Optical correlation of images with signal-dependent noise using constrained-modulation filter devices

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Images with signal-dependent noise present challenges beyond those of images with additive white or colored signal-independent noise in terms of designing the optimal 4-f correlation filter that maximizes correlation-peak signal-to-noise ratio, or combinations of correlation-peak metrics. Determining the proper design becomes more difficult when the filter is to be implemented on a constrained-modulation spatial light modulator device. The design issues involved for updatable optical filters for images with signal-dependent film-grain noise and speckle noise are examined. It is shown that although design of the optimal linear filter in the Fourier domain is impossible for images with signal-dependent noise, proper nonlinear preprocessing of the images allows the application of previously developed design rules for optimal filters to be implemented on constrained-modulation devices. Thus the nonlinear preprocessing becomes necessary for correlation in optical systems with current spatial light modulator technology. These results are illustrated with computer simulations of images with signal-dependent noise correlated with binary-phase-only filters and ternary-phase-amplitude filters.

1. Introduction

It is well known that many optical image-noise sources are signal-dependent in nature.[1,2] However, much of the work concerned with the correlation of noisy images and the design of filters for implementation in optical correlators has generally assumed that any image noise is additive and signal independent. Recently there has been some interest shown in the correlation of images with signal-dependent noise (SDN) for pattern detection and recognition purposes. For example, Morris has analyzed the correlation process for Poisson point process models and shown the ability to make efficient use of a small number of photons to make accurate classification decisions.[3,4] In other recent work, Javidi *et al.* have investigated the situation of a target object located among disjoint noise and determined the optimum receiver for pattern recognition based on maximum likelihood techniques.[5] Recently, Downie and Walkup have considered the SDN models associated with the phenomena of film-grain noise and speckle and analyzed the correlation performance of images degraded with these noise types.[6] The measure of performance addressed there was the signal-to-noise ratio (SNR) of the correlation-peak value, and two approaches or filter designs for the processing of images with SDN were studied. The first approach was to design the optimal linear filter to operate on the image with SDN, and the second approach was to preprocess the noisy image nonlinearly to make the noise signal independent in nature, and then correlate the transformed image with its classical matched filter. The work of Ref. 6 was concerned with the optimal filter designs without regard to their implementation. Because the optimal frequency-domain filters were, in general, complex valued, it was implicitly assumed that the correlation operation would be performed digitally or that holographic filters could be constructed to encode the full complex designs.

In this paper the implementation of optimal filters designed for images with SDN on spatial light modulators (SLM's) of constrained-modulation capability in an updatable optical correlator is addressed. The two approaches discussed above are again considered, and focus is turned to whether the optimal versions of these filters that can be encoded in the coding domain of the SLM device in the filter plane can be determined. The remainder of this paper is divided into three sections. In Section 2 the models of film-grain noise

and speckle are considered and the optimal complex filters that maximize the correlation-peak SNR are presented. The technique of preprocessing an image with SDN by the use of a nonlinear variance-stabilizing algorithm and then correlating it with a classical matched filter is also discussed. In Section 3, a previously developed algorithm for designing constrained-modulation filters for images with signal-independent noise (SIN) is discussed, and its application to the problem of images with SDN is considered. It is shown that, although the algorithm is readily applicable to the variance-stabilizing approach, it cannot be simply applied to the design of linear constrained-modulation filters for images with SDN. However, two realizable optical systems that allow the straightforward implementation of filters for images with SDN are briefly discussed. A summary and some conclusions are given in Section 4.

