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Conversion-Integration of MSFC Nonlinear Signal Diagnostic Analysis Algorithms for realtime Execution on MSFC's MPP Prototype System

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## FOREWORD

This report was prepared by AI Signal Research, Inc. (ASRI) for the George C. Marshall Space Flight Center, National Aeronautics and Space Administration. The work was performed under contract NAS8-40341, entitled "Conversion-Integration of MSFC Nonlinear Signal Diagnostic Analysis Algorithms for realtime Execution on MSFC's MPP Prototype System " over the time period from June 7, 1995 through December 2, 1996.

The work was carried out by Dr. Jen-Yi Jong serving as Program Manager. Dr. Jong was responsible for the software development, conversion and evaluation of Nonlinear Signal Diagnostic Analysis Algorithms. Ms. Polly Lu serving as programmer was responsible for implementing the software on C40 DSP using Data Translation's DSP-EZ programming language.

Mr. T Zoladz, MSFC/ED32, and Mr. T. Fiorucci, MSFC/23, provided valuable guidance through coordination of computer resources and definition of task requirements and priority. In addition, Mr. T. Bapty at Vanderbilt has provided valuable assistance in establishing the handshake for the conversion from ASRI's DSP algorithms to Vanderbilt's MGA Model Based Programming environment

### 1.0 PROGRAM SUMMARY

NASA's advanced propulsion system SSME/ATD has been undergoing extensive flight certification and developmental testing, which involves large numbers of health monitoring measurements. To enhance engine safety and reliability, detailed analysis and evaluation of the measurement signals are mandatory to assess its dynamic characteristics and operational condition. Efficient and reliable signal detection techniques will reduce the risk of catastrophic system failures and expedite the evaluation of both flight and ground test data, and thereby reduce launch turn-around time. During the development of SSME, ASRI participated in the research and development of several advanced nonlinear signal diagnostic methods for health monitoring and failure prediction in turbomachinery components. However, due to the intensive computational requirement associated with such advanced analysis tasks, current SSME dynamic data analysis and diagnostic evaluation is performed off-line following flight or ground test with a typical diagnostic turnaround time of one to two days. The objective of MSFC's MPP Prototype System is to eliminate such "diagnostic lag time" by achieving signal processing and analysis in real-time. Such an on-line diagnostic system can provide sufficient lead time to initiate corrective action and also to enable efficient scheduling of inspection, maintenance and repair activities.

The major objective of this project was to convert and implement a number of advanced nonlinear diagnostic DSP algorithms in a format consistent with that required for integration into the Vanderbilt Multigraph Architecture (MGA) Model Based Programming environment. This effort will allow the real-time execution of these algorithms using the MSFC MPP Prototype System. ASRI has completed the software conversion and integration of a sequence of nonlinear signal analysis techniques specified in the SOW for real-time execution on MSFC's MPP Prototype. This report documents and summarizes the results of the contract tasks; provides the complete computer source code; including all FORTRAN/C Utilities; and all other utilities/supporting software libraries that are required for operation.

### 2.0 TECHNICAL DISCUSSION

### 2.1 Introduction

Machinery fault detection and diagnosis have always been significant technical challenges in the aeronautics and transportation industries. Yet they remain a necessity since a reliable health monitoring system can prevent catastrophic failures and costly unscheduled down time due to false alarms. As computer information processing technology continues to advance, the major challenge associated with machinery monitoring and diagnosis is shifting from how to obtain machinery vibration data to methods of information extraction and interpretation. Therefore, the incorporation of advanced signal processing element into a health monitoring device has become invaluable. However, due to the intensive computational requirement associated with such advanced analysis tasks, current SSME dynamic data analysis and diagnostic evaluation is performed off-line following flight or ground test with a typical diagnostic turnaround time of one to two days. The objective of MSFC's MPP Prototype System is to eliminate such "diagnostic lag time" by achieving signal processing and analysis in real-time. Such an on-line diagnostic system can provide sufficient lead time to initiate corrective action and also to enable efficient scheduling of inspection, maintenance and repair activities.

