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DEVELOPMENT OF A PROBABILISTIC COMPONENT MODE SYNTHESIS METHOD FOR THE ANALYSIS OF NON-DETERMINISTIC SUBSTRUCTURES

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Abstract

Standard methods of structural dynamic analysis assume that the structural characteristics are deterministic. Recognizing that these characteristics are actually statistical in nature, researchers have recently developed a variety of methods that use this information to determine probabilities of a desired response characteristic, such as natural frequency, without using expensive Monte Carlo simulations. One of the problems in these methods is correctly identifying the statistical properties of primitive variables such as geometry, stiffness, and mass. This paper presents a method where the measured dynamic properties of substructures are used instead as the random variables. The residual flexibility method of component mode synthesis is combined with the probabilistic methods to determine the cumulative distribution function of the system eigenvalues. A simple cantilever beam test problem is presented that illustrates the theory.

Nomenclature

r.v.	random variable
CDF	cumulative distribution function
PDS	probabilistic dynamic synthesis
CMS	component mode synthesis
FPI	Fast Probability Integration
FORM	First Order Reliability Method
AMV	Advanced Mean Value
g	limit state function
X	vector of random variables
X*	design or most probable point (MPP)
pf	probability of failure
{u}	vector of uncorrelated std. normal r.v.'s
{x}'	vector of correlated std. normal r.v.'s
{x}	vector of correlated normal r.v's
$[C]^a$	Correlation matrix of r.v's in substruc. a
$[L]_c{}^a$	Cholesky decomposition of $[C]^a$
α	number of substructure
p	total number of substructures
k	number of kept modes/substructure
dof's	degrees of freedom
i	internal dof's
b	boundary dof's
N	total dof's/substructure

Introduction

Structural analysts have always known that the parameters of the system being modeled are not deterministic due to manufacturing tolerances, material deviation, and other factors. Until recently, the primary way to deal with this knowledge was to use factors of safety, which are qualitative and based primarily on experience. In an effort to account for these variations in the structural parameters in a more quantitative fashion, significant research has been performed on developing methods to actually use the statistical characteristics of the input quantities in the analysis to generate an output value that is also described statistically. Monte Carlo simulations can be performed to calculate these probability distributions, but up to a million runs are required for accurate results. Approximate techniques have therefore been developed that require several orders of magnitude less calculations than Monte Carlo techniques. One such method, the "Fast Probability Integration" (FPI) method[1], has recently been implemented into a new probabilistic finite element code, NESSUS.

Numerical analyses of structural vibration generally use the finite element method (FEM) as the basis for obtaining free and forced response characteristics. A frequent problem when using FEM is that, for large models

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composed of many substructures, the number of degrees of freedom (dof's) is so large that the computational costs are prohibitively expensive for eigenanalysis. This problem is particularly relevent when using probabilistic techniques because the eigenanalysis has to be repeated many times. The favored solution method in industry for the deterministic situation is to apply dynamic component mode synthesis methods (CMS), which reduce substantially the dof's for the system model.

This paper defines a procedure for combining CMS with probabilistic methods to obtain the statistical characteristics in an efficient manner. These characteristic are summarized in the form of the cumulative distrubution function (CDF). The procedure makes use of statistical distribution information of each substructure's dynamical modes and residual flexibility, which are available from experimental data. This information is synthesized into a system model using the residual flexibility method of CMS and the statistics of the system dynamic characteristics are obtained using FPI. An advantage of this method over existing probabilistic structural analysis methods is that, in many cases, the statistics of the substructure dynamic characteristics may be easier to determine than those of primitive random variables like geometry, material stiffness, or density. Final development of the method should allow probabilistic methods to be applied to much larger models than previously possible, such as turbomachinery bladed-discs, which are composed of many almost-identical substructures whose structural characteristics can be described statistically.

Probabilistic Theoretical Background

Research in the field of probabilistic structural mechanics has concentrated in two areas. The first can be described as perturbation methods, and the second as reliability methods. The perturbation method, as developed by Collins and Thomson[3], Kiefling[4], and Collins, Kennedy and Hart[5], is used to derive an analytical expression for the mean and standard deviation of structural eigenvalues and eigenvectors as a function of the derivatives of the statistical characteristics of the mass and stiffness matrix for each input random variable. Hart and Hasselman[6] used component mode synthesis to derive analytical expressions for the system, or global, eigenvalue matrix as a function of the modally reduced substructure stiffness and mass matrices. This approach is presently being studied by Mahadevan[7].

