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Canonical Signed Digit Study

Part II: FIR Digital Filter Simulation Results

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INTRODUCTION

Finite Impulse Response digital filter using Canonical Signed-Digit (CSD) number representation for the coefficients has been studied and its computer simulation results are presented here. Minimum Mean Square Error (MMSE) criterion is employed to optimize filter coefficients into the corresponding CSD numbers. To further improve coefficients optimization process, an extra non-zero bit is added for any filter coefficients exceeding1/2. This technique improves frequency response of filter without increasing filter complexity almost at all. The simulation results shows outstanding performance in bit-error-rate (BER) curve for all CSD implemented digital filters included in this presentation material.

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CSD applications for FIR filters

- Issues and Concerns on high speed digital filter implementation

☞ Problems:

- Digital Finite Impulse Response (FIR) filters are widely used in digital signal processing
- Multipliers in filter are the most complex hardware realization which increase the cost of filters
- Multiplication is the most time and power consuming process limits variety of high speed applications like high data rate digital modem implementation

☞ Solution:

- Eliminate the multipliers from filter by employing CSD represented filter coefficients to increase the speed and reduce hardware complexity

Question: How can CSD filter coefficients help high speed FIR digital filter design?

Answer: Less nonzero digits in a CSD form cuts number of addition stages and thereby also reduce both hardware complexity and power consumption

Note: Number of adder/subtracter required to realize a CSD coefficient is one less than the number of nonzero digits in the code

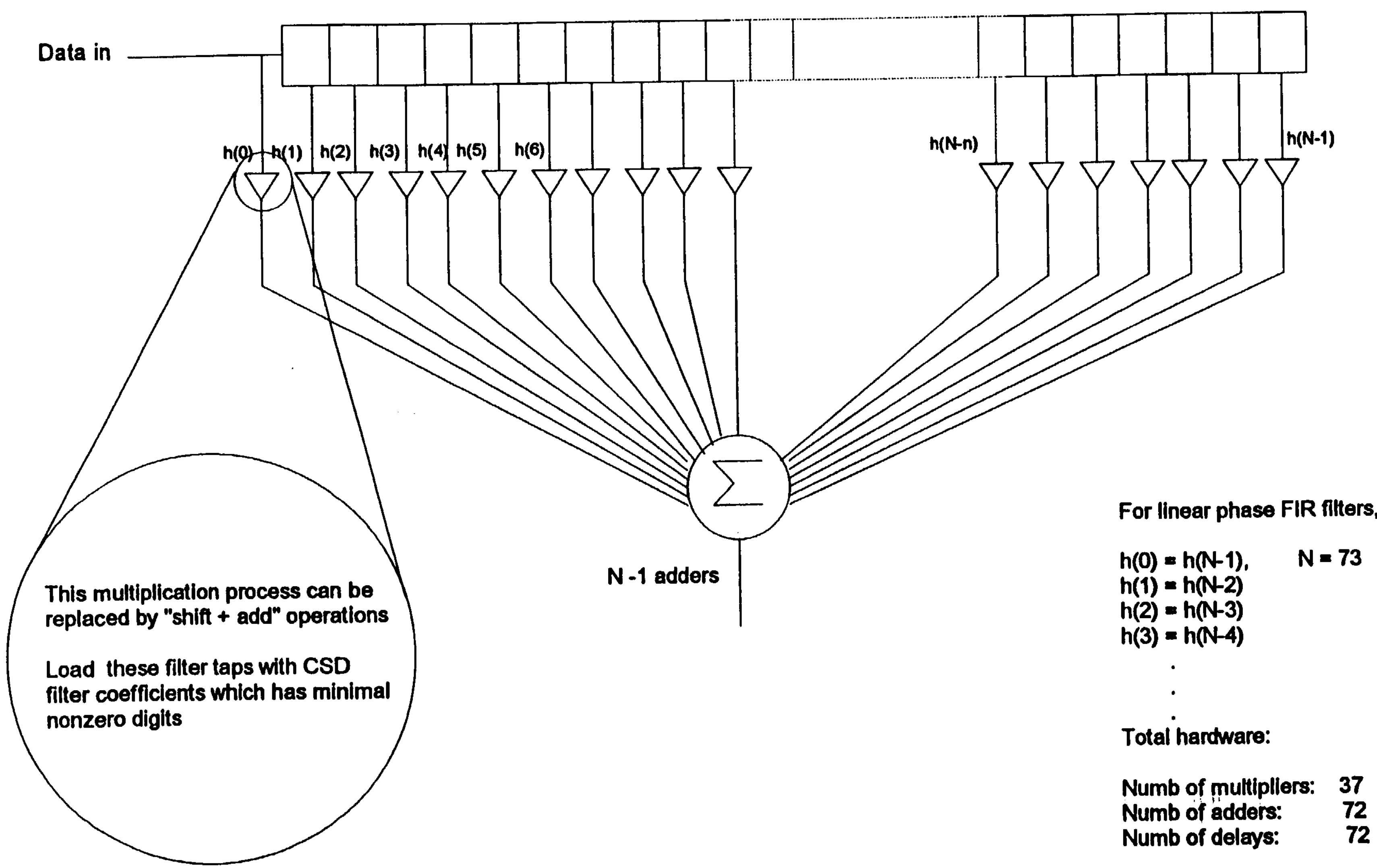
☞ example

Consider a filter coefficients in 8-bit with value = 0.9921875

$0.9921875_{10} = 0.1111111_2 = 1.000000\bar{1}_{csd} = 2^0 - 2^{-7}$

Only one subtracter as opposed to seven adders

General Digital FIR Filter

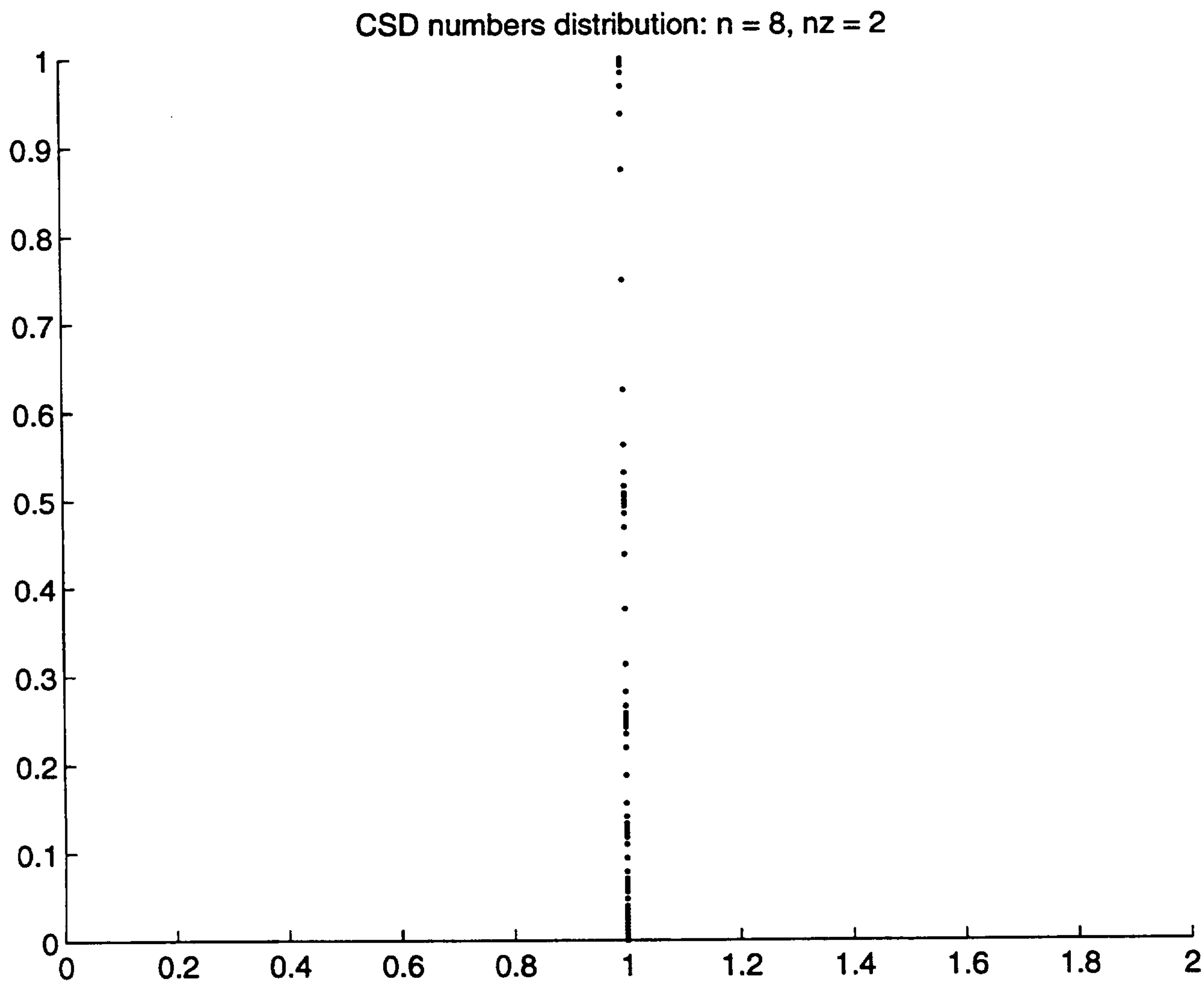


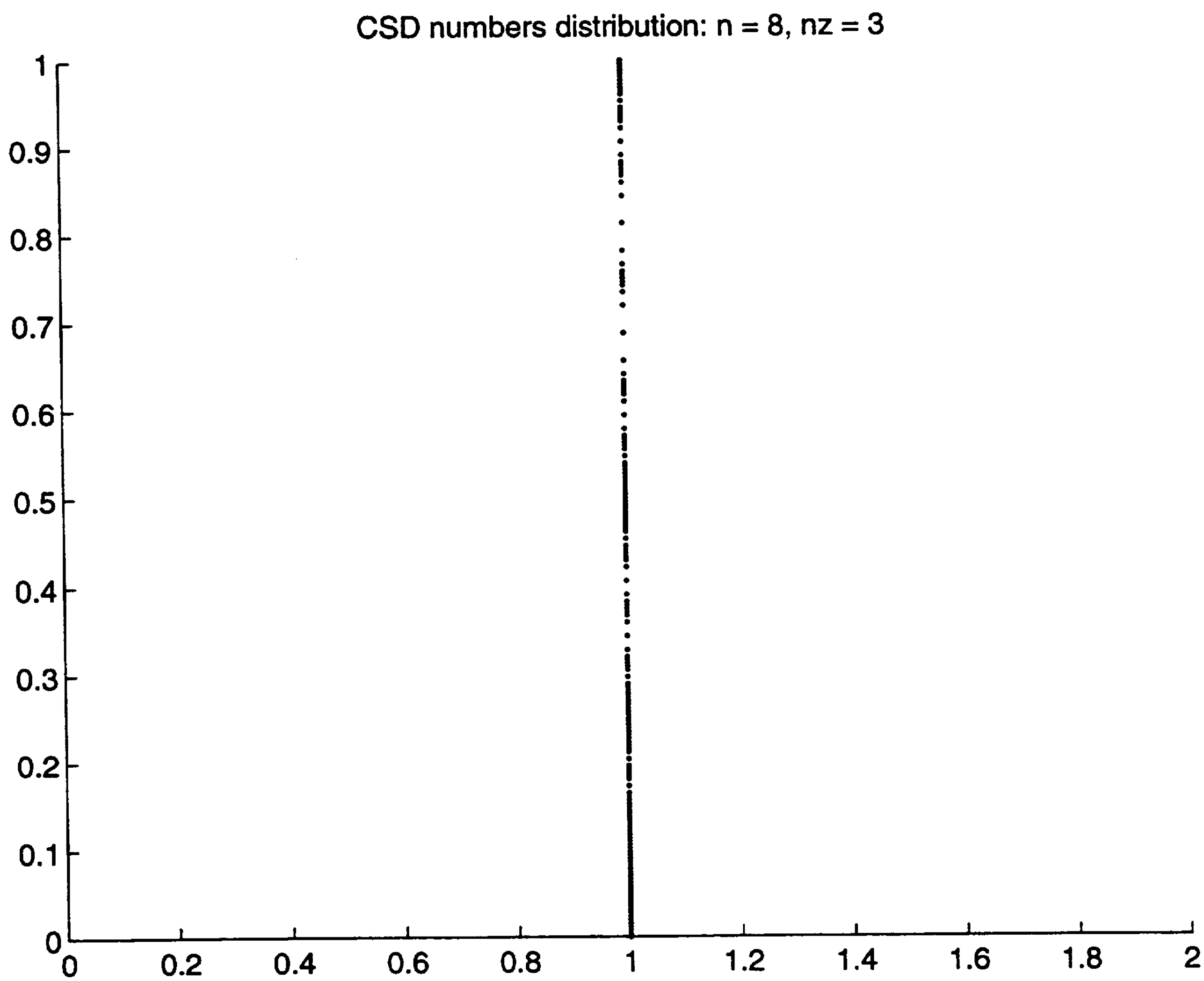
❑ CSD filter coefficients search guidelines:

❶ Select the number of nonzero digits and total code length in CSD format

- Code length (quantization bits) is not very critical for optimizing CSD coefficients
- 8 to 15 were found to satisfactory for most filter specifications
- The number of nonzero digits directly affect hardware complexity and the worst-case coefficient quantization error which results in undesirable frequency response of the filter and also in ISI
- 2 to 4 nonzero digits are satisfactory for most cases
- Rule of thumb: One nonzero digit required in the CSD code for each 20 dB of stopband attenuation
- Quantization errors caused by the nature of nonuniform distribution of CSD coefficients

❷ Optimize CSD filter coefficients through various search algorithms


CSD numbers distribution: n = 8, nz = 2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2


CSD numbers distribution: n = 8, nz = 3
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

❑ Optimization of search algorithm for FIR filter coefficients

■ **Search for an optimal scale factor and bivariate local search to improve frequency response**

Mixed integer linear programming (MILP):

- Very effective but required a lot of computing powers and limited to about 40 filter taps due to exponentially growing CPU time with filter length

Local search:

- Performs nearly as well as the MILP and requires substantially less computing resources

◆ **Optimization Criterion:**

① **FFT approach:** scale the filter coeffs., perform an FFT and compare the frequency responses

② **MMSE approach:** select the scale factor that results in minimum value of coefficient quantization error

③ **Hybrid approach:** coarse time-domain search and then fine-tune with frequency domain search

Method of Choice

1. Assign an extra nonzero digit in CSD form for filter coefficients greater than or equal to 0.5 (optional)

   - will result in substantial improvements in frequency response at the cost of a small increase in hardware complexity

2. Prescale filter coefficients and then convert it into CSD represented filter coefficients

   - will improve ISI substantially as well as frequency response
   - the distribution of CSD coefficients is more dense for smaller valued coefficients

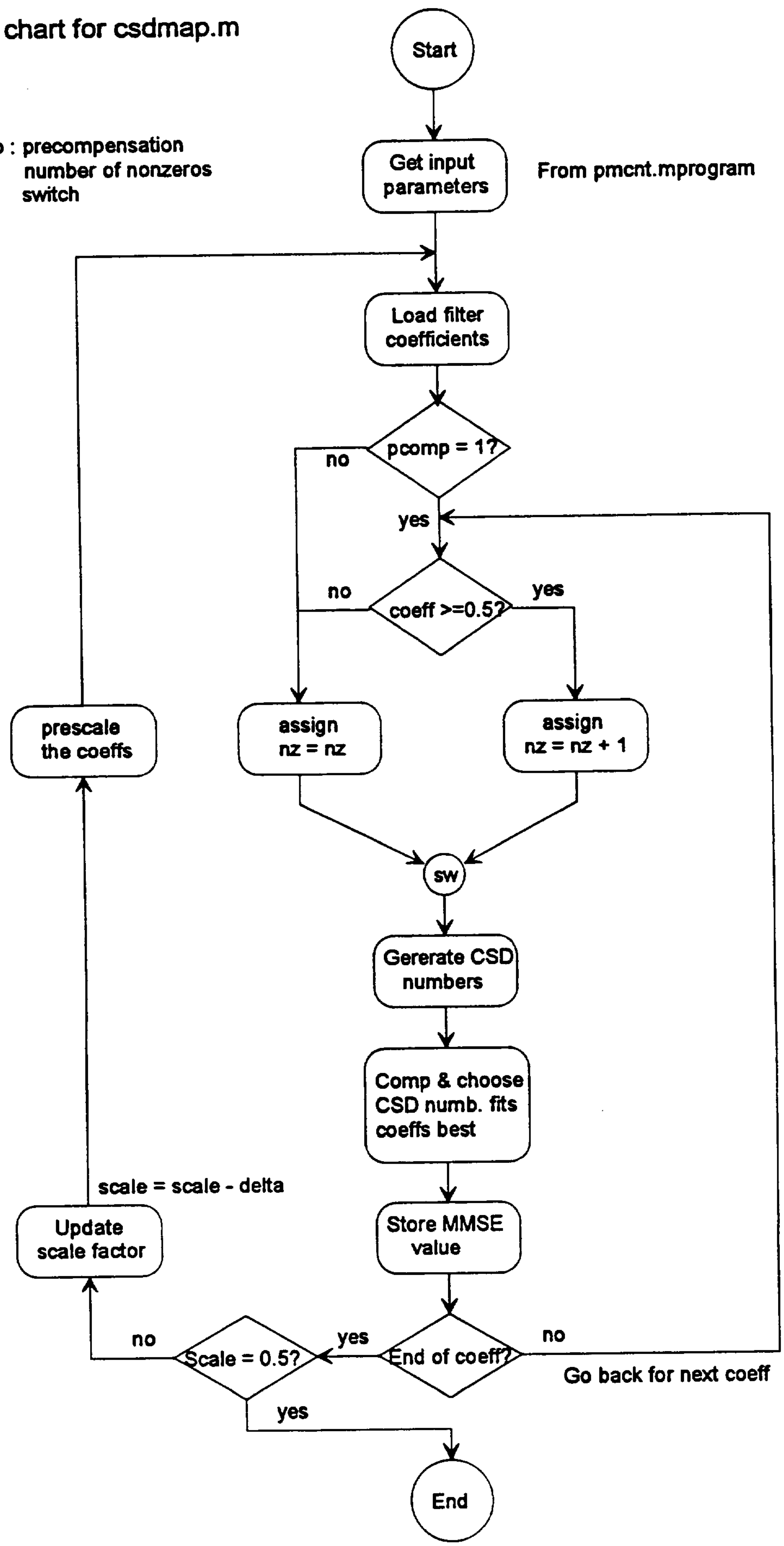
3. Change a prescale factor by one tenth (optional)

