linear terms. Therefore, the first-order approximate mean and variance of Z can be shown, respectively, as

$$\bar{Z} \cong Z(\bar{X}_1, \bar{X}_2, ..., \bar{X}_n) \tag{3}$$

and

$$\sigma_z^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} \left(\frac{\partial Z}{\partial X_i}\right)\left(\frac{\partial Z}{\partial X_j}\right) Cov(X_i, X_j) \tag{4}$$

where $Cov(X_i, X_j)$ is the covariance of X_i and X_j; $\bar{Z}$ = mean of Z; and σ^2 = variance of Z. The partial derivatives of $\partial Z/\partial X_i$ are evaluated at the mean values of the basic random variables. For statistically independent random variables, the variance expression can be simplified as

$$\sigma_z^2 = \sum_{i=1}^{n} \sigma_{x_i}^2 \left(\frac{\partial Z}{\partial X_j}\right)^2 \tag{5}$$

A measure of reliability can be estimated by introducing the *reliability index* or *safety index* β that is based on the mean and standard deviation of Z as

$$\beta = \frac{\bar{Z}}{\sigma_z} \tag{6}$$

If Z is assumed to be normally distributed, then it can be shown that the failure probability P_f is

$$P_f = 1 - \Phi(\beta) \tag{7}$$

where Φ = cumulative distribution function of standard normal variate.

The aforementioned procedure of Eqs. 3 to 7 produces accurate results when the random variables are normally distributed and the performance function Z is linear.

Nonlinear Performance Functions

For nonlinear performance functions, the Taylor series expansion of Z is linearized at some point on the failure surface called *design point* or *checking point* or *the most likely failure point* rather than at the mean. Assuming the original basic variables X_i's are uncorrelated, the following transformation can be used:

$$Y_i = \frac{X_i - \bar{X}}{\sigma_{X_i}} \tag{8}$$

If X_i's are correlated, they need to be transformed to uncorrelated random variables, as described by Thrift-Christensen and Baker (Ref. 33) or Ang and Tang (Ref. 2). The safety index β is defined as the shortest distance to the failure surface from the origin in the reduced Y-coordinate system. The point on the failure surface that corresponds to the shortest distance is the most likely failure point. Using the original X-coordinate system, the safety index β and design point $(X_1^*, X_2^*, ..., X_n^*)$ can be determined by solving the following system of nonlinear equations iteratively for β:

$$\alpha_i = \frac{\left(\frac{\partial Z}{\partial X_i}\right)\sigma_{X_i}}{\left[\sum_{i=1}^{n}\left(\frac{\partial Z}{\partial X_i}\right)^2 \sigma_{X_i}^2\right]^{1/2}} \tag{9}$$

$$X_i^* = \bar{X}_i - \alpha_i \beta \sigma_{x_i} \tag{10}$$

$$Z\left(X_1^*, X_2^*, ..., X_n^*\right) = 0 \tag{11}$$

where α_i = directional cosine; and the partial directives are evaluated at design point. Then, Eq. 7 can be used to evaluate P_f. However, the above formulation is limited to normally distributed random variables.

Equivalent Normal Distributions

If a random variable X is not normally distributed, then it needs to be transformed to an equivalent normally distributed random variable. The parameters of the equivalent normal distribution $\bar{X}_i^N$ and σ_X^N, can be estimated by imposing two conditions (Refs. 27 and 28). The cumulative distribution functions and probability density functions of a non-normal random variable and its equivalent normal variable should be equal at the design point on the failure surface. The first condition can be expressed as

$$\Phi\left(\frac{X_i^* - \bar{X}_i^N}{\sigma_{X_i}^N}\right) = F_i\left(X_i^*\right) \tag{12a}$$

The second condition is

$$\phi\left(\frac{X_i^* - \bar{X}_i^N}{\sigma_{X_i}^N}\right) = f_i\left(X_i^*\right) \tag{12b}$$

where F_i = non-normal cumulative distribution function; f_i = non-normal probability density function; Φ = cumulative distribution function of standard normal variate; and ϕ = probability density function of standard normal variate. The standard deviation and mean of equivalent normal distributions can be shown, respectively, to be

$$\sigma_{X_i}^N = \frac{\phi\left(\Phi^{-1}\left[F_i\left(X_i^*\right)\right]\right)}{f_i\left(X_i^*\right)} \tag{13}$$

and

$$\bar{X}_i^N = X_i^* - \Phi^{-1}\left[F_i\left(X_i^*\right)\right]\sigma_{X_i}^N \tag{14}$$

Having determined $\sigma_{X_i}^N$ and $\bar{X}_i^N$ for each random variable, β can be solved using the same procedure of Eqs. 9 to 11.

The advanced second moment method is capable of dealing with nonlinear performance functions and non-normal probability distributions. However, the accuracy of the solution and the convergence of the procedure depends on the nonlinearity of the performance function in the vicinity of design point and the origin. If there are several local minimum distances to the origin, the solution process may not converge onto the global minimum. The probability of failure is calculated from the safety index β using Eq. 7 which is based on normally distributed performance functions. Therefore, the resulting failure probability P_f based on the ASM is approximate except for linear performance functions because it does not account for any nonlinearity in the performance functions.

SOURCES AND TYPES OF UNCERTAINTY

The following two viewgraphs show example sources of uncertainty, and a classification of uncertainty types.

OBJECTIVES

The objectives of this presentation were to generalize structural reliability assessment methods to account for ambiguity and vagueness sources of uncertainty, and demonstrate the developed methods using ship structures. A viewgraph is provided with a statement of objectives.

Models and methods for merging different uncertainty sources in structural reliability assessment were described. The methods were presented in a finite element analysis framework. Also, intelligence in reliability computations with applications to marine vessels were discussed.

OBJECTIVES

- **Develop methods for structural reliability assessment based on a generalized treatment of uncertainty.**
- **Define failure events over a damage spectrum.**
- **Provide the reliability of the structure over the damage spectrum.**

METHODOLOGY

The following figure shows a procedure for an automated failure classification that can be implemented in a simulation algorithm for reliability assessment for ship structures as an example. The failure classification is based on matching a deformation or stress field with a record within a knowledge base of response and failure classes. In cases of no match, a list of approximate matches is provided, with assessed applicability factors. The user is prompted for any changes to the approximate matches and their applicability factors. In the case of a poor match, the user has the option of activating the failure recognition algorithm shown in the next figure to establish a new record in the knowledge base. The adaptive or neural nature of this algorithm allows the updating of the knowledge base of responses and failure classes. The failure recognition and classification algorithm shown in the figure evaluates the impact of the computed deformation or stress field on several systems of a structure. The impact assessment includes evaluating the remaining strength, stability, repair criticality, propulsion and power systems, combat systems, and hydrodynamic performance. The input of experts in ship performance is needed to make these evaluations using either numeric or linguistic measures. Then, the assessed impacts need to be aggregated and combined to obtain an overall failure recognition and classification within the established failure classes. The result of this process is then used to update the knowledge base.

The development of a methodology for the reliability assessment of continuum ship structural components or systems requires the consideration of the following three components: (1) loads, (2) structural strength, and (3) methods of reliability analysis. Also, the reliability analysis requires knowing the probabilistic characteristics of the operational-sea profile of a ship, failure modes, and failure definitions. A reliability assessment methodology can be developed in the form of the following modules: operational-sea profile and loads; nonlinear structural analysis; extreme analysis and stochastic load combination; failure modes, their load effects, load combinations, and structural strength; library of probability distributions; reliability assessment methods; uncertainty modeling and analysis; failure definitions; and system analysis. Each module can be independently investigated and developed, although some knowledge about the details of other modules is needed for the development of a module. These modules are described by Ayyub, Beach and Packard (Ref. 6).

Prediction of structural failure modes of continuum ship structural components or systems requires the use of nonlinear structural analysis. Therefore, failure definitions need to be expressed using deformations rather than forces or stresses. Also, the recognition and proper classification of failures based on a structural response within the simulation process need to be performed based on deformations. The process of failure classification and recognition needs to be automated in order to facilitate its use in a simulation algorithm for structural reliability assessment. The first figure shows a procedure for an automated failure classification that can be implemented in a simulation algorithm for reliability assessment. The failure classification is based on matching a deformation or stress field with a record within a knowledge base of response and failure classes. In cases of no match, a list of approximate matches is provided with assessed applicability factors. The user can then be prompted for any changes to the approximate matches and their applicability factors. In the case of a poor match, the user can have the option of activating the failure recognition algorithm shown in the second figure to establish a new record in the knowledge base. The adaptive or neural nature of this algorithm allows the updating of the knowledge base of responses and failure classes. The failure recognition and classification algorithm shown in the figure evaluates the impact of the computed deformation or stress field on several systems of a ship. The impact assessment includes evaluating the remaining strength, stability, repair criticality, propulsion and power systems, combat systems, and hydrodynamic performance. The input of experts in ship performance is needed to make these evaluations using either numeric or linguistic measures. Then, the assessed impacts need to be aggregated and combined to obtain an overall failure recognition and classification within the established failure classes. The result of this process is then used to update the knowledge base.

A prototype computational methodology for reliability assessment of continuum structures using finite element analysis with instability failure modes is described in this report. Examples were used to illustrate and test the methodology. Geometric and material uncertainties were considered in the finite element model. A computer program was developed to implement this methodology by integrating uncertainty formulations to create a finite element input file, and to conduct the reliability assessment on a machine level. A commercial finite element package was used as a basis for the strength assessment in the presented procedure. A parametric study for a stiffened panel strength was also carried out. The finite element model was based on the eight-node doubly curved shell element, which can provide the nonlinear behavior prediction of the stiffened panel. The mesh was designed to ensure the convergence of eigenvalue estimates. Failure modes were predicted on the basis of elastic nonlinear analysis using the finite element model.

Reliability assessment was performed using Monte Carlo simulation with variance reduction techniques that consisted of the conditional expectation method. According to Monte Carlo methods, the applied load was randomly generated, finite element analysis was used to predict the response of the structure under the generated loads in the form of a deformation field. A crude simulation procedure can be applied to compare the response with a specified failure definition, and failures can then be counted. By repeating the simulation procedure several times, the failure probability according to the specified failure definition is estimated as the failure fraction of simulation repetitions. Alternatively, conditional expectation was used to estimate the failure probability in each simulation cycle in this study; then the average failure probability and its statistical error were computed.

The developed method is expected to have significant impact on the reliability assessment of structural components and systems; more specifically, the safety and reliability evaluation of continuum structures, the formulation of associated design criteria, evaluation of important variables that influence failures, the possibility of revising some codes of practice, reducing the number of required costly experiments in structural testing, and the safety evaluation of existing structures for the purpose of life extension. The impact of this study can extend beyond structural reliability into the generalized field of engineering mechanics.

STRUCTURAL RELIABILITY ASSESSMENT

The general performance function of a structural component or system according to a specified performance criterion is expressed as follows:

$$Z = \text{strength} - \text{load effect}$$

$$Z = g(X_1, X_2, \cdots, X_n)$$

where X_i = basic random variable

$g(\cdot) > 0$: survival event

$g(\cdot) = 0$: limit state

$g(\cdot) < 0$: failure event

The probability of failure is determined by solving the following integral:

$$P_f = \int \int \cdots \int f_{\underline{X}}(X_1, X_2, \cdots, X_n) \; dx_1 dx_2 \cdots dx_n$$

where $f_{\underline{X}}$ is the joint probability density function of $\underline{X} = \{X_1, X_2, \cdots, X_n)$ and the integration is performed over the range where $g(\cdot) < 0$

UNCERTAINTIES

I. Ambiguity: (1) Physical randomness
(2) Statistical uncertainty
(3) Model uncertainty

II. Vagueness: (1) Definition of parameters
(2) Inter-relationships among the parameters

CRISP FAILURE MODEL

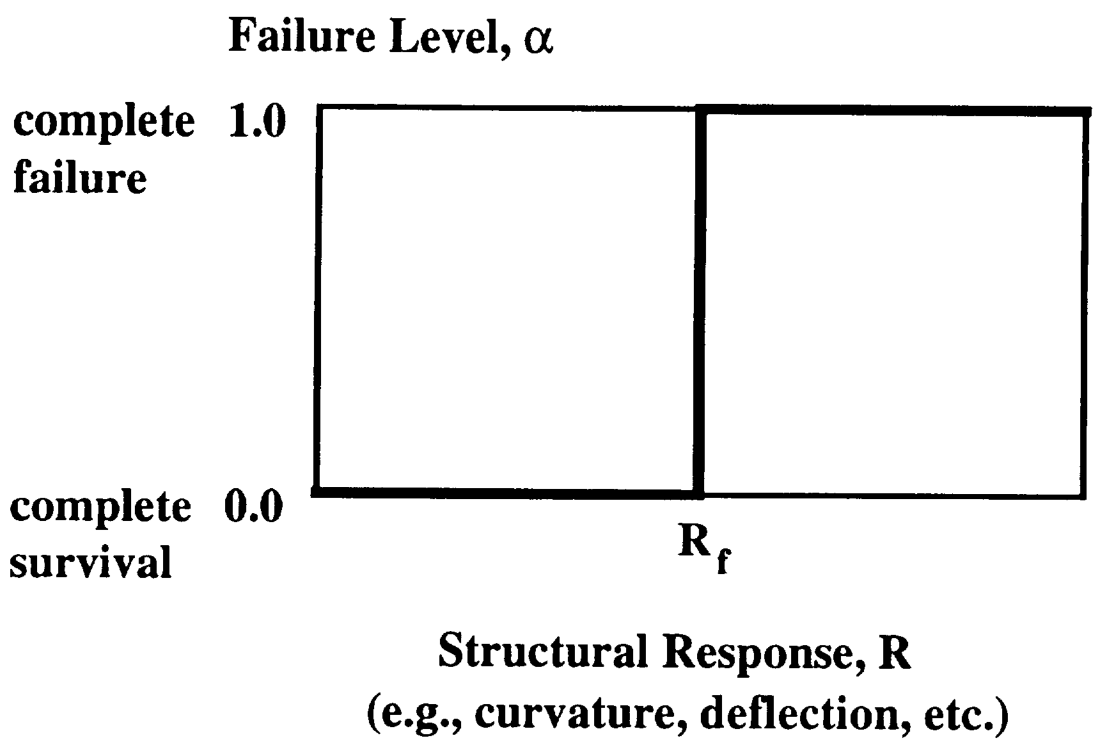
Only two basic, mutually exclusive events, *complete survival* and *complete failure*, are considered, i.e.,

$$U \rightarrow \{0, 1\}$$

where U = the universe of all possible outcomes

0 = failure level of the event *complete survival*

1 = failure level of the event *complete failure*



R_f = structural response at failure

$R < R_f$ ($\alpha=0$) : complete survival

$R = R_f$: limit state

$R > R_f$ ($\alpha=1$) : complete failure

FUZZY (CONTINUOUS) FAILURE MODEL

A subjective index, *failure level* α, is introduced to represent the intermediate levels of damage, i.e.,

$$U \rightarrow A = \{ \alpha : \alpha \in [0, 1] \}$$

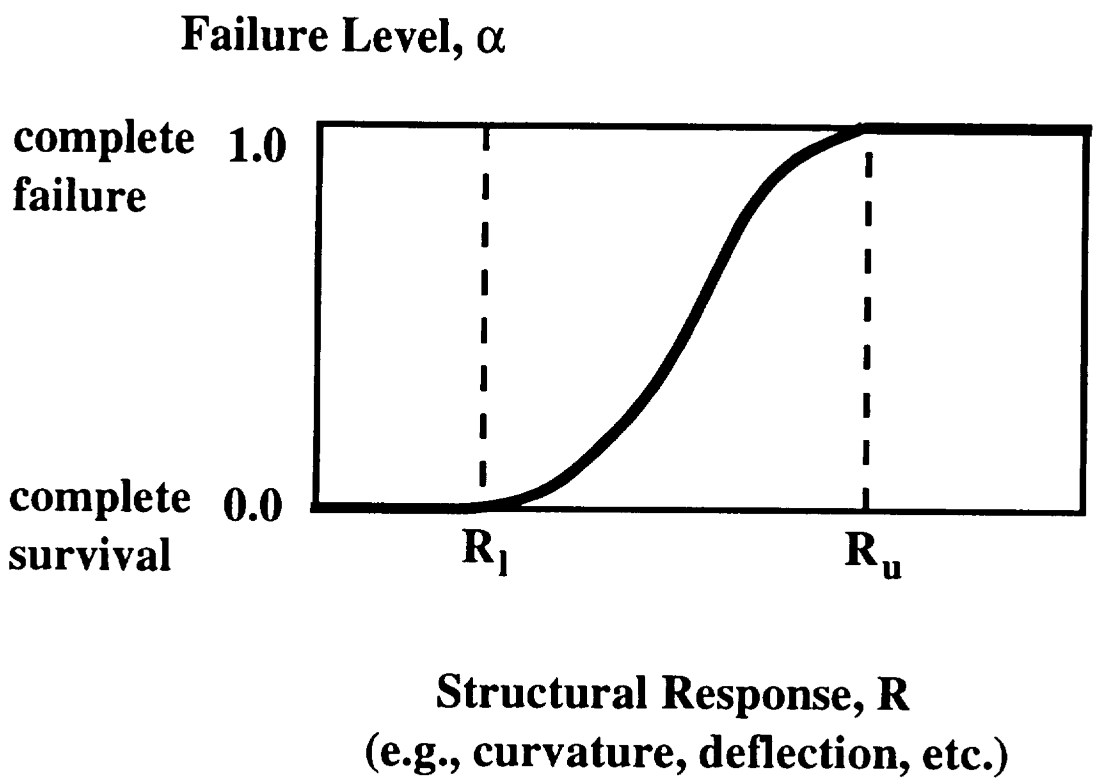
where U = the universe of all possible outcomes

$\alpha = 0$: complete survival

$0 < \alpha < 1$: partial failure

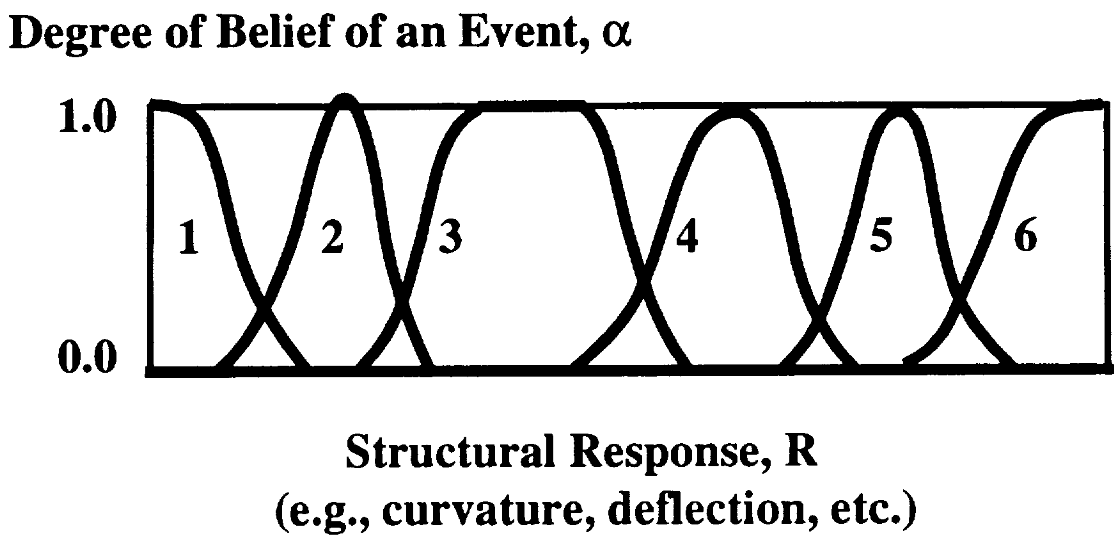
$\alpha = 1$: complete failure

α can be interpreted as the degree of belief of a failure condition.



