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P6.9 PROBABILITY DENSITY FUNCTIONS OF OBSERVED RAINFALL IN MONTANA

Scott D. Larsen* and L. Ronald Johnson and Paul L. Smith

U. S. Department of Agriculture-NRCS
9025 Chevrolet Drive, Suite J
Ellicott City, MD 21042-4093

Institute of Atmospheric Sciences
South Dakota School of Mines and Technology
501 E. St. Joseph Street
Rapid City, SD 57701-3995

1. INTRODUCTION

The determination of convective rainfall volumes independent of rain gage networks is important in areas where such coverage is sparse. A procedure utilizing a relationship between volume and horizontal areal coverage of precipitation would provide a method for estimating rainfall based solely on areal coverage. Doneaud *et al.* (1981) developed one such procedure at the South Dakota School of Mines and Technology. Their work explicitly defined a coverage-based method that eventually came to be known as the Area Time Integral, or ATI. If the average rain rate R is known, the rain volume V follows from

$$V = \bar{R} \iint_{At} dA\,dt \quad , \qquad (1)$$

where the ATI is the double integral over both area A and time t:

$$ATI = \iint_{At} dA\,dt \quad . \qquad (2)$$

The ratio of Radar Estimated Rain Volume (RERV) to the ATI gives the average rain rate. There is no need to determine the structure for the area of precipitation activity, provided an average rain rate is known (Doneaud *et al.*, 1984). The area of integration can be specified with respect to the precipitation itself ("floating target") or a fixed surface area (Doneaud *et al.*, 1984). Doneaud *et al.* (1981) initially presented the fixed surface integration, while Johnson *et al.* (1994) examined the floating target integration.

The ATI depends, in theory, on a combination of a deterministic and statistical distribution of rain rate over area (Atlas *et al.*, 1990). The rain rate probability density function (PDF) is believed to be unimportant as long as the distribution remains constant -- either for all rain events or for a uniform variation between events. The exact PDF has never been explicitly presented, although studies have shown a log-normal distribution (Doneaud *et al.*, 1984; Atlas *et al.* 1990; Sauvageot, 1994) and modified gamma distribution (Doneaud *et al.*, 1979) provide reasonable rain rate distribution fits.

In the ATI sense, a universal PDF (one that fits all cases) would show a relationship between rain rate and area-time coverage. For a low ATI value, the probability of heavy rain would be small. An increase in the probability would be supported by a corresponding increase in area and/or duration of the event (Larsen, 1994).

This paper examines the question of whether a rain rate PDF can vary uniformly between precipitation events (hereafter called "storms" with the understanding that precipitation is falling although it may not be severe). Image analysis on large samples of radar echoes is possible because of advances in technology. The data provided by such an analysis easily allow development of radar reflectivity factor (and by extension rain rate) distributions. Finding a PDF becomes a matter of finding a function that describes the curve approximating the resulting distributions. Ideally, one PDF would exist for all cases; or many PDF's that have the same functional form with only systematic variations in parameters (such as size or shape) exist. Satisfying either of these cases will, according to Atlas *et al.* (1990) validate the theoretical basis of the ATI.

2. ANALYSIS

The floating target analysis requires following a storm through its entire lifetime. This was not practical in the past because the storm lifetime could last several hours and contain thousands of reflectivity factor values. With faster digital equipment, true image processing has become a reality and the

**Corresponding author address*: Scott D. Larsen, U.S. Dept. Of Agriculture-NRCS, 9025 Chevrolet Drive-Suite J, Ellicott City, MD 21042-4093.

capability to easily study storm lifetime distributions now exists.

The Interactive Radar Analysis Software running on Sun workstations (Sun-IRAS) (Priegnitz and Hjelmfelt, 1993) was used to generate the storm distributions. The Sun-IRAS echo area statistics option allows distribution function analysis. The Sun-IRAS user selects a radar echo on a Plan Position Indicator (PPI) display and outlines the area to be analyzed. Sun-IRAS then analyzes the pixels contained in the enclosed area and determines echo region properties such as area above a reflectivity threshold, maximum and mean reflectivities, and corresponding maximum and mean rain rates. A frequency distribution is generated for each echo region showing the number of pixels for a particular range of reflectivity factor. A sample echo-area statistics display is shown in Fig. 1.

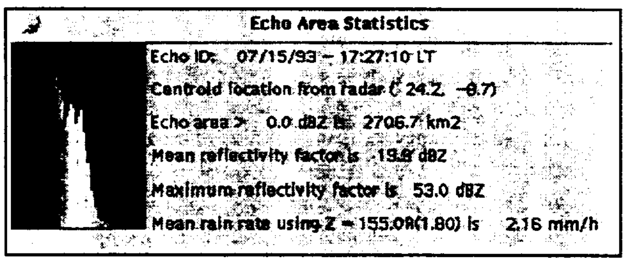

Fig. 1. Sun-IRAS Echo-Area Statistics Display.

The total number of scans analyzed depends on the length of the storm. Once all scans had been analyzed, the time dependency was removed. This involved totaling the number of pixels at a given reflectivity factor value throughout the storm duration and dividing by the total number of pixels encountered throughout the storm's life. The result was a lifetime frequency distribution of reflectivity factor values.

The data for this research came from the CCOPE experiment of 1981 (Knight, 1982). Radar data from the Skywater 5-cm radar located near Miles City, MT, was used because it produced the full-volume scans necessary for studying the storms through their lifetime (Johnson and Hjelmfelt, 1990).