   - choose the step size, Δ, between scale factors to achieve a reasonable tradeoff between search time and frequency response

4. Optimize CSD coefficients in Minimum Mean Square Error (MMSE)

Flow chart for csdmap.m

pcomp : precompensation
nz : number of nonzeros
sw : switch



❑ Root-Raised-Cosine Filter equation,

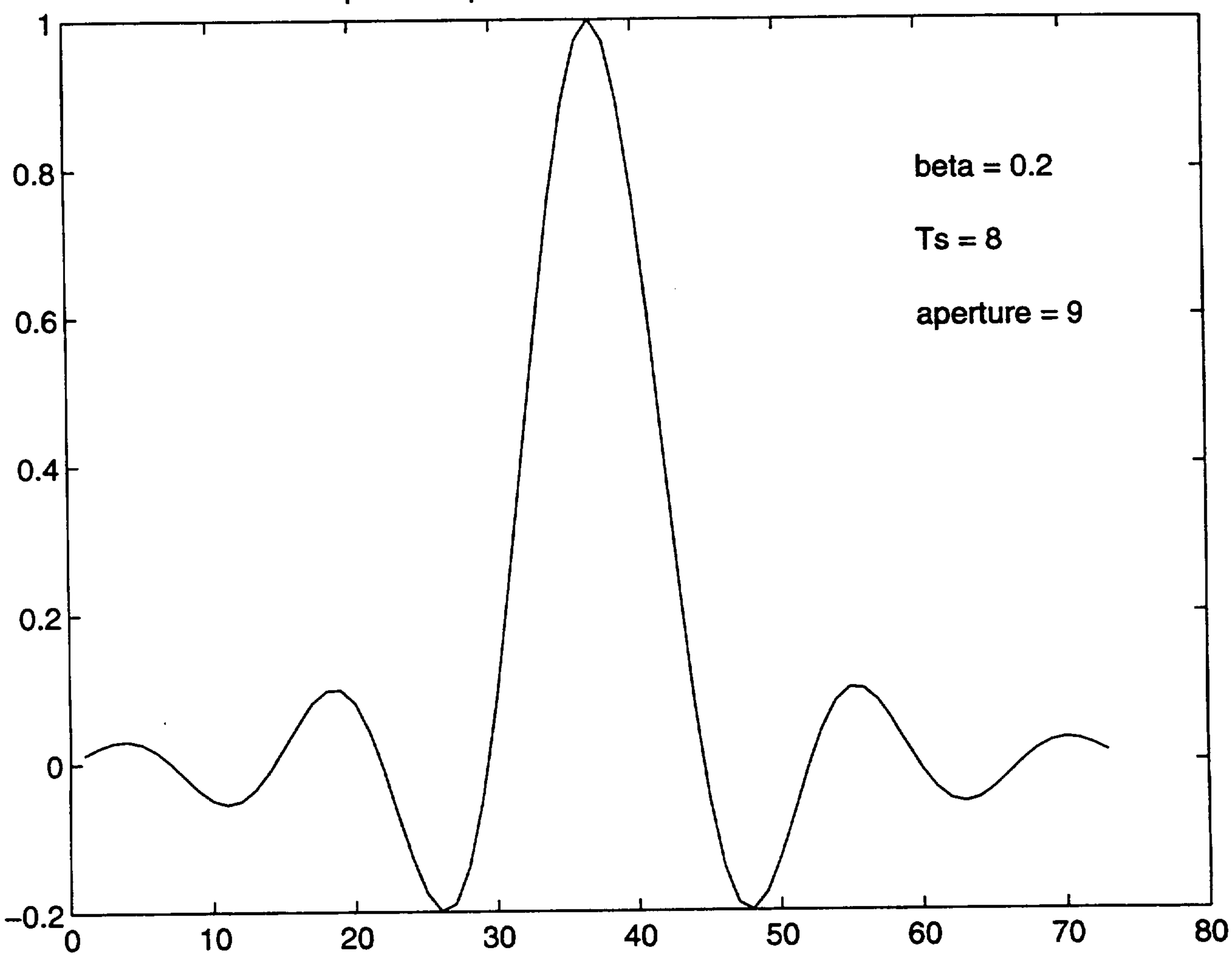
$$h(t) = \left(\frac{4\beta}{\pi(1-\beta) + 4\beta}\right)\left(\frac{T_s \sin\left(\frac{\pi}{T_s}(1-\beta)t\right)}{4\beta t} + \frac{\cos\left(\frac{\pi}{T_s}(1+\beta)t\right) + \frac{4\beta t}{T_s}\sin\left(\frac{\pi}{T_s}(1-\beta)t\right)}{1 - \left(\frac{4\beta t}{T_s}\right)^2}\right), \quad t \neq 0, \pm\frac{T_s}{4\beta}$$

$$h\left(\pm\frac{T_s}{4\beta}\right) = \left(\frac{4\beta}{\pi(1-\beta) + 4\beta}\right)\left(\frac{T_s \sin\left(\frac{\pi}{4\beta}(1-\beta)t\right)}{2} + \frac{\pi\cos\left(\frac{\pi}{4\beta}\right) + \pi\sin\left(\frac{\pi}{4\beta}\right)}{4\sqrt{2}}\right)$$

h(t = 0) = 1 and

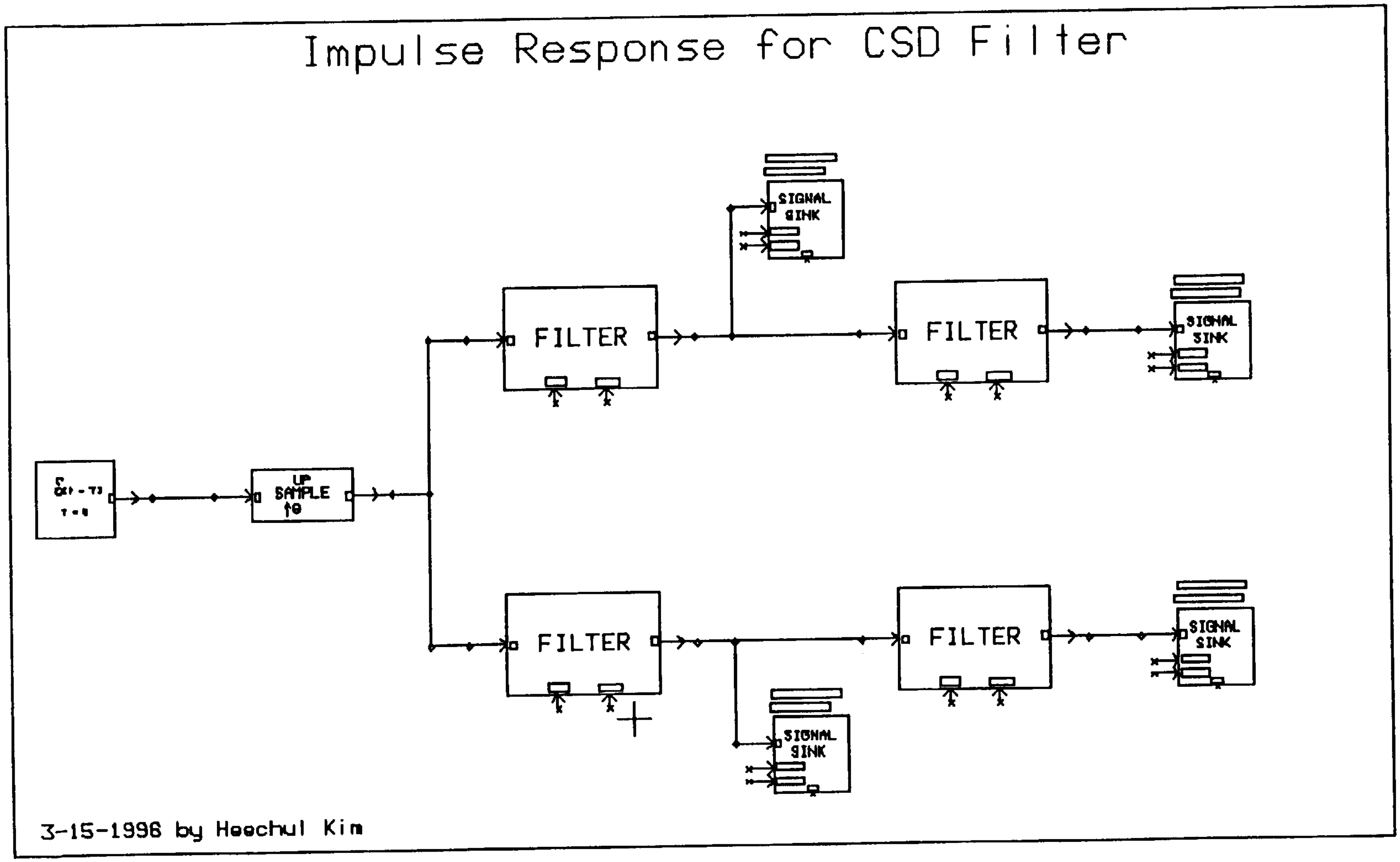
where Ts is the symbol period in seconds, β is the rolloff constant

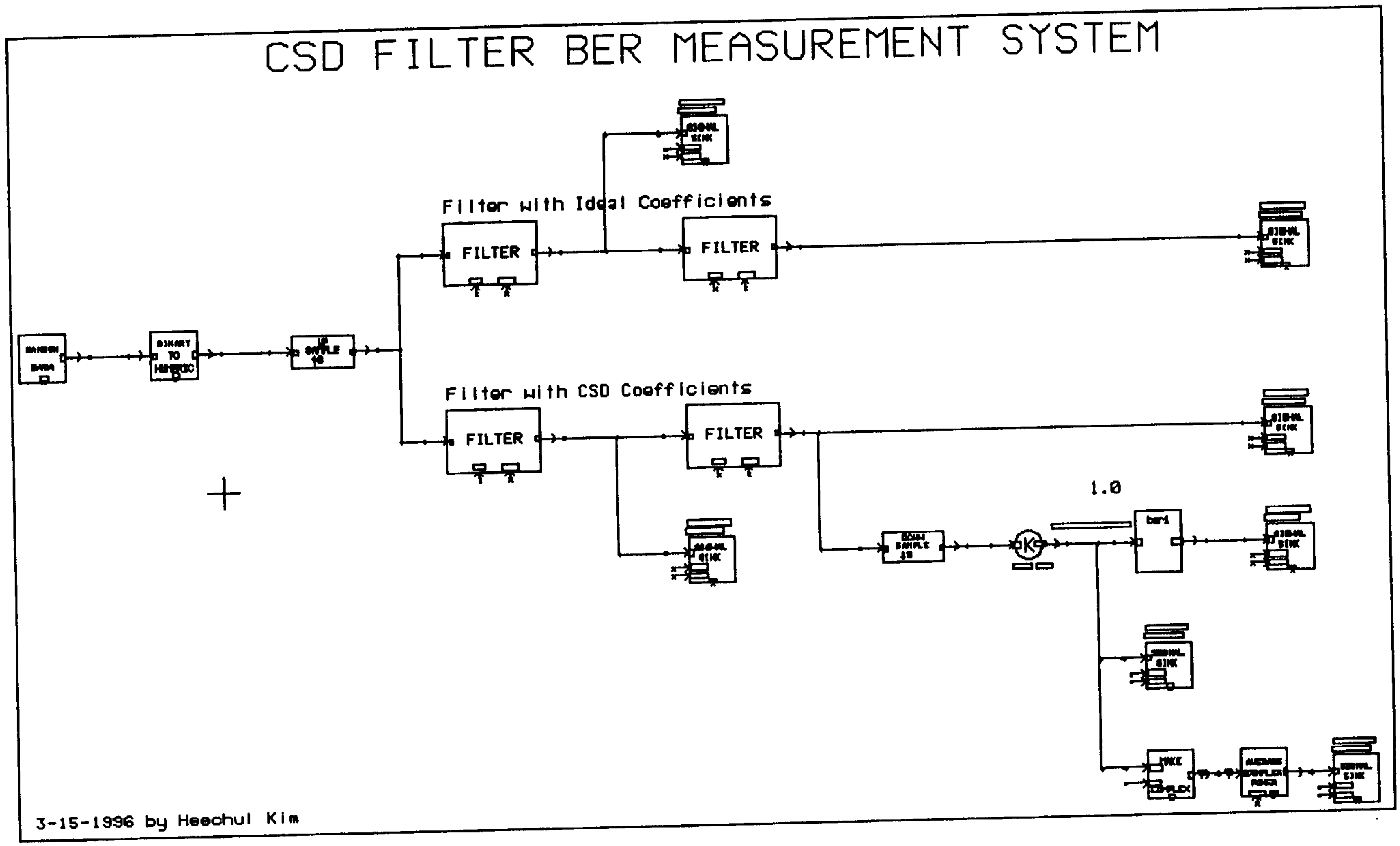
Impulse response of Root_Raised_Cosine filter

