

# MULTILEVEL CONCATENATED BLOCK MODULATION CODES FOR THE FREQUENCY NON-SELECTIVE RAYLEIGH FADING CHANNEL

Dojun Rhee and Shu Lin<sup>1</sup>

LSI LOGIC CORPORATION  
MS G-815  
1525 McCarthy Blvd.  
Milpitas, California 95035  
E-mail : drhee@lsil.com

Dept. of Electrical Engineering  
2540 Dole Street, Holmes Hall #483  
University of Hawaii at Manoa  
Honolulu, Hawaii 96822  
slin@spectra.eng.hawaii.edu

*Abstract* — This paper is concerned with construction of multilevel concatenated block modulation codes using a multi-level concatenation scheme [1] for the frequency non-selective Rayleigh fading channel. In the construction of multilevel concatenated modulation code, block modulation codes are used as the inner codes. Various types of codes ( block or convolutional, binary or nonbinary ) are being considered as the outer codes. In particular, we focus on the special case for which Reed-Solomon (RS) codes are used as the outer codes. For this special case, a systematic algebraic technique for constructing  $q$ -level concatenated block modulation codes is proposed. Codes have been constructed for certain specific values of  $q$  and compared with the single-level concatenated block modulation codes using the same inner codes. A multilevel closest coset decoding scheme for these codes is proposed.

## I. INTRODUCTION

Single-level concatenated trellis coded modulation (TCM) for AWGN channel was first introduced by Deng and Costello in 1989 [2, 3]. Almost at the same time, Kasami et. al. presented a single-level concatenated block coded modulation (BCM) scheme for reliable data transmission over the AWGN channel [4]. Error performance of the single-level concatenated TCM and BCM schemes for the Rayleigh fading channel was first investigated by Vucetic and Lin in 1991 [5] and then by Vucetic in 1993 [6].

In the single-level concatenated BCM scheme [4], every information bits of inner code are protected by the same degree. In the AWGN channel, it is possible to design the inner code such that the bit error rate of each information bit is almost same. However, in the Rayleigh fading channel, it is not easy to design the inner code such that the bit error rate of each information bit is almost same. Therefore, in the single-level

concatenated BCM system, bit-error-rate of some information bits dominates the overall BER of the coded system and results in poor bit error performance. However, in a multilevel concatenated BCM scheme, it is possible to provide different degree of protection to information bits with different bit error rates. Outer codes in each level are designed to stop the error propagation to the next level of decoding.

Multilevel concatenated BCM codes constructed in this paper are designed to achieve better bit error performance than the single-level concatenated BCM code by stopping the error propagation to the next level decoding. Simulation results show that these codes achieve very impressive real coding gains over the uncoded reference system and single-level concatenated BCM codes using the same inner codes.

## II. MULTILEVEL CONCATENATED BLOCK CODED MODULATION SCHEME

The proposed multi-level concatenated coded modulation schemes are constructed using a multi-level concatenation approach [1].

In a  $q$ -level concatenated coded modulation system,  $q$  pairs of outer and inner codes are used as shown in Figure 1. In block coding, Reed-Solomon (RS) codes are used as the outer codes, and coset codes constructed from a block modulation code and its subcodes are used as the inner codes. The encoding and decoding are accomplished in  $q$  levels respectively.

### Outer Code Construction

For  $1 \leq i \leq q$  and  $1 \leq j \leq m_i$ , let  $B_{i,j}$  be an  $(N, K_i)$  RS (or shortened RS) code over  $GF(2^p)$  with minimum Hamming distance  $D_i = N - K_i + 1$ . In the  $i$ -th level outer code encoder, a  $(N, K_i, D_i)$  RS code is interleaved with depth  $m_i$ . For  $1 \leq j \leq m_i$ , let  $B_{i,j}$  represent the  $j$ -th code among  $m_i$  interleaved RS codes. After  $i$ -th level outer code encoding, symbols from  $GF(2^p)$  in each RS code are converted into  $p$ -bits binary representation. After conversion,  $m_i N p$ -bits are stored in the  $m_i$  by  $Np$  array such that every column has  $m_i$  bits and each bit in this column is selected from each RS code  $B_{i,j}$  for  $1 \leq j \leq m_i$ . Since  $B_{i,j} = B_{i,k}$

