AIR-CORED LINEAR INDUCTION MOTOR FOR EARTH-TO-ORBIT SYSTEMS

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SUMMARY

The need for lowering the cost of Earth-to-Orbit (ETO) launches has prompted consideration of electromagnetic launchers. A preliminary design based on the experience gained in an advanced type of coilgun and on innovative ideas shows that such a launcher is technically feasible with almost off-the-shelf components.

INTRODUCTION

In 1990, a seminar was held in Arlington, VA[1] on the subject of Earth-to-Orbit electromagnetic launchers. It was based on the following set of launcher parameters: velocity = 5 to 6 km/s; acceleration = < 1500 Gee's; mass = 500 to 2,000 kg; cycle time = 10 minutes (~ 500 launches per week).

If the proposed limitation on the allowable acceleration a, that is, $a < 1500$ Gee's, is accepted, then the resultant length of the barrel l_b must exceed 833 m.[2] It has been suggested that such a launcher should be built on a mountain having the proper slope and orientation - not a very practical proposition! Probably for this reason the idea was not pursued further at the time. The idea has been revived recently by the Maglev (magnetic levitation) community under a specially coined name, Maglift (or Maglifter*).

The problem of an extremely long barrel, however, will remain, unless a much larger acceleration is allowed. The reason for the original limitation on the acceleration was the low tolerance of the delicate electronic components in the payload, but hardened electronics, as used in artillery shells, can take a = 30,000 Gee's. NASA, in a proposed Advanced Hypervelocity Aerophysics Facility, would have allowed a much larger acceleration, a = 50,500 Gee's, in the fully instrumented models.

Taking the lower value, a = 30,000 Gee's, would reduce the length of the barrel to less than 50 m, and this would allow vertical take-off.

At the Polytechnic, we have developed a linear induction launcher (LIL), the principle of which we now propose be employed for earth-to-orbit launches: The launch vehicle is driven by a set of long air-cored linear induction motors, positioned vertically, or inclined from the vertical, and symmetrically placed around the axis of the vehicle launch path. These motors provide the necessary guidance and levitation forces as well as the propulsion force to the launcher vehicle, to which the motor secondaries are attached during the launch phase.

*Editors note: Maglifter, as currently envisioned by NASA Headquarters, requires relatively low "release" velocities, with chemical propulsion assistance for the climb to orbit.

Each air-cored linear induction motor has a primary winding consisting of a linear array of coaxial circular coils, and a secondary which is a cylindrical conductive sleeve concentric with the primary. Each primary is installed along the entire length of the launch structure, but is divided into sections. These sections are energized by polyphase electric currents, thereby producing a traveling wave of magnetic flux density. This flux is coupled to the passive secondary and induces in it an azimuthal system of currents. The interaction between the primary and secondary currents creates a longitudinal force component used for propulsion, and a strong radial centering force component used for levitation and guidance. The frequency of the primary currents increases from one section to the other to provide constant acceleration of the launch vehicle. The energy is supplied by flywheel motor/generator sets.

The main feature of the concentric arrangement of the primary and secondary is that propulsion, guidance and levitation are provided by the same set of drive coils. Also the magnetic flux is confined, being carried by the inner core and closing mainly in the cylindrical gap between primary and secondary. This permits the elimination of the iron cores without increasing unduly the magnitude of the magnetization current needed for the establishment of the magnetic field. Another feature deriving from the cylindrical symmetry of the primary and secondary is that all portions of the current-carrying conductors contribute to the generation of useful forces. This tends to give high efficiency with reduced material stress and small physical dimensions, and to minimize the cost of the apparatus. As another feature of this system, operation in the asynchronous mode eliminates the need for synchronization between the moving vehicle and the traveling magnetic wave.

The LIL, then, operates as a linear induction motor; hence, its name. A prototype assembled at the Polytechnic with components borrowed from the U.S. Army Electronics Technology and Devices (ET&D) Lab in Fort Monmouth, New Jersey, achieved design performance in 1993, accelerating a 137-gram aluminum (sleeve) cylindrical tube to a velocity of 476 m/s with an acceleration of 19 kGee's, thus validating our computer simulation codes.[3]

Also, in 1993, two of the authors of this paper (ZZ, EL) obtained a U.S. Patent on a spin-off of the LIL, a novel air-cored motor for magnetically levitated (Maglev) trains.[4] Unlike the LIL, the energized coils are inside the sleeve, which is split longitudinally, parallel to the axis, to allow for mechanical support of the coils. The coils are energized at industrial frequency (60 Hz, for example) and provide levitation and guidance, as well as propulsion. The system, of which a close-up view is shown in Fig. 1, is compatible with ordinary steel-wheel rail railroads. The vehicle uses two aluminum rail guides, and it may have four air core motors.