### 2.2. Signal Analysis Techniques

Over the past decade, significant progress has been made in the aerospace, transportation and industrial communities to enhance performance of machinery failure detection and diagnosis through advances in instrumentation, modeling, and signal analysis techniques. ASRI has participated in the development of a hierarchy of advanced nonlinear diagnostic signal analysis techniques for turbomachinery system. These techniques can extract and identify intelligent information hidden in a measurement signal which is often unidentifiable using conventional signal analysis methods. By providing additional insight into the system response, these analysis tools can better identify well-hidden defect symptoms as well as false-alarm signatures. As a result, false-alarm $/ \mathrm{mis}$ interpretation rates are reduced and greatly improve system reliability. These techniques have been applied to day-to-day Space Shuttle Main Engine (SSME) hot-firing test/flight data, and appear to be highly promising for turbomachinery health monitoring and failure diagnostics.

In the following subsections, the technical discussion of several nonlinear diagnostic signal analysis to be converted/integrated into MSFC MPP prototype system will be discussed. These techniques include:

1. TOPO: Topographical Joint Time-Frequency Mapping Algorithm
2. Hyper-Coherence Analysis - Nonlinear correlation characteristics between integer spectral components
3. Bi-Spectral Analysis -Nonlinear correlation characteristics between three spectral components with frequency sum and difference
4. Tri-Spectral Analysis - higher order coherence estimations among four spectral components and modulation sideband
5. Envelope Detection Method - Demodulation of envelope signal using Hilbert Transform
6. GHC-Generalized Hyper-Coherence using instantaneous phase information

Software implementation of these techniques has been performed on a TI-C40 DSP, which are able to run in real-time on ASRI's C40 DSP board and display the results on a PC computer system. Since the Multigraph Architecture (MGA) is not available at ASRI, a handshake point and format for the conversion from ASRI's DSP algorithms to Vanderbilt MGA Model Based Programming environment must be utilized. The algorithm and codes of the programs documented in this report utilize such a conversion format in order to reach the handshake point acceptable by Vanderbilt.

### 2.2.1 TOPO: Topographical Joint Time-Frequency Mapping Algorithm

### 2.2.1.1 General background, algorithm and simulation example TOPO

The Topo spectral time/frequency display algorithm has been implemented on a TI-C40 DSP running in real-time and display Topo on a PC computer system. This format has been proved to be acceptable by Vanderbilt as a feasible handshake point during the conversion of the bi-coherence program. This format utilizes both Data Translation's DSP-EZ language to program signal processing application directly on a TI C40 DSP board, and the Visual Basic language to program the tasks on PC computer. . Detailed information of this Topo program is discussed below:

The 3-D PSD isoplot or waterfall plot (as shown in figure 1.1) has been used extensively for vibration signal analysis for engine diagnostics. It can display the spectral information in a joint frequency, time and amplitude format. This format is especially useful for nonstationary dynamic signal to allow clear visual inspection and identification of the trace of any particular spectral component as a function of time and its relationship with respect to some other frequency components. However, there exist several drawback about this format. First of all, a threshold level must be chosen, so that only the spectral components above the threshold level will be plotted. While the rest peaks below that level will be ignored ( as shown in Figure 1.2-a). However, for machinery diagnostic application, these low amplitude spectral components could still represent some important defect signature and should not be left out. secondly, if the threshold level is set too low in order to include the low-amplitude PSD peaks, then the fluctuating noise floor with larger amplitude will smear the isoplot in its corresponding frequency region ( as shown in Figure 1.2-b). Thirdly, for a nonstationary data, when there are too many spectral peaks present and cross-over occur among different spectral components, then it would become very difficult to identify the trace of those spectral components in the isoplot.