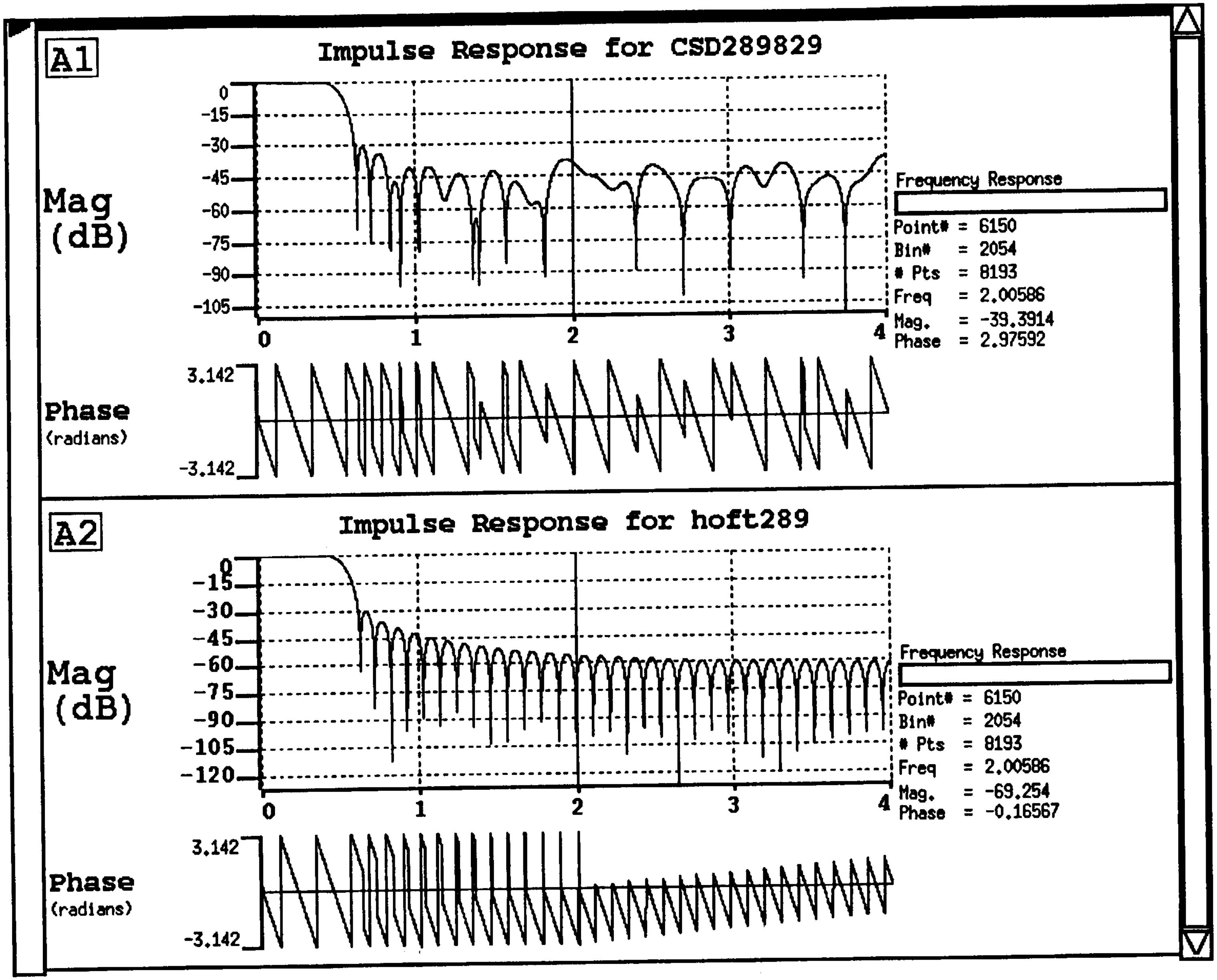


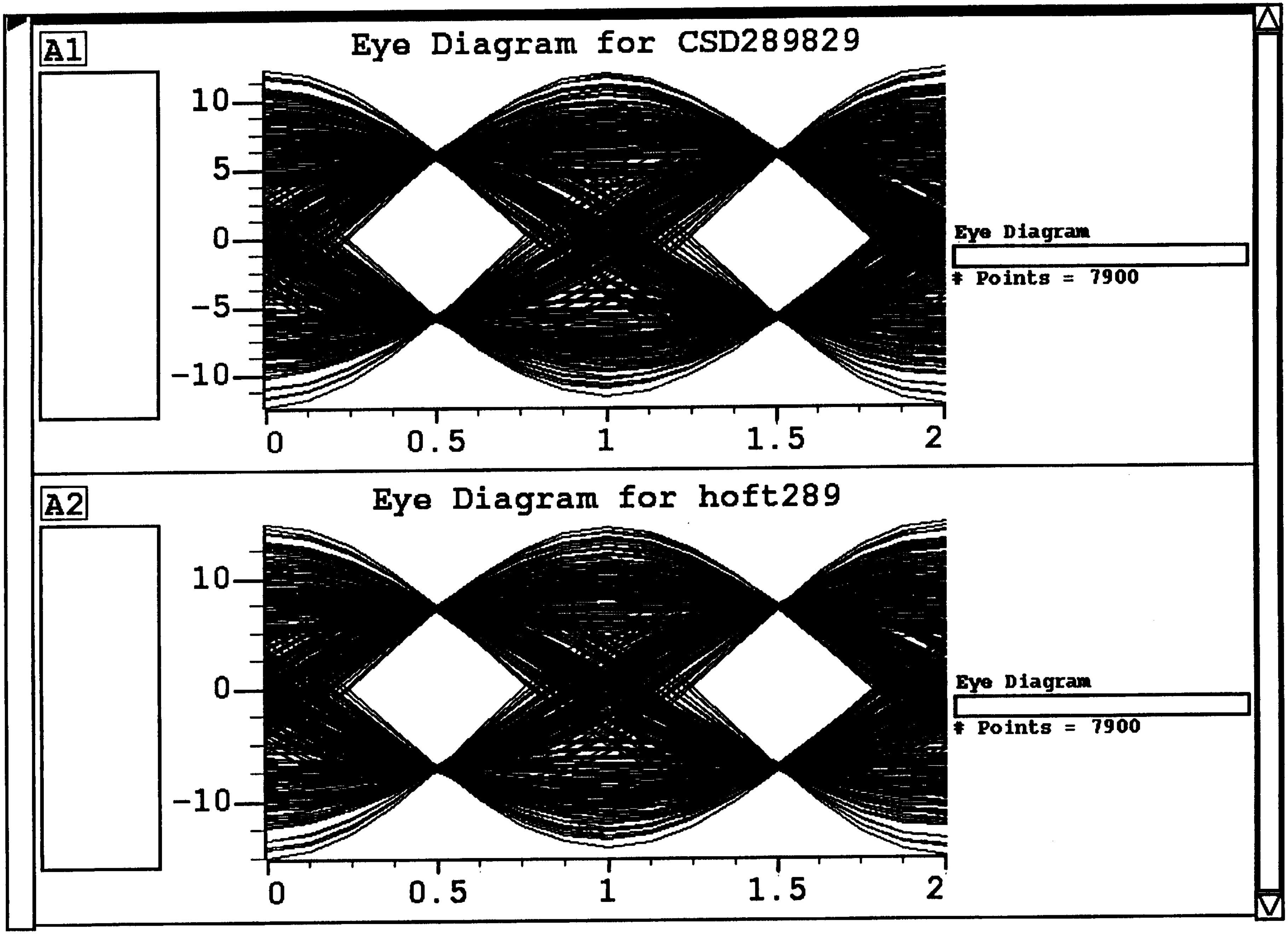
❑ *SPW Simulation Results*

- Simulation Models for Filter Systems
- CSD Filter Frequency Responses
- Eye Diagrams
- Comparison of CSD Filter Frequency Responses
- Bit-Error-Rate Performance Curves

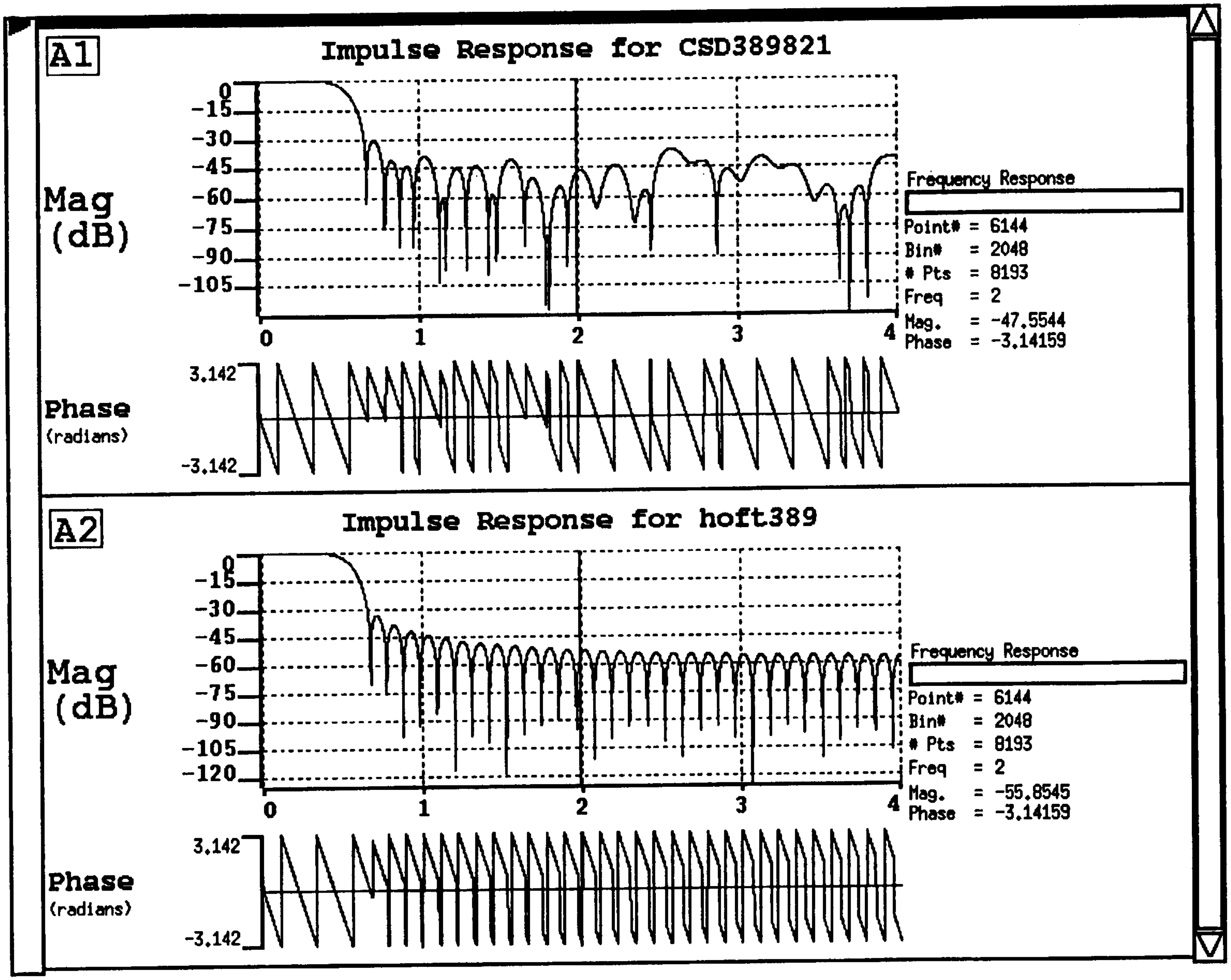

Impulse Response for CSD Filter
UP SAMPLE
FILTER
FILTER
FILTER
FILTER
SIGNAL SINK
SIGNAL SINK
SIGNAL SINK
SIGNAL SINK
3-15-1996 by Heechul Kim

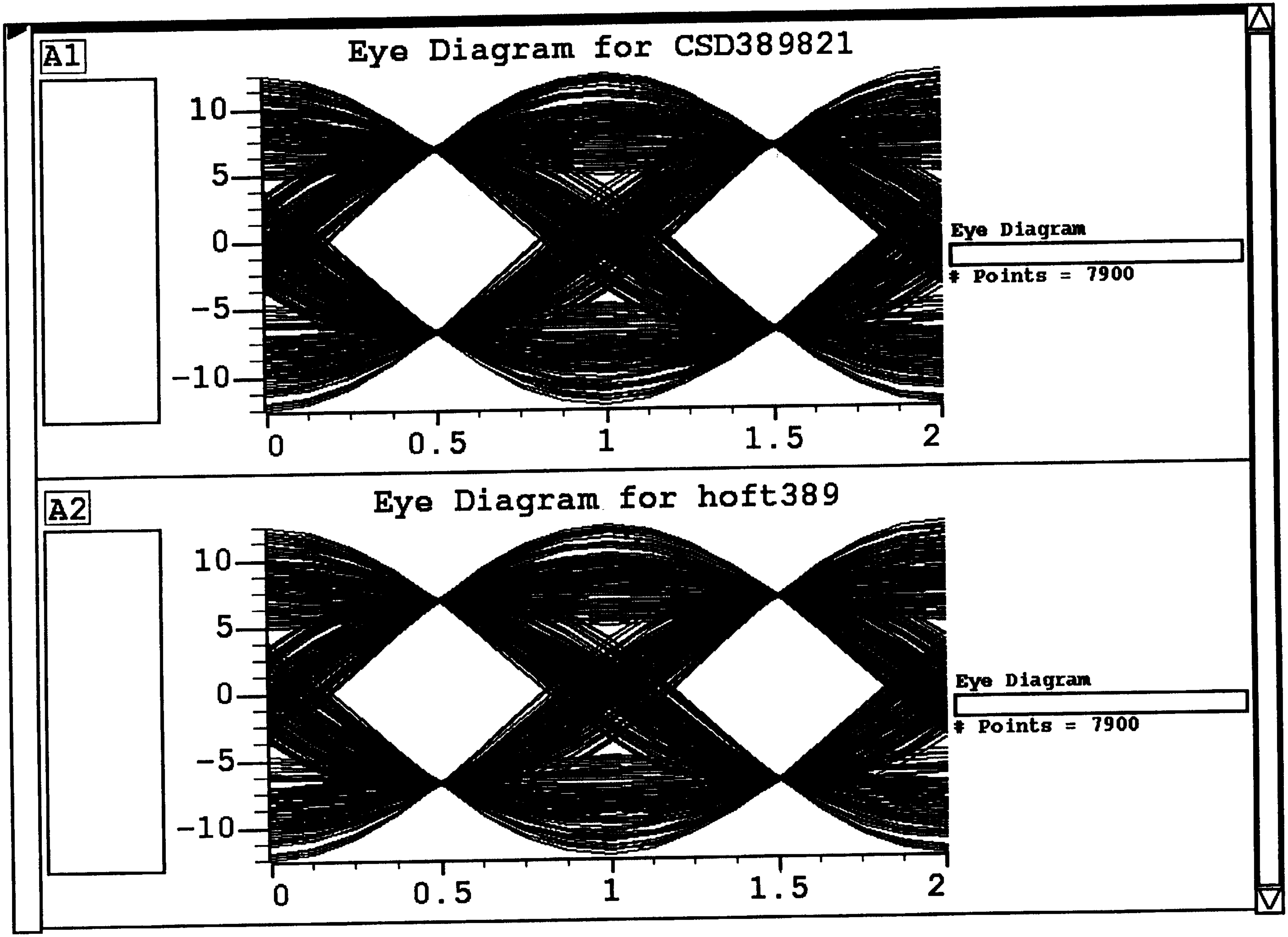

CSD FILTER BER MEASUREMENT SYSTEM
Filter with Ideal Coefficients
FILTER
FILTER
Filter with CSD Coefficients
FILTER
FILTER
1.0
3-15-1996 by Heechul Kim

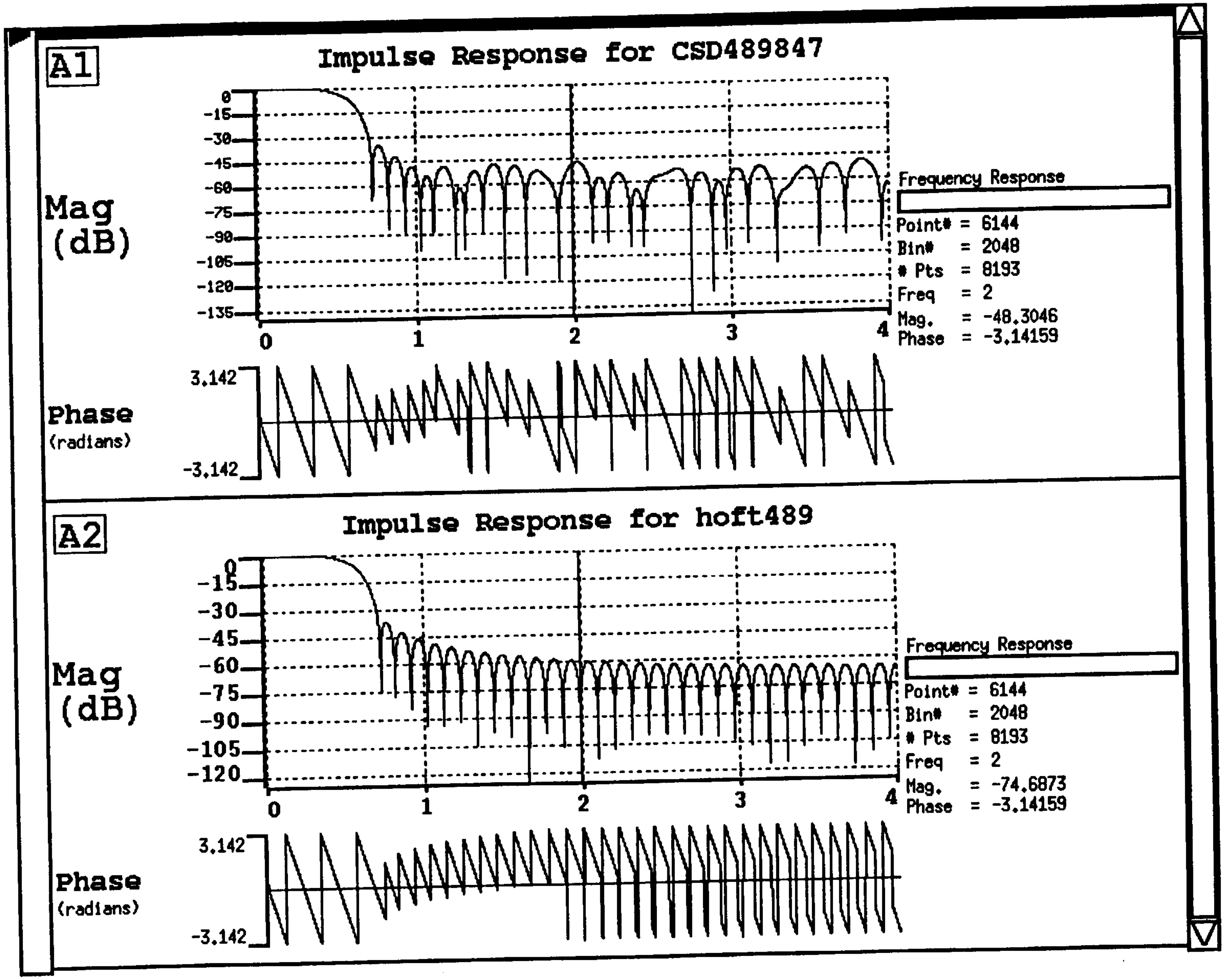

A1
Impulse Response for CSD289829
Mag (dB)
0
-15
-30
-45
-60
-75
-90
-105
0
1
2
3
4
Phase (radians)
3.142
-3.142
Frequency Response
Point# = 6150
Bin# = 2054
# Pts = 8193
Freq = 2.00586
Mag. = -39.3914
Phase = 2.97592
A2
Impulse Response for hoft289
Mag (dB)
0
-15
-30
-45
-60
-75
-90
-105
-120
0
1
2
3
4
Phase (radians)
3.142
-3.142
Frequency Response
Point# = 6150
Bin# = 2054
# Pts = 8193
Freq = 2.00586
Mag. = -69.254
Phase = -0.16567

