# DAMPING ROTOR NUTATION OSCILLATIONS
# IN A GYROSCOPE WITH MAGNETIC SUSPENSION[1]

Valentine N. Komarov
Research Institute for Applied Mathematics and Cybernetics
of Nizhny Novgorod State University, Nizhny Novgorod, Russia

## SUMMARY

A possibility of an effective damping of rotor nutations by modulating the field of the moment transducers in synchronism with the nutation frequency is considered. The algorithms for forming the control moments are proposed and their application is discussed.

## INTRODUCTION

Practical use of noncontact suspensions in gyroscopy is possible if the following requirements, at least, are fulfilled: the rotor must be stably suspended and stably rotate with given parameters; information about the angular motion of the body must be available.

The second problem of making the rotor rotate is, in turn, divided into three other problems: providing a necessary angular momentum for the rotor, which, most frequently, is equivalent to providing a certain rotation speed $\Omega$ for the rotor, providing a small, zero in the limit, nutation angle $\vartheta$ , and maintaining the achieved motion parameters within the given limits in the further operation of the unit. A way for solving these problems with respect to a gyro with a magnetoresonance suspension can be found in (ref. 1).

The small dissipative moments in a noncontact gyro leads to the fact that after acceleration the rotor rotates with a nutation angle defined mainly by the initial conditions. The damping of the nutation oscillations over a finite time can be achieved if, for the formation of the control moments (not only for nutation damping), we use modulation or self-modulation methods, when parameters of the suspension field or of the additional coil field (magnitude, direction, rotation speed, etc.) vary depending on an angular position of the aspheric rotor (ref. 2, 3, 4), due to which a nonconservative component appears in the moment effect on the rotor, which also provides an active damping of nutation. Let us consider one of the possible methods for an effective damping of the rotor nutation relative to a gyro with magnetic suspension.

---

Assuming the gyro rotor to be stably suspended in the suspension field, we represent the equations of its angular motions in the form of six first order equations with respect to the following phase variables (ref. 5): the value of angular momentum $K$ , two angles $\rho$ and $\sigma$ , which determine its orientation (trihedron $OY_j$) relative to the suspension (trihedron $OZ_k$), and The Euler angles $\varphi$ , $\psi$ and $\vartheta$ , which prescribe the position of the rotor (trihedron $(OX_i)$) relative to the angular momentum $\vec{K}$ ; $\varphi$ is the angle of the proper rotation of the rotor about the axis of dynamic symmetry $(OX_3)$ ; $\psi$ is the angle of precession of this axis about the angular momentum $\vec{K}$ ; $\vartheta$ is the nutation angle or the angle between the angular momentum and the axis $OX_3$.

Using the main equation of gyroscopy

$$\frac{d\vec{K}}{dt} + \vec{\Omega} \times \vec{K} = \vec{M} \tag{01}$$

written in the coordinate system $OY_j$ and projecting the vector $\vec{K}$ on the rotor axes $OX_i$

$$\left(\vec{K} \cdot \vec{x}_i\right) = K \cdot b_{i3} = I_i \cdot \Omega_i , \tag{02}$$

where $I_i$ is the rotor's moment of inertia along $OX_i$, and $\Omega_i$ is the projection of the rotor's absolute angular velocity on the same axis, for an axisymmetric rotor $(I_1 = I_2 = A \neq I_3 = C)$ and the arbitrary moments we obtain

$$K\left(\frac{d\rho}{dt} + \Omega_{s2}\right) = M_1 , \quad K\left(\sin\rho\frac{d\sigma}{dt} - \Omega_{s1}\right) = M_2 , \quad \frac{dK}{dt} = M_3 , \tag{03}$$

$$K\frac{d\vartheta}{dt} = -(M_1 \cos\psi + M_2 \sin\psi) = -M_{con} ,$$

$$\frac{d\varphi}{dt} - K\left(\frac{1}{C} - \frac{1}{A}\right)\cos\vartheta = \frac{M_2 \cos\psi - M_1 \sin\psi}{K \sin\vartheta} ,$$

$$\frac{d\psi}{dt} - \frac{K}{A} = -\frac{M_2 \cos\psi - M_1 \sin\psi}{K}\cot an\vartheta - \Omega_3 ,$$

where $M_j = \left(\vec{M} \cdot \vec{y}_j\right)$ , $\Omega_3 = \Omega_{s3} + (M_2/K)\cot an\rho$ , and $\Omega_{sj}$ are the projections onto the axes $OY_j$ of the absolute angular velocity $\vec{\Omega}_s$ of the coordinate system $OZ_k$. This velocity can be determined by the earth's rotation, by the rotation of an object on which the gyro is installed, by a forced rotation of the device casing performed to make its interaction with the rotor symmetric, etc.