2. Correlation of Images with Signal-Dependent Noise

A. Image-Noise Models and Optimal Filters

For the purposes of this paper, two different noise models are considered to illustrate design issues involved with constrained-modulation filters for images with SDN. The first is the case of film-grain image noise. Images captured on photographic film, especially in relatively low-light conditions, have been found to suffer from noise with spatially varying second-order statistics that are dependent on the local signal intensity.[1,2] With one-dimensional notation for convenience, if $s(x)$ is the original object film-density distribution, then the observed density image $r_{fg}(x)$ is often modeled as

$$r_{fg}(x) = s(x) + \kappa s^P(x)n_1(x), \tag{1}$$

where $0 < p < 1$, $n_1(x)$ represents a random noise process and κ is a constant that we assume is equal to 1.0 from now on without limitation to the analysis. The second term in Eq. (1) represents the noise that is clearly dependent on the signal strength. Although not necessary, for convenience we assume that $n_1(x)$ is white noise. The power p is commonly held to be $p = 0.5$ for film-grain noise,[1,2] but we treat it generally for now.

The second noise model analyzed is that of speckle. Laser speckle noise occurs during imaging with spatially coherent light when the object of interest has a random surface roughness of the order of a wavelength, and the imaging system cannot resolve the microscale of the object's roughness. This type of speckle pattern is also referred to as Gaussian speckle because the complex amplitude of the image is a circular complex Gaussian process, which produces an intensity distribution with a negative exponential probability density function.[7] An image model that has been widely used to describe speckle images is the multiplicative model,

$$r_{sp}(x) = \kappa s(x)n_{sp}(x), \tag{2}$$

where $r_{sp}(x)$ is the noisy speckle image, $s(x)$ is the object intensity function, $n_{sp}(x)$ is the noise function, and κ is a proportionality factor dependent on system parameters.[8-10] We again assume that κ is equal to 1 without loss of generality. It should be noted that Eq. (2) is strictly true only for spatially uniform areas of the object and is a less accurate model for image regions that contain spatial details smaller than the resolution of the coherent system.[11] However, we assume that Eq. (2) represents a reasonable model for speckle images in the analysis of the correlation of such images for pattern-recognition purposes. Assuming the model of Eq. 2 and negative exponential statistics for $n_{sp}(x, y)$, {i.e., the probability density function of $n_{sp}(x)$ equals $\exp[-n_{sp}(x)]$}, it is easy to show that both the mean and the standard deviation of $r_{sp}(x, y)$ are proportional to $s(x, y)$, which clearly makes the noise signal dependent in nature.

For pattern recognition and detection, we are interested in the correlation peak of a noisy image $r(x)$, as given in Eq. (1) or Eq. (2), with a reference function or space-domain filter, $h(x)$. Given a signal $s(x)$ that is centered, the central pixel in the correlation function $c(x)$ is denoted as c_0 and can be written as

$$c_0 = \int r(x)h(x)dx. \tag{3}$$

Because we intend to implement correlations optically, we can also often express c_0 in the Fourier domain as

$$c_0 = \int R(u)H^*(u)du, \tag{4}$$

where $R(u)$ is the Fourier transform of $r(x)$ and $H(u)$ is the Fourier-plane filter, which is, of course, the Fourier transform of $h(x)$. The correlation performance quantity of interest is the SNR of c_0, defined as

$$SNR = \frac{|E[c_0]|^2}{var[c_0]}. \tag{5}$$

The expressions for the correlation-peak SNR for both film-grain images and speckle images were derived in Ref. 6. For the case of film-grain noise, we have

$$SNR_{fg} = \frac{\left|\int S(u)H^*(u)du\right|^2}{\int P_{n_1}(u)|G(u) \otimes H(u)|^2du}. \tag{6}$$

where $S(u)$ is the Fourier transform of $s(x)$, $G(u)$ is the Fourier transform of $s^P(x)$, $P_{n_1}(u)$ is the power spectral density of $n_1(x)$, and $\otimes$ represents the convolution operation. For images with speckle noise, the SNR

result is

$$\mathrm{SNR_{sp}} = \frac{\left| \int S(u)H^*(u)du \right|^2}{\int |S(u) \otimes H(u)|^2 du} . \qquad (7)$$

The result in Eq. (7) assumes that the speckle noise $n_{sp}(x)$ is spatially uncorrelated. In the general case, $n_{sp}(x)$ may actually be colored, but the simpler model employed here is sufficient for our purposes of comparing the two approaches. Both Eqs. (6) and (7) are written in the Fourier domain because, in an optical correlator, we design $H(u)$ to be placed in the Fourier plane of a 4-f system. In this case we wish to design $H(u)$ to maximize the correlation-peak SNR. However, by inspection of Eqs. (6) and (7), we find that it is impractical to solve for the filter $H(u)$ to maximize those expressions because of the convolution terms in the denominators.