If desired, the same concept could be applied to Maglift for ETO.

As was already mentioned, we propose vertical take-off. However, in order to decouple the diameter of the payload from that of the barrel and in order to limit the voltage impressed on the coils, we propose to use a cluster of barrels and split sleeves, as shown in Fig. 2, instead of a single barrel. The payload is accelerated by means of several, three in Fig. 2, split sleeves, affixed to it, forming a rigid assembly, and arranged in a star-shaped, geometry. The direction of movement of this assembly -- the payload, the housing, and its three split sleeves -- is intended, in the Figure, to be directed into the page (i.e., away from the reader).

In view of the general trend towards lighter satellites, "smallsats," we have also modified the specifications to be more in line with those of a minimal craft, such as Clementine, which carries a 235 kg payload and 223 kg of fuel.

According to a preliminary design, these modified specifications can be met with a launcher having the dimensions and design parameters shown in the next sections.

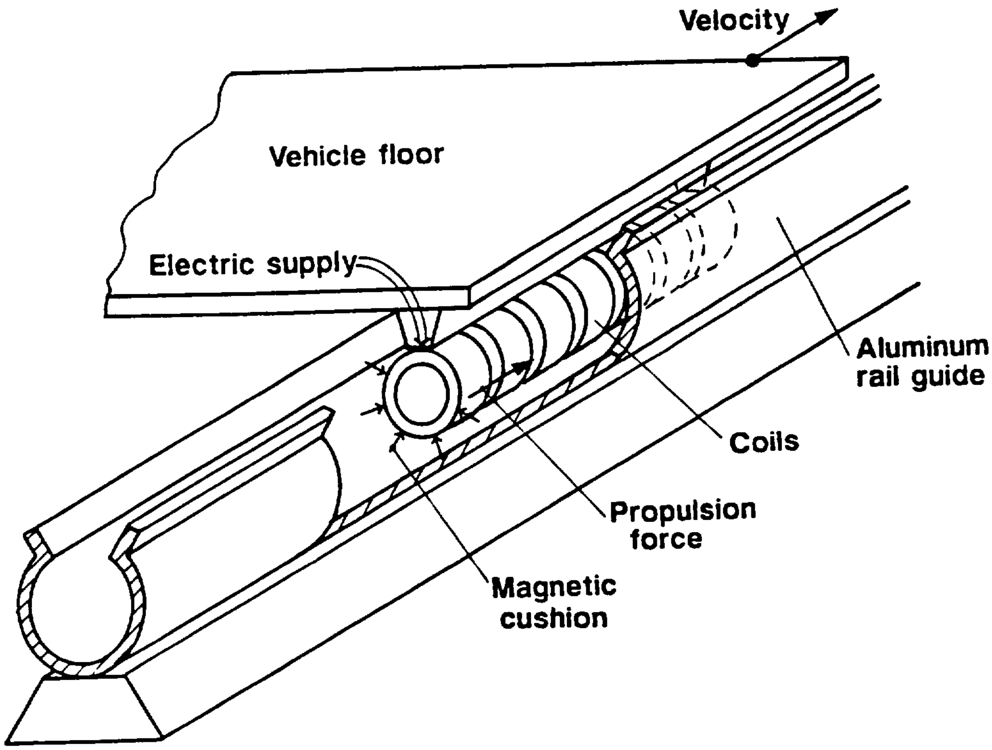

Figure 1. Close-up view of propulsion, suspension and guidance system.