The presence of different time-scale motions in real gyros, such as a "slow" precession motion of the angular momentum and a "fast" rotor rotation allows us to use, for studying the gyro dynamics, the averaging method in which even the first approximation reveals the basic laws of gyro behavior (ref. 5).

Omitting the standard procedure for normalizing the equations and finding in the latter a small parameter $\varepsilon$, which characterizes the ratio of the work of external forces per one rotation of the rotor to the kinetic energy accumulated by the gyro rotor, we write their standard form, which allows the asymptotic methods to be applied:

$$\rho^{\bullet} = \varepsilon \left( \frac{m_1}{k} - \omega_{s2} \right), \quad \sigma^{\bullet} \sin \rho = \varepsilon \left( \frac{m_2}{k} + \omega_{s1} \right), \qquad k^{\bullet} = \varepsilon m_3, \tag{04}$$

$$k\vartheta^{\bullet} = -\varepsilon \left( m_1 \cos \psi + m_2 \sin \psi \right),$$

$$\varphi^{\bullet} + \gamma k \cos \vartheta = \varepsilon \left( \frac{m_2 \cos \psi - m_1 \sin \psi}{k \sin \vartheta} \right),$$

$$\psi^{\bullet} - k = -\varepsilon \left( \frac{m_2 \cos \psi - m_1 \sin \psi}{K} \cotan \vartheta - \omega_3 \right).$$

It is evident from the equations that free motion of the rotor (generating solution) represents a regular precession or the Euler–Poinsot motion with the constant angular velocities of the proper rotation $\varphi_0^{\bullet}$ and precession $\psi_0^{\bullet}$ for a constant nutation angle $\vartheta$, such that

$$\psi_0^{\bullet} = k_0, \qquad \varphi_0^{\bullet} = -\gamma k_0 \cos \vartheta_0, \qquad \left( \gamma = \frac{C - A}{A} \right), \tag{05}$$

where $k_0$ is the constant value of the angular momentum $\vec{k}$ which is immobile in the inertial space. For the periodic dependence of the right-hand sides of the equations of motion on the fast variables $\psi$ and $\varphi$, to obtain the first approximation over the small parameter $\varepsilon$, the right-hand sides of the equations for the slow variables are averaged over the variables $\psi$ and $\varphi$, whose dependences on $\tau$ are assumed to be the same as those in the unperturbed motion.

Let us now specify the requirements to be met by the moment controlling the rotor motion to perform the damping of the rotor's nutation oscillations over a finite period of time.

After the averaging of the equations of motion over the fast variables $\psi$ and $\varphi$, the nutation angle varies according to

$$k\vartheta^{\bullet} = -\varepsilon m_{con} \left( \rho, \sigma, \vartheta, k \right). \tag{06}$$

Since the initial values of the nutation angle $\vartheta$ are closer to 0 rather than $\pi/2$, and this angle must still decrease with time, we introduce variable $x = \sin \vartheta$ and linearize Eq. (06) with respect to this variable

$$kx^{\bullet} = -\varepsilon \left[ m^0(\rho, \sigma, \vartheta, k) + x m^1(\rho, \sigma, \vartheta, k) \right],$$

where $m^0(\rho, \sigma, \vartheta, k) = m_{con}(\rho, \sigma, 0, k)$, $\quad m^1 = \partial m_{con}/\partial x|_{x=0}$.

Therefore, a monotonic decrease of the nutation angle $\vartheta$ to zero over a finite period of time is possible only if the condition $m^0 > |m^1|$ is fulfilled. In this case, there is no equilibrium state for the angle $\theta$ or $x$, and their variation rates are always negative, which allows the required effect to be achieved.

## THE FORMATION OF THE CONTROL MOMENT

Let us discuss a possibility for damping the rotor nutations through its interaction with the field of a transducer for the angular position of the rotor. In a gyro with a magnetoresonance suspension, this transducer is embodied as a system of two four-leg magnetic circuits located on the same axis but on the different sides of the rotor with a coil (ref. 1) at each leg. An axial opening in the rotor leads to the fact that from the moment standpoint, the rotor will behave as anaspheric body, whose surface is described by a set of even harmonics (even Legendre polynomials), the first of which is the second harmonic responsible for the ellipsoidality or the uniaxial anisotropy. The moment defined by this harmonic will be considered below.