To avoid this problem, we can use Parseval's theorem and the Power theorem to rewrite Eqs. (6) and (7) in the spatial domain and then solve for the optimal space-domain filter $h(x)$ in each case. For images with film-grain noise, the result is

$$h_{fg}(x) = \frac{s(x)}{s^{2p}(x)} = s^{1-2p}(x). \qquad (8)$$

Assuming that $p = 0.5$, we can simplify Eq. (8) to

$$h_{fg}(x) = s^0(x) = b(x), \qquad (9)$$

where $b(x)$ is the silhouette of $s(x)$, i.e.,

$$b(x) = \begin{cases} 1, & s(x) > 0 \\ 0, & \text{otherwise} \end{cases} . \qquad (10)$$

For speckle images, the optimal filter is

$$h_{sp}(x) = \frac{b(x)}{s(x)} \equiv \begin{cases} \dfrac{1}{s(x)}, & s(x) > 0 \\ 0, & \text{otherwise} \end{cases} . \qquad (11)$$

The filters given in Eqs. (9) and (11) are optimal in the sense that they will maximize the SNR of the central correlation peak for images with film-grain noise and speckle noise, respectively. This implies that all other filter designs will necessarily produce SNR values less than or equal to those of these filters. Thus the classical matched filter, i.e., $h_{mf}(x) = s(x)$ or $H_{mf}(u) = S(u)$, will also be inferior in this sense. We note that the filters given in Eqs. (9) and (11) are not optimized for other correlation metrics such as discrimination.

B. Variance-Stabilizing Method

The second approach to dealing with the correlation of images with SDN is motivated by previous work by researchers working in the field of image processing

and restoration. There it was found that by the use of the proper nonlinear transformations, images with SDN could first be converted into images with SIN.[2,12,13] Then traditional signal-processing techniques designed for SIN could be applied to the transformed image for enhancement or restoration.[14] In a like manner, in this section the application of variance-stabilizing transformations to images with film-grain and speckle SDN and their subsequent correlation for target-recognition purposes are discussed. After transformation, the images will have SIN properties, and thus the optimal filter to apply for correlation is the classical matched filter designed for the transformed signal.

As above, let $r(x)$ represent a noisy image with SDN. For a given x, let the mean value of $r(x)$ be denoted as μ_r with standard deviation σ_r. Because the noise is signal dependent, the standard deviation of $r(x)$ is a known function of the mean value, and so we can write

$$\sigma_r = D(\mu_r), \qquad (12)$$

where $D(\cdot)$ represents the signal-dependence functionality of σ_r. According to previous work,[12,13] we can transform the noisy image $r(x)$ in a pointwise fashion into another function $y(x)$ such that $y(x)$ will contain statistically SIN (i.e., constant-variance noise) over a wide range of image values. This nonlinear transformation is expressed as

$$y = g(r) = K \int \frac{dr}{D(r)} \qquad (13)$$

for an arbitrary constant K. Strictly speaking, Eq. (13) will produce SIN only if $\sigma_r \ll \mu_r$, although it has been found to work well even when this condition is significantly relaxed, such as with speckle noise.[15]

For images with film-grain noise modeled in Eq. (1), it is straightforward to show that the proper transformation to apply to the noisy image is

$$y(x) = \frac{K}{\sigma_{n_1}(1 - p)} r^{1-p}(x). \qquad (14)$$

Assuming that $p = 0.5$, we then obtain the result

$$y(x) = \frac{2K}{\sigma_{n_1}} r^{1/2}(x), \qquad (15)$$

and thus the proper preprocessing transformation to apply to images with film-grain noise is the square-root operation in order to make the noise signal independent.