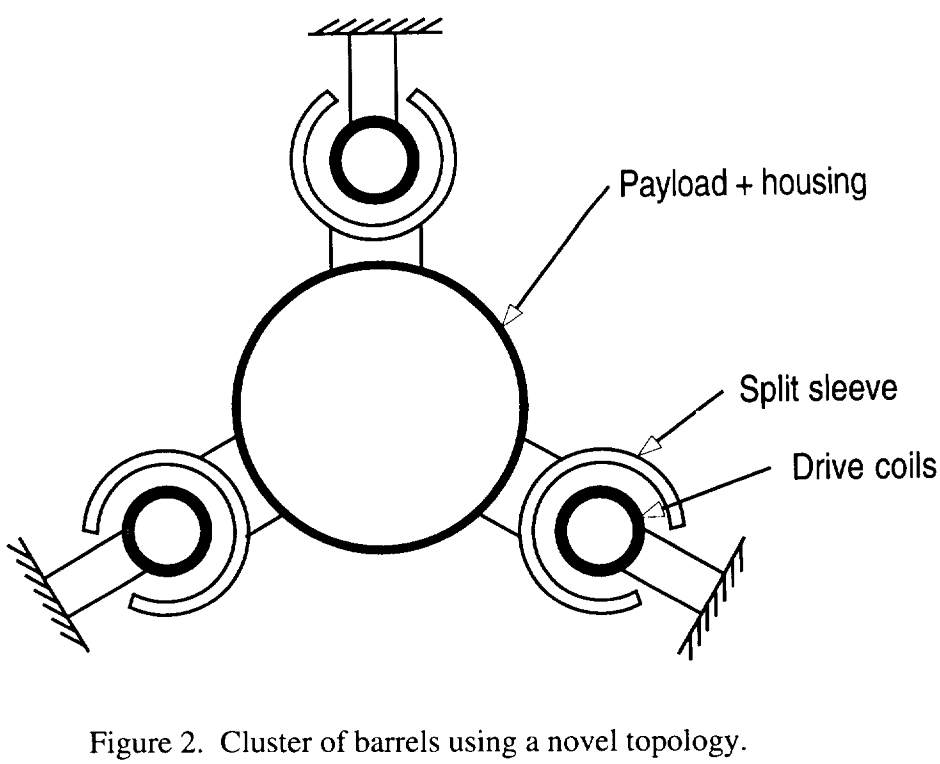

Figure 2. Cluster of barrels using a novel topology.

VERTICAL TAKE-OFF ETO LAUNCHER

A preliminary design (see Table 1 below) made according to Refs. 5-8 and Appendix 1 indicates that a launcher for a final velocity of 5 km/s and a weight payload + housing + sleeves of 500 kg should be visualized as a tower about 50 meters high, consisting of six columns (the six barrels) and 38 levels (one for each section of the launcher). On each level would be placed six flywheel motor/generator sets, which energize the appropriate sections (of the six barrels) that correspond to that level. Initially, in preparation for the launch, all of the flywheels would be brought up to speed using each of the synchronous machines on each level as a motor powered by an adjustable speed drive. Then, during launch, the same machines, working now as generators, would be sequentially switched on at each level, from the lowest to the uppermost, to energize the 38 sections of each barrel.

In the table below is given a set of ETO specifications, followed by a list of the results of the preliminary design of an ETO launcher.

Table 1. ETO Specifications and Preliminary Design for Vertical Take-Off

ETO specifications:

Final velocity:	5	km/sec
Acceleration:	25,000	Gee's
Weight, payload + housing + sleeves:	500	kg
Payload diameter:	as needed	
Armature fraction: (sleeve weight/total weight)	66%	
Cycle time:	~ 500	per week

ETO preliminary design parameters:

Structure:	vertical	(cluster of barrels)
Length of barrel:	50.4	m (height of tower)
No. of barrels:	6	
No. of sections:	38	(one per tower level)
No. of phases:	12	
Pole pitch:	0.36	m
Ampere turns per coil:	$6.78 \cdot 10^6$	AT
Peak volt per turn:	77	kV *
OD of each barrel:	0.192	m
Air-gap clearance:	0.025	m

Length of sleeve:	0.72	m
OD of each sleeve:	0.237	m

* A change in the number of barrels and of their dimensions would reduce the voltage per turn to more acceptable values.