This moment is described by

$$\vec{M} = -2a \left(\vec{s} \cdot \vec{h}\right) \left[\vec{s} \times \vec{h}\right],\tag{07}$$

where $a > 0$ is the interaction amplitude determined, in particular, by the field amplitude: $\vec{s}$ and $\vec{h}$ are the unit vectors of the symmetry axes of the ellipsoid and the field. (Since we allow for the anisotropic properties of the rotor determined by its opening, from the magnetic viewpoint it should be considered as an oblate ellipsoid tending to position itself with its axis perpendicular to the field's axis, which is reflected by the minus sign in Eq. (07).)

It can be assumed that the unit vector $\vec{s}$ coincides with the unit vector of the dynamic symmetry axis $OX_3$ of the rotor, and, thus,

$$\vec{s} = \vec{x}_3 = \vec{y}_1 \sin \vartheta \cos \psi + \vec{y}_2 \sin \vartheta \sin \psi + \vec{y}_3 \cos \vartheta .$$

Using the spherical angles $\alpha_n$ and $\beta_n$, we prescribe the position of the unit vector $\vec{h}$, which determines the symmetry axis of the magnetic field generated by the $n$-th pair of the coils belonging to different magnetic circuits but located on the same axis that passes through the center of the rotor

$$\vec{h} = \vec{z}_1 \sin \alpha_n \cos \beta_n + \vec{z}_2 \sin \alpha_n \sin \beta_n + \vec{z}_n \cos \alpha_n ,$$

where $\alpha_n = \alpha$ is the same for all $n$ and $\beta_n = \beta_0 + \pi n/2$.

Let us substitute these unit vectors into the expression for the moment (07) and find its component $m_{con} = m_1 \cos \psi + m_2 \sin \psi$ responsible for the variation of the nutation angle $\vartheta$

$$M_{con} = 2a_n \left(h_1 \sin \psi - h_2 \cos \psi\right) h_3 \cos \vartheta -\tag{08}$$

$$-\frac{1}{4} \left[2h_1 h_2 \cos 2\psi + \left(h_2^2 - h_1^2\right) \sin 2\psi\right] \sin 2\vartheta ,$$

where

$$h_j = \left(\vec{h} \cdot \vec{y}_j\right), \qquad h_1 = \cos \rho \sin \alpha \cos (\sigma - \beta_n) - \sin \rho \cos \alpha .$$

$$h_2 = -\sin \alpha \sin (\sigma - \beta_n), \qquad h_3 = \sin \rho \sin \alpha \cos (\sigma - \beta_n) + \cos \rho \cos \alpha .$$

From Eq. (08) it follows that $M_{con}$ is a periodic function of the precession angle $\psi$ without the constant component $M_{con}^0 (\rho, \sigma, 0, K)$, which can appear only in the case of a pulsed

switching-on of the field over the time periods, during which the moment does not change its sign. As it is evident from Eq. (08), the moment has a harmonic component with frequency $\psi^\bullet$ proportional to $\cos^2 \vartheta$ and a component with double frequency $2\psi^\bullet$ proportional to $\sin 2\vartheta$. For $\vartheta = 0$ this results in

$$M_{con} = 2a_n \left( h_1 \sin \psi - h_2 \cos \psi \right) h_3 \ .$$

Apparently, for the effective damping of nutation, i.e., dropping the angle $\vartheta$ down to zero over a finite period of time, the coil field with frequency $\psi^\bullet$ must be changed. (The speed of the rotor rotation about the symmetry axis $OX_3$ is equal to $\Omega_0 = K/C$ , the nutation frequency $\psi^\bullet = K/A$ , and, consequently, $\psi^\bullet = \Omega_0 C/A$ ). It should be noted that nutation oscillations of a cryogenic gyro can be damped in a similar manner (ref. 3).

Let us write $h_1 \sin \psi - h_2 \cos \psi$ in the form:

$$h_1 \sin \psi - h_2 \cos \psi = - \sin \rho \cos \alpha \sin \psi +$$

$$+ \frac{1}{2} \left[ (1 + \cos \rho) \sin (\psi + \sigma - \beta_n) - (1 - \cos \rho) \sin (\psi - \sigma + \beta_n) \right] \sin \alpha \ .$$

From the above expression it follows that the optimal switching-on of the field should follow the algorithm

$$a(\psi) = \begin{cases} a, & 2k\pi \le \psi + \sigma - \beta_n \le (2k+1)\pi \\[2mm] 0, & (2k+1)\pi \le \psi + \sigma - \beta_n \le 2(k+1)\pi \ , \end{cases} \tag{09}$$

i.e., the field is switched on over half the period of the nutation oscillations with frequency $\psi^\bullet$.