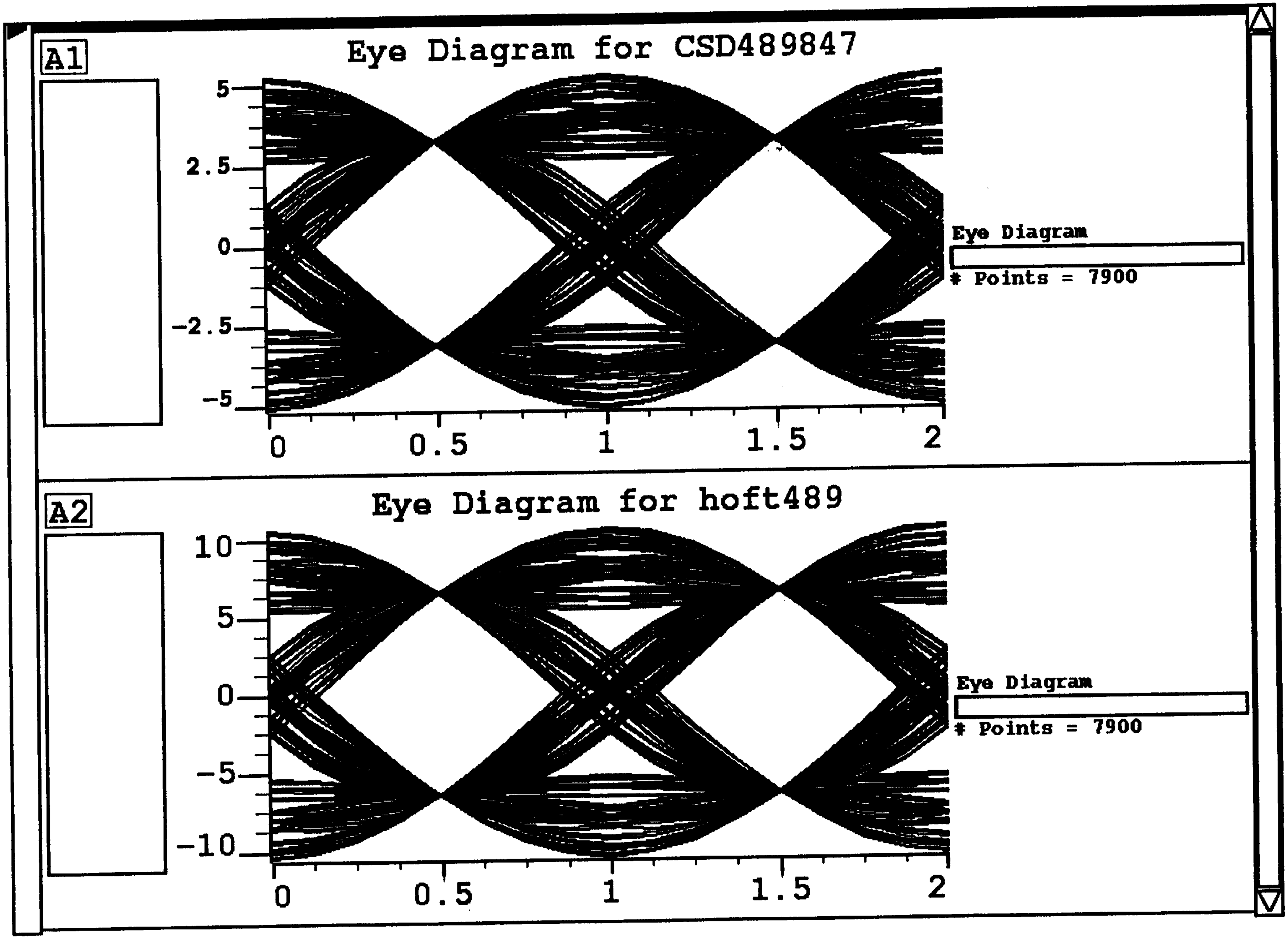


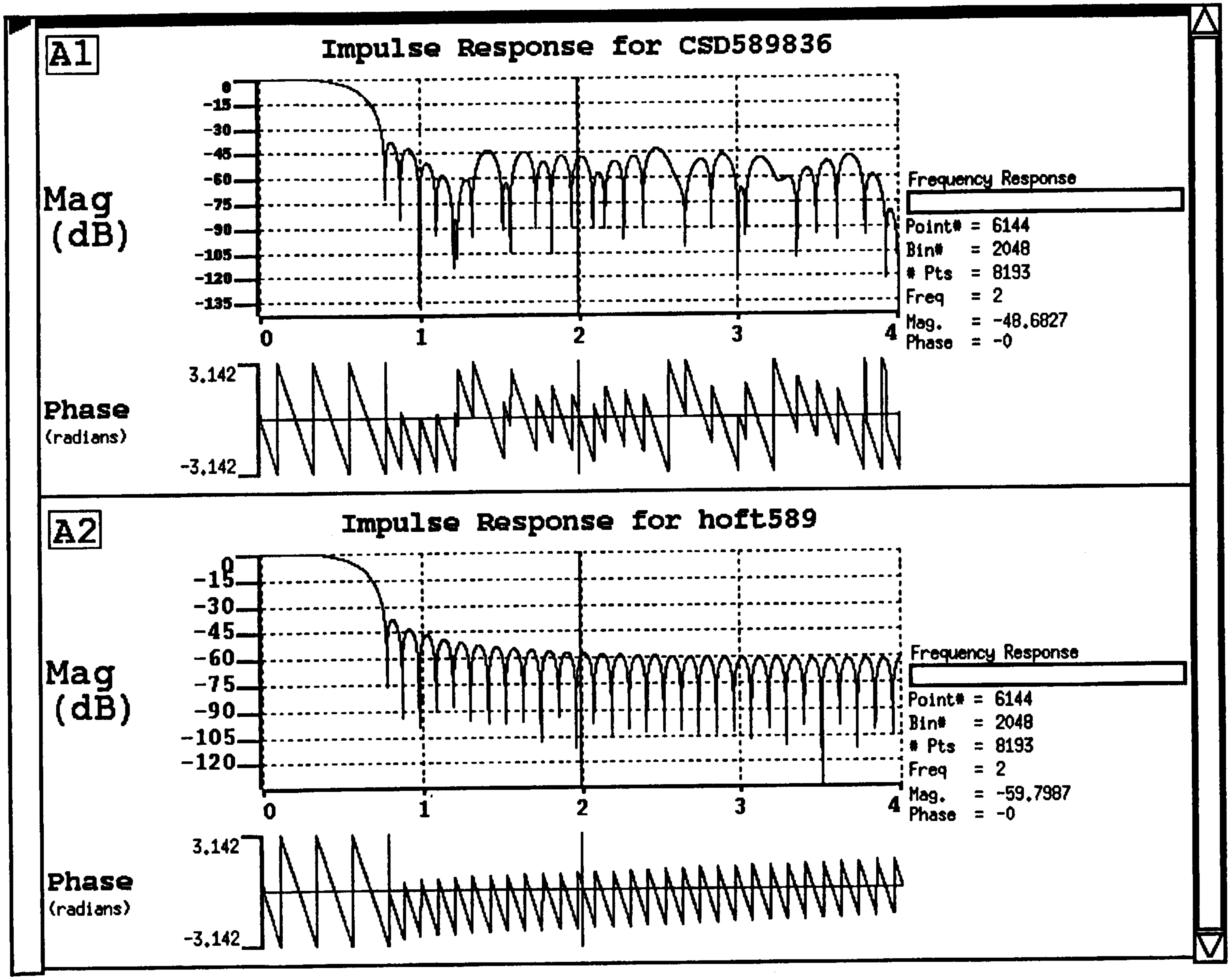
A1
Eye Diagram for CSD289829
10
5
0
-5
-10
0
0.5
1
1.5
2
Eye Diagram
Points = 7900
A2
Eye Diagram for hoft289
10
0
-10
0
0.5
1
1.5
2
Eye Diagram
Points = 7900

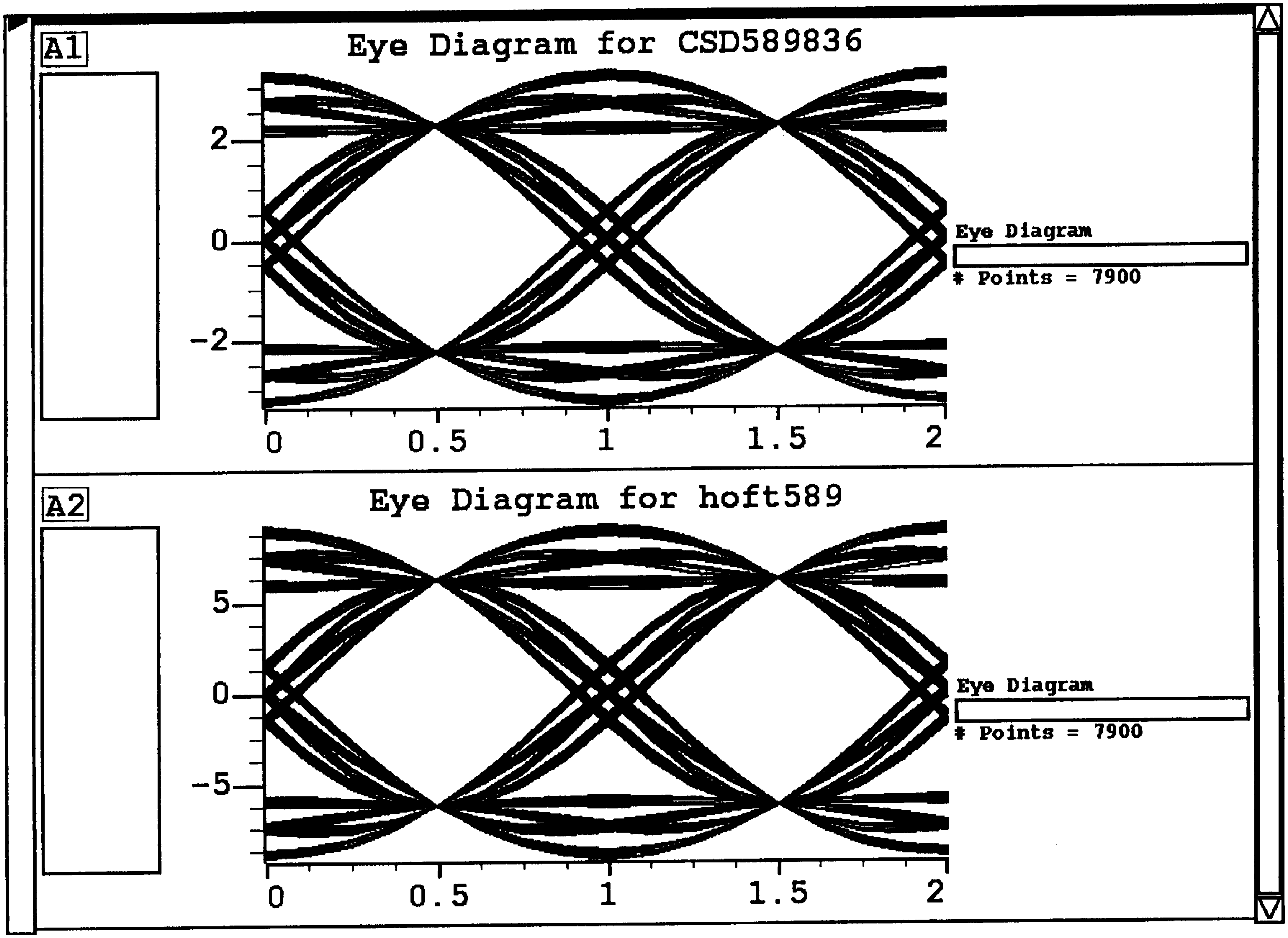

A1
Impulse Response for CSD389821
Mag (dB)
0
-15
-30
-45
-60
-75
-90
-105
0
1
2
3
4
Frequency Response
Point# = 6144
Bin# = 2048
# Pts = 8193
Freq = 2
Mag. = -47.5544
Phase = -3.14159
Phase (radians)
3.142
-3.142
A2
Impulse Response for hoft389
Mag (dB)
0
-15
-30
-45
-60
-75
-90
-105
-120
0
1
2
3
4
Frequency Response
Point# = 6144
Bin# = 2048
# Pts = 8193
Freq = 2
Mag. = -55.8545
Phase = -3.14159
Phase (radians)
3.142
-3.142

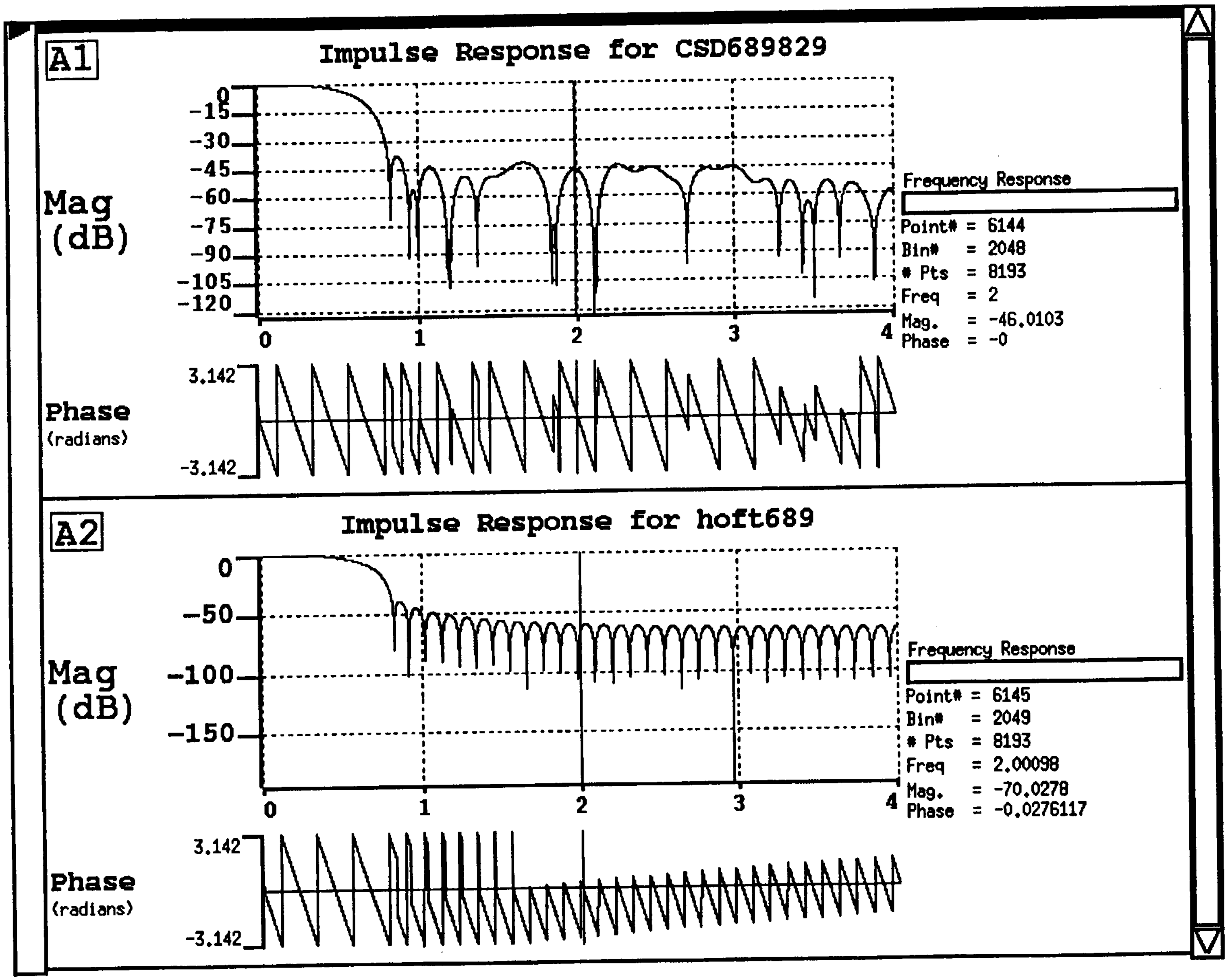

A1
Eye Diagram for CSD389821
10
5
0
-5
-10
0
0.5
1
1.5
2
Eye Diagram
Points = 7900
A2
Eye Diagram for hoft389
10
5
0
-5
-10
0
0.5
1
1.5
2
Eye Diagram
Points = 7900