The research described in this paper employs the reliability method approach to determining the statistical structural response characteristics. To briefly review this technique, consider the scalar limit state function g(**X**), defined as

$$g(\mathbf{X}) = Y(\mathbf{X}) - y \qquad (1)$$

where the limit state value y is some desired surface in the random variable space and the performance function Y(**X**) is a function of the random variables. This formulation divides the space into two parts, $g < 0$ ($Y < y$) and $g > 0$ ($Y > y$). The probability that the function Y not exceed the desired value y is the probability that $g<0$. For example, if y equals an eigenvalue of interest, $p(g<0)$ would be the probability that the actual eigenvalue obtained is less than the desired value.

Cornell[8], using the reliability approach, developed what would later be called the First Order Reliability Method (FORM) by truncating the higher order terms (hot's) from the series:

$$g(\mathbf{X}) = g(\mu_X) + \sum_{i=1}^{n} \frac{\partial g}{\partial x_i} (x_i - \mu_i) + \text{hot's} \qquad (2)$$

where μ_X is the mean value of **X**. This resulted in the following simple approximations for the mean value and standard deviation of g:

$$\mu_g \approx g(\mu_X) \ ; \qquad (3)$$

$$\sigma_g \approx \sqrt{\sum \left(\frac{\partial g}{\partial x_i}\right)^2 \sigma_{x_i}^2} \ . \qquad (4)$$

Now assume that both the original **X** distribution and the resulting g distribution are normal, and define $g(\mathbf{X}) < 0$ to be the failure region. A transformation of g to standard normal coordinates can therefore be performed to obtain the probability of failure:

$$Z = \frac{g(\mathbf{X}) - \mu_g}{\sigma_g} \ . \qquad (5)$$

The probability distribution function of a standard normal variable has a mean of zero and a standard deviation of one, and its

cumulative distribution function $\Phi(Z)$ is tabulated in statistics textbooks and software packages. Since failure is defined as $g(\mathbf{X})<0$, then if this value is substituted into equation (5), the equivalent failure region in standard normal coordinates is

$$Z < -\beta \tag{6}$$

where the parameter β is defined as

$$\beta = \frac{\mu_g}{\sigma_g} \quad . \tag{7}$$

The probability of failure will therefore be the area under the probability density function to the left of $-\beta$, which is equal to the value of the cumulative distribution function Φ at that point. Since negative values for β are not tabulated, the relationship

$$p_f = \Phi(-\beta) = 1 - \Phi(\beta) \tag{8}$$

is used instead to calculate this probability.

Hasofer and Lind [9] refined this method further. They introduced an initial reduction of each of the primitive normal r.v.'s x_i into standard normal r.v.'s v_i using

$$v_i = \frac{x_i - \mu_{x_i}}{\sigma_{x_i}} \quad . \tag{9}$$

In terms of the standard normal r.v.'s v_i, the joint probability function is bell shaped and symmetric about the origin. If the limit state is defined to be $g(\mathbf{V}) = g(v_1, v_2)$, then the minimum distance from the line

$$g(v_1, v_2) = 0 \tag{10}$$

to the origin can be shown to be equal to β. The point along the line $g = 0$ that is closest to the origin is called the design point $\mathbf{X}^*$. It is also termed the most probable point because it is the point along the line $g=0$ that has maximum probability density.

The reliability method was expanded by Rackwitz[10] to multi-dimensional problems for which the limit state curve $g=0$ is an explicit function of the r.v's. Wu and Wirshing[11] developed the Advanced Mean Value (AMV) method, a procedure for using the FORM with a minimum number of calculations; this is vital for non-explicit limit states, such as finite element solutions. The limit state is approximated as a linear function about the means of the r.v.'s (equation 2) and the partial derivatives are approximately obtained by numerically differentiating the limit state with respect to each r.v. Values of β and X^* are obtained for each desired limit state by using the "Fast Probability Integration(FPI)" method, which is a compilation of the improvements to the FORM made by Wu, Rackwitz, and others. At this point in the procedure, an exact solution for each of the limit states (usually a finite element solution) is found by plugging in these most probable points. These results and their associated β values can then be used to create an entire CDF, which is shown in Wu's paper to be very accurate in comparison to Monte Carlo simulations for several examples. Further iterations can be performed by expanding the limit state about the new design points instead of about the means as was performed in equation (2). The FPI and AMV methods have been incorporated in NESSUS, a probabilistic finite element program presently under development by NASA/Lewis and Southwest Research Institute.