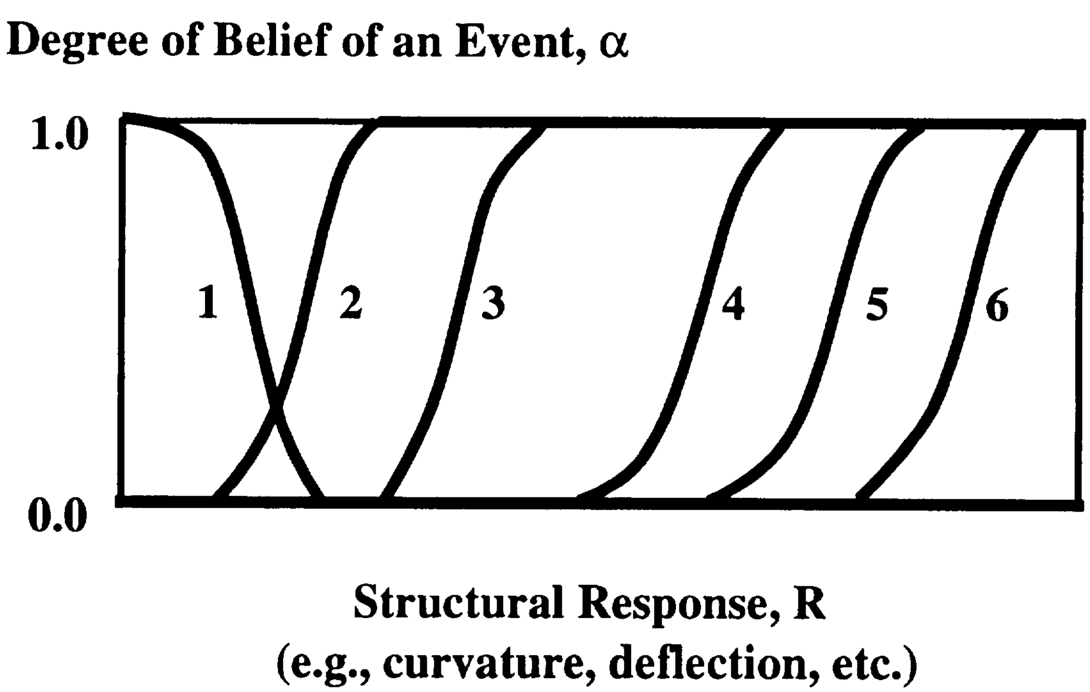
R_l = lower bound of structural response

R_u = upper bound of structural response

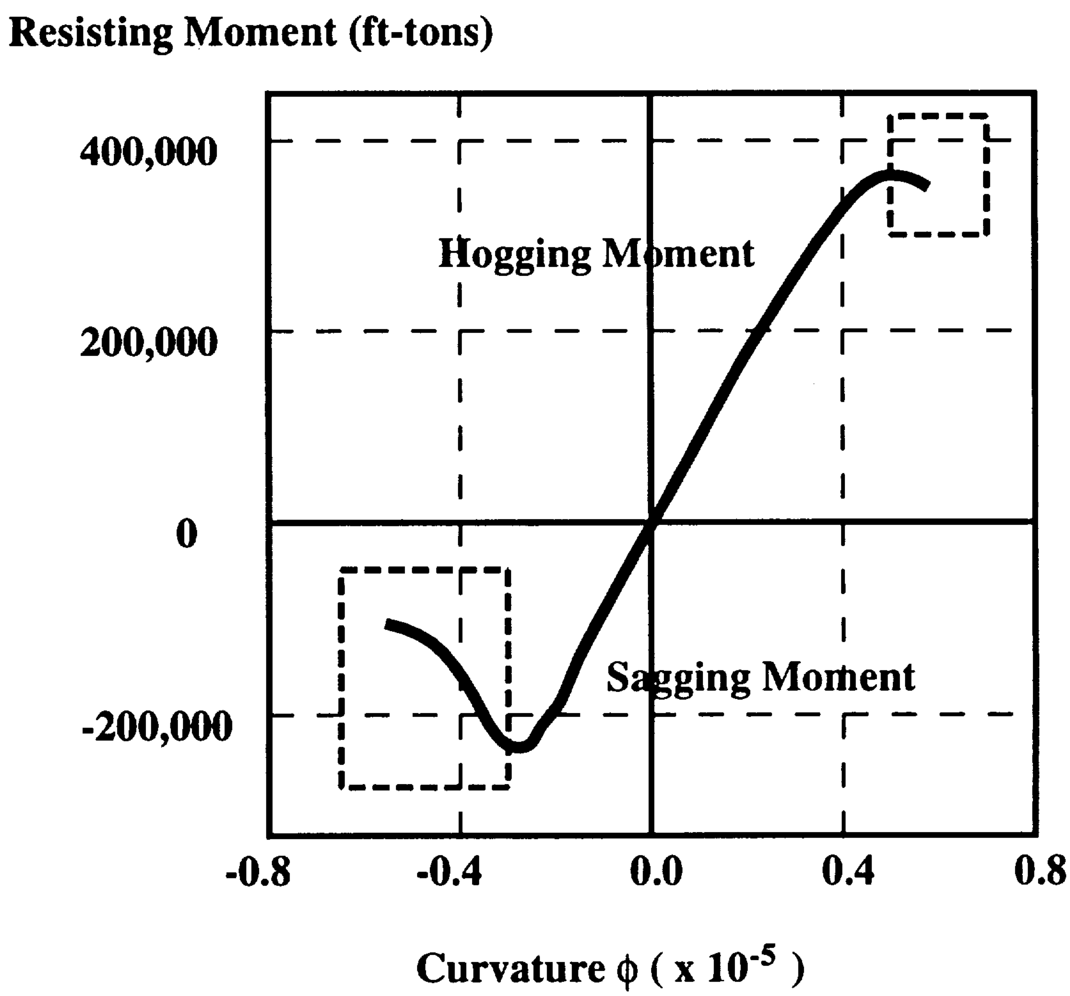


Event Number	Definition
1	complete survival
2	low serviceability failure
3	serviceability failure
4	high serviceability failure
5	partial collapse
6	complete collapse

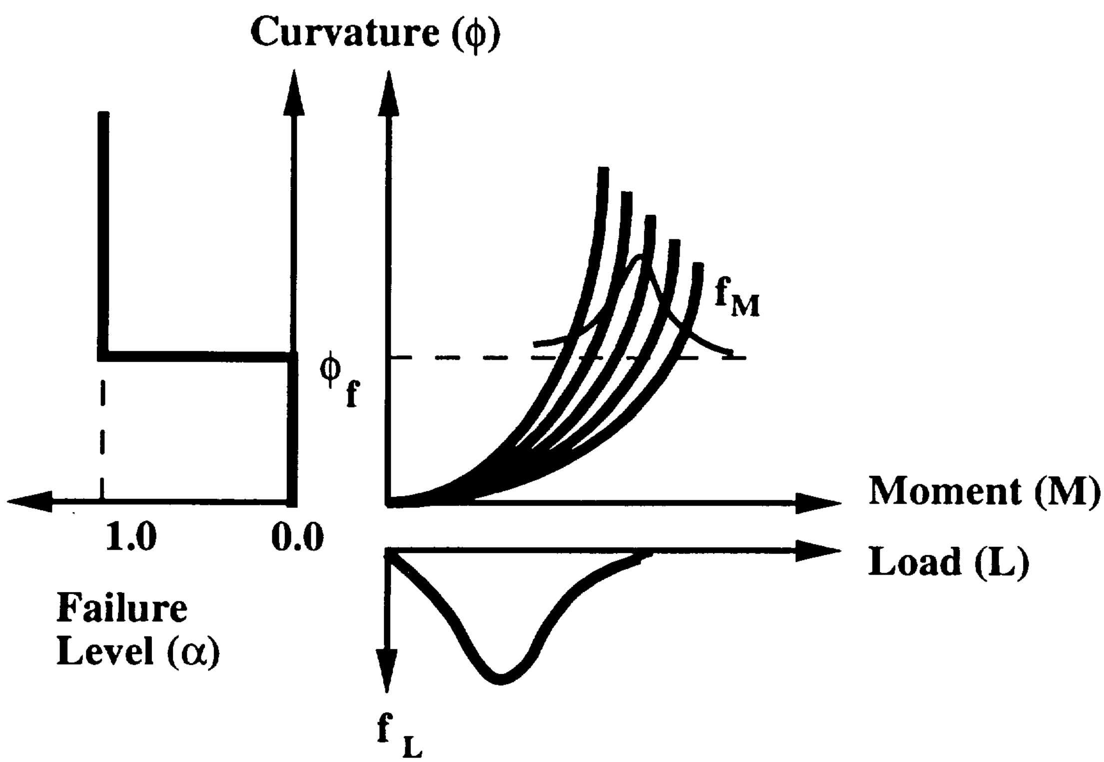
If definitions of failure events are interpreted as "*at least* low serviceability failure, serviceability failure, ···, or complete collapse," the above figure is modified to



STRUCTURAL PERFORMANCE CURVE



CRISP FAILURE MODEL
FOR STOCHASTIC M-Φ RELATIONSHIP



f_L = probability density function (pdf) of L
f_M = conditional pdf of M at $\phi = \phi_f$

The probability of failure is evaluated as

$$P_f = \text{Prob}\{\alpha = 1\}$$

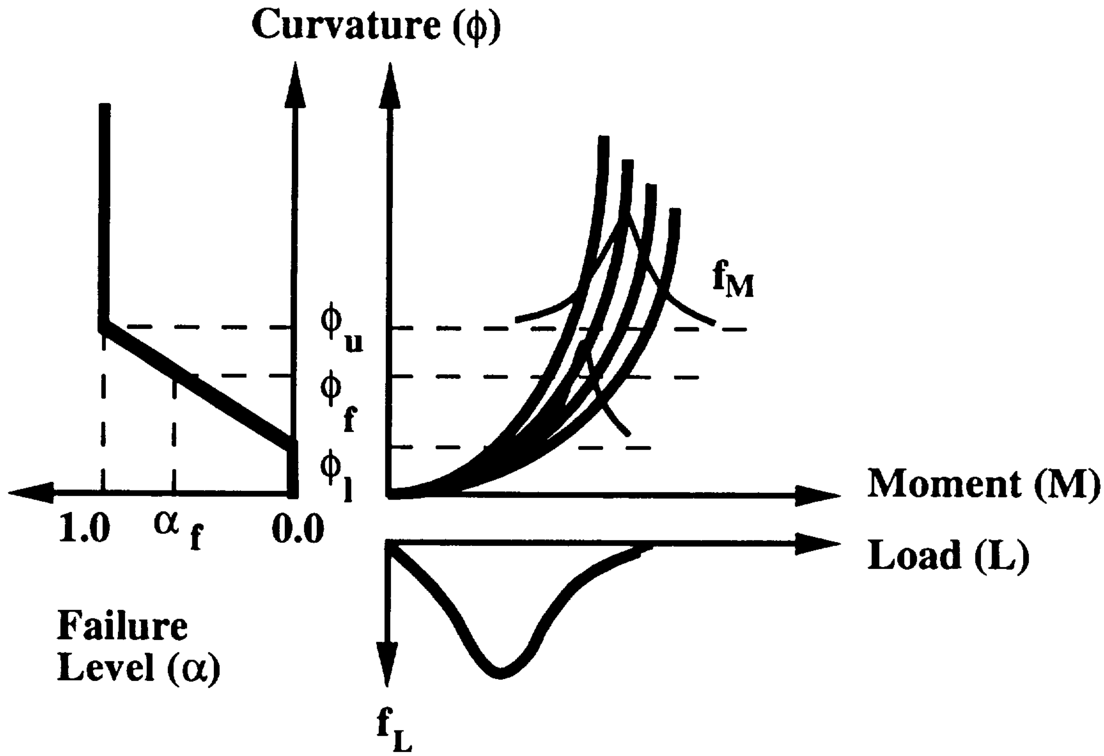
$$= \text{Prob}\{L > (M \text{ at } \phi = \phi_f)\}$$

$$= \int_0^\infty \text{Prob}\{L > (m \text{ at } \phi = \phi_f)\}\, f_M(m)\, dm$$

$$= \int_0^\infty \{1 - F_L(m)\}\, f_M(m)\, dm$$

where F_L = cumulative distribution function of L

FUZZY FAILURE MODEL
FOR STOCHASTIC M-Φ RELATIONSHIP



f_L = probability density function (pdf) of L
f_M = conditional pdf of M at $\phi = \phi_f$

The probability of failure is evaluated as

$$
\begin{aligned}
P_f(\alpha_f) &= \text{Prob}\{\, Z|_{\alpha=\alpha_f} < 0 \,\} \\
&= \text{Prob}\{\, M(\phi_f) - L < 0 \,\} \\
&= \int_0^{\infty} \text{Prob}\{L > (m \text{ at } \phi=\phi_f)\}\, f_M(m)\, dm \\
&= \int_0^{\infty} \{1 - F_L(m)\}\, f_M(m)\, dm
\end{aligned}
$$

where F_L = cumulative distribution function of L

AVERAGE PROBABILITY OF FAILURE

I. Crisp Failure Model:

$$P_{f,\,avg} = P_f$$

II. Fuzzy Failure Model:

- **Arithmetic average:**

$$P_{f_a} = \frac{\int_0^1 P_f(\alpha)\, d\alpha}{\int_0^1 d\alpha}$$

- **Geometric average:**

$$\log_{10}(P_{f_g}) = \frac{\int_0^1 \log_{10}\left(P_f(\alpha)\right) d\alpha}{\int_0^1 d\alpha}$$

EXAMPLE I

Consider the following performance function:

$$Z = M - L = M(\phi) - L$$

where M = resisting moment (ft-tons)
L = external load (ft-tons)

- **Crisp Failure Model**

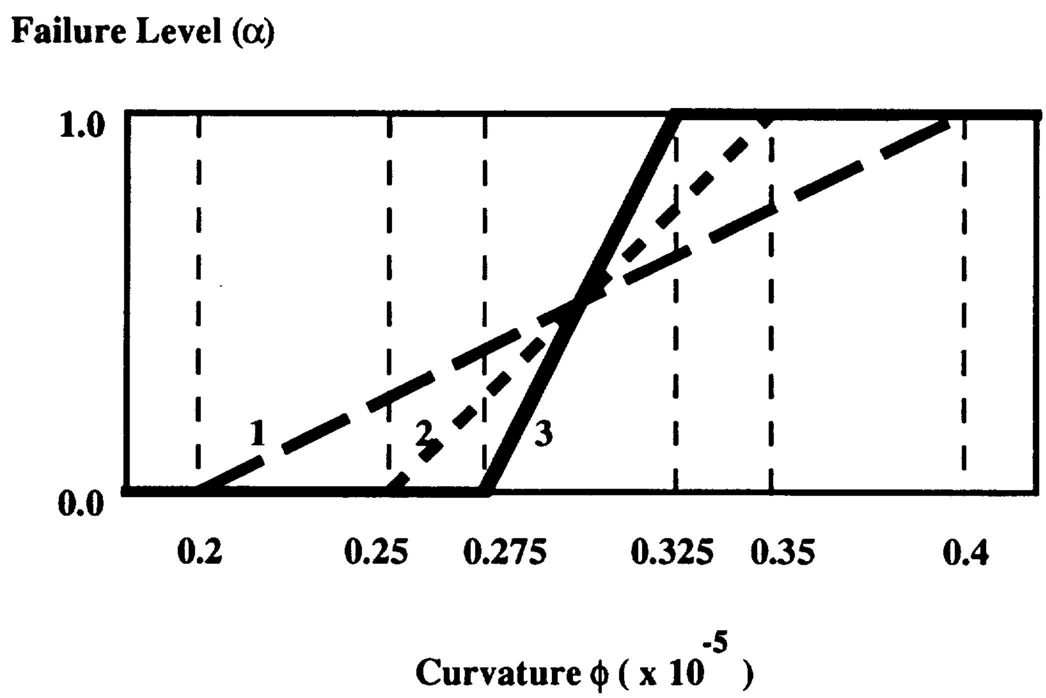
The curvature at failure is specified as