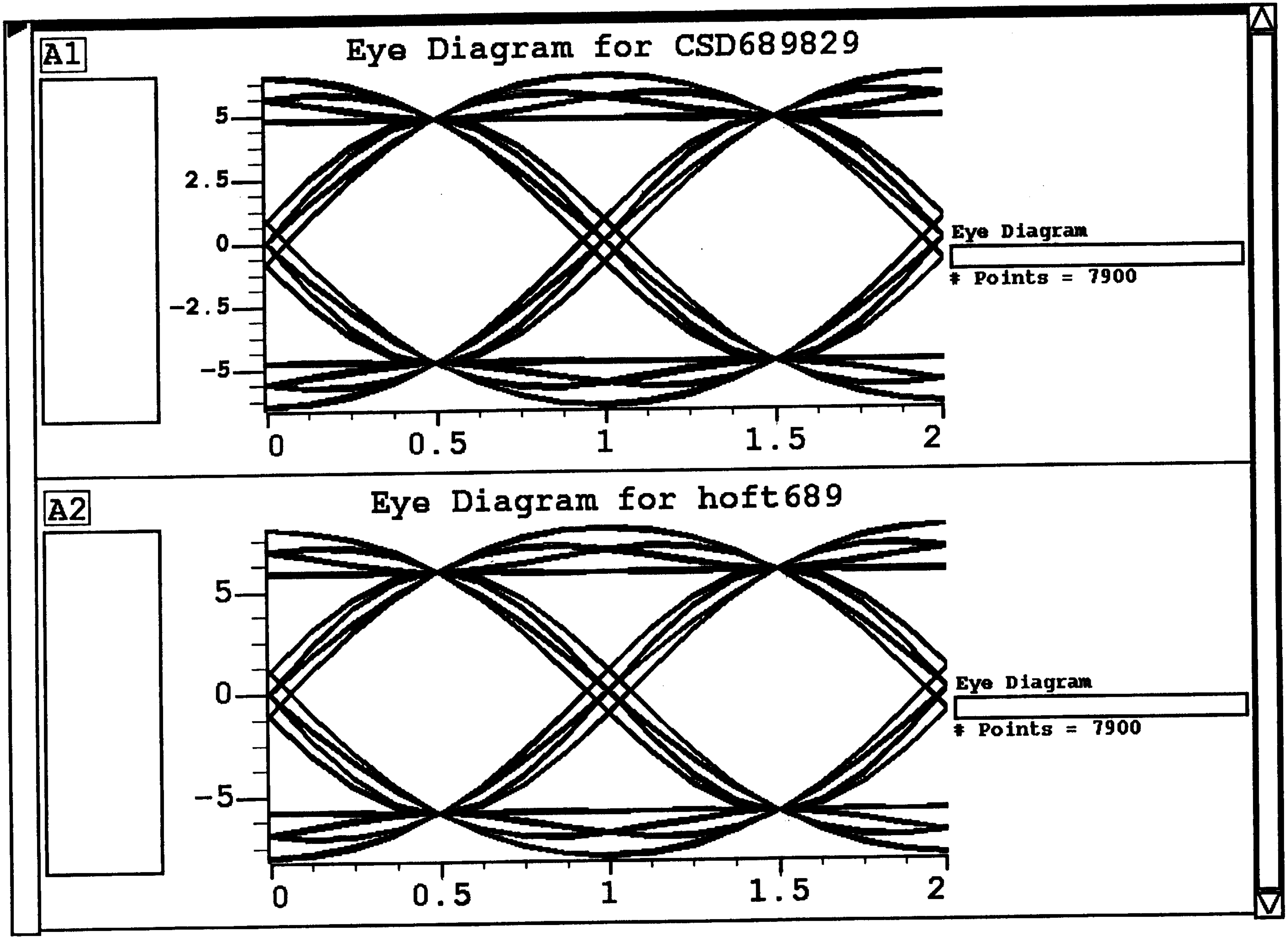

A1
Impulse Response for CSD489847
Mag
(dB)
0
-15
-30
-45
-60
-75
-90
-105
-120
-135
0
1
2
3
4
Frequency Response
Point# = 6144
Bin# = 2048
# Pts = 8193
Freq = 2
Mag. = -48.3046
Phase = -3.14159
Phase
(radians)
3.142
-3.142
A2
Impulse Response for hoft489
Mag
(dB)
0
-15
-30
-45
-60
-75
-90
-105
-120
0
1
2
3
4
Frequency Response
Point# = 6144
Bin# = 2048
# Pts = 8193
Freq = 2
Mag. = -74.6873
Phase = -3.14159
Phase
(radians)
3.142
-3.142

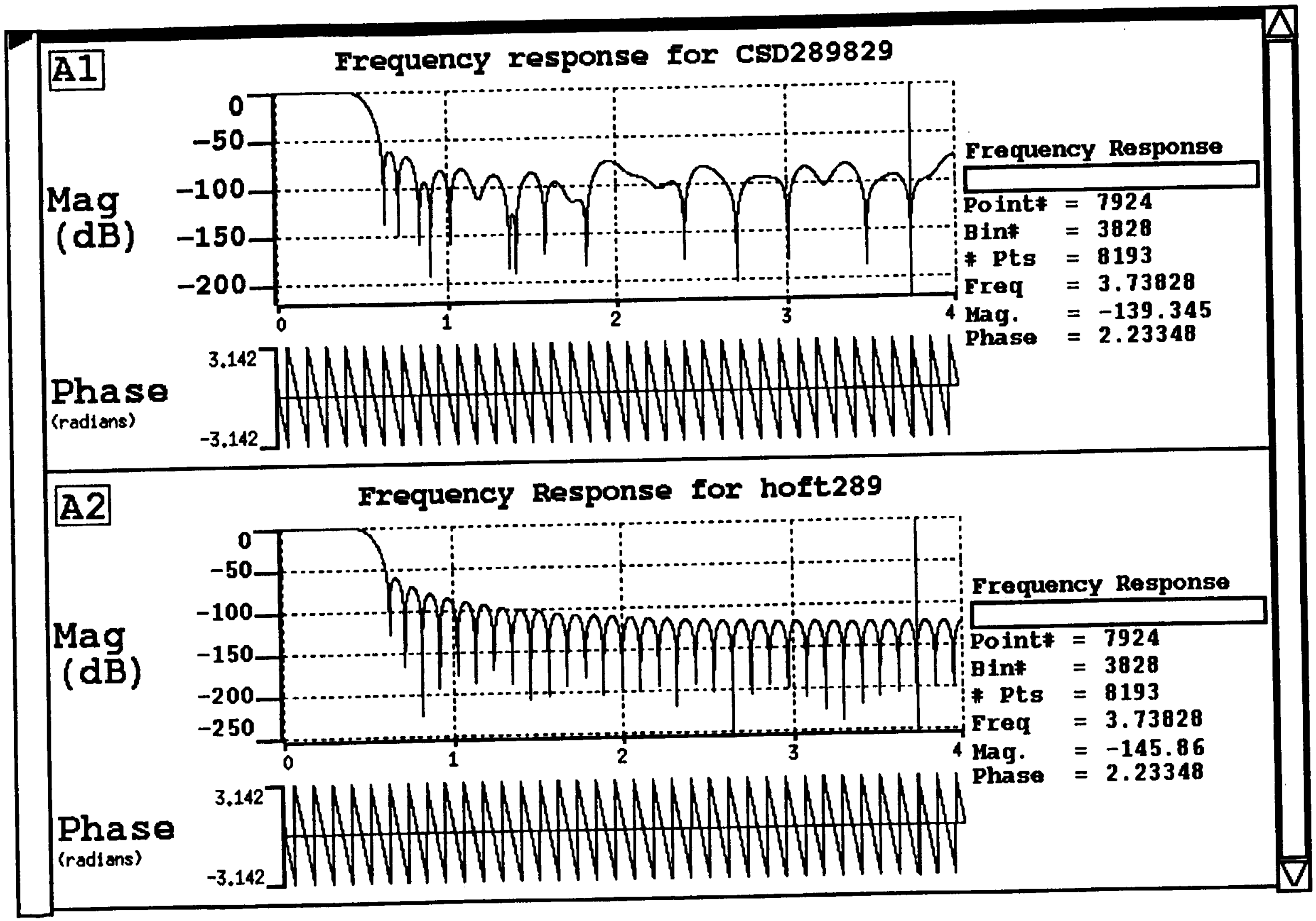

A1
Eye Diagram for CSD489847
5
2.5
0
−2.5
−5
0
0.5
1
1.5
2
Eye Diagram
Points = 7900
A2
Eye Diagram for hoft489
10
5
0
−5
−10
0
0.5
1
1.5
2
Eye Diagram
Points = 7900

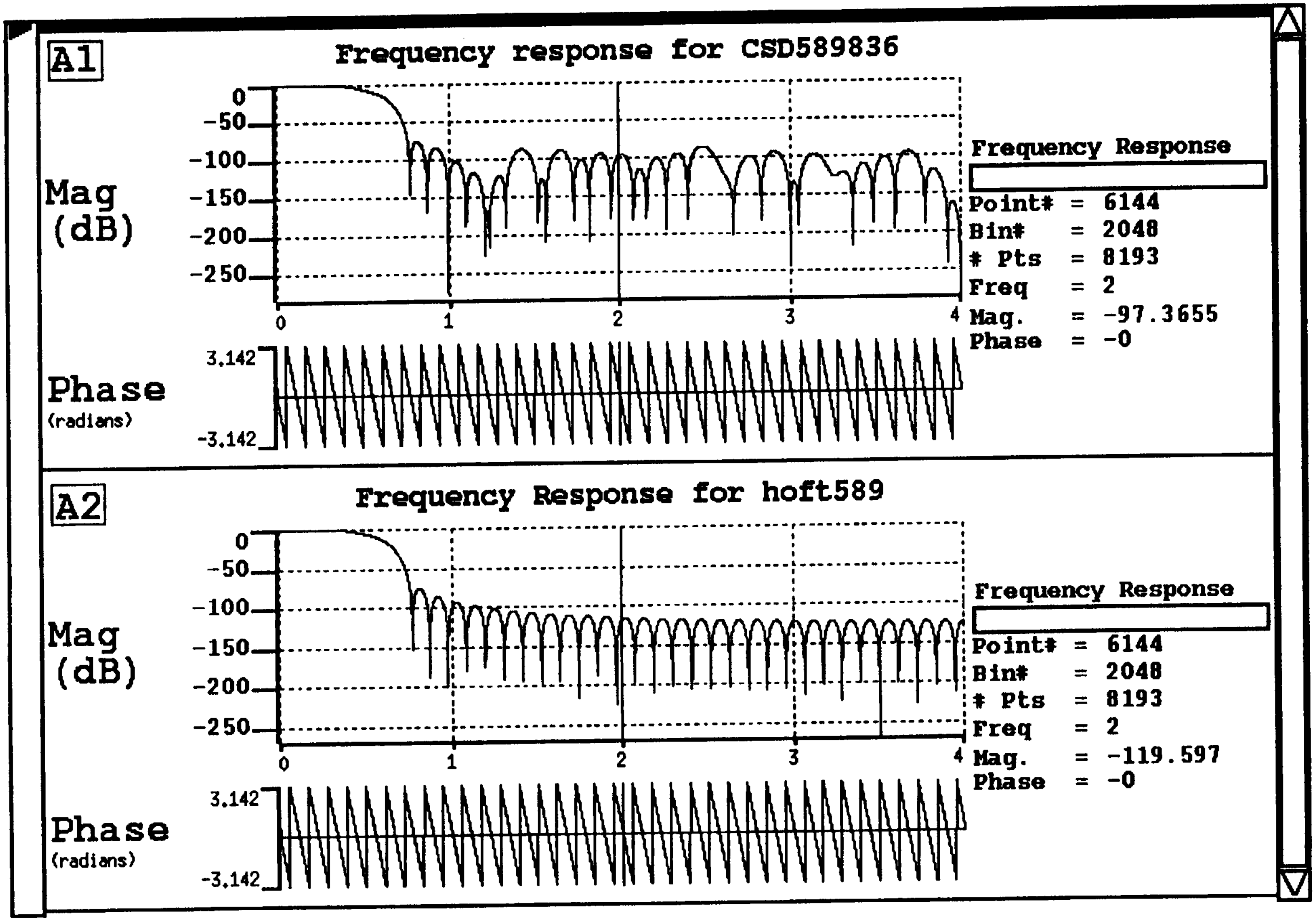

A1
Impulse Response for CSD589836
Mag (dB)
0
-15
-30
-45
-60
-75
-90
-105
-120
-135
0
1
2
3
4
Phase (radians)
3.142
-3.142
Frequency Response
Point# = 6144
Bin# = 2048
# Pts = 8193
Freq = 2
Mag. = -48.6827
Phase = -0
A2
Impulse Response for hoft589
Mag (dB)
0
-15
-30
-45
-60
-75
-90
-105
-120
0
1
2
3
4
Phase (radians)
3.142
-3.142
Frequency Response
Point# = 6144
Bin# = 2048
# Pts = 8193
Freq = 2
Mag. = -59.7987
Phase = -0

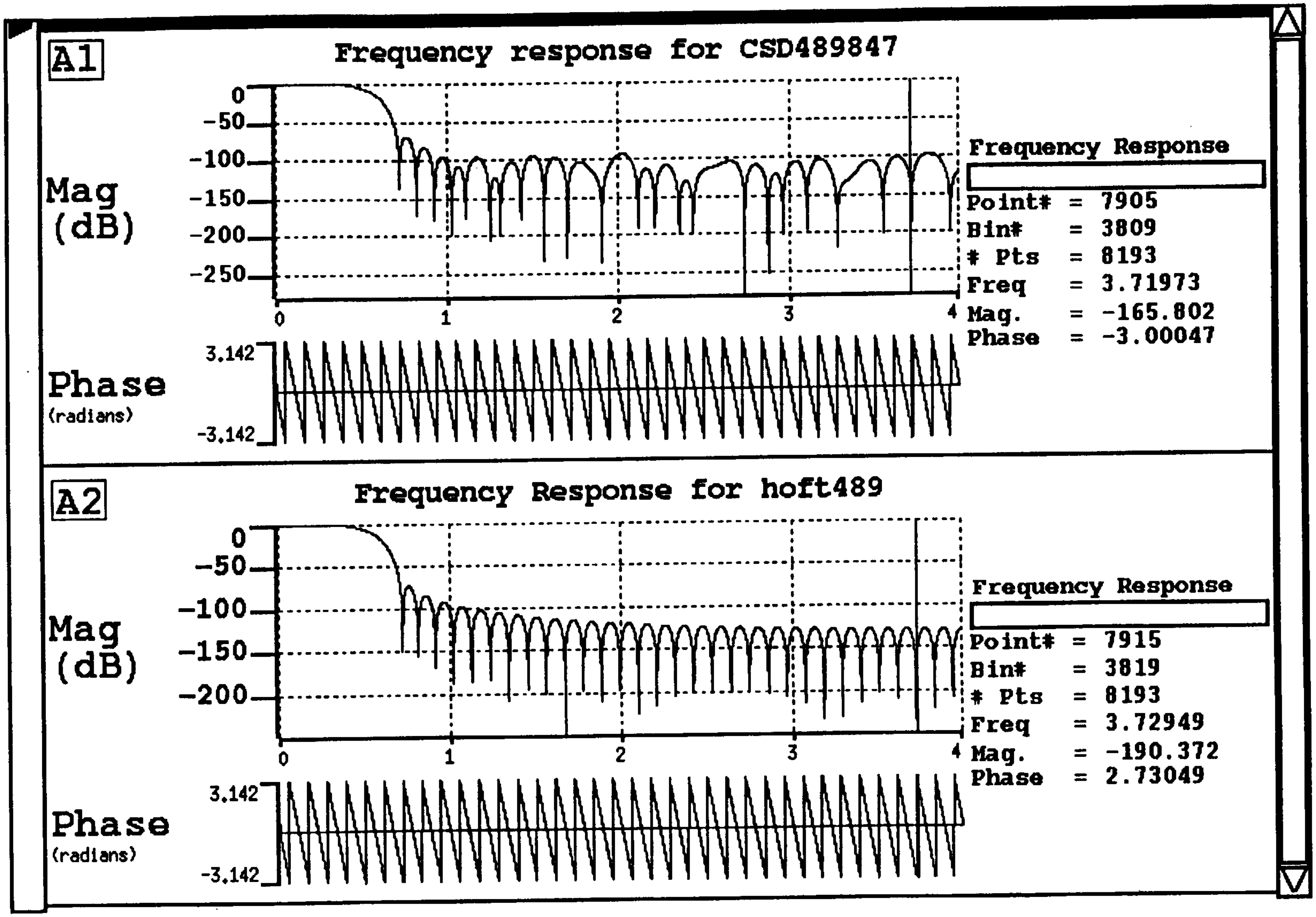

A1
Eye Diagram for CSD589836
2
0
-2
0
0.5
1
1.5
2
Eye Diagram
Points = 7900
A2
Eye Diagram for hoft589
5
0
-5
0
0.5
1
1.5
2
Eye Diagram
Points = 7900

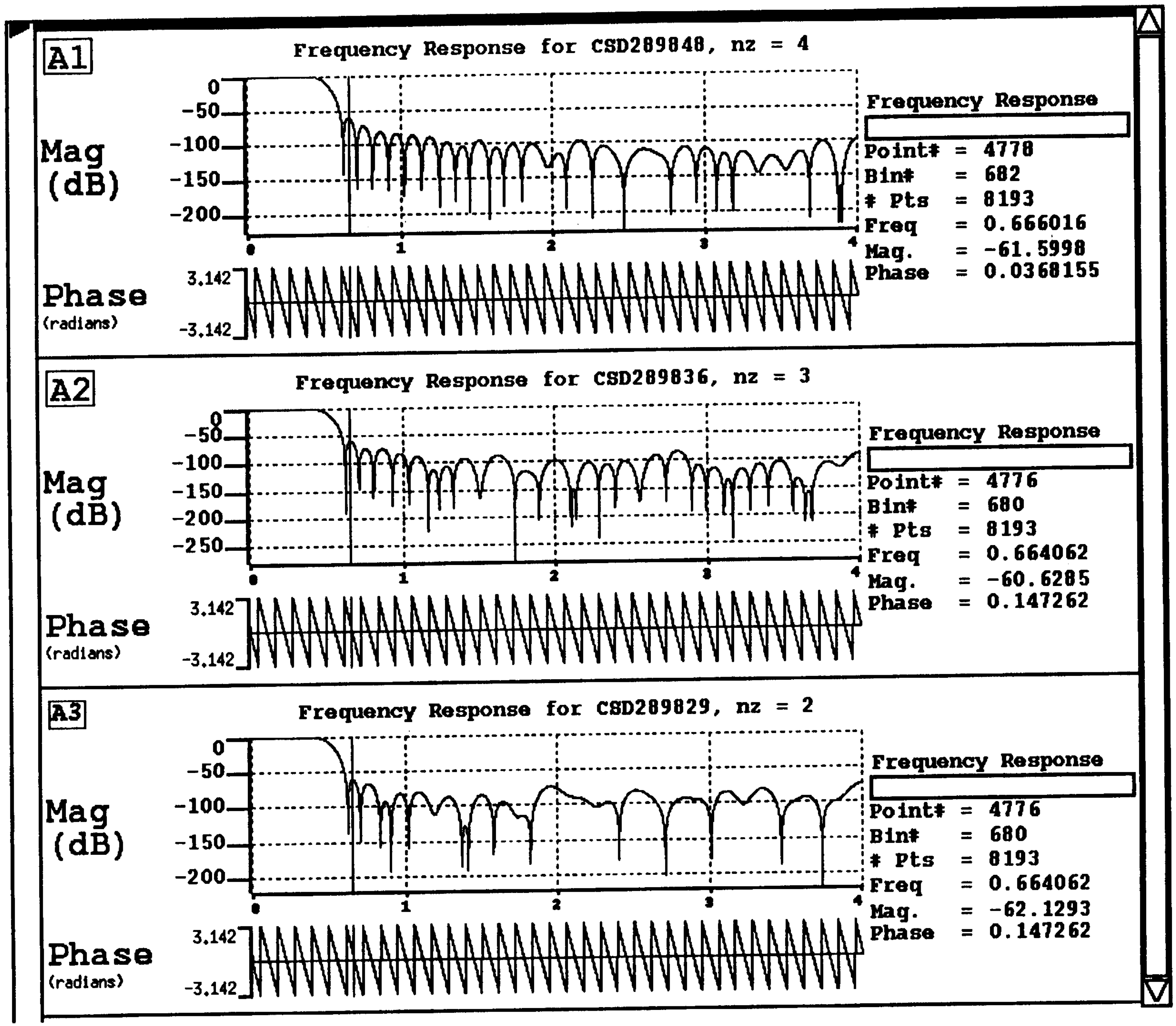

A1
Impulse Response for CSD689829
Mag (dB)
0
-15
-30
-45
-60
-75
-90
-105
-120
0
1
2
3
4
Phase (radians)
3.142
-3.142
Frequency Response
Point# = 6144
Bin# = 2048
# Pts = 8193
Freq = 2
Mag. = -46.0103
Phase = -0
A2
Impulse Response for hoft689
Mag (dB)
0
-50
-100
-150
0
1
2
3
4
Phase (radians)
3.142
-3.142
Frequency Response
Point# = 6145
Bin# = 2049
# Pts = 8193
Freq = 2.00098
Mag. = -70.0278
Phase = -0.0276117