Probabilistic Dynamic Synthesis

The proposed methodology makes use of the residual flexibility method of component mode synthesis. This method has been developed by MacNeil[12], Craig and Chang[13], and Martinez[14]. The essential idea in CMS is that substructure modes are truncated since their higher modes will not have a major effect on the system modes. The residual flexibility method incorporates the effects of the higher modes by determining their flexibility. A side benefit is that all the elements of the system stiffness matrix can be obtained from test and that the mass matrix can be closely approximated by a unity matrix in the non-boundary partition (equal in size to the number of kept modes k). Since all the substructure information can be obtained from test, probabilistic data can be completely incorporated into the system matrices to obtain the system modes.

The first step of the probabilistic dynamic synthesis (PDS) method developed in this paper is to divide the model of a structure into substructures $\alpha = a,b,\ldots,p$. The physical displacement vectors of each substructure, which have either a subscript i denoting internal

dof's or a subscript b denoting boundary dof's, can be written as

$$\begin{Bmatrix} x_i \\ x_b \end{Bmatrix}^a, \begin{Bmatrix} x_i \\ x_b \end{Bmatrix}^b, \ldots, \begin{Bmatrix} x_i \\ x_b \end{Bmatrix}^p \qquad (11)$$

where dim x_i + dim x_b = N total dof's for that substructure.

Each substructure is represented by n samples. Each sample is modally tested individually in a configuration such that the interface locations with other substructures are in a free condition. For substructure α, sample i, the test will yield eigenvalues λ_i^α and eigenvectors $\{\varphi\}_i^\alpha$. In addition, the boundary partition of the residual flexibility matrix $[G_{bb}]_i^\alpha$ are obtained from the measured boundary drive point frequency response functions of the boundary coordinates[15]. For use in the PDS method, only the kept (non-truncated) eigenvalues, the boundary coordinates of the kept eigenvectors, and the boundary partion of the residual flexibility matrix are needed. These values can be combined into a single vector $\{x\}_i^\alpha$, defined as

$$\{x\}_i^\alpha = \begin{Bmatrix} \{\varphi_b\}^1 \\ \vdots \\ \{\varphi_b\}^k \\ \lambda^1 \\ \vdots \\ \lambda^k \\ \{G_{bb}\} \end{Bmatrix}_i^\alpha \qquad (12)$$

where $\{\varphi_b\}^j$ is a vector of the boundary node modal displacements for the j'th mode, and λ^j is the j'th eigenvalue of substructure α.

If the entire sample of substructure α is tested, $\{x\}^\alpha$ can therefore be defined as a vector composed of elements that are each a normally distributed random variable with measured mean and standard deviation. Using equation (9), this vector is now converted to $\{x\}'^\alpha$, a vector of standard normally distributed r.v.'s. In addition, there will be some degree of correlation between each of the random variables. These correlation values range from zero, or no correlation, to +/- 1, or fully correlated, and can be easily calculated from the measured data. The values are placed in a correlation matrix $[C]^\alpha$ relating each element with every other element. For the probabilistic analysis, a set of independent standard normal random variables $\{u\}^\alpha$ will have to be obtained. This can be accomplished by making an orthogonal transformation of $[C]^\alpha$ with its Cholesky Decomposition lower triangular matrix $[L]_c^\alpha$ to uncouple the $\{x\}'$ coordinates, thereby creating $\{u\}^\alpha$ [16]. This can be expressed for substructures α =a,b,...,p as

$$\{x\}'^\alpha = [L]_c^\alpha \{u\}^\alpha . \qquad (13)$$

The FPI algorithm requires that each independent random variable be varied individually by some percentage of its standard deviation σ, which was chosen to be 50 percent for this development, while the other r.v.'s are kept constant at their mean values. Each of these cases is then back-transformed to form a corresponding case of the original correlated random variables. These are then plugged into the model to generate the limit state approximation (equation 2) of the response value, which is used to obtain a CDF and the design points $\mathbf{X}^*$.