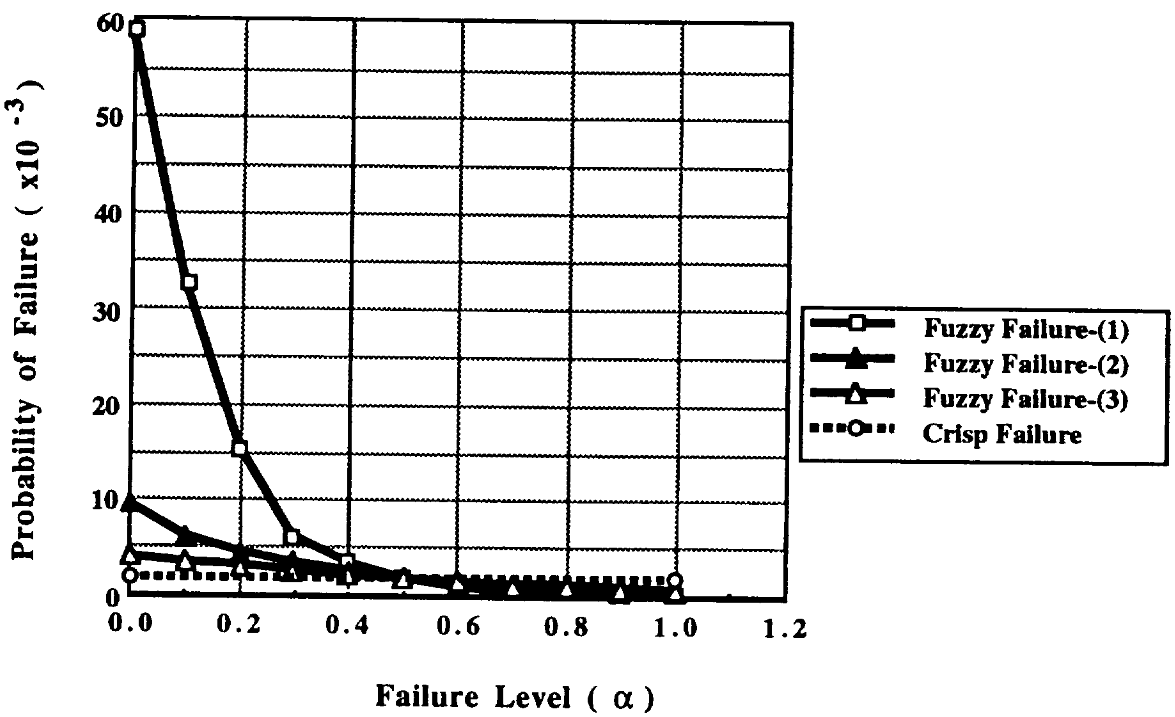
$$\phi_f = 0.30 \times 10^{-5}$$

The statistical characteristics of external moment (L) and resisting moment (M_f):

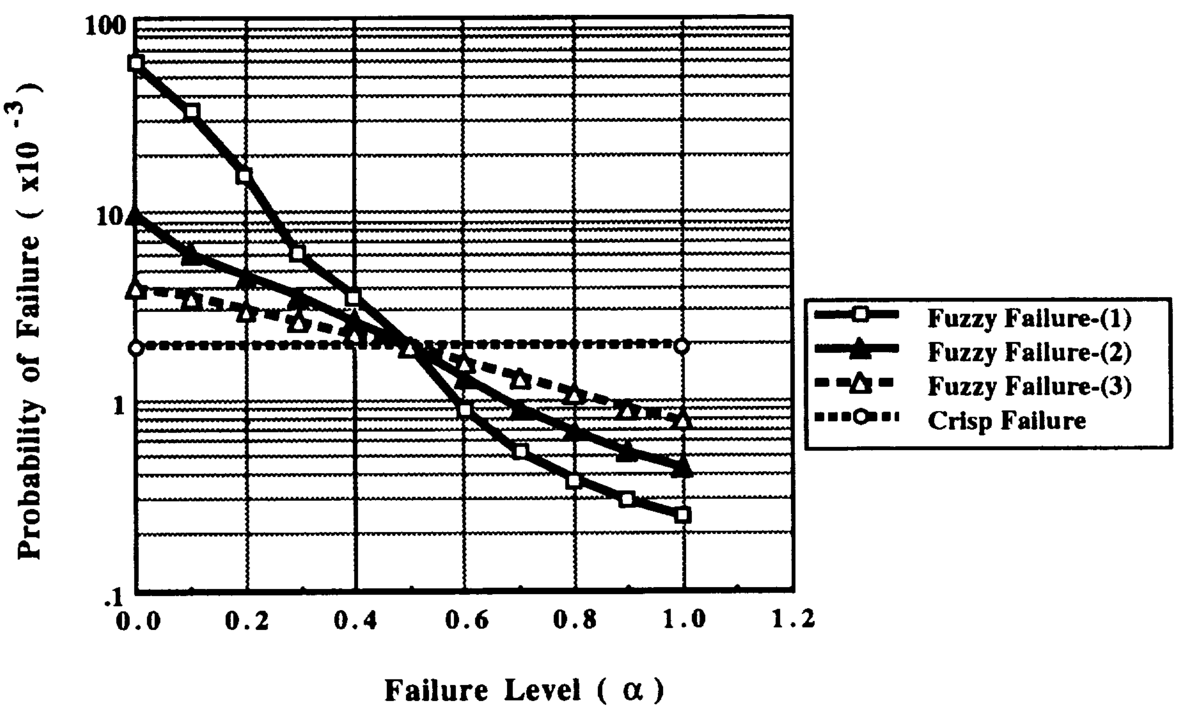
Random Variable	Mean Value	Coefficient of Variation (COV)	Probability Distribution Type
L	100×10^3 ft-tons	0.30	Extreme Value Type I
M_f	244×10^3 ft-tons	0.10	Normal

- **Fuzzy Failure Model**

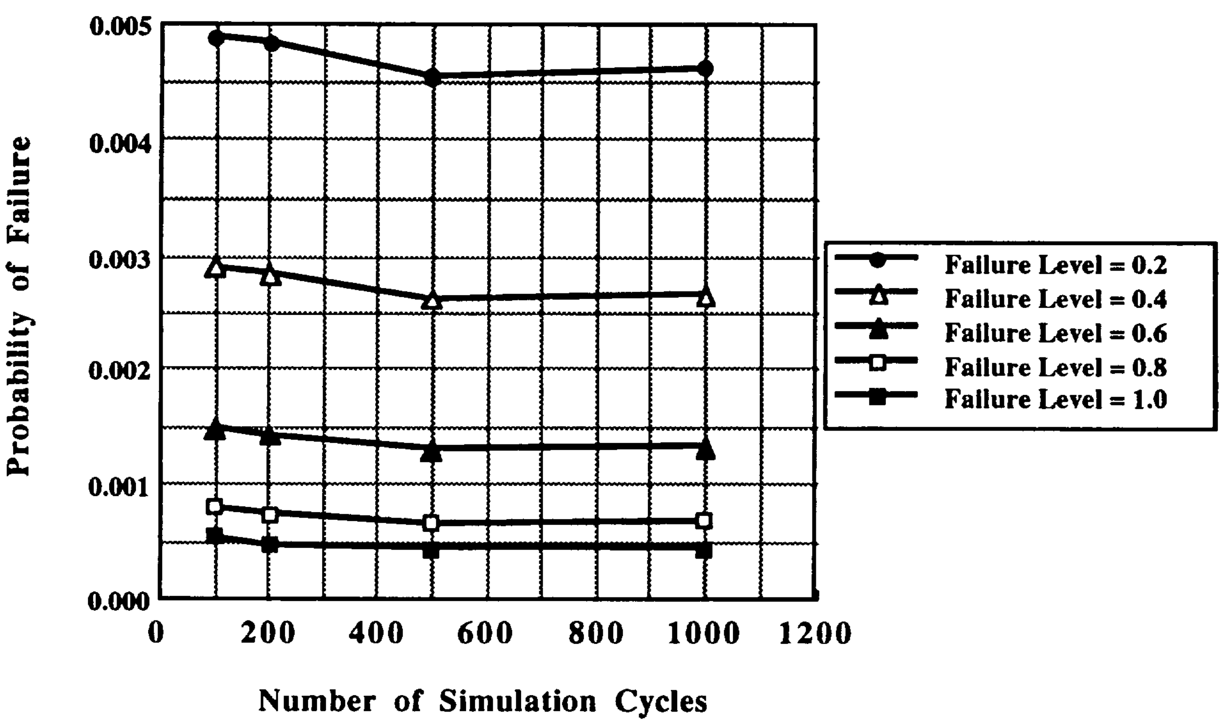




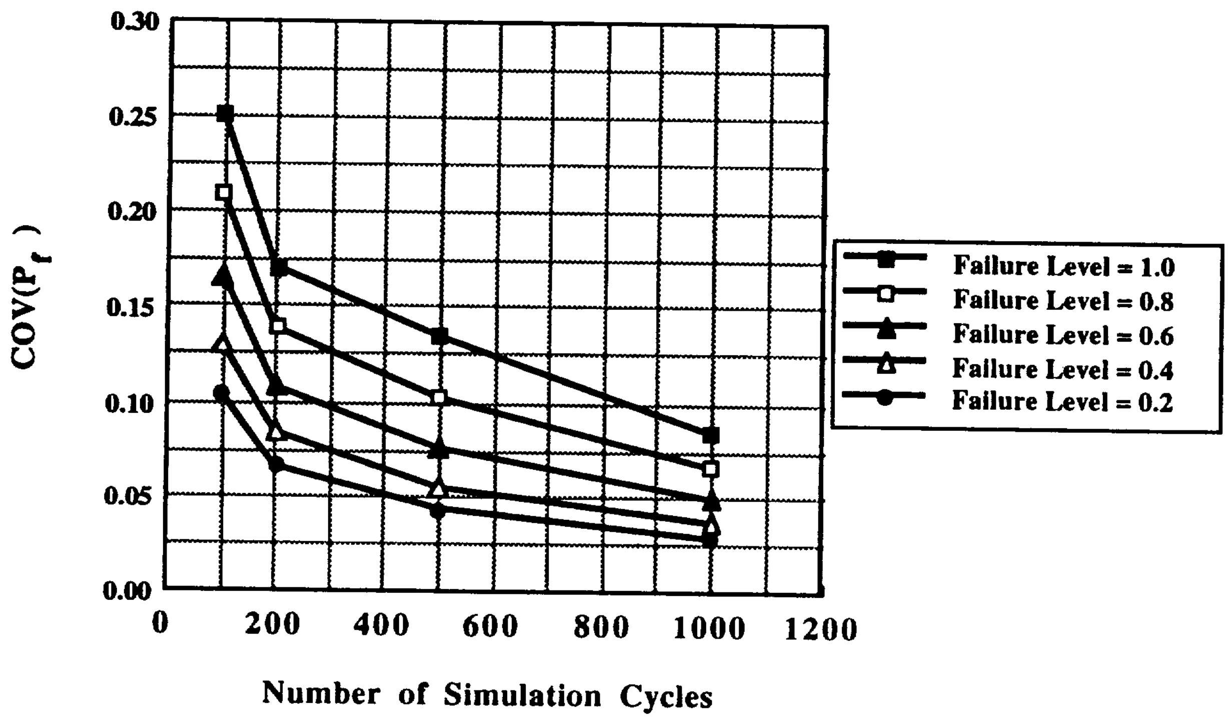
The values of P_f were calculated using 1000 simulation cycles.



The values of P_f were calculated using 1000 simulation cycles.



Probability of Failure
0.005
0.004
0.003
0.002
0.001
0.000
0
200
400
600
800
1000
1200
Number of Simulation Cycles
Failure Level = 0.2
Failure Level = 0.4
Failure Level = 0.6
Failure Level = 0.8
Failure Level = 1.0

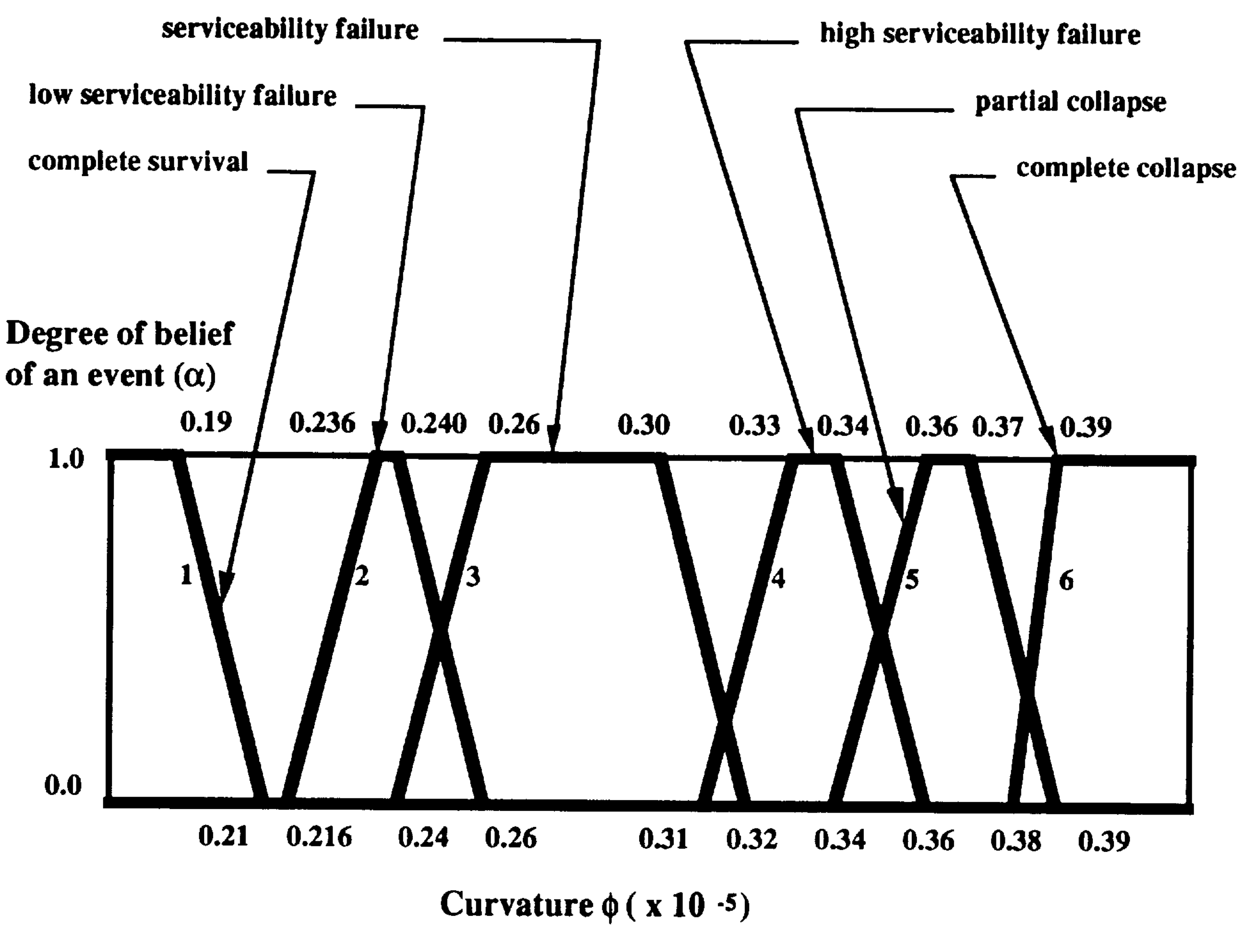


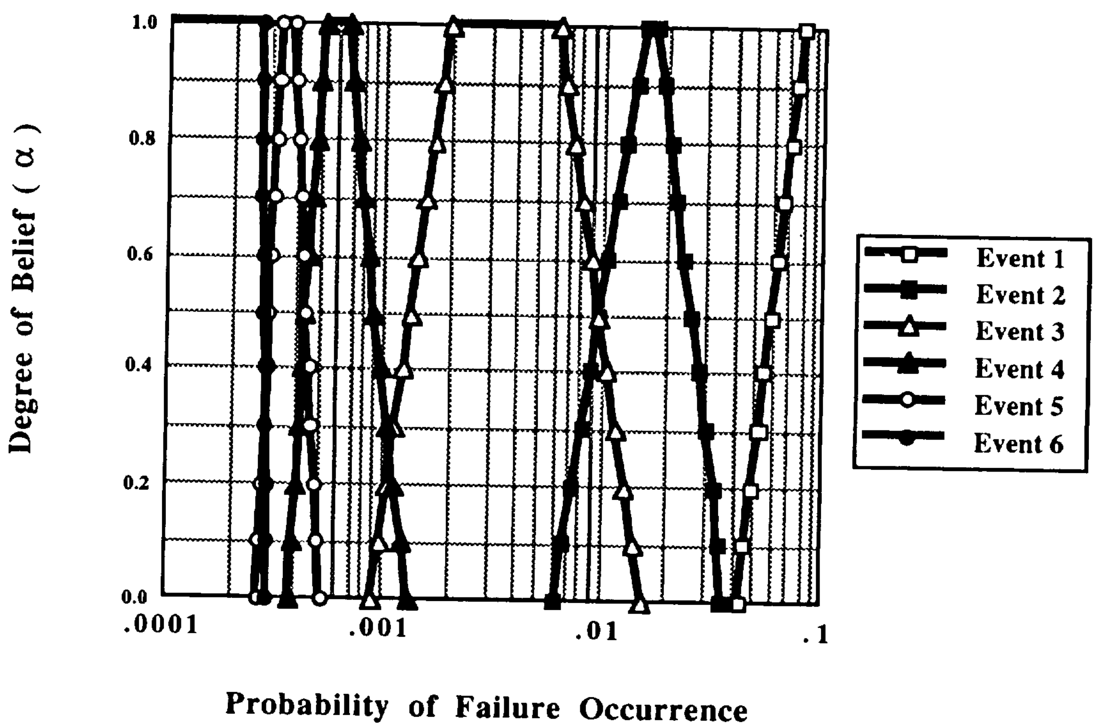
0.30
0.25
0.20
0.15
0.10
0.05
0.00
$COV(P_f)$
0
200
400
600
800
1000
1200
Number of Simulation Cycles
Failure Level = 1.0
Failure Level = 0.8
Failure Level = 0.6
Failure Level = 0.4
Failure Level = 0.2

Average Probability of Failure for Example I

Failure Model	Curvature at Failure ϕ_f	Arithmetic Average of Probability of Failure	Geometric Average of Probability of Failure
Fuzzy 1	0.2×10^{-5} to 0.4×10^{-5}	9.137×10^{-3}	2.320×10^{-3}
Fuzzy 2	0.25×10^{-5} to 0.35×10^{-5}	2.752×10^{-3}	1.851×10^{-3}
Fuzzy 3	0.275×10^{-5} to 0.325×10^{-5}	2.086×10^{-3}	1.854×10^{-3}
Crisp	0.3×10^{-5}	1.973×10^{-3}	1.973×10^{-3}