ELECTROMECHANICAL STORAGE

As already mentioned, it is proposed to use electromechanical storage to power the ETO launcher. The mechanism consists of a flywheel/motor-generator set in which the flywheel would be brought up to speed using the synchronous machine as a motor, powered by a variable frequency drive. The density of kinetic energy stored in a cylindrical flywheel is $e_{kin} = \left(\frac{v}{2}\right)^2 \frac{\text{joule}}{\text{kg}}$ where v is the peripheral velocity of the flywheel. The present speed record is held by a flywheel built at Oak Ridge with a peripheral velocity of 1370 m/s.[9] This yields an energy density $e_{kin} = 4.7 \cdot 10^5 \frac{\text{joule}}{\text{kg}}$ which for ETO, requiring a stored energy of $10.4 \cdot 10^3$ MJ, corresponds to a total mass $m = 44$ tonnes. When this mass is divided by the number of barrels, which is 6, and the number of sections, which is 38, the dimensions of the individual flywheels become quite reasonable. Each flywheel must serve also as a motor/generator set. In the first low-energy sections, the flywheels could consist of a set of permanent magnets embedded in a carbon-fiber composite, similar to those being developed by American Flywheel Systems Inc.[10] for use in electrical vehicles. In the high energy sections, however, the cost of the permanent magnets would become prohibitive. There are other ways to provide excitation to the rotor without the need to resort to sliding contacts and brushes which, due to the high speed, would not be reliable (see Refs. 11, 12).

In the 1970's the authors of this paper developed a homopolar inductor motor for Maglev under sponsorship of the U.S. Department of Transportation[11, 12]. They designed a full-scale motoring unit which was built and successfully tested by the General Electric Company in Schenectady, NY. The unit was mounted on a flywheel rotating at a peripheral velocity of 134 m/s, which corresponds to a train speed of 300 miles/h. In view of the experience gained with this project, we propose to adopt this type of machine in the last sections of the ETO launcher.

Although flywheels are quite adequate for storing large amounts of energy, its delivery at the extremely large rate, which is required by the generator, presents a challenge. To illustrate the fundamental problem in electrical terms, one can look at the equivalent circuit of a unit mass of the flywheel in a motor generator set (Fig. 3).[13, 14] The equivalent dielectric constant $\varepsilon_{eq} = \xi/B^2$ where ξ is the mass density and B is the magnetic flux density, is so huge that also the discharge time constant $\tau = \xi/\gamma B^2$ of the material is very large. Here γ is the conductivity, J is the current density, f_m is the mechanical force density, E is the field intensity and v is the linear velocity.

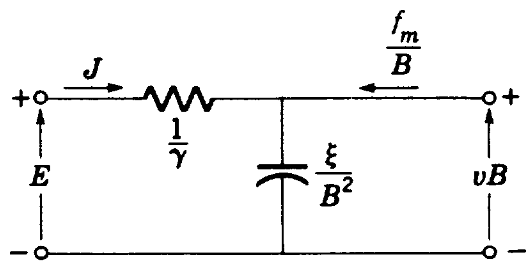

Figure 3. Equivalent circuit of unit volume of conducting material.

INCLINED TAKE-OFF ETO LAUNCHER

The power level required for heavy payloads limits the acceleration rates and, therefore, leads to unacceptable heights for vertical take-off launching towers. The solution is a Maglift, a spin-off of the magnetic levitation systems for high-speed transportation. It envisages an inclined racetrack on the slope of a mountain. Again, the centering forces in the motor elements and their symmetrical distribution around the payload afford the lateral stability that other systems lack.

Using preliminary calculations and design given in Appendix 2, in concept, a low acceleration launcher for a final velocity of 5 km/sec and an assembly weight of 2,000 kg should be visualized as a straight 900-meter-long track.

In Table 2 below is given a set of Maglift ETO specifications, followed by a list of the results of the preliminary design of an ETO launcher.

Table 2. ETO Specifications and Preliminary Design of Maglift ETO

Maglift ETO specifications:

Final velocity:	5	km/sec
Acceleration:	1,420	Gee's
Assembly weight:	2,000	kg
Payload diameter:	as needed	
Armature fraction: (sleeve weight/total weight)	66%	
Cycle time:	~ 500	per week

ETO preliminary design parameters:

Structure:	inclined	
Length of barrels:	900	m
No. of barrels:	4	
No. of sections:	46	
No. of phases:	12	
Pole pitch:	2	m
Ampere turns per coil:	$8.26 \cdot 10^5$	AT
Peak volt per turn:	18.8	kV
OD of each barrel:	0.21	m
Air-gap clearance:	0.025	m
Length of sleeve:	0.72	m
OD of each sleeve:	0.255	m

ETO power supply:

Flywheel/motor-generator sets		
Total stored energy	$41.6 \cdot 10^3$	MJ
Total mass of all sets	176	tonnes
Total number of sets	184 on 46 levels; 4 sets per level	

CONCLUDING REMARKS

The development of electromagnetic ETO launchers still presents challenging tasks. The preliminary designs contained in this paper indicate, however, that there do not appear to be any problems that cannot be surmounted with existing technology.