<sup>1</sup>This research was supported by NSF Grant NCR-911540, BCS-9020435 and NASA Grant NAG 5-931

for  $1 \leq j, k \leq m_i$ , let  $\mathbf{B}_i = \{B_{i,1}, B_{i,2}, \dots, B_{i,m_i}\}$  represents an  $i$ -th level outer code for  $1 \leq i \leq q$ . The  $i$ -th level outer code encoder is shown in Figure 2. Later, these  $q$ -sets of RS codes will be used as the outer codes in  $q$  levels of concatenation.

### Inner Coset Code Construction

Let  $\Lambda_0$  be a block modulation code over a certain elementary signal set  $S$  with length  $n$ , dimension  $k_0$  and minimum squared Euclidean distance  $\Delta_0$ . We require that

$$k_0 = m_1 + m_2 + \dots + m_q \quad (1)$$

From  $\Lambda_0$ , we form a sequence of subcodes,  $\Lambda_0, \Lambda_1, \Lambda_2, \dots, \Lambda_q$ , where  $\Lambda_q$  consists of the all-zero codeword, i.e.  $\Lambda_q = \{0\}$ . The dimension of these subcodes satisfy the following conditions: For  $1 \leq i \leq q$ ,  $\Lambda_i$  is a linear subcode of  $\Lambda_{i-1}$  with dimension

$$k_i = k_{i-1} - m_i \quad (2)$$

and minimum squared Euclidean distance  $\Delta_i$ . From 1 and 2, we have

$$\begin{aligned} k_1 &= m_2 + m_3 + \dots + m_{q-1} + m_q \\ k_2 &= m_3 + m_4 + \dots + m_q \\ &\vdots \\ k_{q-1} &= m_q \\ k_q &= 0 \end{aligned} \quad (3)$$

We also note that  $\Delta_0 \leq \Delta_1 \leq \dots \leq \Delta_q$  and that  $\Lambda_q$  consists of only the all-zero codeword with  $\Delta_q = \infty$ .

Now we are going to construct  $q$  coset codes from  $\Lambda_0, \Lambda_1, \dots, \Lambda_q$ . These  $q$  coset codes will be used as the inner codes in the proposed  $q$ -level concatenated coded modulation scheme. First we partition  $\Lambda_0$  into  $2^{m_1}$  cosets modulo  $\Lambda_1$ . Let  $\Lambda_0/\Lambda_1$  denote the set of cosets of  $\Lambda_0$  modulo  $\Lambda_1$ . The minimum squared Euclidean distance of each coset in  $\Lambda_0/\Lambda_1$  is  $\Delta_1$ . The minimum squared distance between two cosets in  $\Lambda_0/\Lambda_1$  is  $\Delta_0$ .  $\Lambda_0/\Lambda_1$  is called the **coset code of  $\Lambda_0$  modulo  $\Lambda_1$** . Next we partition each coset in  $\Lambda_0/\Lambda_1$  into  $2^{m_2}$  cosets modulo  $\Lambda_2$ . Let  $\Lambda_0/\Lambda_1/\Lambda_2$  denote the set of cosets of a coset in  $\Lambda_0/\Lambda_1$  modulo  $\Lambda_2$ . It is clear that the minimum squared Euclidean distance of a coset in  $\Lambda_0/\Lambda_1/\Lambda_2$  is  $\Delta_2$ , and the minimum squared distance among the cosets of a coset in  $\Lambda_0/\Lambda_1$  modulo  $\Lambda_2$  is  $\Delta_1$ . We call  $\Lambda_0/\Lambda_1/\Lambda_2$  the **coset code of  $\Lambda_0/\Lambda_1$  modulo  $\Lambda_2$** . We continue the above partition process to form coset codes. For  $1 \leq i \leq q$ , let  $\Lambda_0/\Lambda_1/\dots/\Lambda_{i-1}$  be the coset code of  $\Lambda_0/\Lambda_1/\dots/\Lambda_{i-2}$  modulo  $\Lambda_{i-1}$ . We partition each coset in  $\Lambda_0/\Lambda_1/\dots/\Lambda_{i-1}$  into  $2^{m_i}$  cosets modulo  $\Lambda_i$ . Then  $\Lambda_0/\Lambda_1/\dots/\Lambda_i$  is the coset code of  $\Lambda_0/\Lambda_1/\dots/\Lambda_{i-1}$  modulo  $\Lambda_i$ . The minimum squared Euclidean distance of a coset in  $\Lambda_0/\Lambda_1/\dots/\Lambda_i$  is  $\Delta_i$ , and the minimum squared distance among the cosets of a coset in  $\Lambda_0/\Lambda_1/\dots/\Lambda_{i-1}$  modulo  $\Lambda_i$  is