The transformation for images with speckle noise is also obtained with Eqs. (12) and (13). Letting the observed speckle image be represented as given in Eq. (2) with $n_{sp}(x)$ having a negative exponential probability density function, we find that the proper transformation in this case becomes

$$y(x) = K \ln[r(x)]. \qquad (16)$$

Of course the logarithmic transformation also makes the noise additive as well as signal independent.

Given the image-transformation algorithms of Eqs. (15) and (16), the second approach for pattern recognition by the correlation of images suffering film-grain or speckle SDN is thus a two-step process. We first nonlinearly transform the noisy input image $r(x)$ into $y(x)$ with statistically white, constant-variance noise. The optimal filter for this type of image is of course the classical matched filter. Thus we create a matched filter for the transformed signal and correlate $y(x)$ with it. The matched filter for the transformed signal is found by the simple substitution of $s(x)$ for $r(x)$ in either Eq. (15) or Eq. (16). For film-grain images, the classical filter for the transformed image is

$$h'_{fg}(x) = s^{1\,2}(x),\tag{17}$$

where we have set the constants in front to 1.0 for convenience. The prime symbol denotes that this is the matched filter for the transformed signal. For speckle images, the classical filter is

$$h'_{sp}(x) = \begin{cases} \ln[s(x)], & s(x) > 0 \\ 0, & \text{otherwise} \end{cases}.\tag{18}$$

Theoretical results that compare the correlation peak SNR performances for the two approaches to the detection of images with film-grain and speckle SDN have been derived previously.[6] For film-grain images, the variance-stabilization approach always yields a peak SNR value 6 dB higher than that of the optimal linear filter, regardless of the object function. The comparison is object dependent for speckle images, but it has been found that the variance-stabilization approach usually is again significantly superior. For a quantitative comparison, the SNR results for the gray-scale object in Fig. 1 were calculated. Table 1 contains the results for the optimal linear filter, the classical matched filter (derived assuming SIN) applied directly to the image with SDN, and the variance-stabilization approach in which
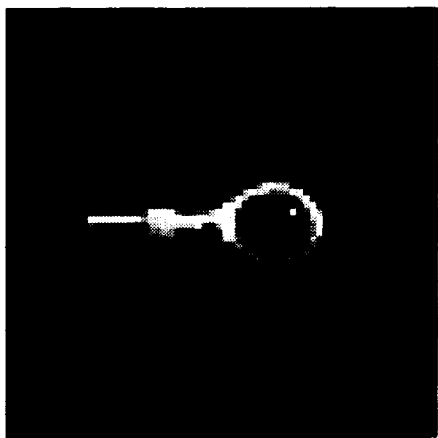

Fig. 1. 64 × 64 pixel image of an eyebolt with 256 gray levels used for simulations.

Type of Noise	$h_{opt}(x)$ [a]	$h_{mf}(x)$ [b]	$h'_{opt}(x)$ [c]
Film grain ($\sigma_{n_i} = 5.0$)	32.5	32.1	38.5
Speckle	24.7	22.7	35.2

[a]Optimal linear filter for SDN.
[b]Classical matched filter for SIN.
[c]Filter matched to transformed image.

the image is preprocessed and then correlated with the matched filter for the transformed object. The last approach is clearly the best for performance in terms of SNR. We note that the optimal linear filters for film-grain noise and speckle indeed produce better SNR's than the classical matched filter for the original object, but the improvements are fairly small for this object.