EXAMPLE II - CASE A



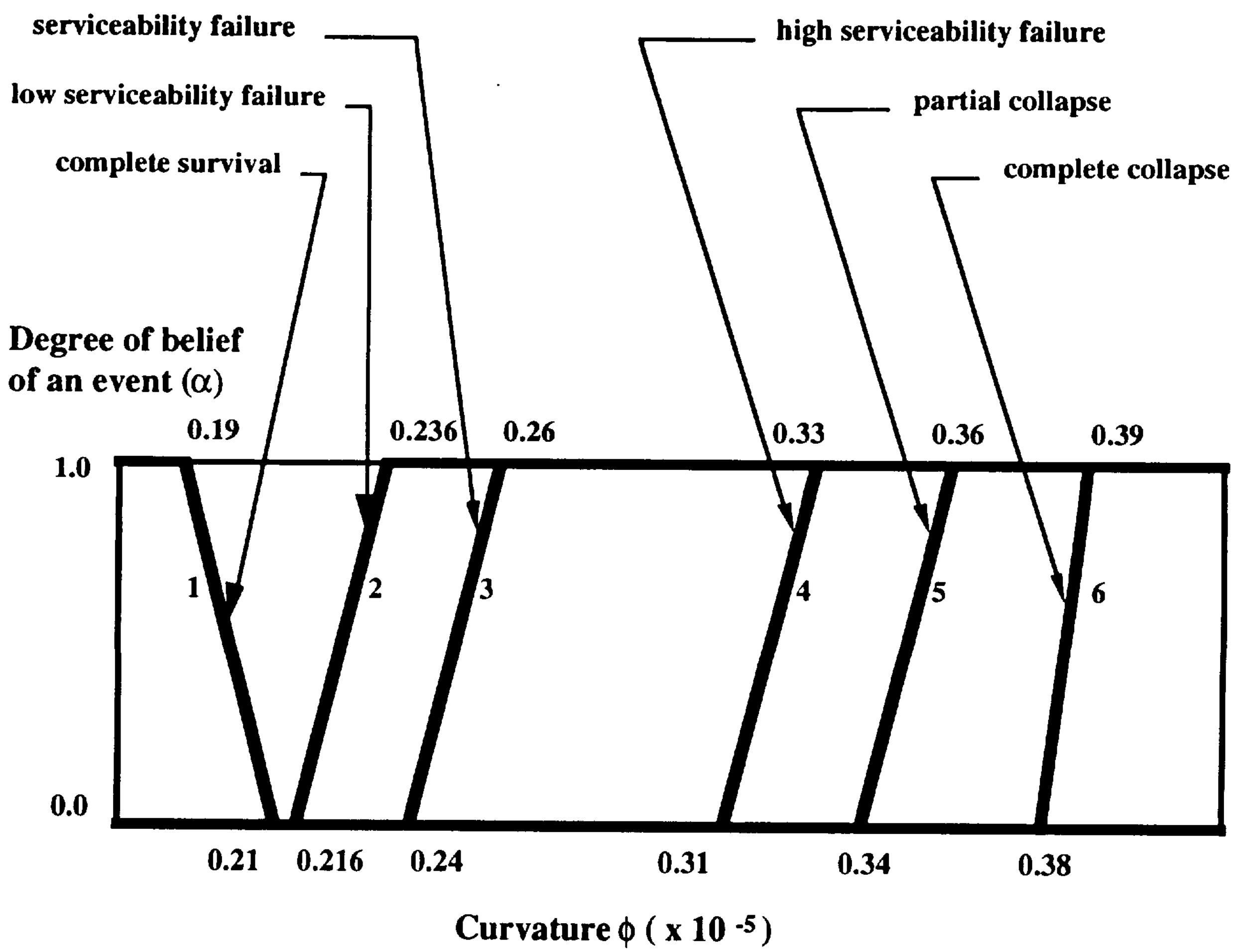


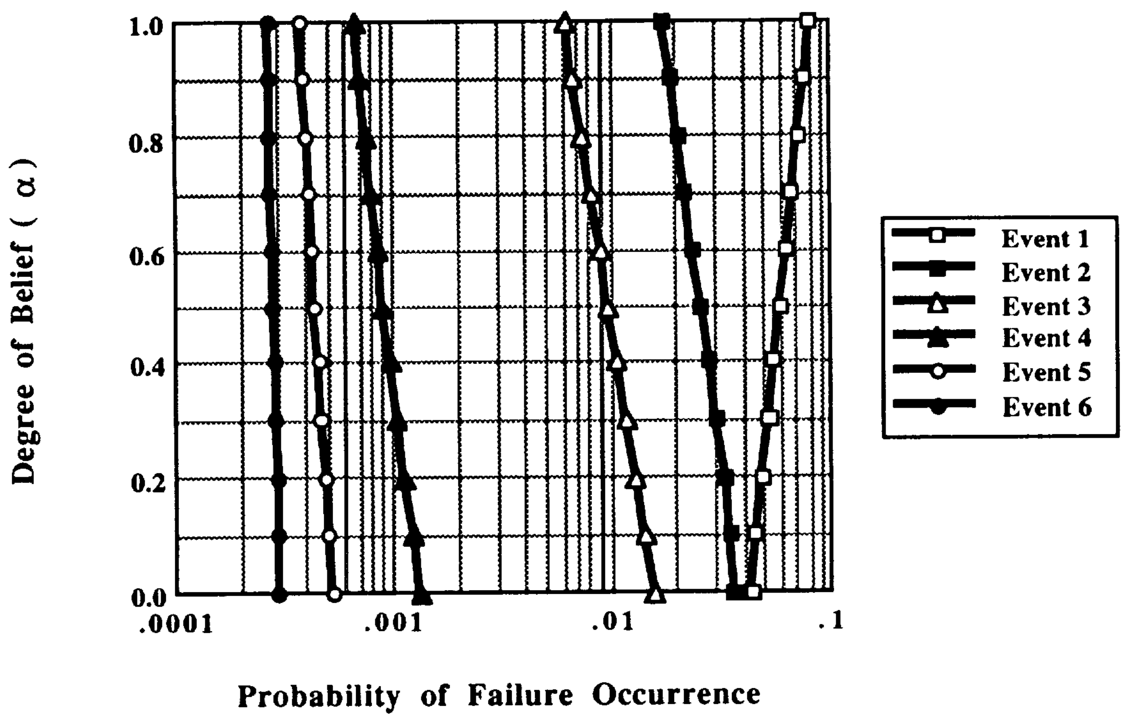
The values of P_f were calculated using 1000 simulation cycles.

Average Probability of Occurrence - Case A

Event No.	Definition	Arithmetic Average of Probability of Occurrence	Geometric Average of Probability of Occurrence
1	complete survival	0.940	0.940
2	low serviceability failure	1.611×10^{-2}	1.583×10^{-2}
3	serviceability failure	8.718×10^{-3}	8.396×10^{-3}
4	high serviceability failure	4.944×10^{-4}	4.782×10^{-4}
5	partial collapse	1.439×10^{-4}	1.424×10^{-4}
6	complete collapse	2.847×10^{-4}	2.846×10^{-4}

EXAMPLE II - CASE B





The values of P_f were calculated using 1000 simulation cycles.

Average Probability of Occurrence - Case B

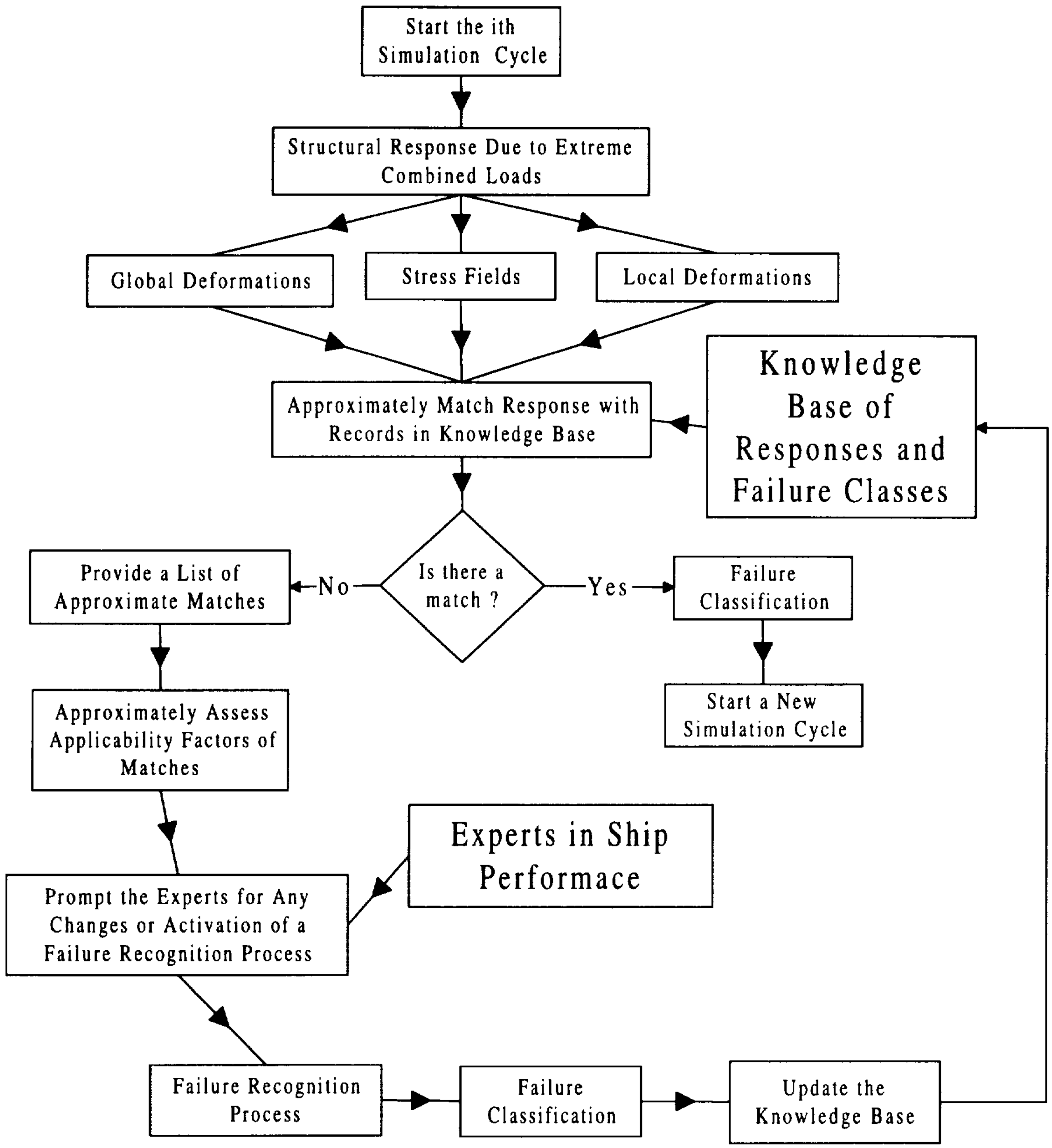
Event No.	Definition	Arithmetic Average of Probability of Occurrence	Geometric Average of Probability of Occurrence
1	complete survival	0.940	0.940
2	low serviceability failure	2.619×10^{-2}	2.555×10^{-2}
3	serviceability failure	1.008×10^{-2}	9.722×10^{-3}
4	high serviceability failure	9.388×10^{-4}	9.206×10^{-4}
5	partial collapse	4.444×10^{-4}	4.424×10^{-4}
6	complete collapse	2.847×10^{-4}	2.846×10^{-4}

UNCERTAINTY MEASURES

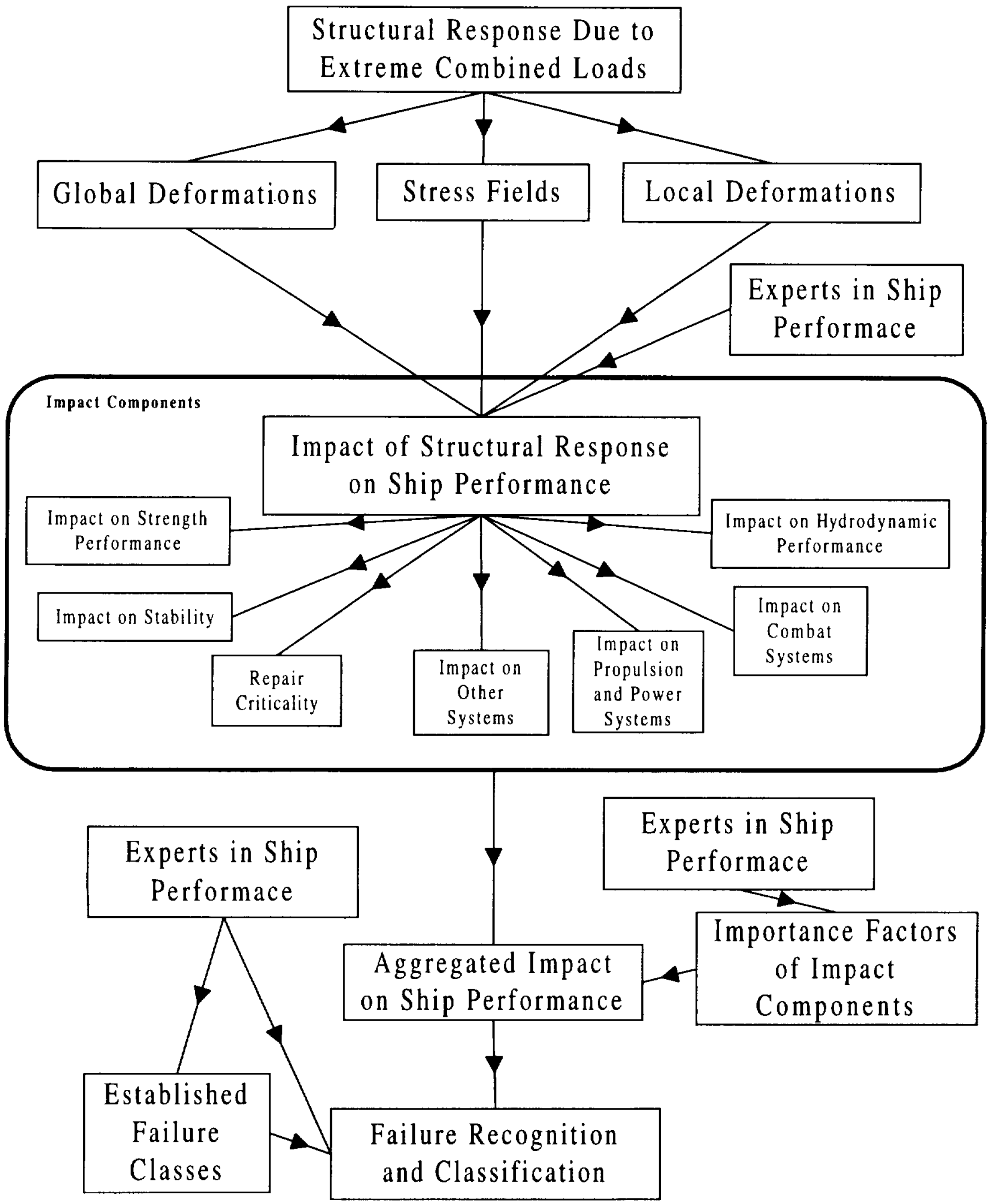
- **Hartley Measure: Set theory**
- **Shannon Entropy: Probability theory**
- **Measure of Fuzziness: Fuzzy set theory**
- **U-Uncertainty: Possibility theory**
- **Measure of Dissonance: Theory of evidence**
- **Measure of Confusion: Theory of evidence**

UNCERTAINTY MEASURES

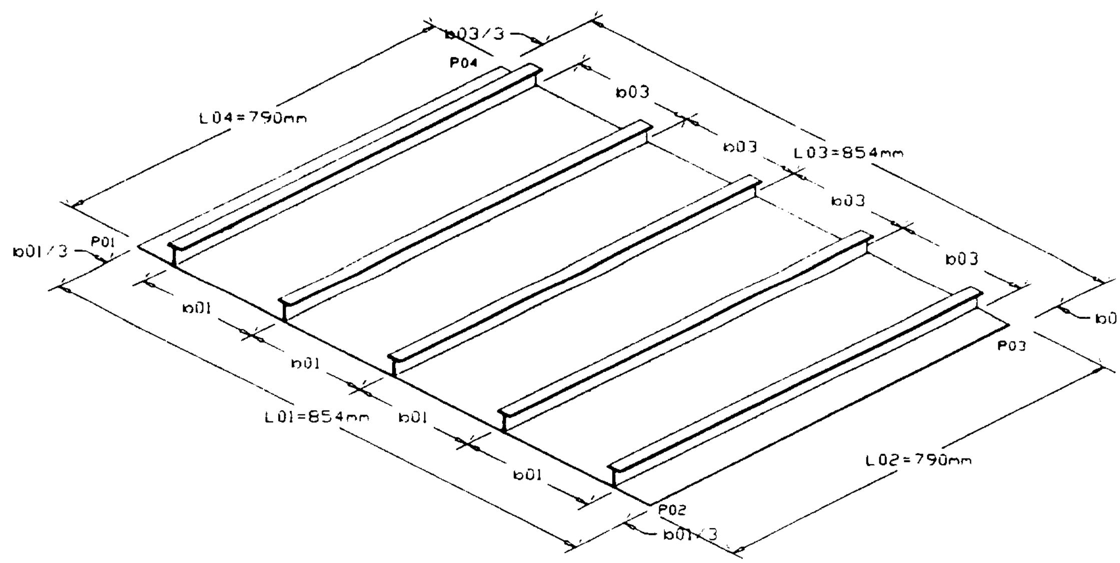
	A	B	C	D	E	F	G
1	Uncertainty measure	Type of uncertainty	Type of sets or events	Theory type	Comments	Uncertainty range	Reference
2					A basic discrete measure.		
3	Hartley	ambiguity	crisp	set	A larger number of outcomes	$[0,\infty)$	Hartley [1928]
4					means larger uncertainty		
5				set	The closer the outcomes		
6	Shannon Entropy	ambiguity	crisp	and	to an equal liklihood, the	$[0,\infty)$	Shannon [1948]
7				probability	larger the uncertainty		
8				set	Possibilistic counterpart		
9	U-uncertainty	ambiguity	crisp	and	to Shannon entropy and	$[0,\infty)$	Higashi and Klir [1983]
10				possibility	generalization to Hartley		
11					measure		
12		vagueness		set	Measures the lack of		Deluca and Termini
13	Fuzziness measure	and	fuzzy	and	distinction between a set	$[0,\infty)$	[1972,1974,1977]
14		ambiguity		fuzziness	and its complement		
15		conflict		set	Measures conflict of		
16	Dissonance measure	and	crisp	and	evidence using theory of	$[0,\infty)$	Yager [1983]
17		ambiguity		evidence	evidence		
18		confusion		set	Measures confusion of		
19	Confusion measure	and	crisp	and	evidence using theory of	$[0,\infty)$	Hohle [1981]
20		ambiguity		evidence	evidence		



Failure Classification



Failure Recognition and Classification



STIFFENED PANEL

Assumptions: **b01 = L01 / 4.666667** **b03 = L03 / 4.666667**

Stiffened Panel (dimensions and assumptions)