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APPENDIX 1

Preliminary Design for Vertical ETO Launcher

<u>Specifications</u>: muzzle velocity: $v_m = 5 \times 10^3$ m/s; acceleration: $a = 2.5 \times 10^5$ m/s^2

assembly weight: $w_{pr} = 500$ kg.; armature fraction: $\frac{1}{\nu} = \frac{2}{3}$

The length of the barrels is: $l_b = \frac{1}{2}\frac{v_m^2}{a} = \frac{1}{2}\frac{25 \times 10^6}{2.5 \times 10^5} = 50 \text{ m}$.

We assume a pole pitch: $\tau = 0.36$ m and we round the length of the barrel to $l_b = 50.4$ in order to have an integer number of pole pairs. We choose $l_b = 50.4$ m, in order to have an even number of pole pairs. We choose a cluster of 6 barrels, so that we have available for each sleeve a weight $w_s = \frac{1}{\nu}\frac{w_{pr}}{6} = 55.55$ kg. We assume a sleeve length $l_s = 2\tau = 0.72$ m and a thickness $a_s = 2$ cm, so that the average radius is

$$r_s^{ave} = \frac{w_s}{\xi_s \times 2\pi \times a_s \times l_s} = \frac{55.55}{2.7 \times 10^3 \times 2\pi \times 2 \times 10^{-2} \times 0.72} = 0.227\text{m}.$$

The inside radius is then: $r_s^i = r_s^{ave} - \frac{a_s}{2} = 0.227 - \frac{0.02}{2} = 0.217$ m.

We assume an air-gap length $g_a = 0.025$ m, so that the outside radius of the barrel is $r_b^0 = r_s^i - g_a = 0.217 - 0.025 = 0.192$. We assume that the thickness of the barrel coils is $a_b = 0.04$ m. The effective air gap is then:

$$g = \frac{2 + 4}{2} + 2.5 = 5.5 \text{ cm};\ \beta g = \frac{\pi}{0.36} \times 5.5 \times 10^{-2} = 0.48;\ coth\,\beta g = 2.241$$

Letting the critical slip be s_c and the synchronous speed be v_s we have

$$s_c v_s = \frac{1 + coth\,\beta g}{a_s\,\gamma_{400}\,\mu_o} = \frac{1 + 2.241}{2 \times 10^{-2} \times 1.338 \times 4\pi} = 9.63 \text{ m/s}.$$

Letting θ be the temperature rise and Γ the ratio of peak to minimum power, we then have

$$\theta/\Gamma = \frac{\nu\xi_s\, v_m\, s_c\, v_s}{c} = \frac{1.5 \times 2.7 \times 10^3 \times 5 \times 10^3 \times 9.63}{2.68 \times 10^6} = 72.76.$$

where ν is the inverse of the armature fraction, ξ_s is the specific weight of the sleeve material and c is the heat coefficient per-unit volume.

Let $\theta = 500K$, so that $\Gamma = 6.84$ and the number of sections is

$$n = \frac{v_m}{2\, s_c\, v_s \sqrt{\Gamma^2 - 1}} = \frac{5 \times 10^3}{2 \times 9.63 \sqrt{6.84^2 - 1}} = 38 .$$

The kinetic energy is $E_{kin} = \frac{1}{2} w_{pr}\, v_m^2 = \frac{1}{2} \times 500 \times 5^2 \times 10^6 = 6.25 \times 10^9\,\mathrm{J}.$

The average force is $F_{ave} = \frac{E_{kin}}{l_b} = \frac{6.25 \times 10^9}{50.4} = 1.24 \times 10^8\,\mathrm{N}.$