$\Delta_{i-1}$ . Note that each coset in  $\Lambda_0/\Lambda_1/\dots/\Lambda_{q-1}$  consists of  $2^{m_q}$  codewords in  $\Lambda_0$ . Since  $\Delta_q = \{0\}$ , each coset in  $\Lambda_0/\Lambda_1/\dots/\Lambda_q$  consists of only one codeword in  $\Lambda_0$ . Hence the minimum squared Euclidean distance of each coset is  $\Delta_q = \infty$ . The minimum squared distance among the cosets of a coset in  $\Lambda_0/\Lambda_1/\dots/\Lambda_{q-1}$  modulo  $\Lambda_q$  is  $\Delta_{q-1}$ . The above partition process results in a sequence of  $q$  coset codes,

$$A_1 = \Lambda_0/\Lambda_1 \quad (4)$$

$$A_2 = \Lambda_0/\Lambda_1/\Lambda_2$$

$$\vdots$$

$$A_q = \Lambda_0/\Lambda_1/\dots/\Lambda_q \quad (5)$$

These  $q$  coset codes are used as inner codes in the proposed  $q$ -level concatenated coded modulation scheme. This  $q$ -level concatenated modulation code  $\mathbf{C}$  is denoted as follows:

$$\mathbf{C} \triangleq \{\mathbf{B}_1, \mathbf{B}_2, \dots, \mathbf{B}_q\} * \{A_1, A_2, \dots, A_q\} \quad (6)$$

If  $\Lambda_0, \Lambda_1, \dots, \Lambda_{q-1}$  have simple trellis diagrams, the coset inner codes,  $A_1, A_2, \dots, A_q$ , also have simple trellis diagrams. If the coset inner codes have simple trellis structure, then we can use Viterbi decoding to decode coset inner codes. This will decrease decoding complexity of inner codes drastically.

### III. MULTILEVEL CLOSEST COSET DECODING

A multilevel closest coset decoding for the proposed scheme is presented in this section. Each level decoding consists of the inner closest coset decoding and the outer code decoding. At  $i$ -th level decoding, the inner coset code  $A_i$  is decoded by using decoded estimates from first level decoder to  $i-1$  th level decoder. In the multilevel concatenated scheme, the  $i-1$  th level outer code is designed to reduce the error propagation from  $i-1$  th level decoder to  $i$ -th level decoder. Since decoded information at each level is passed to next level, decoding at each level depends on decoded information from the preceding level. Therefore, error propagation may occur. To reduce the probability of error propagation, outer codes must be selected by considering the specific channel characteristic. In following sections, multilevel concatenated codes are constructed by following rule. In the Rayleigh fading channel, strong outer codes must be used for levels where inner codes have small minimum symbol and product distances.