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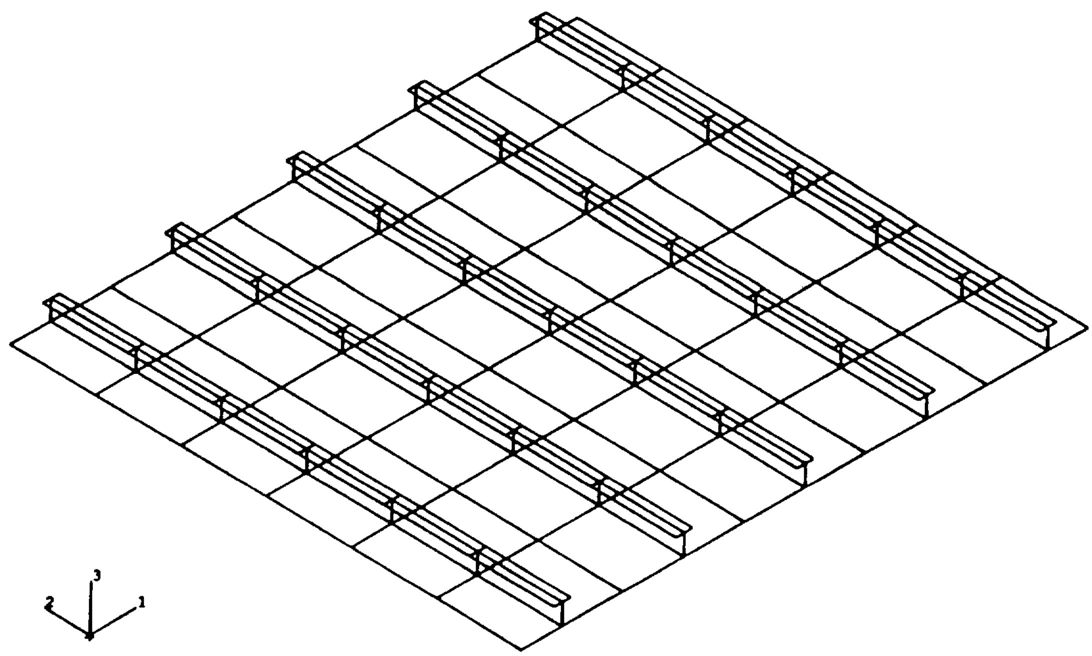
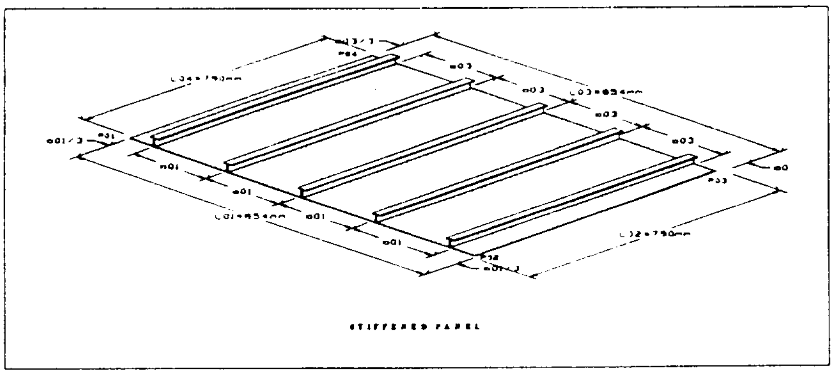
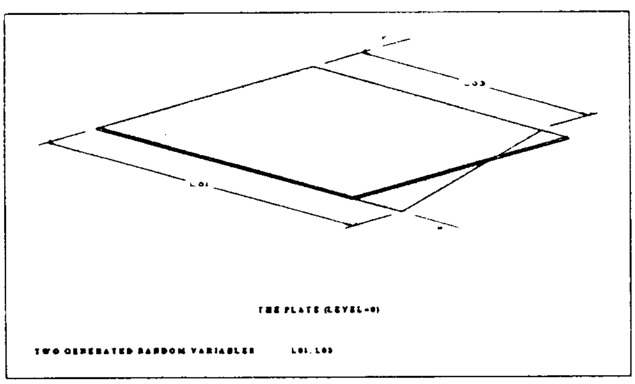


Finite Element Mesh of the Stiffened Panel





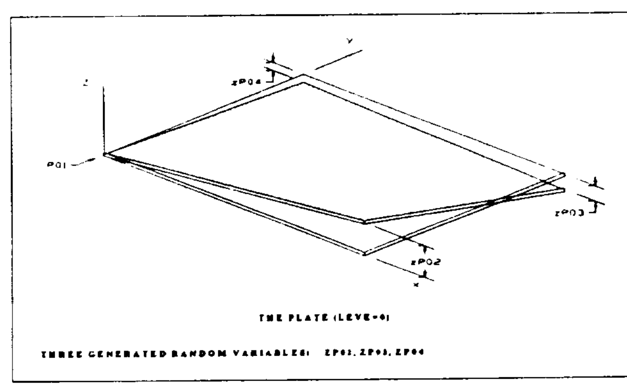
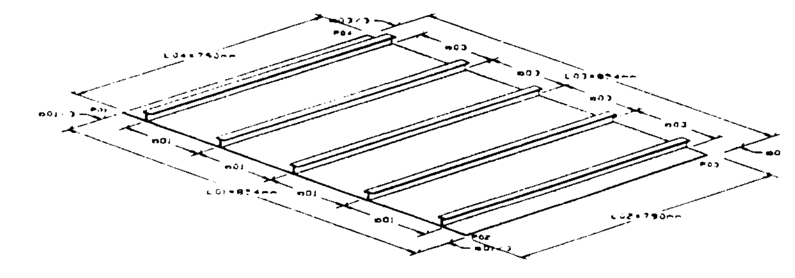
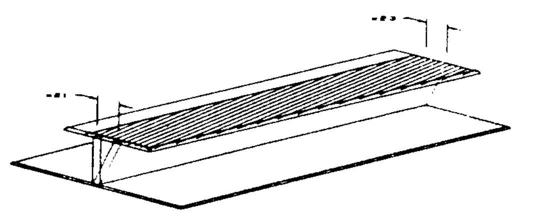


Plate Width Variability and Plate Distortion

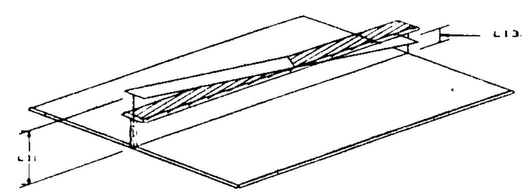


STIFFENED PANEL



WEB TILTING (LEVEL-1)

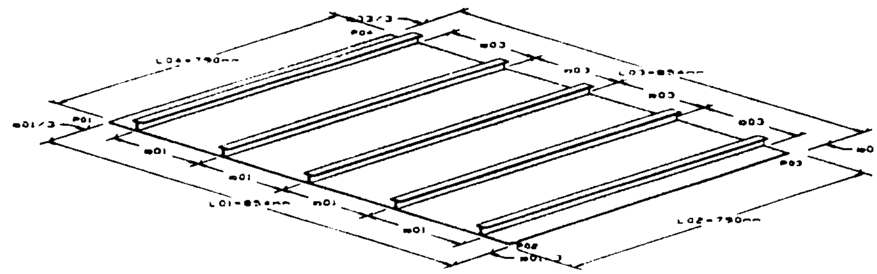
TEN GENERATED RANDOM VARIABLES: X211, X212, X213, X214, X215
X231, X232, X233, X234, X235



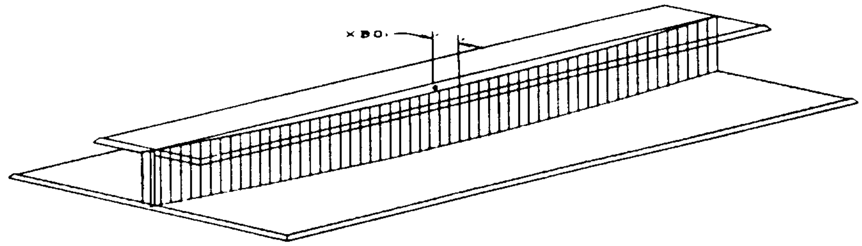
WEB HEIGHT (LEVEL=1).

TEN GENERATED RANDOM VARIABLES: L111, L112, L113, L114, L115
L131, L132, L133, L134, L135

Web Height Variability and Web Tilting



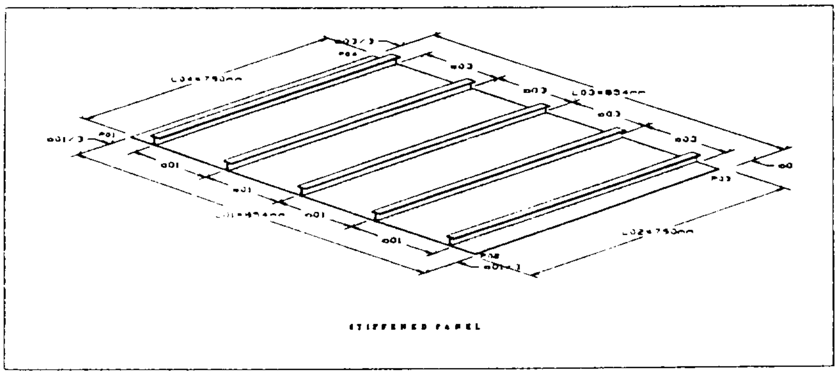
STIFFENED PANEL

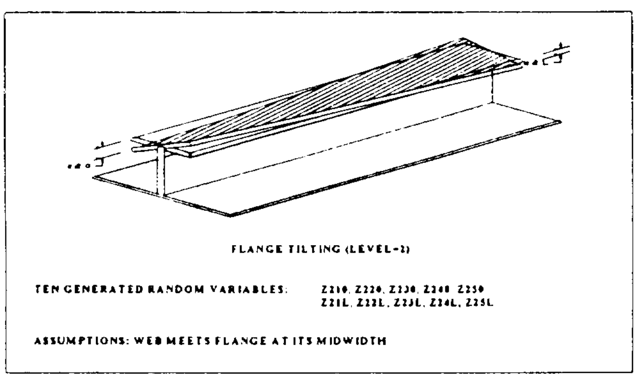


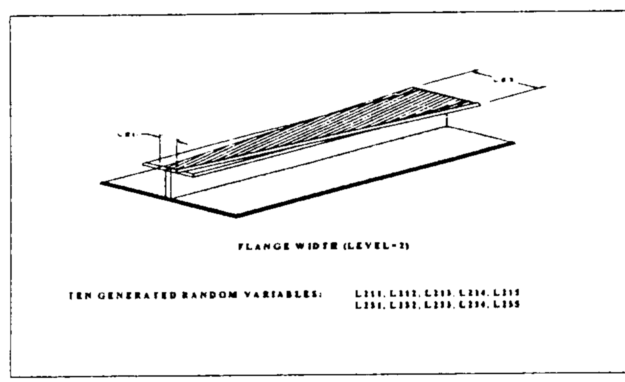
WEB BOWING (LEVELS = 0, 1, and 2)