3. Constrained Modulation Filters for Images with Signal-Dependent Noise

The filter designs presented in Eqs. (9) and (11) (for the optimal linear filter) and Eqs. (17) and (18) (for the nonlinear image-transformation approach) are given as purely mathematical functions, without limitations to the filter values in either the space or the frequency domains. However, to implement the correlation process optically, we must obviously consider the modulation capabilities of the SLM in the frequency domain that encodes the filter function. At this point we assume the implementation of a VanderLugt, or 4-f, optical correlator. Currently available updatable SLM's all have constrained-modulation characteristics, leading to filter types such as binary-phase-only filters[16] (BPOF's), phase-only filters,[17] and ternary phase-amplitude filters[18] (TPAF's), among others. Thus we have to consider how to map optimally the desired filter function as expressed with full complex values onto the SLM with distinct modulation constraints in a way that maximizes or minimizes the performance criterion of interest. This problem has been previously addressed for situations that include the presence of additive colored signal-independent image noise, notably in the papers by Juday[19] and Laude and Réfrégier.[20] Juday examined a generalized correlation metric that involved a designer-defined balance between peak intensity, SNR, and peak-to-correlation energy. Laude and Réfrégier extended the analysis to the design of optimal trade-off filters (OTOF's) involving those metrics to compare better the performance of different types of coding-domain SLM's. In both studies and with different mathematical approaches, the optimal filters encoded in the domain of the SLM were found to result from first designing the optimal fully complex filter and then taking the Euclidean projection of that optimal complex filter onto the coding domain. If $H^+(u)$ is the optimal complex filter, then the optimal realizable filter is given by

$$H(u) = P[\gamma H^+(u)\exp(i\phi)]\tag{19}$$

where ϕ is a variable that is searched (over a 2π range) to find the optimal design, γ is a scale factor that may also need to be searched, depending on the coding domain, and $P[\ ]$ represents the projection operation. Thus the important general result of these previous studies is that the optimal filter value at pixel u is found by the choice of the realizable SLM value that is closest to the optimal complex filter value. This Euclidean distance minimization is of course performed in the frequency domain of the filter, as the filter is to be used in the Fourier plane of the optical correlator.

The question then arises as to whether we can apply the same design rule to the design of filters for images with SDN such as film-grain noise or speckle noise. We are interested in studying the implementation of both approaches developed, i.e., the optimal linear filters that correlate with the images with SDN and the transformation technique in which we transform the noisy image and correlate it with the matched filter for the transformed object function.

Let us first consider the second approach. After applying the proper transformation to the image $r(x)$ with SDN, we obtain in general

$$y(x) = s'(x) + n(x), \qquad (20)$$

where $n(x)$ is now signal independent. For example, for speckle images we have

$$y(x) = \ln[s(x)] + \ln[n_{\mathrm{sp}}(x)], \qquad (21)$$

and for film-grain images we have

$$y(x) \approx [s(x)]^{1/2} + \frac{n_1(x)}{2}, \qquad (22)$$

using the image models of Eqs. (1) and (2). Assuming in general that $n(x)$ in Eq. (20) is colored noise, the optimal Fourier plane filter to apply is of course the classical matched filter,

$$H^+(u) = \frac{S'(u)}{P_n(u)}, \qquad (23)$$

where $S'(u)$ is the Fourier transform of $s'(x)$ and $P_n(u)$ is the noise power spectral density. Regarding Eqs. (20) and (23), it is clear that these are the same conditions of additive, SIN that were assumed in the earlier studies of constrained-modulation filter design. Thus the same design approach of Euclidean projection will also be optimal here to maximize the peak SNR. The necessary design steps are as follows:

(1) Transform the object $s(x)$ to produce $s'(x)$ according to the proper transformation rule.
(2) Calculate the optimal complex filter to maximize the SNR as in Eq. (23).
(3) Calculate the optimal constrained-modulation filter according to the minimum Euclidean distance rule as in Eq. (20).

Of course this same basic algorithm also applies for a generalized metric including SNR, or an optimal trade-off between the SNR and another performance criterion.