### IV. EXAMPLE

Consider a two-level concatenated coded 8-PSK modulation system for the frequency non-selective Rayleigh fading channel. In this system, two 3-level 8-PSK modulation codes of length 8 are used as the inner codes and two RS codes over the Galois field  $GF(2^9)$  are used as the outer codes as shown in Figure 3. The two

inner codes are constructed from two 3-level 8-PSK modulation codes,  $\Lambda_0 = \lambda[(8, 4, 4) * (8, 7, 2) * (8, 7, 2)]$  and  $\Lambda_1 = \lambda[(8, 1, 8) * (8, 4, 4) * (8, 4, 4)]$ .  $\Lambda_0$  has dimension 18, minimum symbol distance 2, minimum product distance 4, and minimum squared Euclidean distance 2.344. And  $\Lambda_1$  has dimension 9, minimum symbol distance 4, minimum product distance 16, and minimum squared Euclidean distance 4.688.  $\Lambda_1$  has a very simple trellis structure and can be decoded in either 3 stages, 2 stages, or one stage. Either way, the decoding complexity is very simple.

Since (8,4,4) code is a subcode of (8,7,2) and (8,1,8) is subcode of (8,4,4),  $\Lambda_1$  is a subspace of  $\Lambda_0$ . Now, partition  $\Lambda_0$  with respect to  $\Lambda_1$ . Let  $\Lambda_0/\Lambda_1$  denote this partition. Then  $\Lambda_0/\Lambda_1$  consists of  $2^9$  cosets, denoted  $\Omega_i$  with  $1 \leq i \leq 2^9$ , modulo  $\Lambda_1$ . Each coset in  $\Lambda_0/\Lambda_1$  has a trellis structure identical to that of  $\Lambda_1$ . Let  $[\Lambda_0/\Lambda_1]$  denote the set of coset representatives of the cosets in  $\Lambda_0/\Lambda_1$ .  $[\Lambda_0/\Lambda_1]$  is called a coset code. In the proposed two-level concatenated coded 8-PSK modulation system,  $[\Lambda_0/\Lambda_1]$  is used as the first-level inner code and  $\Lambda_1$  is used as the second-level inner code. Let  $C$  denote this two-level concatenated coded modulation system.

The two outer codes used in the proposed system are the (511,411,101) and (511,499,13) RS codes with symbols from the Galois field  $GF(2^9)$ . The (511,411,101) RS code is used as the first-level outer code and the (511,499,13) RS code is used as the second-level outer code. Each code symbol can be represented by a binary 9-bits.

The spectral efficiency of the above two-level concatenated coded 8-PSK system is 2.0034 bits/symbol. Its bit error performance over the frequency non-selective Rayleigh fading channel using two-level closest coset decoding is simulated and shown in Figure 4.

For comparison purpose, we construct a single-level concatenated modulation code with  $\Lambda_0$  as inner code. Since  $\Lambda_0$  has 18 information bits, (511,455,57) RS code is used as outer code with interleaving depth 2. Each symbols over  $GF(2^9)$  in outer code is converted into an 9-bits. Then form an 511 by 18 array where each column consists of 18 bits. Each column is then encoded into a codeword in  $\Lambda_0$ . Let  $C(S)$  denote the above single-level concatenated BCM code. The spectral efficiency of the code  $C(S)$  is 2.0034 bits/symbol. The error performance of the coherently detected 8-PSK single-level concatenated modulation code  $C(S)$  over the Rayleigh fading channel is also shown in Figure 4. The two-level concatenated modulation code  $C$  achieves a 1.4757 dB real coding gain over the single-level concatenated modulation code  $C(S)$  at a bit error rate (BER)  $10^{-5}$  with the same spectral efficiency. Also, two-level concatenated code achieves a 32.259 dB real coding gain at a BER  $10^{-5}$  over the uncoded QPSK modulation without bandwidth expansion.

Other two-, three- and six-level concatenated BCM schemes have been devised for the Rayleigh fading channel and they all achieve very impressive real coding gains over the uncoded reference system and single-level concatenated BCM schemes using same inner codes.