TEN GENERATED RANDOM VARIABLES: XB01, XB02, XB03, XB04, XB05

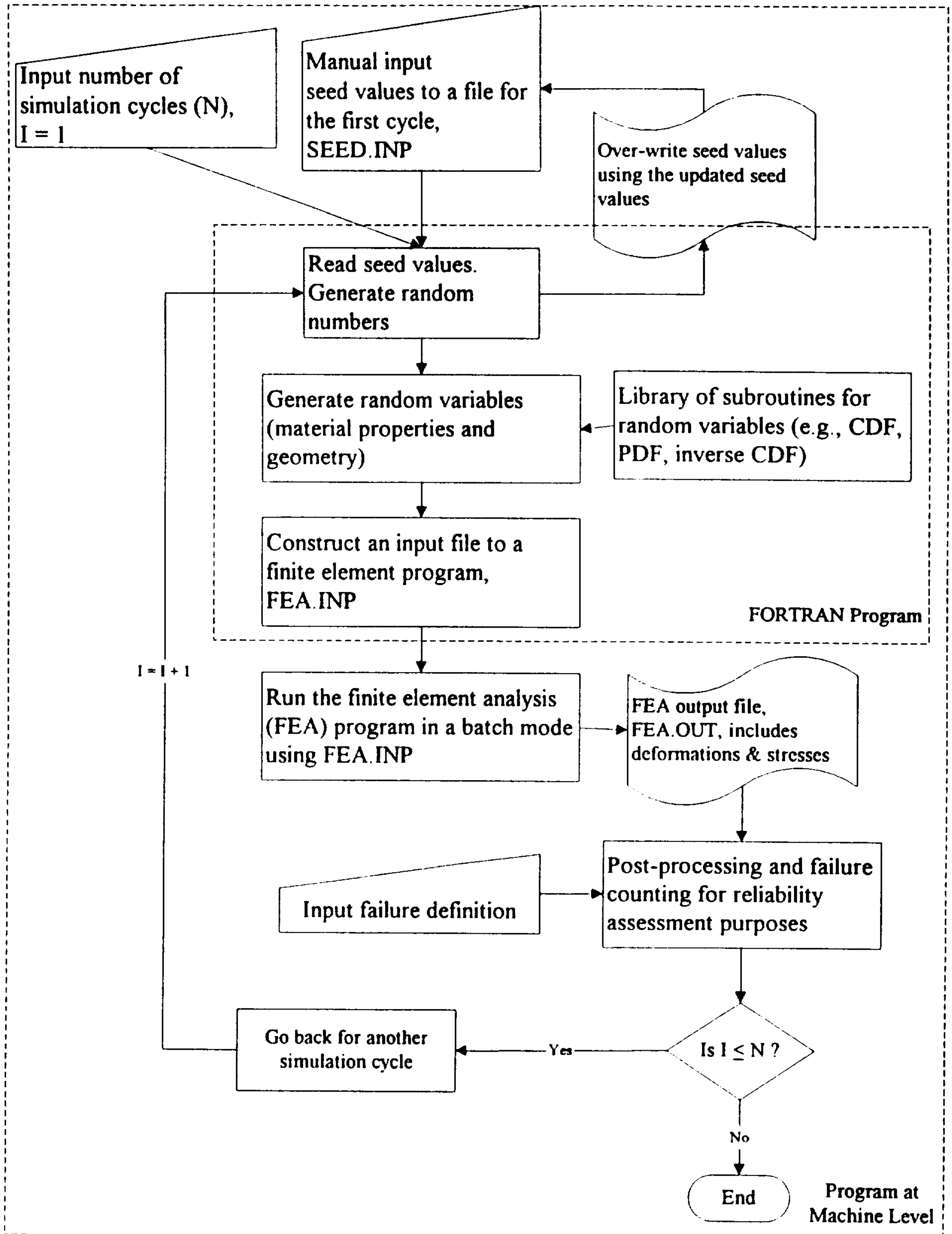
Web Bowing



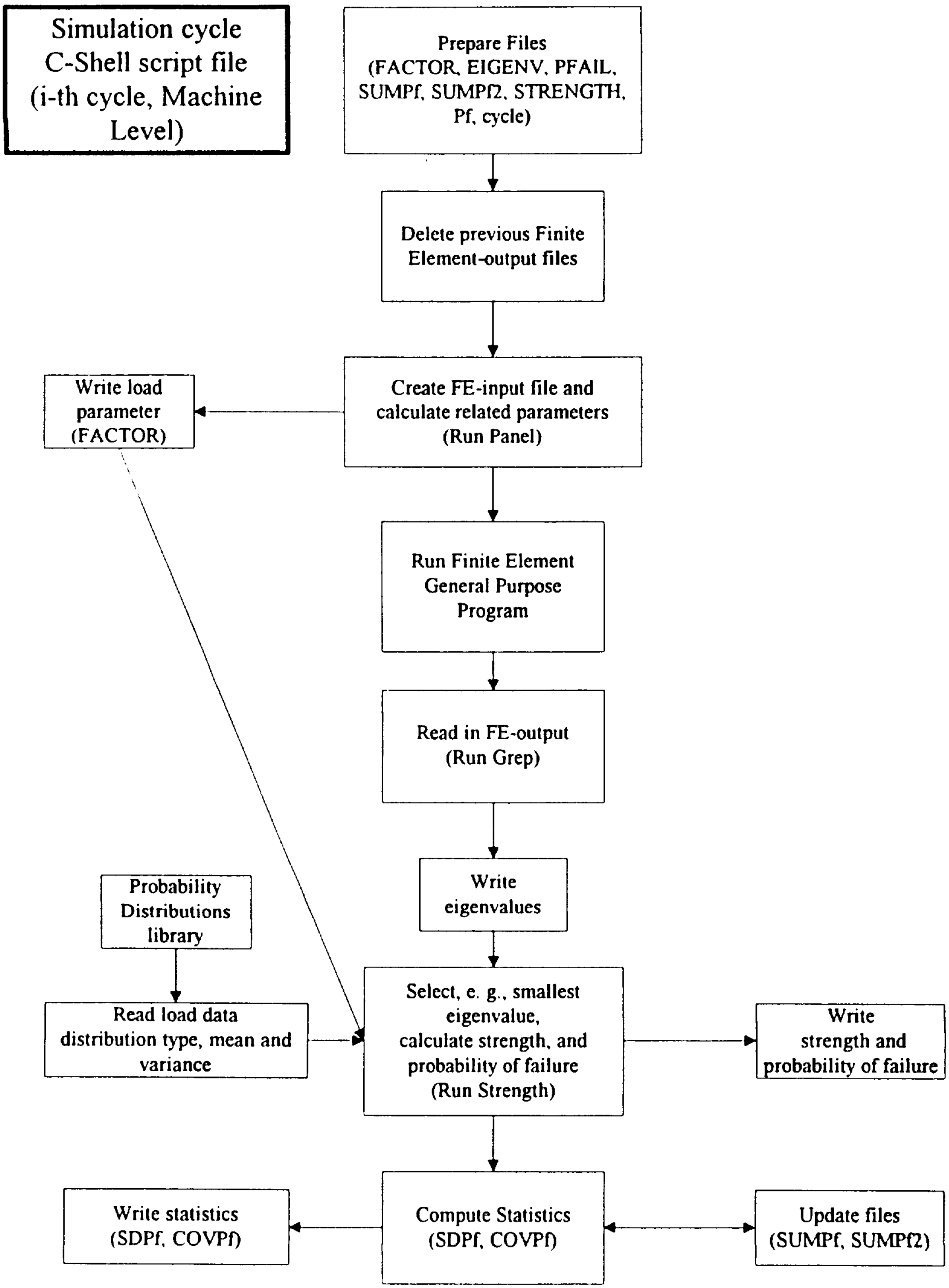




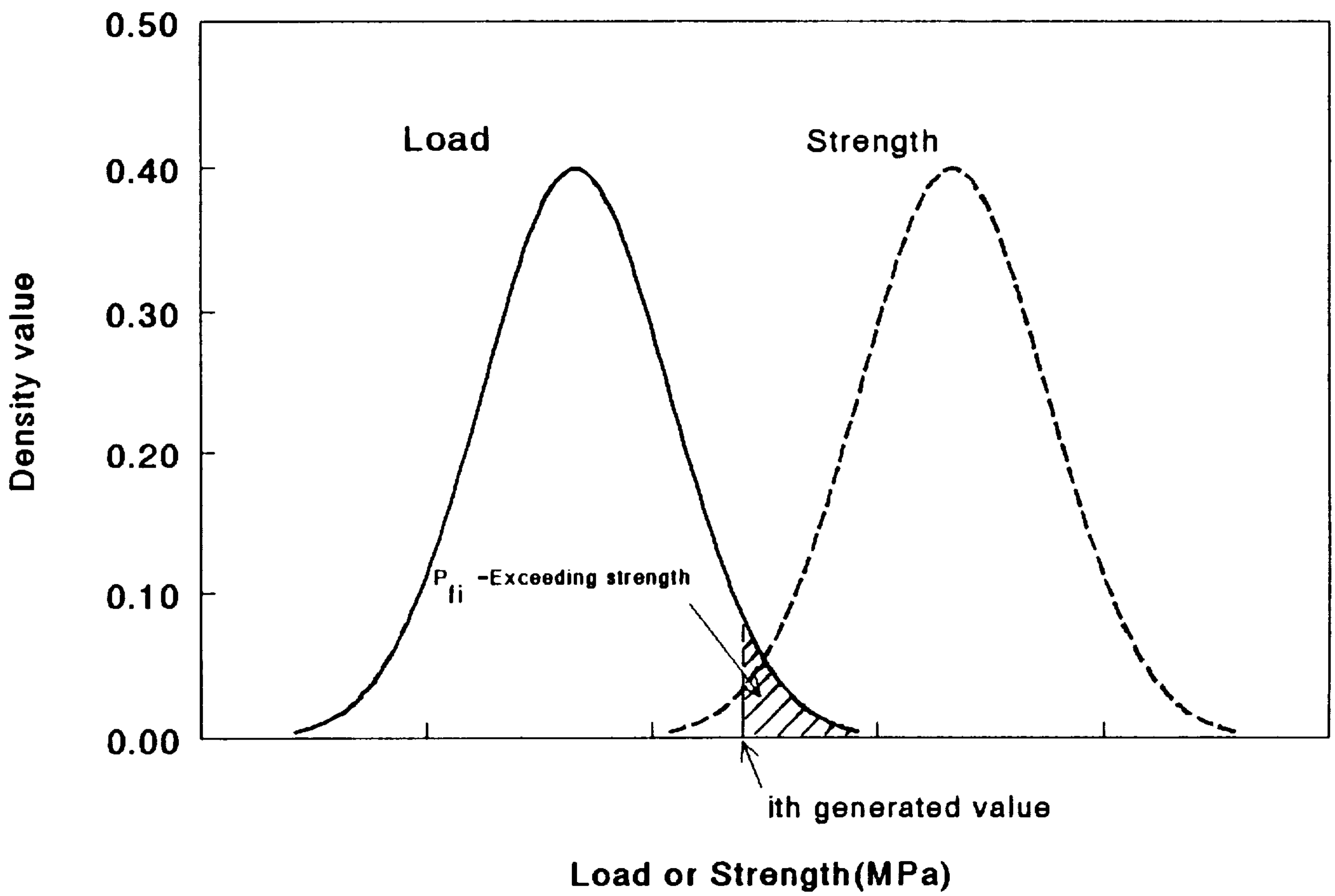
Flange Width Variability and Flange Tilting



Methodology for *ith* Simulation Cycle



C-Shell Script Flow Chart for *ith* Simulation Cycle-Machine Level



Conditional Expectation for Probability of Failure

Geometric And Material Random Variables for the Stiffened Panel

Variable no	Geometrical variables	Notation	Mean value	Coefficient of variation (COV)	Standard deviation
1	Plate size (mm)	L0i	854		4.0
2	Plate thickness (mm)	t_0	3.0	4%	0.12
3	Web thickness (mm)	t_1	4.9	4%	0.196
4	Flange thickness (mm)	t_2	5.84	4%	0.234
5	Plate-out of plane distortion (mm)	zP0i	0.0		1.0
6	Web height (mm)	L1ji	31.08	2.5%	0.77
7	Web tilting (mm)	X2ji	0.0		0.5
8	Web bowing (mm)	XB0i	0.0		0.1
9	Flange width (mm)	L2ji	25.4	2.5%	0.635
10	Flange tilting (mm)	Z2i0, Z2iL	0.0		0.2
11	Modulus of elasticity (MPa)	E	208000	4%	8320
12	Poisson's ratio	ν			
13	Yield stress (KPa)[1]	F_y	250000	7%	17500

[1] Nominal yield stress = 240000 kPa

Thicknesses and Plate Geometric Variables

Variable no.		Geometrical variables	Notation	Mean value (mm)	Coefficient of variation (COV)	Standard deviation (mm)
global	local					
1	1	Panel width (side 1)	L_{01}	854.0		4.0
2	2	Panel width (side 3)	L_{03}	854.0		4.0
3	3	Plate thickness	t_p	3.0	4%	0.12
4	4	Web thickness	t_w	4.9	4%	0.196
5	5	Flange thickness	t_f	5.84	4%	0.234
6	6	Plate-out of plane distortion (corner2)	Z_{P02}	0.0		1.0
7	7	Plate-out of plane distortion (corner3)	Z_{P03}	0.0		1.0
8	8	Plate-out of plane distortion (corner4)	Z_{P04}	0.0		1.0

Web Height Variables

Variable no.		Geometrical variables	Notation	Mean value (mm)	Coefficient of variation (COV)	Standard deviation (mm)
global	local					
9	1	Height of web no. 1 (side 1)	L_{111}	31.08	2.5%	0.77
10	2	Height of web no. 2 (side 1)	L_{112}	31.08	2.5%	0.77
11	3	Height of web no. 3 (side 1)	L_{113}	31.08	2.5%	0.77
13	4	Height of web no. 4 (side 1)	L_{114}	31.08	2.5%	0.77
14	5	Height of web no. 5 (side 1)	L_{115}	31.08	2.5%	0.77
15	6	Height of web no.1 (side 3)	L_{131}	31.08	2.5%	0.77
16	7	Height of web no.2 (side 3)	L_{132}	31.08	2.5%	0.77
17	8	Height of web no.3 (side 3)	L_{133}	31.08	2.5%	0.77
18	9	Height of web no.4 (side 3)	L_{134}	31.08	2.5%	0.77
19	10	Height of web no.5 (side 3)	L_{135}	31.08	2.5%	0.77

Web Tilting Variables

Variable no. global	Variable no. local	Geometrical variables	Notation	Mean value (mm)	Coefficient of variation (COV)	Standard deviation (mm)
20	1	Tilting of web no.1 (side 1)	X_{211}	0.0		0.5
21	2	Tilting of web no.2 (side 1)	X_{212}	0.0		0.5
22	3	Tilting of web no.3 (side 1)	X_{213}	0.0		0.5
23	4	Tilting of web no.4 (side 1)	X_{214}	0.0		0.5
24	5	Tilting of web no.5 (side 1)	X_{215}	0.0		0.5
25	6	Tilting of web no.1 (side 3)	X_{231}	0.0		0.5
26	7	Tilting of web no.2 (side 3)	X_{232}	0.0		0.5
27	8	Tilting of web no.3 (side 3)	X_{233}	0.0		0.5
28	9	Tilting of web no.4 (side 3)	X_{234}	0.0		0.5
29	10	Tilting of web no.5 (side 3)	X_{235}	0.0		0.5

Web Bowing Variables

Variable no. global	Variable no. local	Geometrical variables	Notation	Mean value (mm)	Coefficient of variation (COV)	Standard deviation (mm)
45	1	Bowing of web no. 1 (side 3)	X_{B01}	0.0		0.1
46	2	Bowing of web no. 2 (side 3)	X_{B02}	0.0		0.1
47	3	Bowing of web no. 3 (side 3)	X_{B03}	0.0		0.1
48	4	Bowing of web no. 4 (side 3)	X_{B04}	0.0		0.1
49	5	Bowing of web no. 5 (side 3)	X_{B05}	0.0		0.1

Flange Width Variables

Variable no.		Geometrical variables	Notation	Mean value (mm)	Coefficient of variation (COV)	Standard deviation (mm)
global	local					
30	1	Width of flange no. 1 (side 1)	L_{211}	25.4	2.5%	0.635
31	2	Width of flange no. 2 (side 1)	L_{212}	25.4	2.5%	0.635
32	3	Width of flange no.3 (side 1)	L_{213}	25.4	2.5%	0.635
33	4	Width of flange no. 4 (side 1)	L_{214}	25.4	2.5%	0.635
34	5	Width of flange no. 5 (side 1)	L_{215}	25.4	2.5%	0.635
35	6	Width of flange no.1 (side 3)	L_{231}	25.4	2.5%	0.635
36	7	Width of flange no.2 (side 3)	L_{232}	25.4	2.5%	0.635
37	8	Width of flange no.3 (side 3)	L_{233}	25.4	2.5%	0.635
38	9	Width of flange no.4 (side 3)	L_{234}	25.4	2.5%	0.635
39	10	Width of flange no.5 (side 3)	L_{235}	25.4	2.5%	0.635

Flange Tilting Variables

Variable no.		Geometrical variables	Notation	Mean value (mm)	Coefficient of variation (COV)	**Standard deviation (mm)**
global	local					
40	1	Tilting of flange no.1 (side 1)	Z_{210}	0.0		0.2
41	2	Tilting of flange no.2 (side 1)	Z_{220}	0.0		0.2
42	3	Tilting of flange no.3 (side 1)	Z_{230}	0.0		0.2
43	4	Tilting of flange no.4 (side 1)	Z_{240}	0.0		0.2
44	5	Tilting of flange no.5 (side 1)	Z_{250}	0.0		0.2
45	6	Tilting of flange no.1 (side 3)	Z_{21L}	0.0		0.2
46	7	Tilting of flange no.2 (side 3)	Z_{22L}	0.0		0.2
47	8	Tilting of flange no.3 (side 3)	Z_{23L}	0.0		0.2
48	9	Tilting of flange no.4 (side 3)	Z_{24L}	0.0		0.2
49	10	Tilting of flange no.5 (side 3)	Z_{25L}	0.0		0.2

Material Variability

Variable no.		Material variables	Notation	Mean value	Coefficient of variation (COV)	**Standard deviation**
global	local					
50	1	Modulus of elasticity of plate material (MPa)	E_0	208000	4%	**8320**
51	2	Modulus of elasticity of web material (MPa)	E_1	208000	4%	**8320**
52	3	Modulus of elasticity of flange material (MPa)	E_2	208000	4%	**8320**
53	4	Poisson's ratio of plate	ν_0			
54	5	Poisson's ratio of web	ν_1			
55	6	Poisson's ratio of flange	ν_2			
56	7	Yield stress of plate (kPa) [1]	F_{y0}	250000	7%	**17500**
57	8	Yield stress of web (kPa) [1]	F_{y1}	250000	7%	**17500**
57	9	Yield stress of flange (kPa) [1]	F_{y2}	250000	7%	**17500**

[1] Nominal yield stress = 240000 kPa

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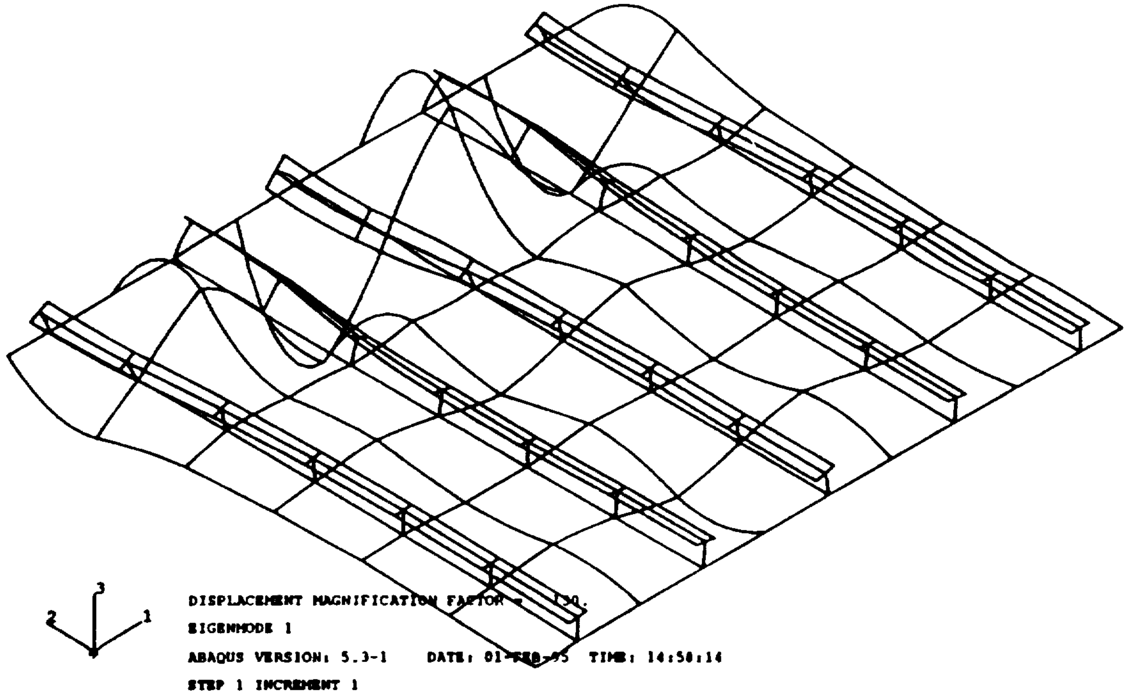
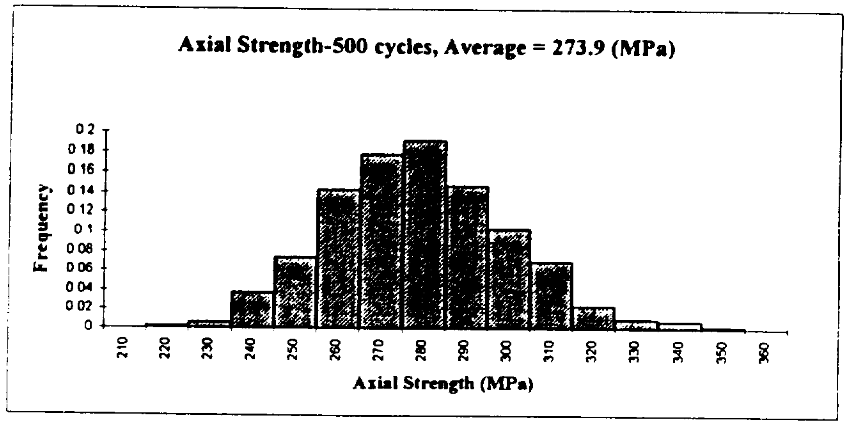
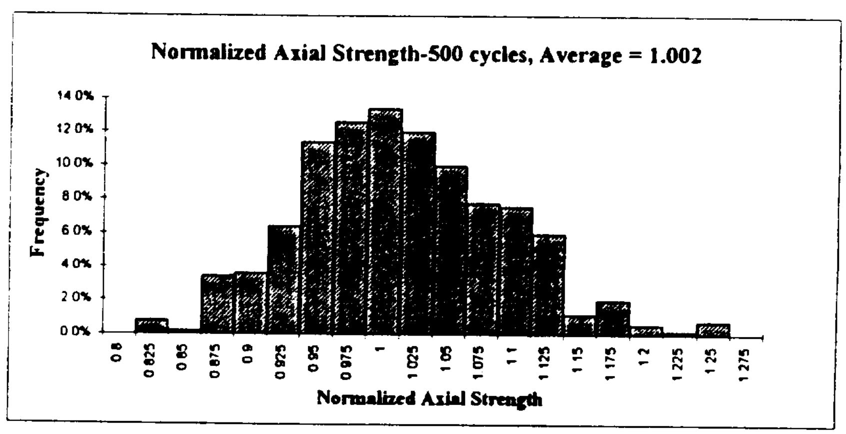


Buckling Shape of the Stiffened Panel

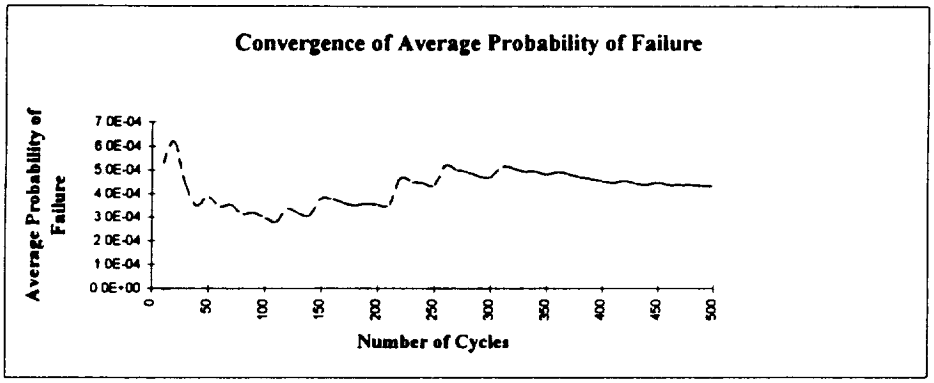
Statistical Measures	Axial Strength
Mean	273.9064
Standard Error	0.933275
Median	272.1663
Standard Deviation	20.86865
Sample Variance	435.5007
Kurtosis	0.031126
Skewness	0.278253
Range	121.3818
Minimum	219.5003
Maximum	340.8821
Sum	136953.2
Count	500
Confidence Level(95%)	1.829182



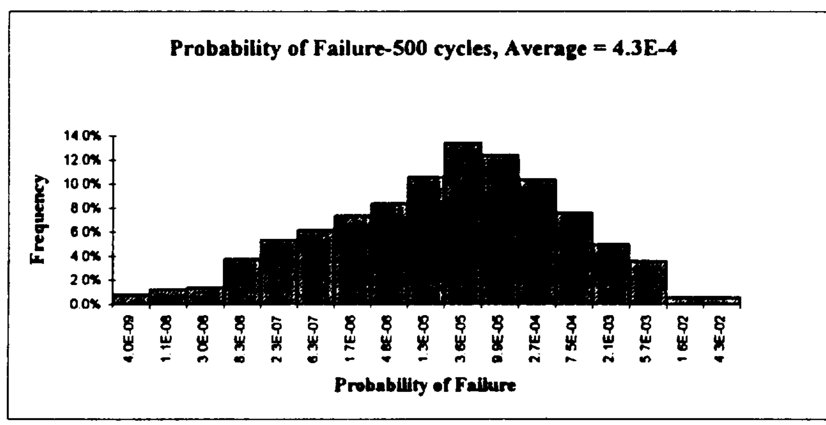
Statistical Measures	Normalized Axial Strength
Mean	1.00221868
Standard Error	0.00341484
Median	0.99585163
Standard Deviation	0.07635806
Sample Variance	0.00583055
Kurtosis	0.03112581
Skewness	0.27825334
Range	0.44413392
Minimum	0.80314782
Maximum	1.24728174
Sum	501.109341
Count	500
Confidence Level(95%)	0.006692946