For this operation mode, two pairs of the control coils are switched on at any instant, which greatly improves the efficiency but leading, however, to a higher heat release that can be reduced due to a decrease in either the field amplitude, or the switching-on duration.

$$a(\psi) = \begin{cases} a, & \left(2k + \frac{1}{2}\right)\pi - \frac{\tau}{2} \le \psi + \sigma - \beta_n \le \left(2k + \frac{1}{2}\right)\pi + \frac{\tau}{2} \\[2mm] 0, & \text{outside the intervals} \end{cases} \tag{10}$$

Here $\tau < \pi$ is the switching-on duration. Apparently, the algorithm (09) is a particular case of (10) for $\tau = \pi$.

If all the coil pairs and their switching-on algorithms are identical, they create a total control moment

$$\langle M_{con} \rangle = 2 \frac{a}{\pi} (1 + \cos \rho) (2 \cos \rho - 1) \sin 2\alpha \sin \frac{\tau}{2} \cos^2 \vartheta \ , \tag{11}$$

which leads to equation

$$K\vartheta^\bullet = - 2 \frac{a}{\pi} (1 + \cos \rho) (2 \cos \rho - 1) \sin 2\alpha \sin \frac{\tau}{2} \cos^2 \vartheta \ . \tag{12}$$

331

Apart from the variable $\vartheta$ , this equation also incorporates other quantities $K(t)$ and $\rho(t)$ that do not remain constant. If we write the rotor-affecting moments via the force function $W$ (ref. 5), then, instead of the corresponding equations in (03), we can obtain

$$K\frac{d\rho}{dt} = \frac{1}{\sin\rho}\left(\frac{\partial W}{\partial\psi}\cos\rho - \frac{\partial W}{\partial\sigma}\right) ,$$

$$\frac{dK}{dt} = \frac{\partial W}{\partial\psi} ,$$

$$\frac{d\vartheta}{dt} = \frac{1}{K\sin\vartheta}\left(\frac{\partial W}{\partial\psi}\cos\vartheta - \frac{\partial W}{\partial\varphi}\right) .$$

Since the rotor opening is symmetric about its dynamic axis, then, consequently, $\partial W/\partial\varphi = 0$ . The summation of the moments from all the pairs of the symmetric coils also results in $\partial W/\partial\sigma = 0$ . These conditions yield the equations

$$K\frac{d\rho}{dt} = \frac{1}{\sin\rho}\frac{\partial W}{\partial\psi}\cos\rho , \qquad \frac{dK}{dt} = \frac{\partial W}{\partial\psi} ,, \qquad \frac{d\vartheta}{dt} = \frac{1}{K\sin\vartheta}\frac{\partial W}{\partial\psi}\cos\vartheta ,$$

from which we obtain two integrals

$$K_{z3} = K\cos\rho = Const , \qquad K_{x3} = K\cos\vartheta = Const , \tag{13}$$

the physical meaning of which is evident, i.e., on the strength of the control system and rotor symmetries, the projections of the angular momentum onto the symmetry axes of the suspension ($K_{z3}$) and the rotor ($K_{x3}$) remain constant. Representing $K$ and $\cos\rho$ from Eq.(13) and substituting them into Eq. (12), we can find the law of the nutation angle $\vartheta$ variation with time. To simplify the solution, let us use the smallness of the misalignment angle $\rho$ , which, along with the use of the second integral, yields

$$\vartheta^{\bullet} = -\nu\cos^3\vartheta , \qquad \left(\nu = \frac{4a}{\pi K_{x3}}\sin 2\alpha\sin\frac{\tau}{2}\right) \tag{14}$$

from which it is evident that the time of the nutation angle decrease from the initial value $\vartheta_0$ to 0 is given by

$$T(\vartheta_0) = \frac{1}{\nu}\left\{\frac{\sin\vartheta_0}{2\cos^2\vartheta_0} + \frac{1}{2}\ln\left[\tan\left(\frac{\pi}{4} + \frac{\vartheta_0}{2}\right)\right]\right\} . \tag{15}$$

At least up to $\vartheta_0 \approx 20^0$ , the dependence $T(\vartheta_0)$ is insignificantly different from the linear dependence $T(\vartheta_0) = \vartheta_0/\nu$ , which simplifies the calculations.

Since on the right side of Eq. (14) there are no terms that are linear in $\sin\vartheta$ , the interaction amplitude $a$ (or the coil field value) can be any value other than zero, which affects only the time of the nutation damping rather than its character.