#### REFERENCES

- [1] V. A. Zinoviev, "Generalized Cascade Codes," *Problemy Peredach i Informatsii*, Vol. 12. 1, pp. 2-9, trans. from Russian by Plenum Publishing Co., 1976.
- [2] R. H. Deng and D. J. Costello, Jr., "High Rate Concatenated Coding Systems Using Bandwidth Efficient Trellis Inner Codes," *IEEE Transactions on Communications*, Vol. COM-37, pp. 420-427, May 1989.
- [3] R. H. Deng and D. J. Costello, Jr., "High Rate Concatenated Coding Systems Using Multi-dimensional Bandwidth Efficient Trellis Inner Codes," *IEEE Transactions on Communications*, Vol. COM-37, pp. 1091-1096, October 1989.
- [4] T. Kasami, T. Takata, T. Fujiwara, and S. Lin, "A Concatenated Coded Modulation Scheme for Error Control," *IEEE Trans. on Communications*, Vol. COM-38, No. 6, pp. 752-763, June 1990.
- [5] B. Vucetic, "Concatenated Multilevel Block Coded Modulation," *IEEE Trans. on Communications*, Vol. 41, No. 1, January 1993.
- [6] B. Vucetic and S. Lin "Block Coded Modulation and Concatenated Schemes for Error Control On Fading Channels," *Journal of Discrete Applied Mathematics*, No. 33, pp. 257-269, 1991.

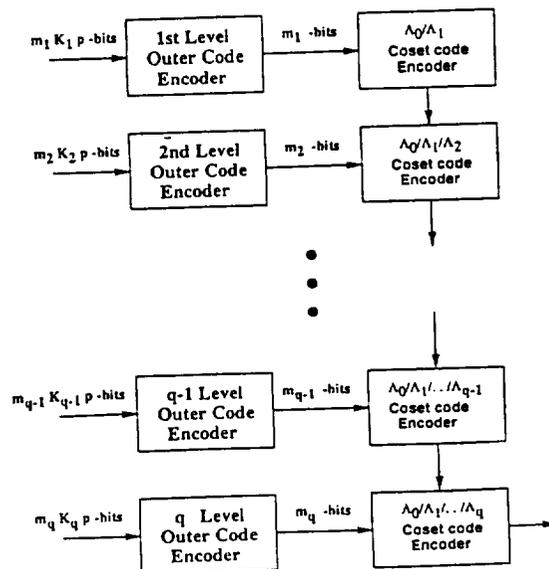


Figure 1 q-level concatenated block modulation code encoder

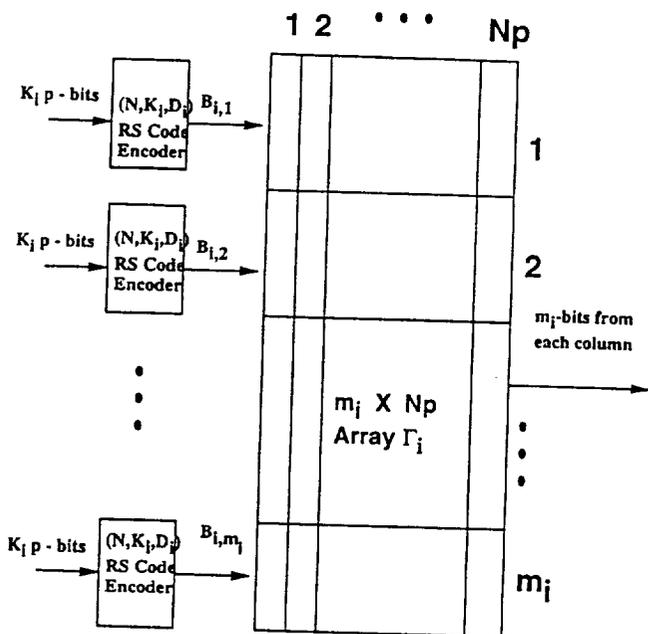
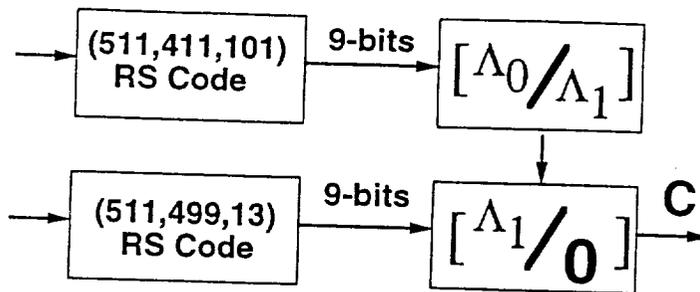