Since the distributions of the r.v.'s are standard normal, .5 σ will simply equal a value of .5 for the r.v. to be varied. The first case is therefore

$$\{u\}^a = \begin{Bmatrix} .5 \\ 0 \\ \vdots \\ 0 \end{Bmatrix}, \{u\}^m = \begin{Bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{Bmatrix}, m = 2,\ldots,p \qquad (14)$$

The next case will consist of the second element in $\{u\}^a$ equaling .5 and all the other elements of $\{u\}^a$ as well as all the elements of the other $\{u\}^m$'s equaling zero, and so on.

For each case the {u} for each substructure is then transformed to the set of correlated standard normal r.v.'s {x}' using the transpose of $[L]_c^\alpha$ and then into the original r.v.'s {x} using equation (9). The new vectors {λ}, [Φ], and

$[G_{bb}]$ are pulled out from {x} and placed in substructure mass and stiffness matrices according to the residual flexibility formulation:

$$K^{\alpha} = \begin{bmatrix} \Lambda + \Phi_{bk}^{T} G_{bb}^{-1} \Phi_{bk} & -\Phi_{bk}^{T} G_{bb}^{-1} \\ sym & G_{bb}^{-1} \end{bmatrix},$$

$$M^{\alpha} = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}. \qquad (15)$$

The system mass and stiffness matrices are now generated by directly coupling the substructure mass and stiffness matrices. This is accomplished by ordering the "kept" dof's of each substructure sequentially in the system matrices and adding the boundary partitions together . These matrices can now be used to form the system equation of motion,

$$[M]_{sys} \begin{Bmatrix} \ddot{q}^{sa} \\ \vdots \\ \ddot{q}^{sn} \\ \ddot{x}_b \end{Bmatrix} + [K]_{sys} \begin{Bmatrix} q^{sa} \\ \vdots \\ q^{sn} \\ x_b \end{Bmatrix} = 0 \quad (16)$$

where the q's are generalized coordinates related to the kept modes for each substructure.

The system eigenvalues are then obtained, and a single eigenvalue of interest is chosen. This eigenvalue corresponds to a single point on the random vector response surface defined in the FPI method. As each independent random variable in the p number of {u} vectors is varied, a new response surface point is obtained. This surface can be directly input into the FPI code and a CDF obtained for the chosen system eigenvalue. This will be only a first approximation to the CDF, however. The MPP's are plugged back into the substructures' mass and stiffness matrices, the system is re-synthesized, and new, updated eigenvalue levels are obtained for each probability level, following the AMV method described by Wu. These levels are then plotted to show the entire CDF.

Test Case

Analysis of a spring-mass system (Figure 1) using the PDS method has been completed. The test system consists of two substructures, a

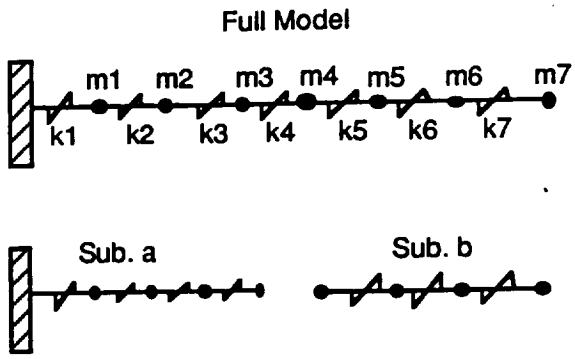

Figure 1 Test Case System

and b, each having four dof's. 5000 samples of each substructure were created initially using standard Monte Carlo techniques. To achieve complete probabilistic generality, each spring in the system was assigned a normal distribution with a mean of 200 and standard deviation of 10, and each mass was assigned a normal distribution with a mean of 1.0 and standard deviation of 0.5. The Monte Carlo random vectors were then used to create the mass and stiffness matrices for the substructures (5000 for each) and a modal analysis run on the substructure samples to obtain their eigenvalues $\{\lambda\}^{\alpha}$ and eigenvectors $[\Phi]^{\alpha}$. Three of the four modes for each substructure were "kept" for the analysis. The boundary partition of the N x N residual flexibility matrix $[G_{bb}]^{\alpha}$ was analytically calculated directly from the modes that had been chosen to be truncated, in this case just the highest one, using