Now we consider the first approach. We have already found the optimal filter functions in the space domain for maximizing SNR when correlating directly with the images with SDN, e.g., $h_{\mathrm{fg}}(x)$ and $h_{\mathrm{sp}}(x)$. Thus we obviously also know the optimal complex frequency-domain filters simply by Fourier transforming the space-domain filters. The question is then whether we can apply the minimum Euclidean distance principle to this filter to obtain the optimal constrained-modulation filter. First recall the expressions for SNR expressed in the frequency domain as given by Eqs. (6) and (7), for film-grain noise and speckle noise, respectively. These expressions are rewritten as discrete summations for convenience as

$$\mathrm{SNR}_{\mathrm{fg}} = \frac{\left|\sum_{u=1}^{N} S(u)H^*(u)\right|^2}{\sum_{u=1}^{N}\left[P_{n_1}(u)\left|\sum_{v=1}^{N} G(u-v)H(v)\right|^2\right]}, \qquad (24)$$

$$\mathrm{SNR}_{\mathrm{sp}} = \frac{\left|\sum_{u=1}^{N} S(u)H^*(u)\right|^2}{\sum_{u=1}^{N}\left|\sum_{v=1}^{N} S(u-v)H(v)\right|^2}, \qquad (25)$$

where $H(\cdot)$ is any filter in general. Of course we know the forms of the general complex filters that maximize Eqs. (24) and (25) by having solved for them in the space domain. With convolutions in the denominators, these two equations clearly do not match in form with the corresponding expression for SNR, assuming SIN, which is given by

$$\mathrm{SNR}_{\mathrm{SIN}} = \frac{\left|\sum_{u=1}^{N} S(u)H^*(u)\right|^2}{\sum_{u=1}^{N} P_n(u)|H(u)|^2}. \qquad (26)$$

Thus we can make the following qualitative argument that the minimum Euclidean distance design rule does not necessarily apply to filters designed for images with SDN: In Eq. (26) (SIN case), we see that the denominator is essentially a summation over u of the weighted values of the squared modulus of $H(u)$. The numerator is a similar weighted summation. Because the numerators are identical in Eqs. (24)–(26), we consider only the denominators now. In the case of SIN in Eq. (26), it makes intuitive sense that minimizing each term of the summation by Euclidean projection of $H^+(u)$ to find a realizable $H(u)$ will also minimize the entire denominator. However, in Eqs. (24) and (25) for the cases of images with SDN, we have convolution terms in the denominator. For

these equations it is not possible to say that minimizing the distance of each pixel of $H(v)$ to $H^+(v)$ independently through Euclidean projection will minimize the whole denominator, as each term in the summation over u is dependent on every filter pixel value $H(v)$. Thus it is highly unlikely that the simple Euclidean projection algorithm will work for filters designed to correlate with images with SDN because the mean square error (SNR denominator) is dependent on the choice of $H(v)$ in a highly nontrivial way through the convolution.

As an example, consider the results for BPOF designs for the bolt image, assumed to be corrupted by film-grain and speckle noise, presented in Table 2. This table lists the SNR values of three BPOF's, which were designed with the Euclidean projection technique from the optimal linear filter for the object with SDN, the classical matched filter that assumes SIN, and the matched filter to the transformed variance-stabilized image. We know that the complex-valued optimal linear filters, as expressed in Eqs. (9) and (11), produce higher SNR values than the classical matched filter does when applied to images with film-grain noise and speckle, respectively. Yet the BPOF versions of $h_{fg}(x)$ and $h_{sp}(x)$ actually yield lower SNR's than the BPOF versions of $h_{mf}(x)$ do, and thus they cannot be optimal. In particular, the BPOF made from $h_{sp}(x)$ produces a peak SNR that is more than 14 dB lower than the BPOF made from $h_{mf}(x)$. This does not imply that the latter BPOF version of $h_{mf}(x)$ is optimal for either film-grain noise or speckle, only that binarizing the optimal linear filter according to the minimum Euclidean distance rule does not work with filters designed for SDN. In fact, we do not know how to design the optimal constrained-modulation frequency-domain filter for images with SDN. On the other hand, the BPOF versions of $h'_{fg}(x)$ and $h'_{sp}(x)$, which are the filters matched to the transformed image, can indeed be designed with the projection algorithm. These BPOF's clearly produce correlation peaks with higher SNR than either of the other designs and are evidence that the variance stabilization of images with SDN is highly beneficial before the correlation process is performed.