Figure 2 The  $i$ -th level outer code encoder



$$\Lambda_0 = (8,4,4)*(8,7,2)*(8,7,2)$$

$$\Lambda_1 = (8,1,8)*(8,4,4)*(8,4,4)$$

Figure 3 Two-level concatenated block modulation code

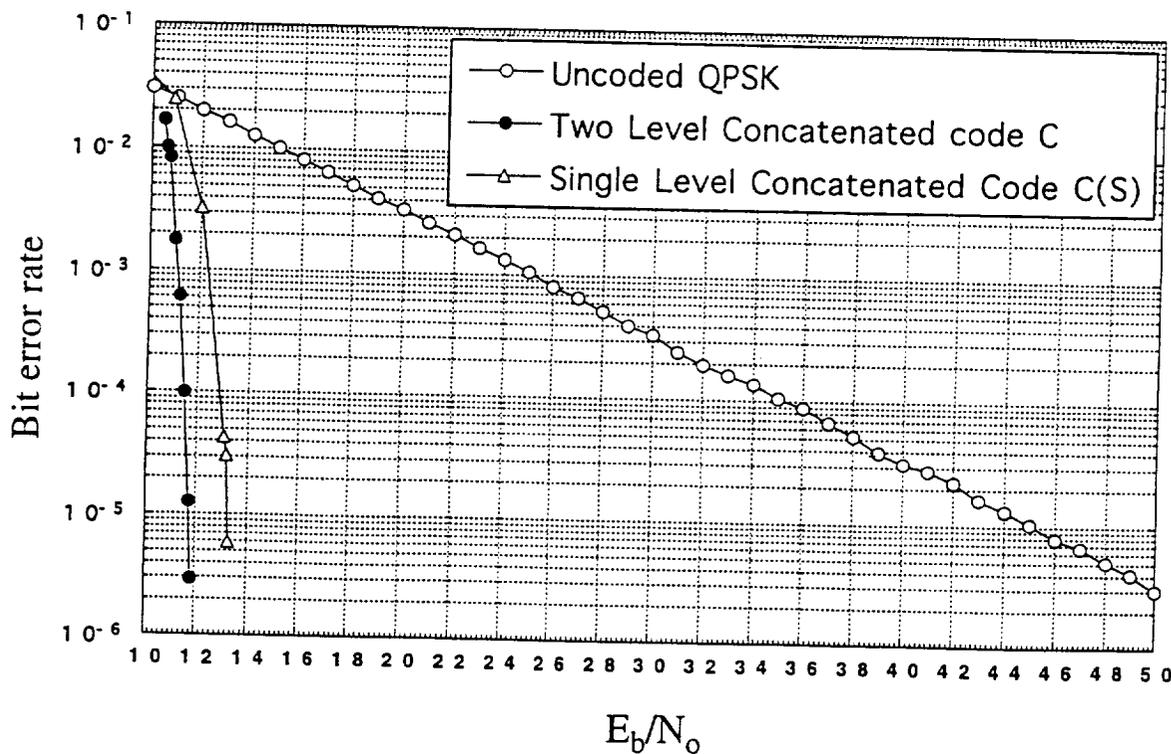


Figure 4. Bit error performance of 8-PSK two-level concatenated block modulation code over the frequency non-selective Rayleigh fading channel.