Storms selected for this study had to be easily identified and followed through their lifetime. They were categorized according to duration and seasonal dependence. Duration classes were short (lasting between 0.5 and 2.0 hours), medium (between 2.0 and 4.0 hours), and long (between 4.0 and 8.0 hours). Seasonal classes were roughly two-week intervals throughout the CCOPE experiment: May (18-31 inclusive), June 1-15, June 16-30, July 1-15, July 16-31, and August (1-7 inclusive). A total of 54 storms were analyzed, with an equal number of storms selected for each class (Larsen, 1994).

3. RESULTS

The method-of-moments procedure was used to produce indicators for selecting a mathematical relationship to fit the distributions. Curve selection criteria based on moment generated parameters (Elderton, 1953) were then applied to determine the form of the PDF equation. A Pearson Type I curve was found to fit 48 of the 54 selected storms. Since this one equation is indicated as the choice for 89% of the cases, the Type I equation appears to be a very good prospect for a universal function. The term universal is used with some caution since only one season-location was examined (Larsen, 1994).

The shape of a Type I curve ranges from a closed, positively skewed curve to an open, asymptotic curve. The shape is J, U, or twisted-J, depending on shape parameters. Typical Type-I curves are shown in Fig. 2. Duration and seasonal curve shape relationships are shown in Table 1 and 2, respectively.

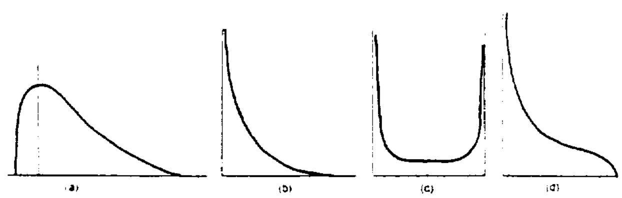

Fig. 2. Typical Pearson Type I curves: a) bell shaped; b) J-shaped; c) U-shaped; d) twisted J-shape.

TABLE 1. Duration-Curve Shape Relationships

	J-Shape	Bell-Shape	U-Shape
Short	14	3	1
Medium	7	9	0
Long	6	9	0

TABLE 2. Seasonal-Curve Shape Relationships

	J-Shape	Bell-Shape	U-Shape
May	5	3	0
1-15 Jun	4	3	0
16-30 Jun	3	6	0
1-15 Jul	6	3	0
16-31 Jul	5	2	1
Aug	3	4	0

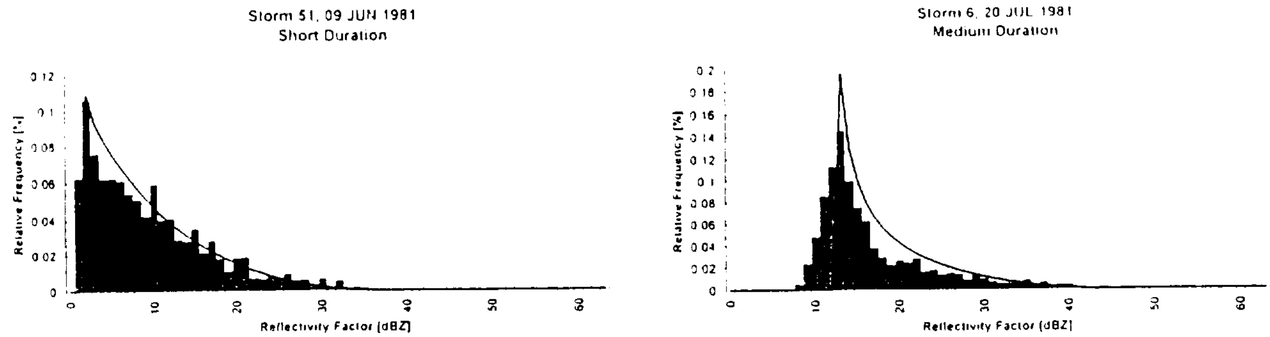

Fig. 3. Type-I curve fit to rain-rate distribution: a) 9 June 1981; b) 20 July 1981.

By inspection, the generql shape of many of the curves is approximated. However, a chi-square goodness-of-fit test indicates the majority of the curves do not provide a fit with 95% confidence. Of the 54 distributions, the chi-square statistic at a 5% level of significance favored rejection of the null hypothesis ("the Type I curve approximates the shape of all the distributions") for 52 cases. One case where the test statistic met the criteria, as well as another example where the criteria was not satisfied, are shown in Fig. 3.

4. CONCLUSION

This paper examined whether a universal PDF of rainfall rate exists, supporting the theoretical basis for the ATI proposed by Atlas *et al.* (1990). Using the method of moments and Elderton's (1953) curve selection criteria, the Pearson Type 1 equation was identified as a potential fit for 89% of the observed distributions. Further analysis indicates that the Type I curve does approximate the shape of the distributions but quantitatively does not produce a great fit (Larsen, 1994).

There is evidence to show that the method-of-moments may not be the most effective estimator of parameters available (Brooks and Carruthers, 1953; Elderton, 1953). With that said, another method may produce better parameters and curve fits. This is a starting point, however, and at the very least, opens the door for further discussion of curve fitting techniques (Larsen, 1994).

Since one type of curve was identified to fit 89% of the storms researched, there does appear to be some type of universal distribution present. Whether the distribution can be applied to all seasons and climates, or simply to the data of the CCOPE project is subject for further study.

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