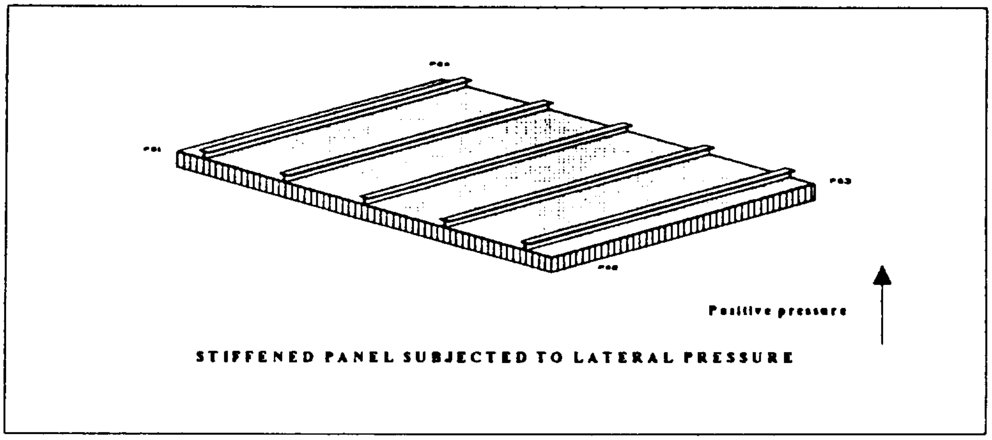
Axial Strength Statistics of the Stiffened Panel-500 Cycles

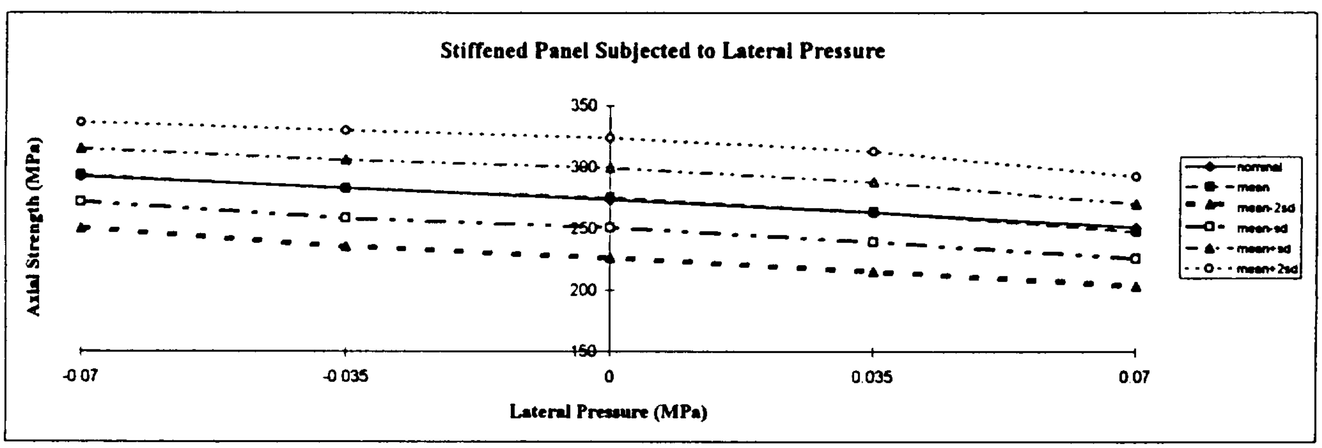


Statistical Measures	Probability of Failure
Mean	0.000431
Standard Error	8.83E-05
Median	1.83E-05
Standard Deviation	0.001973
Sample Variance	3.89E-06
Kurtosis	104.3945
	9.528373
Range	0.025693
Minimum	7.28E-11
Maximum	0.025693
Sum	0.215415
Count	500
Confidence Level (95.0%)	0.000173



Probability of Failure Statistics of the Stiffened Panel-500 Cycles





Stiffened Panel Subjected to Concentric Axial Compression and Lateral Pressure

PARAMETRIC ANALYSIS

A parametric analysis was conducted for the axial strength and failure probability of the panel. The analysis was carried out by individually varying the coefficients of variation or standard deviations of the basic random variables. The notations, mean values, and ranges of COV and standard deviations of the random variables are given in the following table. The following observations were developed based on the results of the parametric analysis using 100 simulation cycles:

- For the plate width, a figure shows that increasing the COV from 0.47% to 0.94%, the normalized strength decreases from 1.007 to 0.988, the COV of the axial strength decreases from 8.87% to 7.79%, and the average of probability of failure decreases from 8.20×10^{-4} to 6.45×10^{-4}.

- For the plate-out of plane distortion, a figure shows that increasing the standard deviation from 1.0 to 3.0, the normalized strength increases from 1.007 to 1.009, the COV of the axial strength decreases from 8.87% to 7.42%, and the average of probability of failure decreases from 8.20×10^{-4} to 6.45×10^{-4}.

- For the web height, a figure shows that increasing the COV from 2.5% to 5.0%, the normalized strength increases from 1.007 to 1.011, the COV of the axial strength decreases from 8.87% to 7.37%, and the average of probability of failure decreases from 8.20×10^{-4} to 3.94×10^{-4}.

- For the web tilting, a figure shows that increasing the standard deviation from 0.2 mm to 0.5 mm, the normalized strength increases from 1.005 to 1.007, the COV of the axial strength increases from 7.17% to 9%, and the average of probability of failure increases from 5.0×10^{-4} to 8.45×10^{-4}.

- For the web bowing, a figure shows that increasing the standard deviation from 0.1 mm to 0.2 mm, the normalized strength decreases from 1.007 to 0.99, the COV of the axial strength decreases from 9.0% to 7.8%, and the average of probability of failure decreases from 8.20×10^{-4} to 4.84×10^{-4}.

- For the flange width, a figure shows that increasing the COV from 2.5% to 5.0%, the normalized strength decreases from 1.007 to 1.004, the COV of the axial strength decreases from 9.0% to 7.26%, and the average of probability of failure decreases from 8.20×10^{-4} to 2.23×10^{-4}.

- For the flange tilting, a figure shows that increasing the standard deviation from 0.2 mm to 0.5 mm, the normalized strength decreases from 1.007 to 0.995, the COV of the axial strength decreases from 9.0% to 8.0%, and the average of failure probability increases from 8.20×10^{-4} to 1.77×10^{-3}.

- For the thicknesses, a figure shows that increasing the COV from 4.0% to 8.0%, the normalized strength decreases from 1.007 to 0.994, the COV of the axial strength increases from 8.87% to 13.0%, and the average of probability of failure decreases from 8.28×10^{-4} to 1.53×10^{-2}.

- For the modulus of elasticity, a figure shows that increasing the COV from 4.0% to 8.0%, the normalized strength decreases from 1.007 to 1.002, the COV of the axial strength remains constant at the value of 8.90%, and the average of probability of failure decreases from 8.20×10^{-4} to 1.49×10^{-3}.

The above failure probability observations were based on results from 100 simulation cycles. The number of simulation cycles might not be adequate for obtaining accurate failure probability results, but it is sufficient for determining the axial strength. The number of cycles was limited to 100 in order to make the study feasible within the planned time frame of the project.

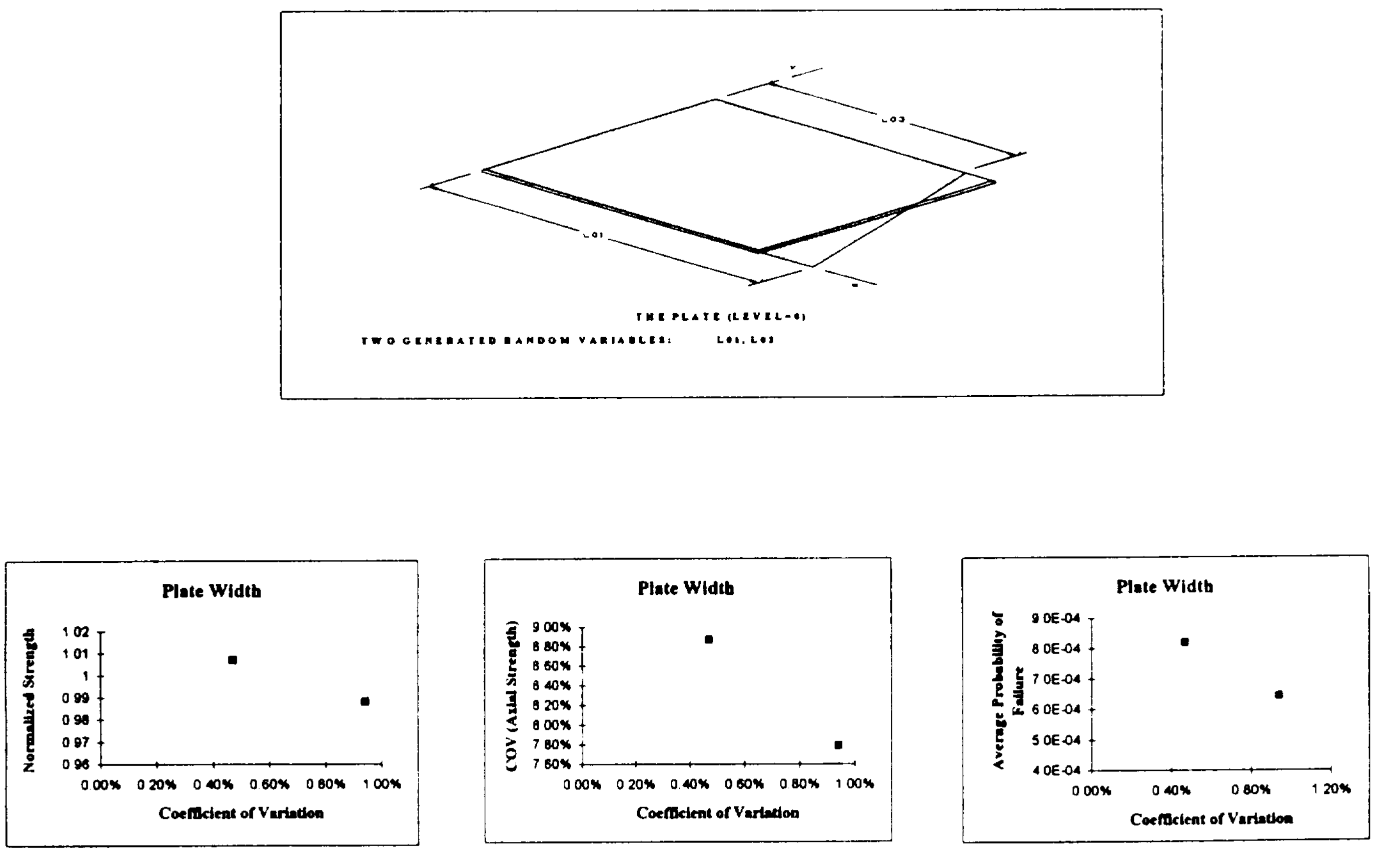
A table shows a summary of the results of the parametric study. According to the table, variations in the variability of plate size and web bowing produced the largest effect on the mean axial strength ratio; whereas variations in the variability of thicknesses of the plate, webs, and flanges produced the largest effect on the coefficient of variation of the axial strength.

Variation of Coefficient of Variation or Standard Deviation

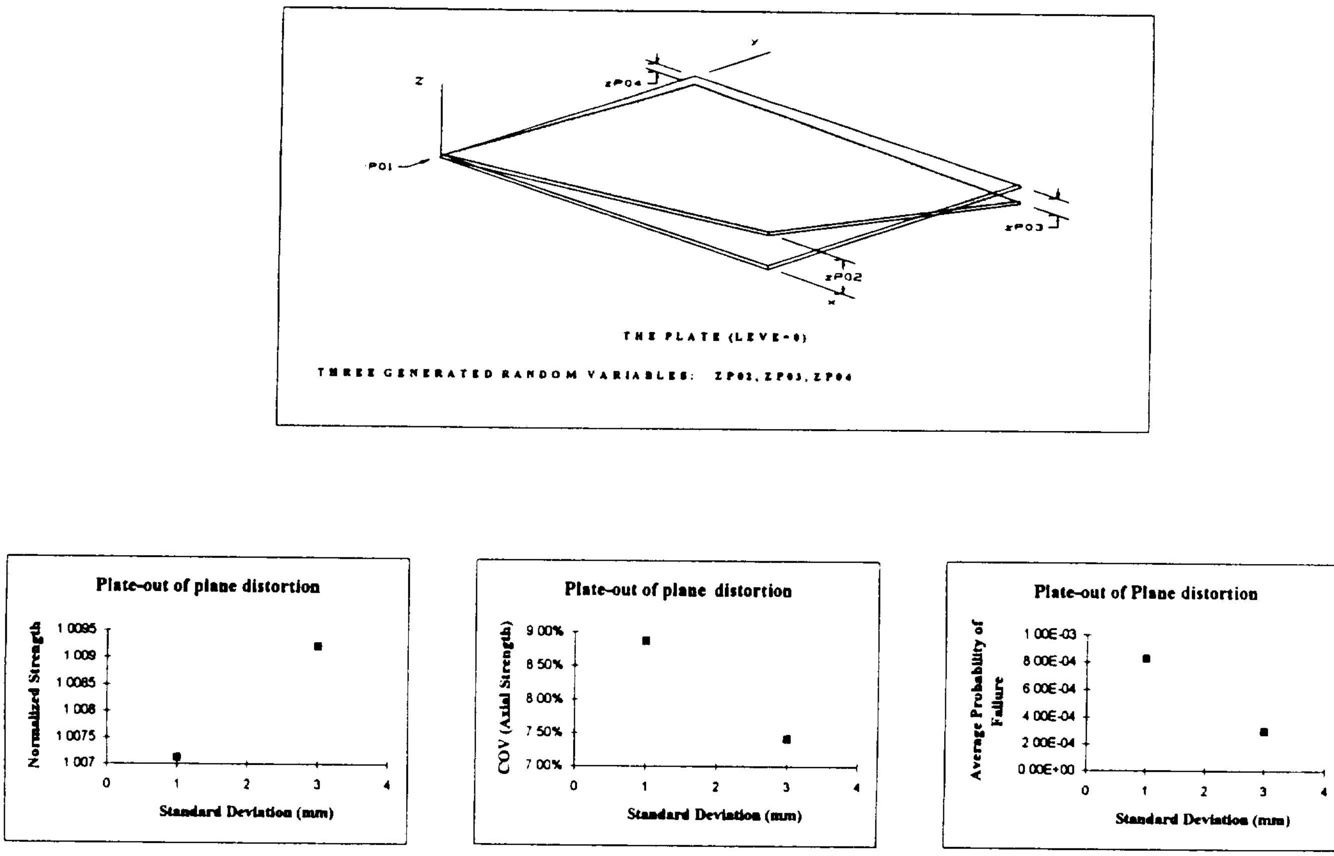
Variable no.	Geometrical variables	Notation	Mean value	Coefficient of variation (COV)	Standard deviation
1	Plate size (mm)	L0i	854		4.0 to 8
2	Plate thickness (mm)	t_0	3.0	4 to 8%	
3	Web thickness (mm)	t_1	4.9	4 to 8%	
4	Flange thickness (mm)	t_2	5.84	4 to 8%	
5	Plate-out of plane distortion (mm)	zP0i	0.0		1.0 to 3.0
6	Web height (mm)	L1ji	31.08	2.5 to 5%	
7	Web tilting (mm)	X2ji	0.0		0.2 to 0.5
8	Web bowing (mm)	XB0i	0.0		0.1 to 0.2
9	Flange width (mm)	L2ji	25.4	2.5 to 5%	
10	Flange tilting (mm)	Z2i0, Z2iL	0.0		0.2 to 0.5
11	Modulus of elasticity (MPa)	E	208000	4 to 8%	

Parametric Analysis Results

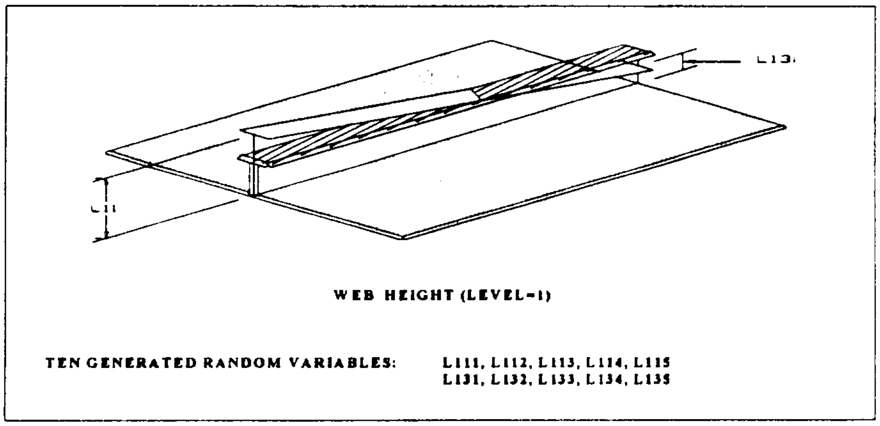
Variable no.	Geometrical Variables	Mean value	Variation of coefficient of variation	Variation of standard deviation	Effect on axial strength ratio	Effect on coefficient of variation of strength
1	Plate size (mm)	854		4.0 to 8.0	High	Medium/Low
2	Plate thickness (mm)	3.0	4 to 8%	0.12 to 0.24	Medium	High
3	Web thickness (mm)	4.9	4 to 8%	0.196 to 0.392	Medium	High
4	Flange thickness (mm)	5.84	4 to 8%	0.234 to 0.468	Medium	High
5	Plate-out of plane distortion (mm)	0.0		1.0 to 3.0	Low	Medium/Low
6	Web height (mm)	31.08	2.5 to 5%	0.77 to 1.54	Low	Medium/Low
7	Web tilting (mm)	0.0		0.2 to 0.5	Low	Medium/Low
8	Web bowing (mm)	0.0		0.1 to 0.2	High	Medium/Low
9	Flange width (mm)	25.4	2.5 to 5%	0.635 to 1.27	Low	Medium/Low
10	Flange tilting (mm)	0.0		0.2 to 0.5	Medium	Medium/Low
11	Modulus of elasticity (MPa)	208000	4 to 8%	8320 to 16640	Low	None