$$[G] = \sum_{i=k+1}^{N} \left[\frac{\{\varphi\}_i \{\varphi\}_i^{T}}{\lambda_i} \right] \qquad (17)$$

where, in this case, k=3 and N=4. The statistics on these dynamic characteristics and the correlation between them were then calculated. These statistics are listed in Table 1. The listed quantities comprise the vectors $\{x\}^a$ and $\{x\}^b$ as described in equation (12).

A distribution characterization routine[17] was also performed on the distributions to see if they could be characterized as normal, which is an assumption of the methodology outlined (several more steps would have to be performed to handle non-normal distributions).[18] Partial

Table 1 Statistics of Dynamic Characteristics

Substructure A

Eigenvalues	Mean	Standard Deviation
1	30.283	2.5256
2	245.90	20.897
3	553.01	47.274

Eigenvectors, boundary location only

Location	Mean	Standard Deviation
1	.7072	.0225
2	-.7066	.05786
3	.70588	.011097

Residual Flexibility (one boundary point only)

Mean	Standard Deviation
6.3848E-04	3.0688E-04

Substructure B

Eigenvalues	Mean	Standard Deviation
1	0.0	0.0
2	150.31	13.402
3	488.99	44.077

Eigenvectors, boundary location only

Location	Mean	Standard Deviation
1	.53517	.014106
2	.75643	.043935
3	-.66730	.36929

Residual Flexibility (one boundary point only)

Mean	Standard Deviation
7.4945E-04	2.9817E-04

Table 2 Partial Results of Distribution Types Routine

Substructure B, Eigenvalue 2

W statistics: (all types included)

Normal:	.00948
Exponential:	.30339
Weibull:	.04448
EVD:	.04323
Lognormal:	.00934

Normal Distrubution Parameters
sample mean = 150.61
sample std. dev. = 6.502

Normal Distribution CDF fit to data

response value	CDF value
143.0	.1210
147.0	.2895
150.0	.4628
155.0	.7503

Lognormal Parameters, base e
mu = 5.014
sigma = 0.43201

Lognormal Distribution CDF fit to data

response value	CDF value
143.0	.1193
147.0	.2947
150.0	.4713
155.0	.7540

results of the distribution characterization routine for one of the r.v.'s are shown in Table 2. The "W" statistic is a goodness-of-fit test developed by Wirshing and Carlson, where a smaller number indicates a closer fit.[19] The results show that the data is well represented by both normal and lognormal distributions, with the lognormal being a little better. The CDF values for each distribution, however, indicate that the curves for the two distributions predict almost exactly the same value, which can be the case for a particular set of lognormal parameters. The assumption of normality was therefore deemed to be accurate.

At this point the procedure follows the outline discussed previously for an actual case, which would use modal testing of physical samples to generate the dynamic characteristics and correlation matrix. For each substructure, a matrix composed of cases of {u} vectors (see equation 14) was generated and multiplied by the correlation Cholesky decomposition matrix $[L]_C$ to obtain {x}', the set of correlated standard normal r.v.'s. These were then converted to their non-standard normal distributions and used in the residual flexibility substructure stiffness matrix. The substructures were then coupled together and a modal analysis was performed on the system matrices. The first system eigenvalue for each case, which was the response value chosen, was then input along with its {u} case into the FPI algorithm routine.

The output of the FPI routine is the "Mean Value Solution," an initial estimate of the CDF of the response variable, and the MPP's for the specified CDF probability levels. One MPP from the output is shown in Table 3. Following the AMV procedure, these MPP's were recorrelated and converted to the original dynamic r.v.'s as

Table 3 Sample MPP Output

Fund. Evalue Response Value = 0.806648E+01
Probability = 0.010000000

Most Probable Point

sub a r.v.	value
u1	-0.5067741E+00
u2	-0.1808949E+00
u3	0.6107269E-01
u4	-0.1579644E+01
u5	-0.1287386E+00
u6	-0.4253657E-01
u7	0.2481191E+00

sub b r.v.	value
u1	-0.1500363E+01
u2	0.2808886E+00
u3	-0.2627451E+00
u4	-0.6609528E-02
u5	-0.2985903E+00
u6	-0.5956755E-01
u7	0.2272107E+00

before, coupled, and a solution obtained for the updated fundamental eigenvalue. This value and its associated probability level were then used to create a new CDF.