A1
Eye Diagram for CSD689829
5
2.5
0
-2.5
-5
0
0.5
1
1.5
2
Eye Diagram
Points = 7900
A2
Eye Diagram for hoft689
5
0
-5
0
0.5
1
1.5
2
Eye Diagram
Points = 7900

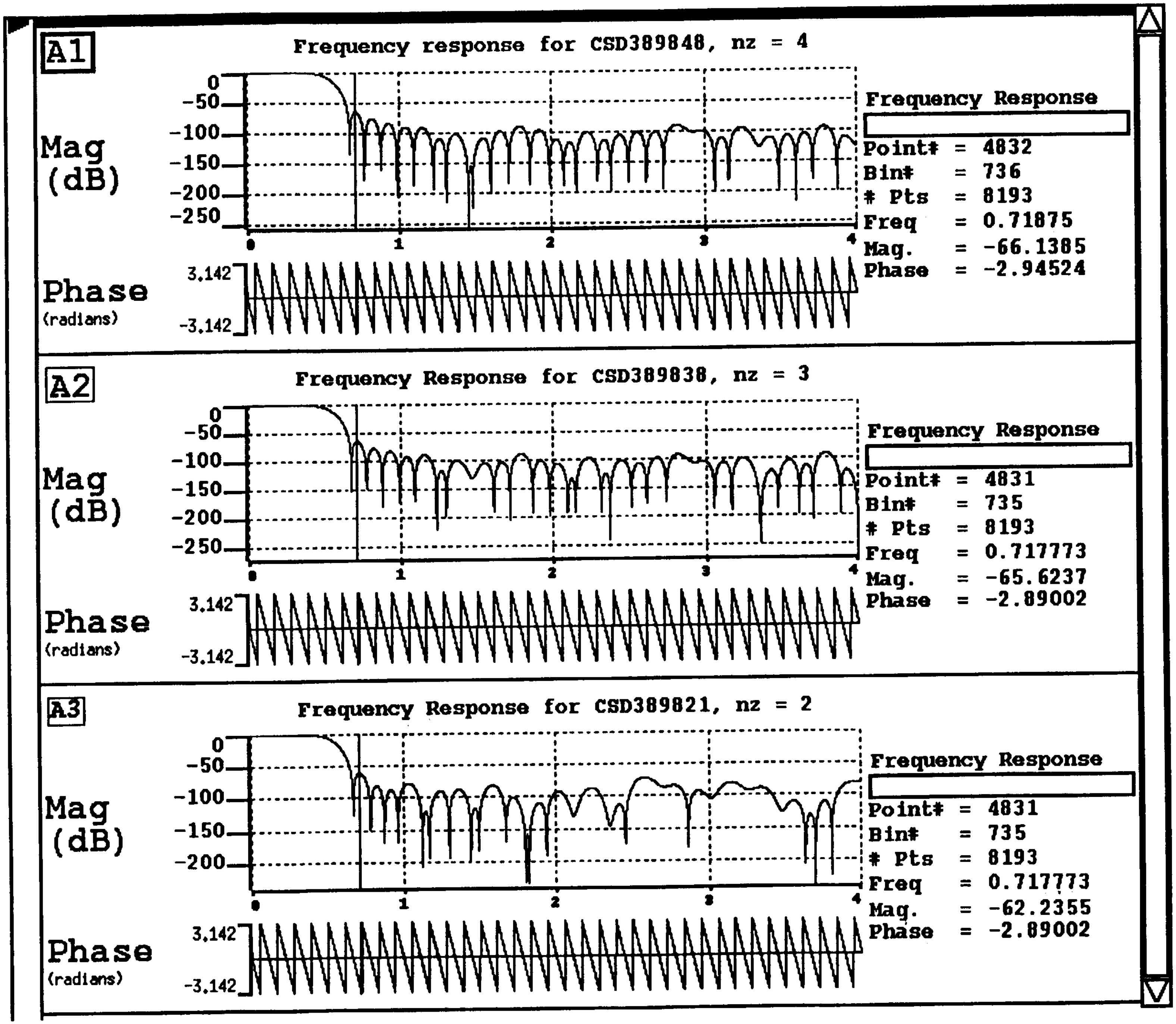

A1
Frequency response for CSD289829
Mag (dB)
0
-50
-100
-150
-200
0
1
2
3
4
Frequency Response
Point# = 7924
Bin# = 3828
# Pts = 8193
Freq = 3.73828
Mag. = -139.345
Phase = 2.23348
Phase (radians)
3.142
-3.142
A2
Frequency Response for hoft289
Mag (dB)
0
-50
-100
-150
-200
-250
0
1
2
3
4
Frequency Response
Point# = 7924
Bin# = 3828
# Pts = 8193
Freq = 3.73828
Mag. = -145.86
Phase = 2.23348
Phase (radians)
3.142
-3.142


A1
Frequency response for CSD589836
Mag (dB)
0
-50
-100
-150
-200
-250
0
1
2
3
4
Phase (radians)
3.142
-3.142
Frequency Response
Point# = 6144
Bin# = 2048
# Pts = 8193
Freq = 2
Mag. = -97.3655
Phase = -0
A2
Frequency Response for hoft589
Mag (dB)
0
-50
-100
-150
-200
-250
0
1
2
3
4
Phase (radians)
3.142
-3.142
Frequency Response
Point# = 6144
Bin# = 2048
# Pts = 8193
Freq = 2
Mag. = -119.597
Phase = -0

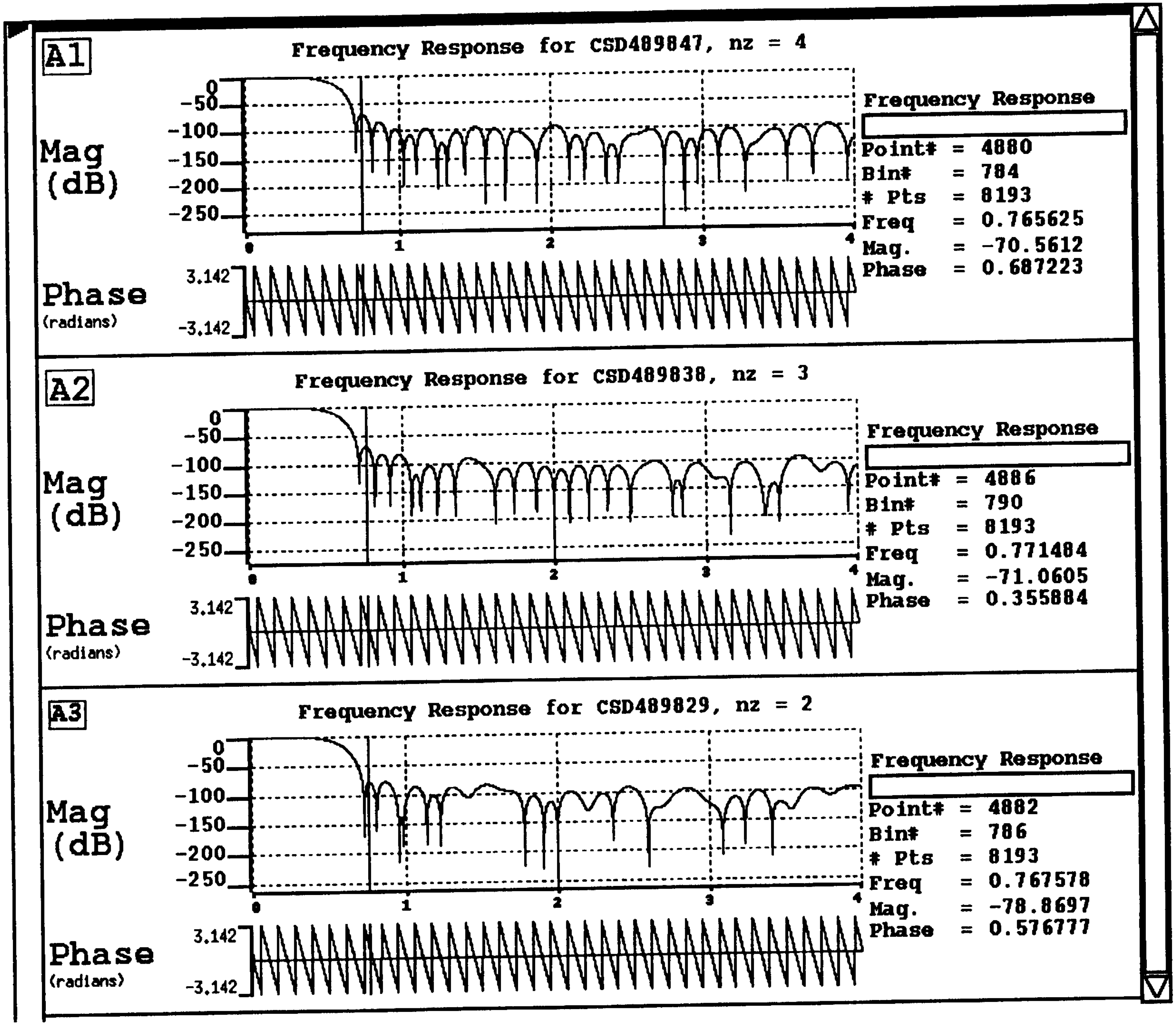

A1
Frequency response for CSD489847
Mag (dB)
0
-50
-100
-150
-200
-250
0
1
2
3
4
Phase (radians)
3.142
-3.142
Frequency Response
Point# = 7905
Bin# = 3809
# Pts = 8193
Freq = 3.71973
Mag. = -165.802
Phase = -3.00047
A2
Frequency Response for hoft489
Mag (dB)
0
-50
-100
-150
-200
0
1
2
3
4
Phase (radians)
3.142
-3.142
Frequency Response
Point# = 7915
Bin# = 3819
# Pts = 8193
Freq = 3.72949
Mag. = -190.372
Phase = 2.73049