The increment of kinetic energy in the last section is

$$\Delta E_{kin} = Pt = P\frac{\Delta v}{a} = \frac{1}{2}(v_f^2 - v_i^2) = \frac{1}{2} m\Delta v\,(2v_f - \Delta v) = ma(v_f - \frac{\Delta v}{2}).$$

Assuming $\Delta v = \mathrm{const} = \frac{v_m}{n} = \frac{5 \times 10^3}{38} = 131\,\mathrm{m/s}$, we get in the last section $P = 1.44 \times 10^8 \times (5 \times 10^3 - \frac{131}{2}) = 7.105 \times 10^{11}.$

Assuming PF $= 0.7$; efficiency $\eta = 0.6$ and 12 phases per barrel, we need switches with a handling capacity of $\frac{7.105 \times 10^{11}}{6 \times 12 \times 0.7 \times 0.6} = 2.35 \times 10^{10}\,\mathrm{VA}.$

Such switches are commercially available. Now we calculate the voltage and current:

$$F_{ave} = \frac{F_m}{\sqrt{\Gamma^2 - 1}}\, ln\,(\Gamma + \sqrt{\Gamma^2 - 1}) = \frac{F_m}{6.76}\, ln\,(6.84 + 6.76) = 0.386\, F_m$$

$$F_m = \frac{F_{ave}}{0.386} = 3.21 \times 10^8\ \mathrm{N}.$$

The average force density is then:

$$< f_m > = \frac{F_m/6}{2\pi\, r_b^o \times l_s} = \frac{3.21 \times 10^8/6}{2\pi \times 0.192 \times 0.72} = 6.16 \times 10^7\,\mathrm{N/m^2}$$

$$K_b = 2\, e^{\beta g} \sqrt{\frac{< f_m >}{\mu_o}} = 2 \times e^{0.48} \sqrt{\frac{6.16 \times 10^7}{4\pi \times 10^{-7}}} = 2.26 \times 10^7\,\mathrm{A/m}$$

Assuming that the width of the coil is $w_c = \frac{0.36}{12} = 0.03$ m

$$NI = 0.03 \times 2.26 \times 10^7 = 6.78 \times 10^5\,\mathrm{AT}$$

$$B_1 = \frac{1}{2}\mu_o K_b = \frac{1}{2} 4\pi \times 10^{-7} \times 2.26 \times 10^7 = 14.2\text{ T}.$$

In the last section we have:

$$E = v_s B = 5.05 \times 10^3 \times 14.2 = 71.7\text{ kV/m}$$

$$\frac{V}{N} = 2\pi \times r_b^{ave} \times E = 2\pi(0.192 - 0.02) \times 71.7 \times 10^3 = 77.48\text{ kV}$$

This voltage is a little on the high side but is acceptable.

APPENDIX 2

Preliminary Design for Inclined ETO Launcher

<u>Specifications</u>: muzzle velocity: $v_m = 5 \times 10^3$ m/s; acceleration $= 1.39 \times 10^4$ m/s

projectile weight: $w_{pr} = 2{,}000$ kg.; armature fraction: $\frac{1}{\nu} = \frac{2}{3}$

The length of the barrels is: $l_b = \frac{1}{2}\frac{v_m^2}{a} = \frac{1}{2}\frac{2.5 \times 10^6}{1.39 \times 10^4} = 900$ m

We assume a pole pitch: $\tau = 2$ m

We choose a cluster of 4 barrels, so that we have available for each sleeve a weight $w_s = \frac{1}{\nu}\frac{w_{pr}}{4} = \frac{2}{3}\frac{2000}{4} = 333$ kg. We assume a sleeve length $l_s = 2\tau = 4$ m and a thickness $a_s = 2$ cm, so that the average radius is

$$r_s^{ave} = \frac{w_s}{\xi_s \times 2\pi \times a_s \times l_s} = \frac{333}{2.7 \times 10^3 \times 2\pi \times 2 \times 10^{-2} \times 4} = 0.245 \text{ m}.$$

The inside radius is then: $r_s^i = r_s^{ave} - \frac{a_s}{2} = 0.245 - \frac{0.02}{2} = 0.235\, m.$