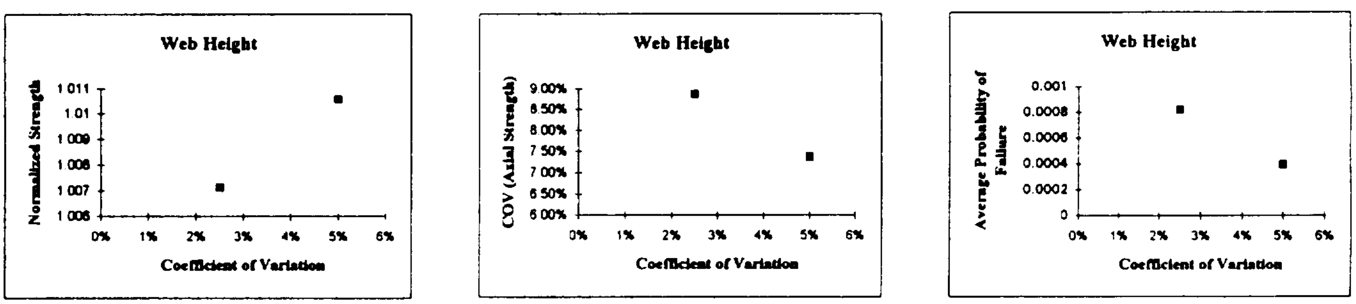


Strength and Probability of Failure Due to Plate Width Variability

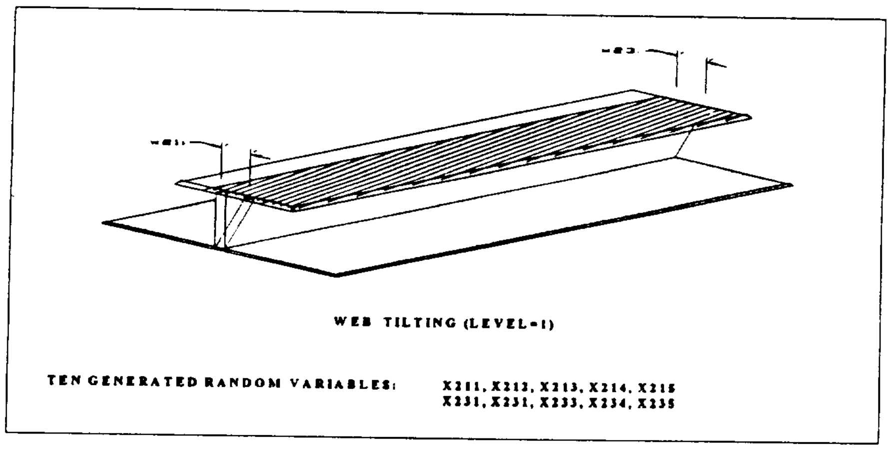


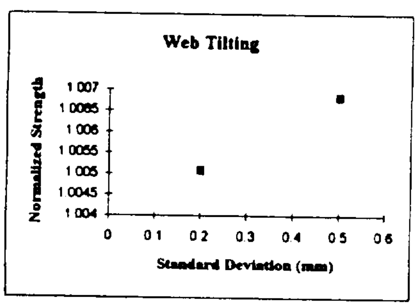
Strength and Probability of Failure Due to Plate Distortion Variability

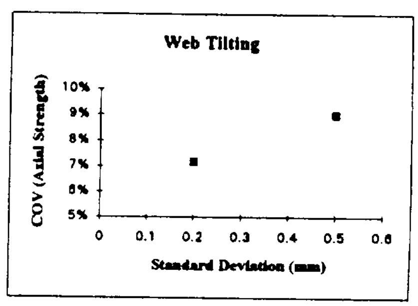


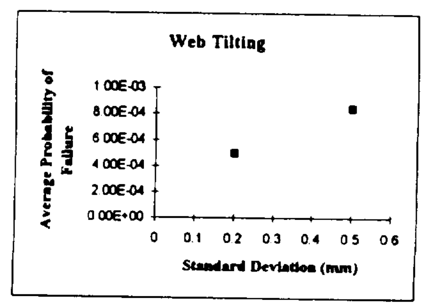


Strength and Probability of Failure Due to Web Height Variability

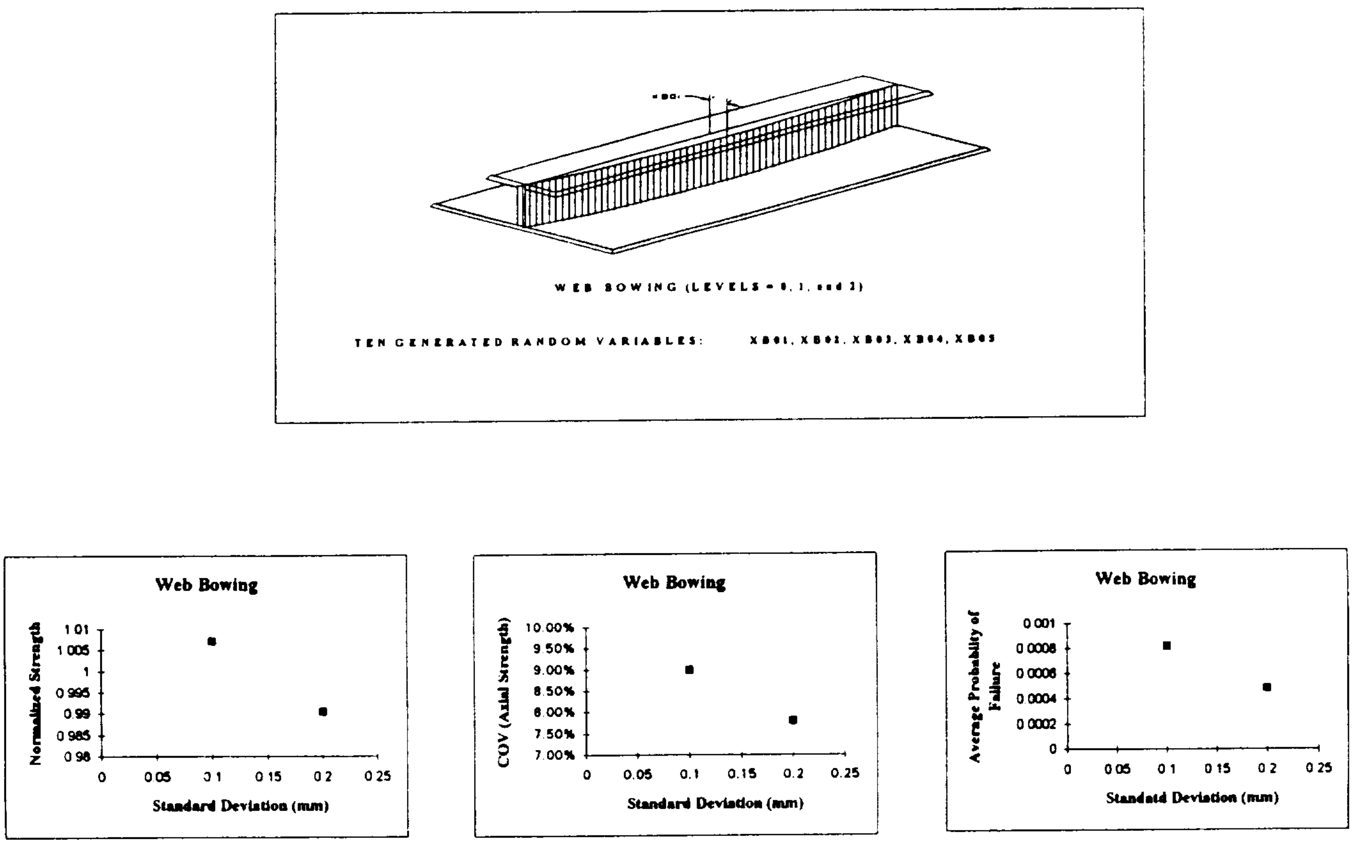




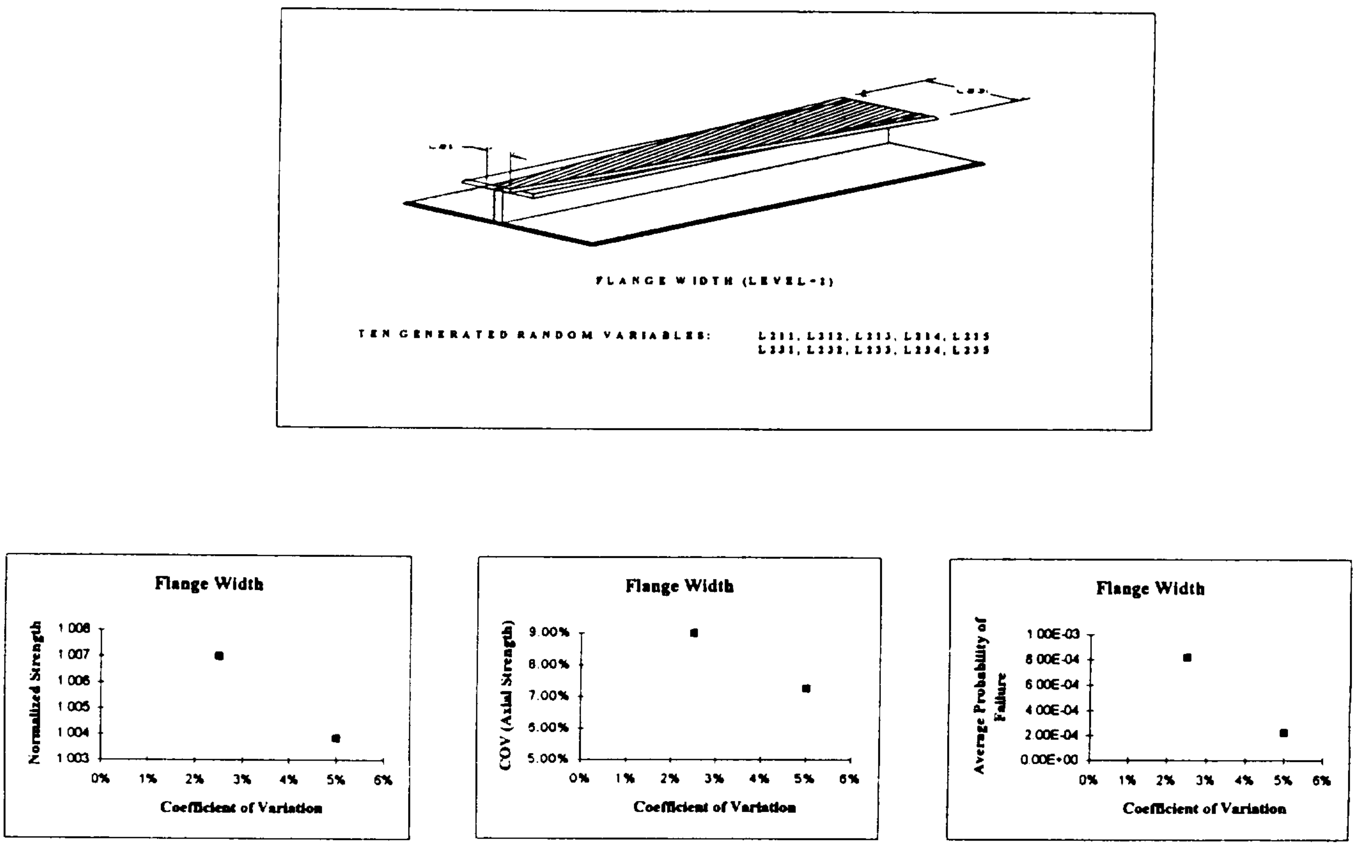




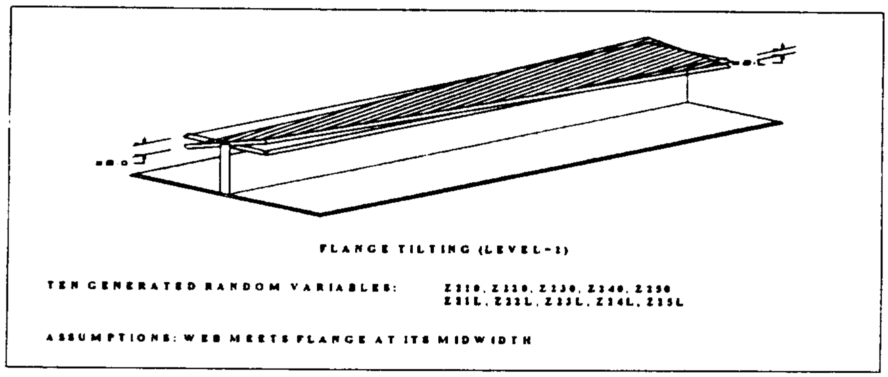
Strength and Probability of Failure Due to Web Tilting Variability

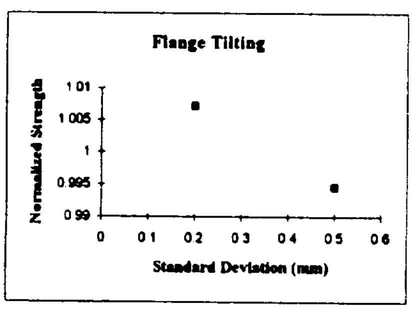


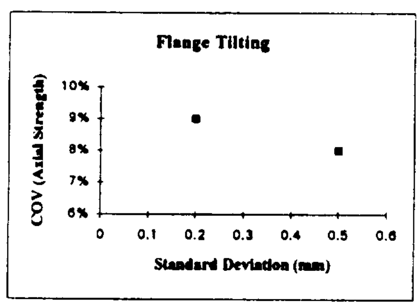
Strength and Probability of Failure Due to Web Bowing Variability

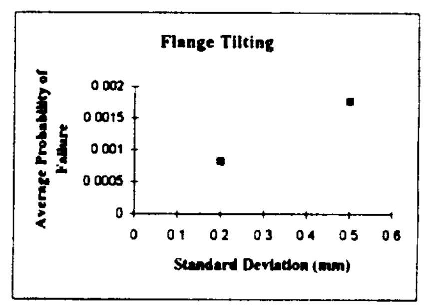


Strength and Probability of Failure Due to Flange Width Variability

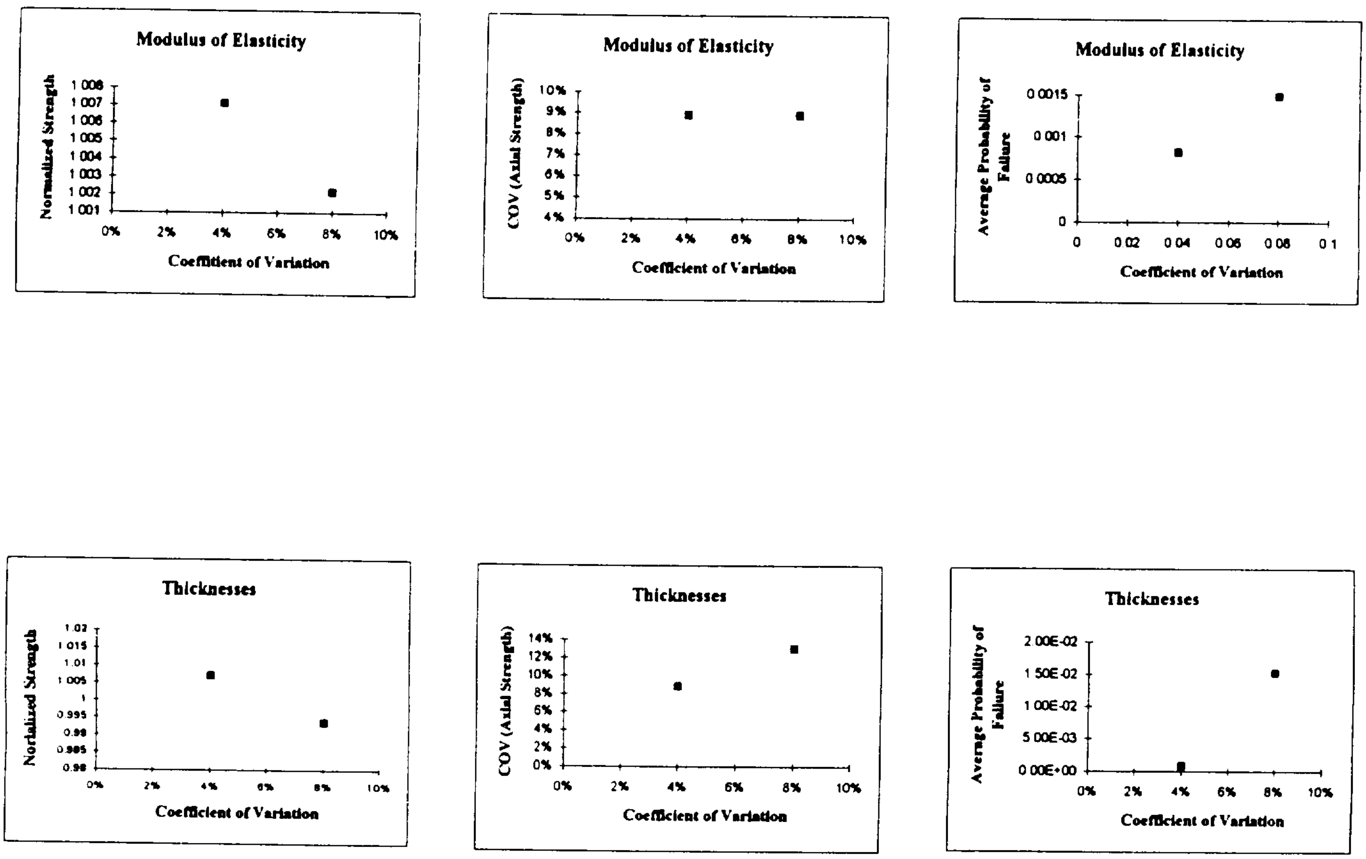








Strength and Probability of Failure Due to Flange Tilting Variability



Strength and Probability of Failure Due to Modulus of Elasticity and Thicknesses Variability

RECOMMENDATIONS FOR FUTURE WORK

Based on this study, the following recommendations for future work are provided:

- The feasibility of using the developed method for complex structures with multiple failure modes needs to be investigated. The structures need to be selected such that methods for failure recognition and classification as previously demonstrated can be developed.
- The effects of failure recognition and classification for continuum structures on reliability estimates need to be studied.

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