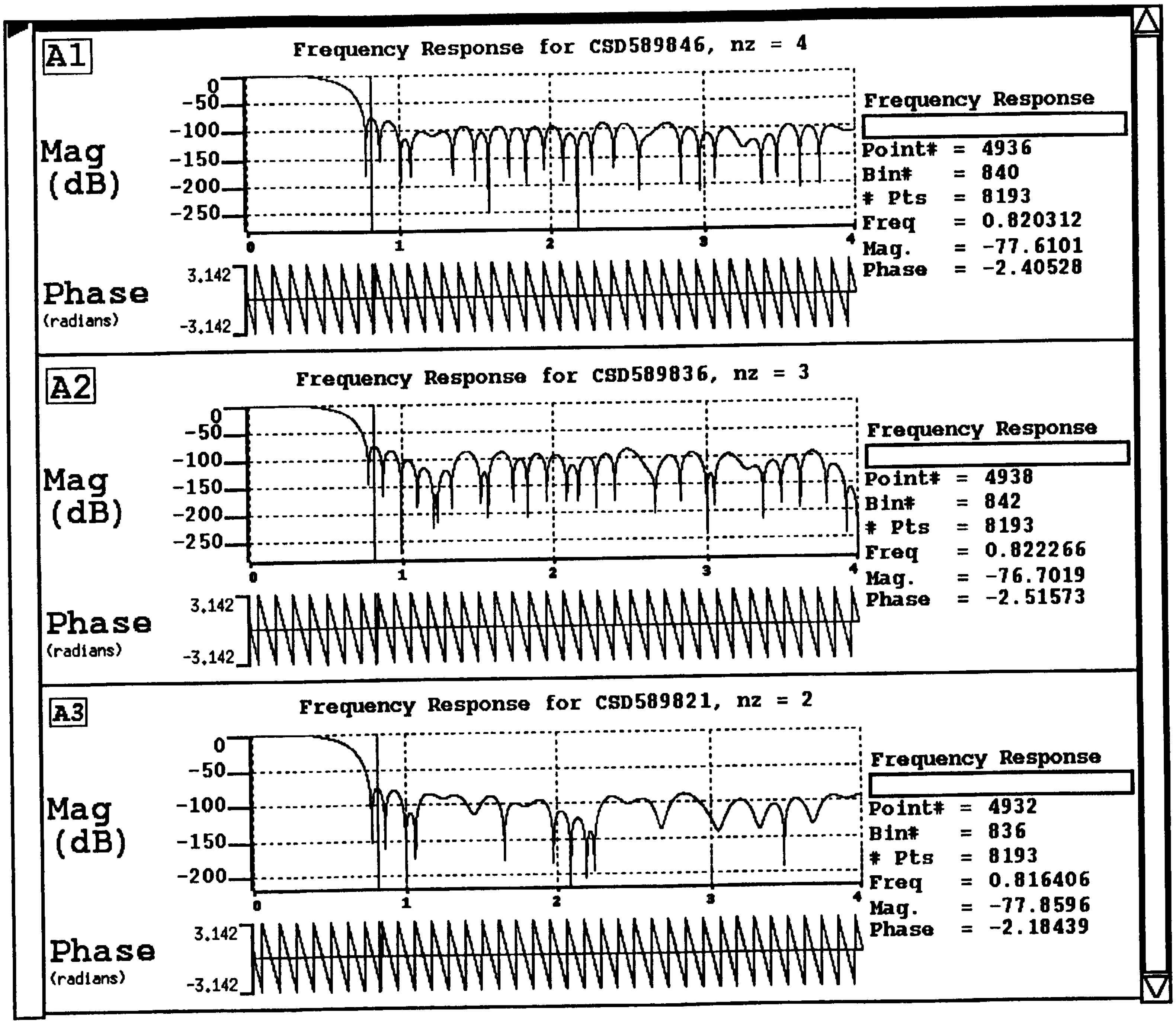


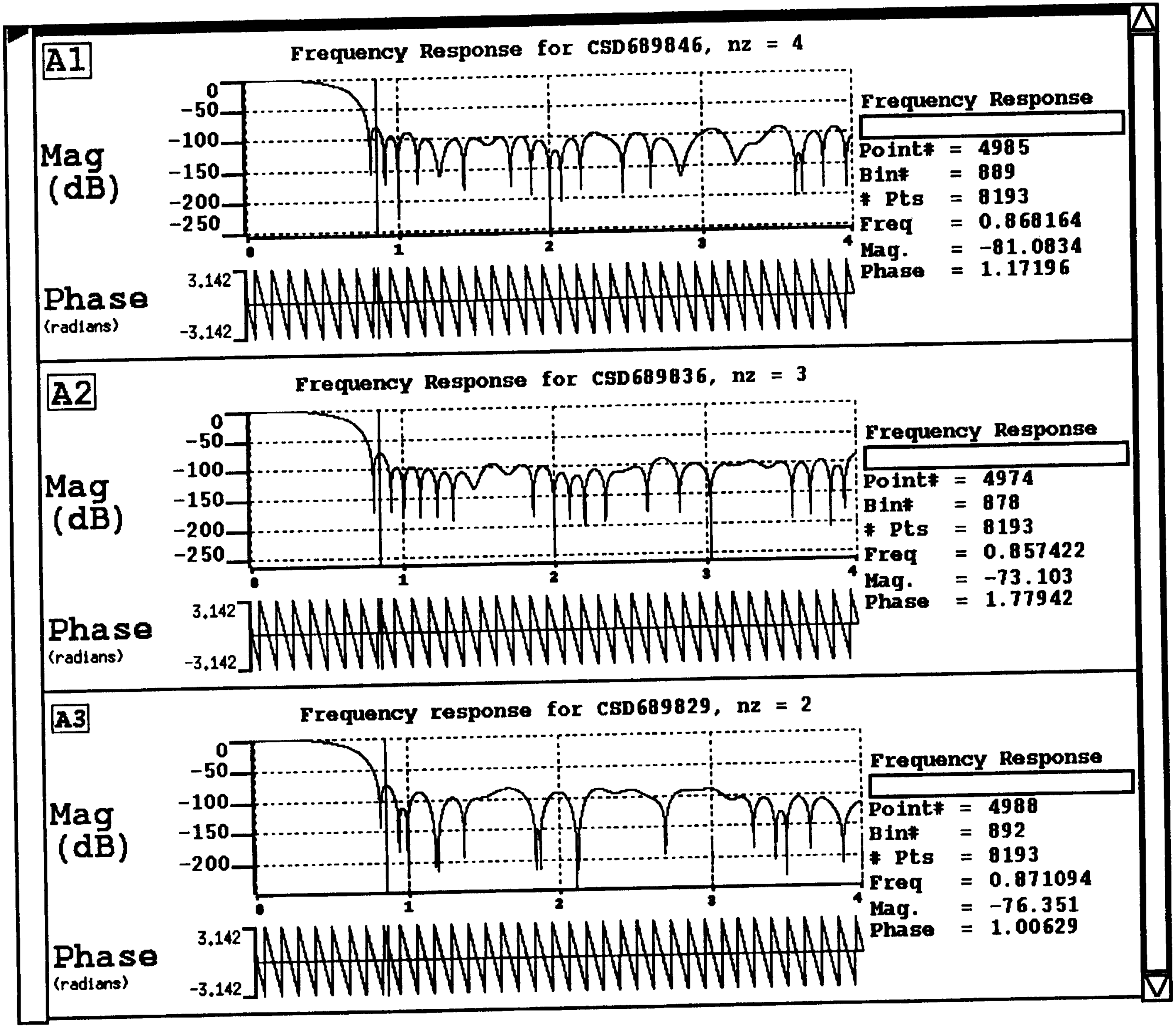
A1
Frequency Response for CSD289848, nz = 4
Mag (dB)
0
-50
-100
-150
-200
0
1
2
3
4
Phase (radians)
3.142
-3.142
Frequency Response
Point# = 4778
Bin# = 682
# Pts = 8193
Freq = 0.666016
Mag. = -61.5998
Phase = 0.0368155
A2
Frequency Response for CSD289836, nz = 3
Mag (dB)
0
-50
-100
-150
-200
-250
0
1
2
3
4
Phase (radians)
3.142
-3.142
Frequency Response
Point# = 4776
Bin# = 680
# Pts = 8193
Freq = 0.664062
Mag. = -60.6285
Phase = 0.147262
A3
Frequency Response for CSD289829, nz = 2
Mag (dB)
0
-50
-100
-150
-200
0
1
2
3
4
Phase (radians)
3.142
-3.142
Frequency Response
Point# = 4776
Bin# = 680
# Pts = 8193
Freq = 0.664062
Mag. = -62.1293
Phase = 0.147262



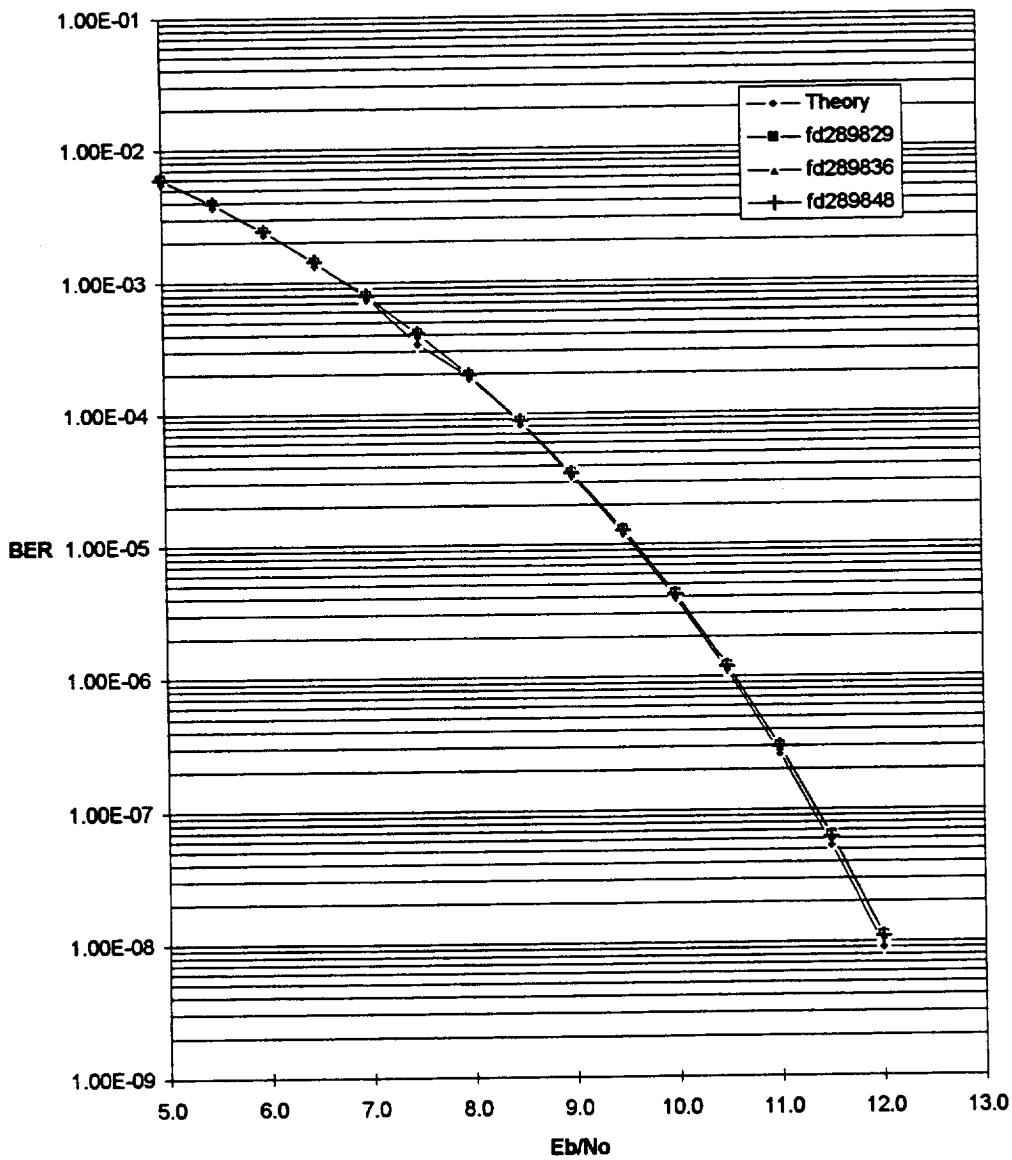
A1
Frequency response for CSD389848, nz = 4
Mag (dB)
0
-50
-100
-150
-200
-250
0
1
2
3
4
Phase (radians)
3.142
-3.142
Frequency Response
Point# = 4832
Bin# = 736
# Pts = 8193
Freq = 0.71875
Mag. = -66.1385
Phase = -2.94524
A2
Frequency Response for CSD389838, nz = 3
Mag (dB)
0
-50
-100
-150
-200
-250
0
1
2
3
4
Phase (radians)
3.142
-3.142
Frequency Response
Point# = 4831
Bin# = 735
# Pts = 8193
Freq = 0.717773
Mag. = -65.6237
Phase = -2.89002
A3
Frequency Response for CSD389821, nz = 2
Mag (dB)
0
-50
-100
-150
-200
0
1
2
3
4
Phase (radians)
3.142
-3.142
Frequency Response
Point# = 4831
Bin# = 735
# Pts = 8193
Freq = 0.717773
Mag. = -62.2355
Phase = -2.89002


A1
Frequency Response for CSD489847, nz = 4
Mag (dB)
0
-50
-100
-150
-200
-250
0
1
2
3
4
Phase (radians)
3.142
-3.142
Frequency Response
Point# = 4880
Bin# = 784
# Pts = 8193
Freq = 0.765625
Mag. = -70.5612
Phase = 0.687223
A2
Frequency Response for CSD489838, nz = 3
Mag (dB)
0
-50
-100
-150
-200
-250
0
1
2
3
4
Phase (radians)
3.142
-3.142
Frequency Response
Point# = 4886
Bin# = 790
# Pts = 8193
Freq = 0.771484
Mag. = -71.0605
Phase = 0.355884
A3
Frequency Response for CSD489829, nz = 2
Mag (dB)
0
-50
-100
-150
-200
-250
0
1
2
3
4
Phase (radians)
3.142
-3.142
Frequency Response
Point# = 4882
Bin# = 786
# Pts = 8193
Freq = 0.767578
Mag. = -78.8697
Phase = 0.576777

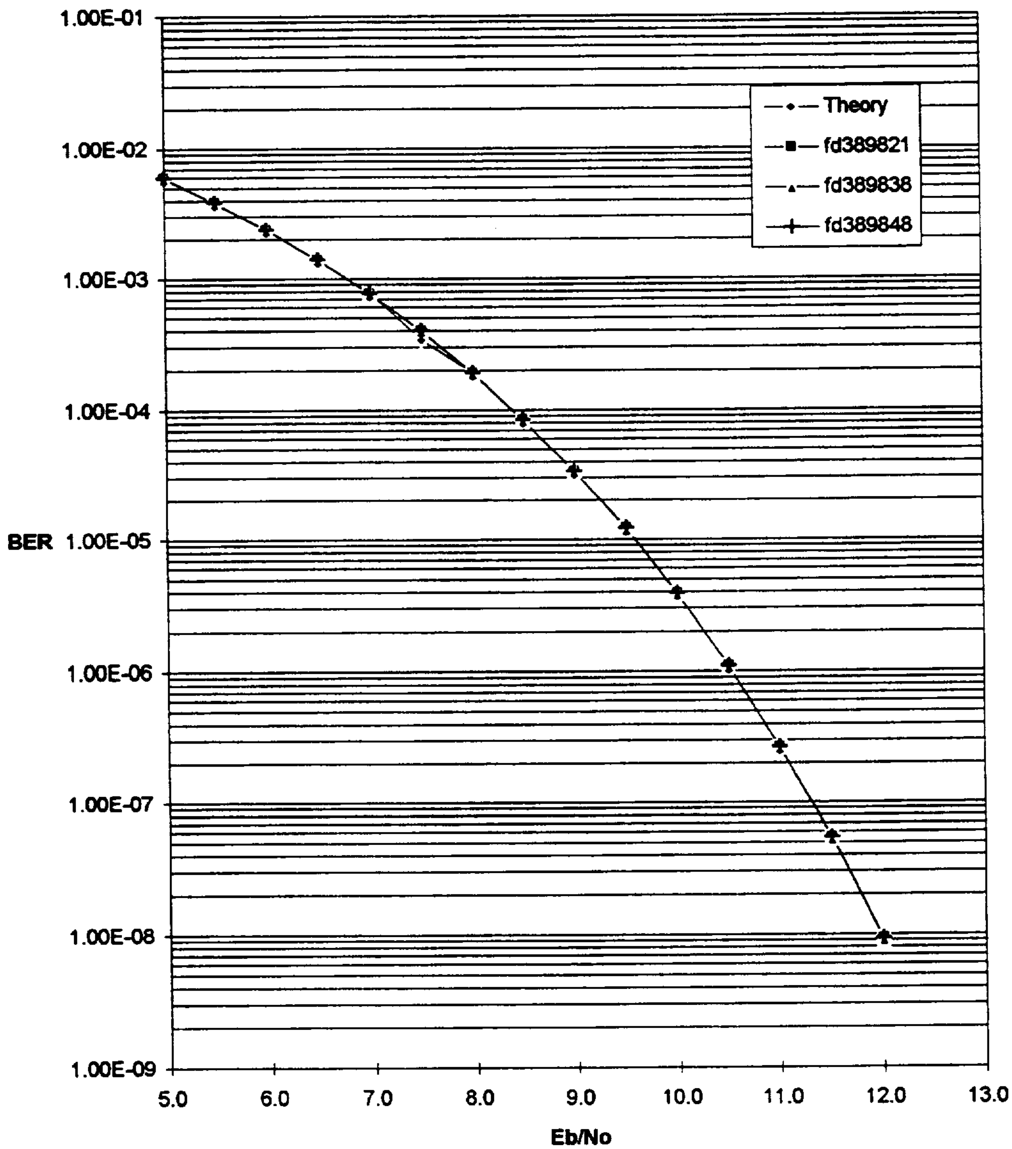

A1
Frequency Response for CSD589846, nz = 4
Mag (dB)
0
-50
-100
-150
-200
-250
0
1
2
3
4
Phase (radians)
3.142
-3.142
Frequency Response
Point# = 4936
Bin# = 840
# Pts = 8193
Freq = 0.820312
Mag. = -77.6101
Phase = -2.40528
A2
Frequency Response for CSD589836, nz = 3
Mag (dB)
0
-50
-100
-150
-200
-250
0
1
2
3
4
Phase (radians)
3.142
-3.142
Frequency Response
Point# = 4938
Bin# = 842
# Pts = 8193
Freq = 0.822266
Mag. = -76.7019
Phase = -2.51573
A3
Frequency Response for CSD589821, nz = 2
Mag (dB)
0
-50
-100
-150
-200
0
1
2
3
4
Phase (radians)
3.142
-3.142
Frequency Response
Point# = 4932
Bin# = 836
# Pts = 8193
Freq = 0.816406
Mag. = -77.8596
Phase = -2.18439


A1
Frequency Response for CSD689846, nz = 4
Mag (dB)
Phase (radians)
0
-50
-100
-150
-200
-250
3.142
-3.142
1
2
3
4
Frequency Response
Point# = 4985
Bin# = 889
# Pts = 8193
Freq = 0.868164
Mag. = -81.0834
Phase = 1.17196
A2
Frequency Response for CSD689836, nz = 3
Mag (dB)
Phase (radians)
Frequency Response
Point# = 4974
Bin# = 878
# Pts = 8193
Freq = 0.857422
Mag. = -73.103
Phase = 1.77942
A3
Frequency response for CSD689829, nz = 2
Mag (dB)
Phase (radians)
Frequency Response
Point# = 4988
Bin# = 892
# Pts = 8193
Freq = 0.871094
Mag. = -76.351
Phase = 1.00629