Similar results for OTOF's[21] balancing SNR and optical efficiency for TPAF designs are presented in Figs. 2 and 3, which again point out the superiority of the image transformation technique and the nonoptimality of the constrained-modulation version of the optimal complex linear filter. We obtain these optimal trade-off curves by varying the scale factor γ
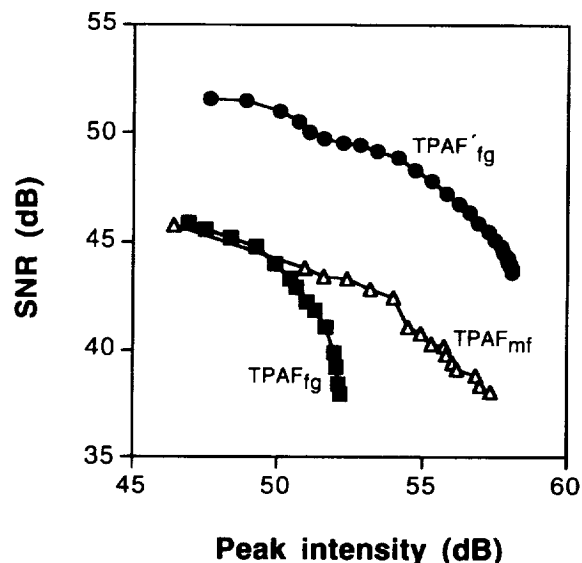

Peak intensity (dB)

Fig. 2. Optimal trade-off curves for SNR versus peak intensity for TPAF's designed for the bolt image with film-grain SDN. TPAF$_{fg}$ is designed from $h_{fg}(x)$, TPAF$_{mf}$ is designed from $h_{mf}(x)$, and TPAF$'_{fg}$ is designed from $h'_{fg}(x)$.

given in Eq. (19) and where $H^+(u)$ was defined in turn as the Fourier transform of $h_{fg}(x)$, $h_{mf}(x)$, and $h'_{fg}(x)$ in Fig. 2 and $h_{sp}(x)$, $h_{mf}(x)$, and $h'_{sp}(x)$ in Fig. 3. The results show that for most values of peak intensity, it is possible to design a ternary filter from $h_{mf}(x)$ that has better SNR performance than the best ternary filter made from the optimal linear filter $h_{opt}(x)$. Furthermore, the TPAF's designed from the variance-stabilized image-matched filters $h'_{opt}(x)$ outperform both other filter types.

We again note that although the minimum Euclidean distance design rule does not necessarily work
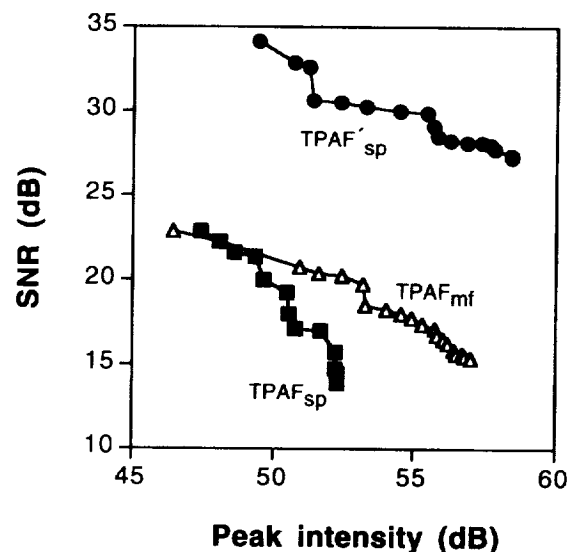

Peak intensity (dB)

Fig. 3. Optimal trade-off curves for SNR versus peak intensity for TPAF's designed for the bolt image with speckle SDN. TPAF$_{sp}$ is designed from $h_{sp}(x)$, TPAF$_{mf}$ is designed from $h_{mf}(x)$, and TAPF$'_{sp}$ is designed from $h'_{sp}(x)$.