Figure 1.1

- SHRESHOLD LEVEL TOO HIGH--- Important Low Amplitude Defect Signature will Be Ignored.
- SHRESHOLD LEVEL TOO LOW --- The Fluctuating Noise floor will Smear The Entlre Isoplot
(a)


- NON-STATIONARY DATA ---

Many PSD Peaks Get Compressed Too Closely Together \& Cross-over Occur Among Different Spectral Components.




To overcome these limitations, the PSD to be display must be Normalized so that the noise floor variation would not smear the isoplot. This can be achieved with the TOPO algorithm by first identifying the noise floor of the PSD which is shown by the solid line near the bottom of the PSD in figure 1.3. The original PSD is then Normalized by removing the noise floor from the raw PSD so that all the spectral peaks will have the same "ground" level. From this normalized PSD, all the spectral peaks can be better identified without the interference of noise floor variation. This noise floor estimation is performed using a "Rainfall" algorithm to be discussed in detail in this report.

Figure 1.4 shows a typical Topo plot of a SSME vibration measurement signal during engine start-up. In this Topo plot, 400 PSD are plotted as a function of frequency and time. Each line or dot in this plot represent a PSD peak location. The width of each line is proportional to the signal-to-noise ration (SNR is defined as the amplitude of spectral peak above noise floor in dB ) of each PSD peak. During this non-stationary operation, detailed spectral information and cross-over between different component can be clearly visualized and identified which would be difficult from an isoplot. The Topo Algorithm can transform a large amount of dynamic signal into a simple image-like pattern, which would allow an easy identification of any special pattern associated with certain particular phenomenon. The examples in figure 1.5 show 4 different SSME dynamics pattern. Figure 1.5 -a shows the Zipper frequency component with a unique periodic frequency variation phenomenon. Figure 1.5 -b shows 12 KHZ anomalous phenomena with a group of spectral peaks not tracking the Sync and Sync harmonics during engine shut-down. Figure 1.5-c shows the development trend of the modulation/sideband pattern associated with bearing defect. And figure $1.5-\mathrm{d}$ shows the subtle difference of frequency variation during PWL change between a rotor related components and a fluid related component.

### 2.2.1.2 RainFall Algorithm for Noise floor Estimation.

The rainfall algorithm forms the basis for noise floor estimation in generating major spectral peaks along with their signal-to-noise ration (SNR) and bandwidth (BW). The following steps demonstrate the Rainfall algorithm for noise floor estimation.

1. The deep valleys are first removed and smoothed (interpolated) with the two neighboring points.
2. All PSD peaks are first identified and counted.
3. The left and right roots of each peak are also identified based on the rainfall trace shown in figure 6-a.
4. Remove Small Peaks by Applying Rainfall roll-over as shown in figure 6-b for any peak amplitude less than the user-specified threshold.
5. Consolidate peaks by merging smaller peaks been rolled-over with larger peaks not been rolled-over. Assign new left and right roots after peak consolidation.
6. calculate noise floor by interpolating all settle-down rainfall drops


Figure 1.6-b

### 2.2.1.3 DSP Code Overview of TOPO using Data Translation's DSP-EZ programming language running on C40 DSP.

The Topo algorithm has been implemented on ASRI's TI-C40 DSP, and has been preliminary tested using some "simple" input analog wave from a signal generator. The complete DSP Codes are listed below:

## $\% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \%$

PARTI: Function ApplyRainFall is implemented in DSP-EZ side.
It takes PSD, rainfall parameter and number of PSD elements as arguments and returns the Noise Floor data and two arrays of data storing all the peaks and indices of the peaks in the array.

| ********************************* |  |  |
| :---: | :---: | :---: |
| ** | Function ApplyRainFall | ${ }_{* *}^{* *}$ |
|  | al: PkLoc( ) and PkAmp() | ** |
|  | rn: Noise Floor | ** |
|  | ******************* |  |