332

To conclude the study of the nutation damping, let us make some final remarks relative to the application of the method proposed:

a) As it is evident, the working part of the control moment of the $n$-th coil pair $\sim \sin(\psi + \sigma - \beta_n)$. At the same time, the projection of the rotor symmetry axis onto the direction $\vec{r}_n = \vec{z}_1 \cos \beta_n + \vec{z}_2 \sin \beta_n$ , that corresponds to this pair is $(\vec{s} \cdot \vec{r}_n) = \vartheta \cos(\psi + \sigma - \beta_n) + \rho \cos(\sigma - \beta_n)$ for small $\vartheta$ and $\rho$ . Since the moment transducer in the gyro incorporates four coil pairs, it is apparent that the control moment of any pair can be formed based on the signal taken from an adjacent pair.

However, this mode of operation, in which the signal is taken from one pair and there are switchings-over in the other pair, can lead to an electrical engagement of both channels disrupting thereby their normal operation. In this respect, we need to obtain a signal with frequency $\psi^\bullet$ by another method, for example, using a transducer to obtain a signal on one side of the rotor and a transducer to ensure control on the other side of the rotor. In this case, however, along with the moment effect, there will also be a force effect on the rotor or tension.

b) Another method is the switching-over mode of operation, in which, for a certain period of time, the transducers are used for obtaining the information and then for control with periodic repetitions of the process. There is no input signal in the control mode, and it results in a decrease of $K$. Consequently, the control signal can be formed by a device of an underexcited oscillator type to the input of which an external signal with frequency $\psi^\bullet$ is initially applied. In this case, control will be efficient only over the time period until the phase difference between the real signal from the rotor with frequency $\psi^\bullet$ and the signal from the oscillator achieves the critical value $(\Delta \varphi = \pi/2)$ .

If at the initial time it was the frequency $\omega_0$ , and the rotor deceleration could be simulated by the viscous friction when $\omega(t) = \omega_0 \exp(-\lambda t)$ , the phase difference between the signals from the rotor and from the oscillator can be calculated as

$$\Delta \varphi = \int_0^t \omega_0 \left(1 - e^{-\lambda \tau}\right) d\tau = \omega \left(t + \frac{1}{\lambda} e^{-\lambda t} - \frac{1}{\lambda}\right) \approx \frac{1}{2} \omega_0 \lambda t^2.$$

If $\quad T_0 = 2\pi/\omega_0 , \quad T_\lambda = \lambda^{-1}$ , then

$$T_{con} = \sqrt{\frac{\Delta \varphi}{\pi} T_0 T_\lambda} \tag{16}$$

Let us consider the following example: if the rotor speed is $\Omega = 1000 \ r.p.s.$ $(\psi^\bullet = 2\pi \cdot 1200 \ sec^{-1})$ , and the damping is such that $T_\lambda = 1 \ hr$ , the nutation can be damped during $T_{con} \approx 1.22 \ sec$ until the phase difference becomes $\Delta \varphi = \pi/2$ , after which we again need to switch over to the regime of signal reception and control moment formation.

c) The proposed mechanism of nutation damping is active for both constant and variable magnetic fields generated by the coils, but the constant field efficiency is greater. The maximum

gain in efficiency of the effect on the rotor can be obtained, provided the transducer coils are used for control alone and the input signal with frequency $\psi^*$ for the control moment formation is taken from other devices.

## REFERENCES

1. Odintsov, A. A.: Magnetic Gyroscope and Inertial System on its Basis. *Sudostroitel'naya promyshlennost'*, *ser. Navigatsiya i giroscopiya*, no. 1, 1991, pp. 51-58 (in Russian).

2. Komarov, V.N.: On the Influence of the Self-Modulation on Nutation of a Noncontact Gyroscope. *Prikladnaya mekhanika*, vol. 20, no. 5, 1984, pp.104-109 (in Russian).

3. Komarov, V.N.; Urman, Yu. M.: Active Damping of Noncontact Gyroscope Rotor Nutation. *Izv. vuzov. Priborostroyeniye*, vol. 25, no.12, 1982, pp.- 44-47 (in Russian).

4. Komarov, V.N.: Regulating the Magnetic Gyroscope's Motion. *The Fourth International Symposium on Magnetic Bearings*. Zurich. Hochschulverlag AG an ETN,1994, pp.19-22.

5. Beletskiy, V.V.: Dynamics of Fast Rotations. *Proc. Res. Inst. for Mechanics. Moscow University*, no. 29, 1973, pp.97-118 (in Russian).

# Session 10 -- Bearing/Actuator Design

Chairman: James Downer
Satcon Technology Corporation