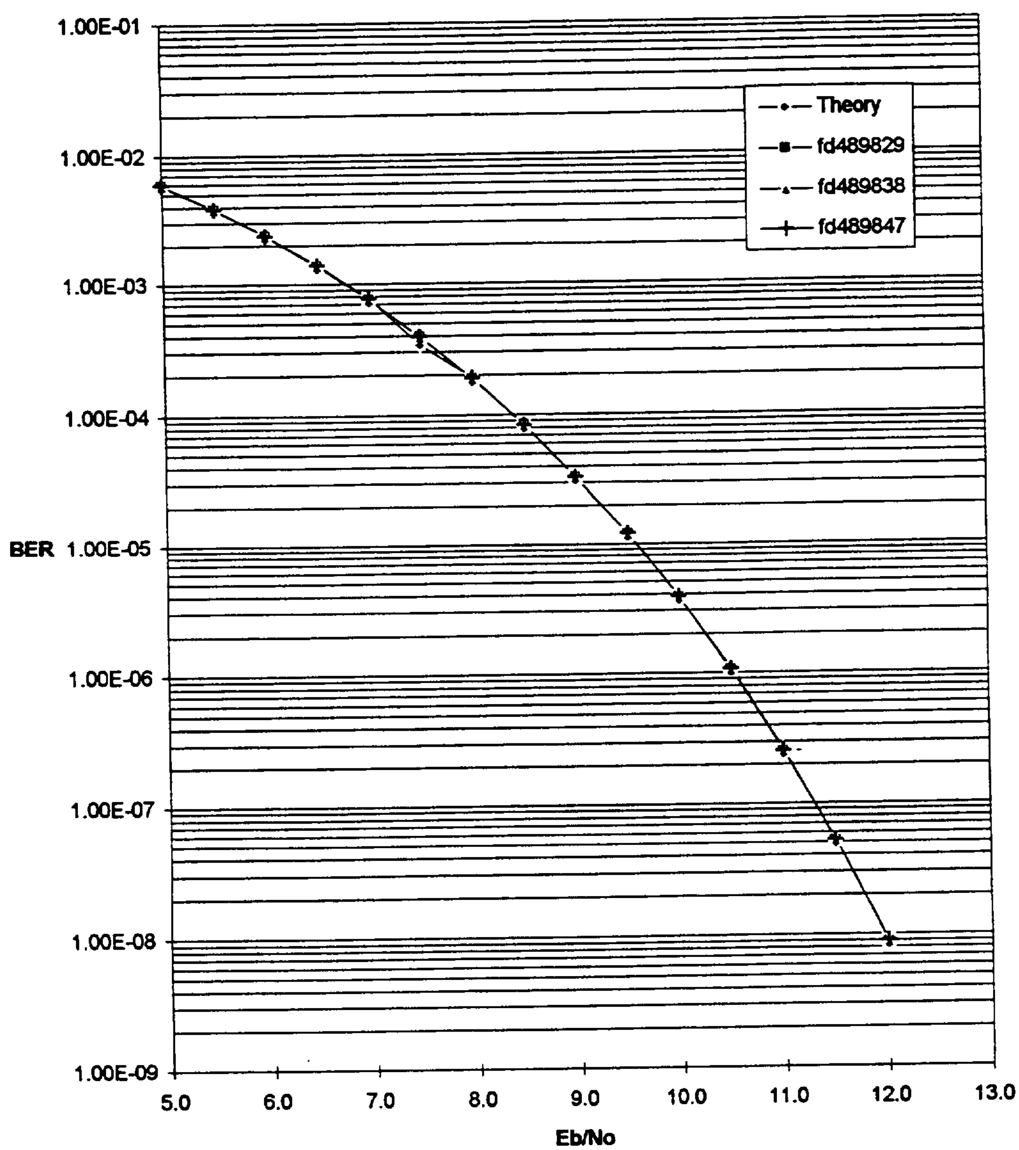
Simulated CSD Filter BER Curves

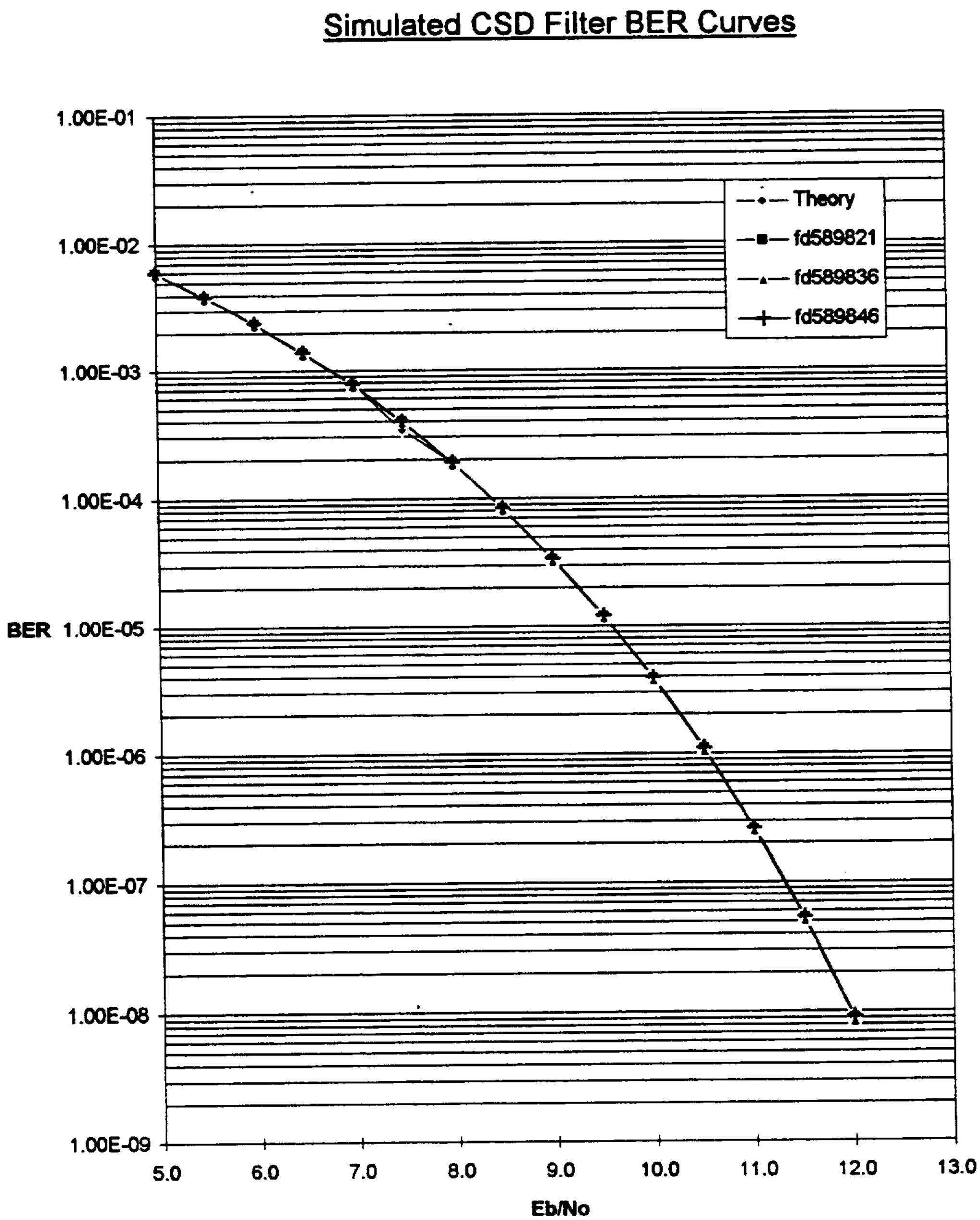


Simulated CSD Filter BER Curves

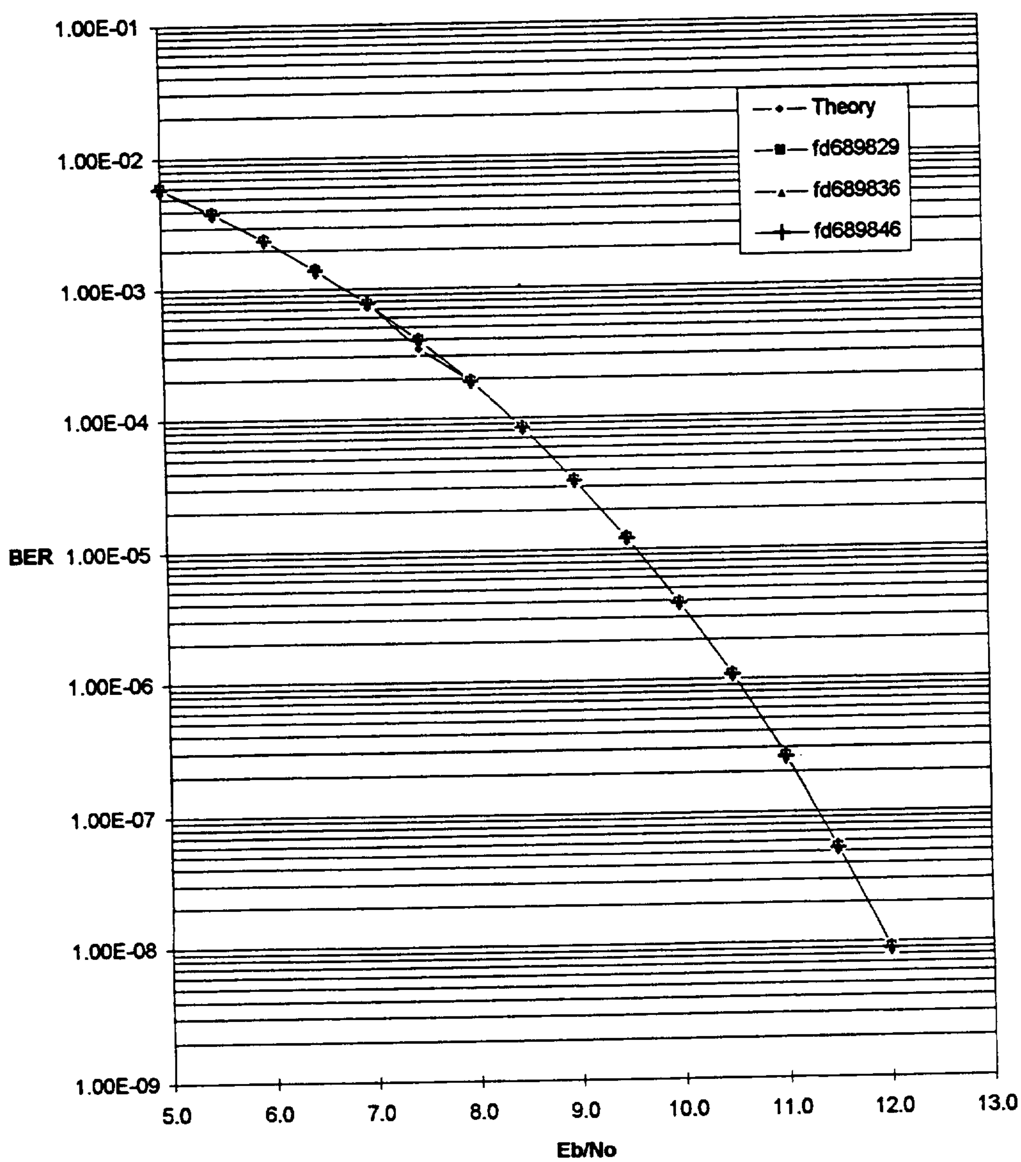


Simulated CSD Filter BER Curves




Simulated CSD Filter BER Curves
BER
1.00E-01
1.00E-02
1.00E-03
1.00E-04
1.00E-05
1.00E-06
1.00E-07
1.00E-08
1.00E-09
5.0
6.0
7.0
8.0
9.0
10.0
11.0
12.0
13.0
Eb/No
Theory
fd589821
fd589836
fd589846

Simulated CSD Filter BER Curves



❑ Hardware Complexity Comparison

☞ example 1 CSD coefficients for 73-tap FIR 289829 filter

$h(0) = 2^{-6} - 2^{-8}$
$h(1) = 2^{-6} + 2^{-8}$
$h(2) = 2^{-5} - 2^{-8}$
$h(3) = 2^{-5} - 2^{-8}$
$h(4) = 2^{-5} - 2^{-7}$
$h(5) = 2^{-6}$
$h(6) = 0$
$h(7) = - 2^{-6}$
$h(8) = - 2^{-5}$
$h(9) = - 2^{-4} + 2^{-6}$
$h(10) = - 2^{-4} + 2^{-6}$
$h(11) = - 2^{-4} + 2^{-6}$

$h(12) = - 2^{-5}$
$h(13) = - 2^{-7}$
$h(14) = 2^{-6} + 2^{-8}$
$h(15) = 2^{-4} - 2^{-6}$
$h(16) = 2^{-4} + 2^{-7}$
$h(17) = 2^{-3} - 2^{-5}$
$h(18) = 2^{-3} - 2^{-5}$
$h(19) = 2^{-4} + 2^{-7}$
$h(20) = 2^{-5} + 2^{-7}$
$h(21) = - 2^{-7}$
$h(22) = - 2^{-4}$
$h(23) = - 2^{-3} + 2^{-7}$

$h(24) = - 2^{-3} - 2^{-5}$
$h(25) = - 2^{-2} + 2^{-4}$
$h(26) = - 2^{-3} - 2^{-6}$
$h(27) = - 2^{-3}$
$h(28) = - 2^{-4} + 2^{-6}$
$h(29) = 2^{-4} + 2^{-7}$
$h(30) = 2^{-2} - 2^{-5}$
$h(31) = 2^{-1} - 2^{-3}$
$h(32) = 2^{-1} + 2^{-5} + 2^{-8}$
$h(33) = 2^{0} - 2^{-3} - 2^{-5}$
$h(34) = 2^{0} - 2^{-3} + 2^{-5}$
$h(35) = 2^{0} - 2^{-4}$
$h(36) = 2^{0} - 2^{-4} + 2^{-6}$

Hardware: for one multiplier, 8 x 37 = 296 adders (On average) - 0.86 adders/tap

	Conventional	CSD Implemented	Difference	Ratio
Adders	72	32 + 72 = 104	264	1 : 3.54
Multipliers	37	0	**	**
Delays	72	72	0	1 : 1

☞ example 2 CSD coefficients for 73-tap FIR 289836 filter

$h(0) = 2^{-7}$
$h(1) = 2^{-6} - 2^{-8}$
$h(2) = 2^{-6}$
$h(3) = 2^{-6} + 2^{-8}$
$h(4) = 2^{-6}$
$h(5) = 2^{-7}$
$h(6) = 0$
$h(7) = -2^{-6} + 2^{-8}$
$h(8) = -2^{-5} + 2^{-7}$
$h(9) = -2^{-5}$
$h(10) = -2^{-5} - 2^{-8}$
$h(11) = -2^{-5}$

$h(12) = -2^{-6} - 2^{-8}$
$h(13) = -2^{-7}$
$h(14) = 2^{-6} - 2^{-8}$
$h(15) = 2^{-6}$
$h(16) = 2^{-4} - 2^{6}$
$h(17) = 2^{-4} - 2^{-8}$
$h(18) = 2^{-4} - 2^{-8}$
$h(19) = 2^{-4} - 2^{-6}$
$h(20) = 2^{-5} - 2^{-8}$
$h(21) = -2^{-8}$
$h(22) = -2^{-4} + 2^{-6} + 2^{-8}$
$h(23) = -2^{-4} - 2^{-6}$

$h(24) = -2^{-3} + 2^{-6} + 2^{-8}$
$h(25) = -2^{-3} + 2^{-8}$
$h(26) = -2^{-3} + 2^{-6} - 2^{-8}$
$h(27) = -2^{-3} + 2^{-5} + 2^{-7}$
$h(28) = -2^{-5}$
$h(29) = 2^{-4} - 2^{-6}$
$h(30) = 2^{-3} + 2^{-6} + 2^{-8}$
$h(31) = 2^{-2}$
$h(32) = 2^{-1} - 2^{-3} - 2^{-6}$
$h(33) = 2^{-1} - 2^{-4} + 2^{-6}$
$h(34) = 2^{-1} + 2^{-5}$
$h(35) = 2^{-1} + 2^{-4} + 2^{-6} + 2^{-8}$
$h(36) = 2^{-1} + 2^{-3} - 2^{-5} + 2^{-7}$

Hardware: for one multiplier, 8 x 37 = 296 adders

	Conventional	CSD Implemented	Difference	Ratio
Adders	72	36 + 72 = 108	260	1 : 3.4
Multipliers	37	0	**	**
Delays	72	72	0	1 : 1

☞ example 3 CSD coefficients for 73-tap FIR 289848 filter

$h(0) = 2^{-7}$
$h(1) = 2^{-6}$
$h(2) = 2^{-5} - 2^{-7}$
$h(3) = 2^{-5} - 2^{-7}$
$h(4) = 2^{-6} + 2^{-8}$
$h(5) = 2^{-6} - 2^{-8}$
$h(6) = 0$
$h(7) = - 2^{-6}$
$h(8) = - 2^{-5} + 2^{-8}$
$h(9) = - 2^{-5} - 2^{-7}$
$h(10) = - 2^{-4} + 2^{-6} + 2^{-8}$
$h(11) = - 2^{-5} - 2^{-7}$

$h(12) = - 2^{-5} + 2^{-8}$
$h(13) = - 2^{-7}$
$h(14) = 2^{-6}$
$h(15) = 2^{-4} - 2^{-6} - 2^{-8}$
$h(16) = 2^{-4} - 2^{8}$
$h(17) = 2^{-4} + 2^{-6}$
$h(18) = 2^{-4} + 2^{-6}$
$h(19) = 2^{-4} - 2^{-8}$
$h(20) = 2^{-5} + 2^{-8}$
$h(21) = - 2^{-7}$
$h(22) = - 2^{-4} + 2^{-7}$
$h(23) = - 2^{-3} + 2^{-5} - 2^{-7}$

$h(24) = - 2^{-3} - 2^{-6}$
$h(25) = - 2^{-3} - 2^{-5} - 2^{-8}$
$h(26) = - 2^{-3} - 2^{-6} + 2^{-8}$
$h(27) = - 2^{-3} + 2^{-6} - 2^{-8}$
$h(28) = - 2^{-5} - 2^{-7}$
$h(29) = 2^{-4}$
$h(30) = 2^{-2} - 2^{-4} + 2^{-8}$
$h(31) = 2^{-2} + 2^{-4} + 2^{-6} + 2^{-8}$
$h(32) = 2^{-1} - 2^{-5} + 2^{-7}$
$h(33) = 2^{-1} + 2^{-3} - 2^{-6} - 2^{-8}$
$h(34) = 2^{0} - 2^{--3} - 2^{-6} - 2^{-8}$
$h(35) = 2^{0} - 2^{-3} + 2^{-6} - 2^{-9}$
$h(36) = 2^{0} - 2^{-3} + 2^{-5} - 2^{-7} + 2^{-9}$

Hardware: for one multiplier, 8 x 37 = 296 adders

	Conventional	CSD Implemented	Difference	Ratio
Adders	72	48 + 72 = 120	248	1 : 3.1
Multipliers	37	0	**	**
Delays	72	72	0	1 : 1

Conclusions

- For CSD implemented root-raised cosine filter with $\beta = 0.2, 0.3, 0.4$ and nonzero = 2, 3, 4 and 72 taps shows that all works nearly as well and the same as ideal one - aperture bounded
- Frequency response shows -30 dB or better stopband for all cases
- Sidelobes improve as the number of nonzero digits increases - also aperture bounded
- Eye diagrams look excellent and no timing jitters at the sampling instance
- Bit-Error-Rate test validates CSD implemented root-raised-cosine filter performs as well as one implemented conventionally
- Hardware saving from CSD implementation of filter becomes working model of Root-Raised-Cosine filter more dominant and powerful with less tap size filters

Future work

- FPGA implementation of CSD filter
- Other optimization criteria, i.e. BER and ACI
- Other search methods, i.e. # of taps

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13. ABSTRACT ***(Maximum 200 words)***

Finite Impulse Response digital filter using Canonical Signed-Digit (CSD) number representation for the coefficients has been studied and its computer simulation results are presented here. Minimum Mean Square Error (MMSE) criterion is employed to optimize filter coefficients into the corresponding CSD numbers. To further improve coefficients optimization process, an extra non-zero bit is added for any filter coefficients exceeding 1/2. This technique improves frequency response of filter without increasing filter complexity almost at all. The simulation results shows outstanding performance in bit-error-rate (BER) curve for all CSD implemented digital filters included in this presentation material.

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