Table 2. SNR (dB) for BPOF's Designed for Bolt with SDN[a]

Type of Noise	$h_{opt}(x)$	$h_{mf}(x)$	$h'_{opt}(x)$
Film grain ($\sigma_{n_1} = 5.0$)	21.6	24.1	29.7
Speckle	0.88	15.2	27.2

[a]Original complex filters are optimal linear filter for SDN $[h_{opt}(x)]$, classical matched filter for SIN $[h_{mf}(x)]$, and filter matched to transformed image $[h'_{opt}(x)]$.

with designing filters for images with SDN (whereas it does work for the transformed images with SIN), we have no way of determining the optimal filter in the coding domain of the SLM. That is, we know what is not the optimal filter, but we do not know what is the optimal filter. This is a strong argument for preprocessing the images with the proper transformation to make the noise signal independent if one plans to correlate images with SDN in an optical correlator with a constrained-modulation SLM in the filter plane.

However, there are at least two cases for which it is possible to implement the optimal linear filters for images with SDN in an optical system, and those are the joint-transform correlator[22] (JTC) and the spatial-domain acousto-optic correlator.[23] This is because, in these systems, the filter is actually encoded in the space domain as a reference function. Because we calculated the optimal filters $h_{fg}(x)$ and $h_{sp}(x)$ in the space domain as real functions, they can be easily implemented on most types of SLM. In fact, the filter for film-grain images, $h_{fg}(x)$, is especially easy because it is a binary function that is the silhouette of the object, as given in Eq. (9). The filter for speckle images, $h_{sp}(x)$, requires continuous gray-scale transmission to implement, but if the available gray levels are limited, we can easily find the optimal constrained-modulation filter $h(x)$ by applying the minimum Euclidean distance algorithm in the space domain. This becomes clear when a direct analogy is made of the SNR equation for speckle images in the space domain to Eq. (28) for images with SIN in the frequency domain. Thus, although it is impossible to find the optimal filter $H(u)$ for speckle images for implementation on a constrained-modulation SLM in the Fourier plane of a 4-f correlator, it is relatively simple to calculate the optimal reference function $h(x)$ for implementation on a constrained-modulation SLM in the input plane of a JTC or in an acousto-optic template-matching correlator.

4. Summary

We have studied the design of optical correlation filters for application to images corrupted with SDN. In particular, the design of filters for implementation on constrained-modulation SLM's in an optical correlator system was addressed. As examples, SDN was analyzed as modeled by film-grain noise and speckle noise and two different approaches for their treatment in the correlation process were studied. The minimum Euclidean distance design rule, which was previously developed by other researchers for general correlation metrics that assumed additive, signal-independent image noise, was reviewed. We found that we can apply the same mapping algorithm to those complex filters designed with the second approach, in which the image is nonlinearly transformed to have SIN. This is because the conditions are then the same as those assumed during the derivation of the minimum Euclidean distance rule. However, we found that we cannot simply apply that

mapping rule to the complex optimal linear filters (approach 1) that maximize SNR when correlating directly with the images with SDN. Qualitatively, this is because the mean square error of the correlation peak contains a convolution involving the Fourier-plane filter, and minimizing the mean square error cannot be accomplished by simple projection of the optimal complex filter onto the coding domain. We are not able to determine the optimal realizable Fourier-plane filter in this situation, but are able to say only that the design rule that works for images with SIN does not produce optimal filters for images with SDN. The conclusion is that the nonlinear transformation preprocessing step must be applied to images with SDN that are to be correlated in an optical system with currently available SLM's, at least if peak SNR is part of the measured correlation metric. Only in this way can one design a filter that is known to be optimal for the defined metric. Two examples of BPOF and TPAF designs that confirmed this point were presented.

Finally, we noted that it is possible to implement the filters optimally for images with SDN (approach 1) in a constrained optical system if the filter is encoded in the spatial domain, such as in a JTC or an acousto-optic correlator. In that case we can indeed apply the minimum Euclidean distance rule to the design of spatial-domain filters that maximize SNR or OTOF's that balance the SNR with another correlation metric, such as optical efficiency. However, even in this case the nonlinear image transformation approach will probably provide superior performance and should be carefully considered.

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