For verification of the PDS method, a Monte Carlo analysis was performed on the same non-deterministic spring-mass system with the system eigenvalue directly obtained from each sample. The CDF for this "full" model is superimposed on the AMV CDF from the PDS method in Figure 2. A very small amount of error is indicated graphically. To identify the error quantitatively , the amount of variation of the fundamental eigenvalue from its mean value at selected probability values for the PDS method was compared to the spread for the full model. The result shown in Table 4 indicates that the deviations from the mean as computed by the AMV and MC methods agree to within 5%. In addition, the mean value of the fundamental eigenvalue computed by the AMV method is 8.727, which is only .2 % higher than that computed using MC, and the AMV standard deviation is .272, which is only 3.2 % less than the MC standard deviation of 0.281.

Concluding Remarks

A new methodology has been presented for performing analysis of structures composed of substructures whose dynamic characteristics can be statistically identified. This method uses the substructure eigenvalues, eigenvectors, and residual flexibility as random vectors for determining the desired response value by combining new probabilistic analysis techniques with the residual flexibility method of component mode synthesis. Results for a test case show the method predicts close to the same cumulative distribution function for the system fundamental eigenvalue as a non-substructured probabilistic Monte Carlo analysis.

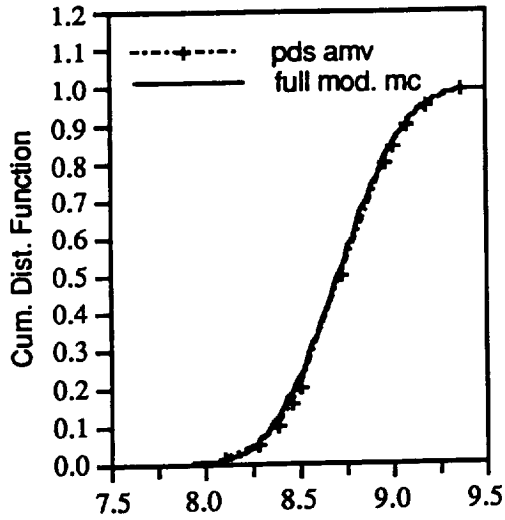

Figure 2

CDF's for fund. evalue using PDS method and M.C. of full model

Future work on this method includes examining some basic conceptual questions on the limitations and applicability of the method. Since some of the dynamic characteristics necessary for the synthesis may be difficult to measure in some circumstances, the formulation of a hybrid method combining analysis and test will be pursued. Other questions include finding cases where the number of random variables can be reduced by perhaps only allowing stiffness or mass to vary, and examining the effect of boundary variability for situations like the fir tree interface between blades and discs, which are neither fixed nor free. In addition, the method will be compared to perturbation methods to determine the areas of most efficient applicability for each one.

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Table 4

Relative Dispersion Error of PDS Method vs Full Model Monte Carlo

CDF Value	AMV Fund. e'value	del from AMV mean	Monte Carlo Fund. e'value	delta from MC mean	% error of deltas
0.010	8.098	-0.629	8.094	-0.613	-2.49%
0.050	8.281	-0.446	8.266	-0.441	-1.22%
0.100	8.378	-0.348	8.352	-0.355	1.92%
0.159	8.455	-0.272	8.426	-0.281	3.27%
0.200	8.497	-0.229	8.467	-0.240	4.70%
0.500	8.727	0.000	8.707	0.000	not applicable
0.800	8.958	0.231	8.944	0.237	2.53%
0.841	9.002	0.275	8.987	0.280	1.84%
0.900	9.080	0.353	9.072	0.365	3.35%
0.950	9.181	0.454	9.169	0.462	1.76%
0.990	9.371	0.644	9.344	0.637	-1.17%

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