We assume an air-gap length $g_a = 0.025$ m, so that the outside radius of the barrel is $r_b^0 = r_s^i - g_a = 0.235 - 0.025 = 0.21$ m. We assume that the thickness of the barrel coils is $a_b = 0.04$ m. The effective air gap is then:

$$g = \frac{2 + 4}{2} + 2.5 = 5.5 \text{ cm} \quad \beta g = \frac{\pi}{2} \times 5.5 \times 10^{-2} = 0.086; \; coth\, \beta g = 11.65$$

$$s_c v_s = \frac{1 + coth\, \beta g}{a_s\, \gamma_{400}\, \mu_o} = \frac{1 + 11.65}{2 \times 10^{-2} \times 1.338 \times 4\pi} = 37.61 \text{ m/s}.$$ We then have

$$\theta/\Gamma = \frac{\nu \xi_s\, v_m\, s_c\, v_s}{c} = \frac{1.5 \times 2.7 \times 10^3 \times 5 \times 10^3 \times 37.61}{2.68 \times 10^6} = 284\,.$$

Let $\theta = 500$K, so that $\Gamma = 1.76$ and

$$n = \frac{v_m}{2\, s_c\, v_s \sqrt{\Gamma^2 - 1}} = \frac{5 \times 10^3}{2 \times 37.61 \times \sqrt{1.76^2 - 1}} = 45.9 \sim 46\,.$$

The kinetic energy is $E_{kin} = \frac{1}{2} w_{pr} v_m^2 = \frac{1}{2} \times 2000 \times 5^2 \times 10^6 = 2.5 \times 10^{10}$ J.

The average force is $F_{ave} = \frac{E_{kin}}{l_b} = \frac{2.5 \times 10^{10}}{900} = 2.77 \times 10^7$ N.

The increment of kinetic energy in the last section is

$$\Delta E_{kin} = Pt = P\frac{\Delta v}{a} = \frac{1}{2}(v_f^2 - v_i^2) = \frac{1}{2} m\Delta v\,(2v_f - \Delta v) = ma(v_f - \frac{\Delta v}{2}).$$

Assuming $\Delta v = \text{const} = \frac{v_m}{n} = \frac{5 \times 10^3}{46} = 108.7$ m/s, we get in the last section

$$P = 2.77 \times 10^7 \times (5 \times 10^3 - \frac{108.7}{2}) = 1.367 \times 10^{11}.$$

Assuming PF = 0.7; η = 0.6 and 12 phases per barrel, we need switches with a handling capacity of $\frac{1.367 \times 10^{11}}{4 \times 12 \times 0.7 \times 0.6} = 6.78 \times 10^9$ VA.

Such switches are commercially available. Now we calculate the voltage and current:

$$F_{ave} = \frac{F_m}{\sqrt{\Gamma^2 - 1}} ln\,(\Gamma + \sqrt{\Gamma^2 - 1}) = \frac{F_m}{1.448} ln\,(1.76 + 1.448) = 0.805\,F_m$$

$$F_m = \frac{F_{ave}}{0.805} = 3.44 \times 10^7 \text{ N}$$

$$< f_m > = \frac{F_m/6}{2\pi r_b^o \times l_s} = \frac{3.44 \times 10^7}{2\pi \times 0.21 \times 4} = 6.5 \times 10^6 \text{ N/m}^2$$

$$K_b = 2\,e^{\beta g}\sqrt{\frac{< f_m >}{\mu_o}} = 2 \times 1.09\sqrt{\frac{6.5 \times 10^6}{4\pi \times 10^{-7}}} = 4.96 \times 10^6 \text{ A/m}$$

Assuming that the width of the coil is $w_c = \frac{2}{12} = 0.167$

$$NI = 0.167 \times 4.96 \times 10^6 = 0.826 \times 10^6 \text{ AT}$$

$$B_1 = \frac{1}{2}\mu_o K_b = \frac{1}{2} 4\pi \times 10^{-7} \times 4.96 \times 10^6 = 3.116 \text{ T}.$$

In the last section we have:

$$E = v_s B = 5.05 \times 10^3 \times 3.116 = 15.73 \text{ kV/m}$$

$$\frac{V}{N} = 2\pi \times r_b^{ave} \times E = 2\pi \times 0.19 \times 15.73 = 18.78 \text{ kV}$$

A quite acceptable voltage.

Session 17 -- Applications of Superconductivity

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