Function ApplyRainFall(A As FloatArray, RainFall As Integer, n As Integer) As FloatArray Dim PSDG As FloatArray
Dim iLeft As FloatArray
Dim iRight As FloatArray
Dim iFlag As FloatArray
Dim Floor As FloatArray
Dim Ad As FloatArray
Dim nPeak, i, j, L, R, iNext, iL, iR, kPeak As Integer
Dim Ampr, AmpL, aL, aR, delta, root As Single
Dim iLeft0 As FloatArray

```
PSDG \(=\operatorname{GenLine}(0,0, \mathrm{n}+1) \quad\) 'allocate memories for array elements
iLeft \(=\operatorname{GenLine}(0,0, \mathrm{n}+1)\)
iRight \(=\operatorname{GenLine}(0,0, \mathrm{n}+1)\)
iFlag \(=\operatorname{GenLine}(0,0, \mathrm{n}+1)\)
PkLoc \(=\) GenLine \((0,0, \mathrm{n}+1)\)
PkAmp \(=\operatorname{GenLine}(0,0, \mathrm{n}+1)\)
Floor \(=\operatorname{GenLine}(0,0, \mathrm{n}+1)\)
iLeft \(0=\operatorname{GenLine}(0,0, \mathrm{n}+1)\)
For \(\mathbf{i}=0\) To \(\mathbf{n}-1\)
        \(\operatorname{PSDG}(\mathrm{i}+1)=\mathrm{A}(\mathrm{i})\)
Next i
For \(\mathrm{i}=2\) To \(\mathrm{n}-1\)
        If \(\operatorname{PSDG}(\mathrm{i})<=\operatorname{PSDG}(\mathrm{i}-1)\) And \(\operatorname{PSDG}(\mathrm{i})<=\operatorname{PSDG}(\mathrm{i}+1)\) Then
            \(\operatorname{PSDG}(\mathrm{i})=0.5 *(\operatorname{PSDG}(\mathrm{i}-1)+\operatorname{PSDG}(\mathrm{i}+1))\)
        End If
Next \(i\)
\(\operatorname{PSDG}(1)=\operatorname{PSDG}(2)+\operatorname{Abs}(\operatorname{PSDG}(2)) * 0.001\)
```

```
PSDG(n)= PSDG(n-1)+\operatorname{Abs}(\operatorname{PSDG}(\textrm{n}-1))}*0.00
npeak =0
i = 2
Do While (i <= n-1)
    If PSDG(i) > PSDG(i-1) And PSDG(i) > PSDG(i+1) Then
                        npeak = npeak+1
                        L = i-1
                            Do While (L> 1 And PSDG(L-1) <= PSDG(L))
                L=L-1
            Loop
            R=i
                    Do While ( }\textrm{R}<\textrm{n}\mathrm{ And PSDG( R+1)<= PSDG(R))
                R=R+1
                    Loop
                    iLeft (nPeak) = L
                    iRight(nPeak) = R
                        PkLoc(nPeak) = i
                iFlag(nPeak) = 1
            i=R
        End If
    i = i + 1
Loop
For i = 2 To npeak -1
        If iFlag(i)=1 Then
            AmpL = PSDG(PkLoc(i)) - PSDG(iRight(i) )
            AmpR = PSDG(PkLoc(i+1)) - PSDG(iLeft(i+1))
            If (AmpR > AmpL) And ( AmpL <= RainFall) Then
                iFlag(i)=-9 'Left merge
            End If
            If (AmpL > AmpR) And (AmpR <= RainFall) Then
                iFlag(i+1)=9 'Right merge
            End If
        End If
Nexti
iFlag(1)=1
iFlag(npeak) =1
iFlag(npeak +1)=1
iLeft(nPeak + 1) = iRight( nPeak)
kPeak=0
For i = 1 To nPeak
    If iFlag(i)=1 Then
        kPeak = kPeak+1
        iL =i
        If i > 1 Then
            iL = iL - 1
            Do While (iL > 1 And iFlag(iL) = -9)
                iL = iL -1
            Loop
            iL = iL +1
            End If
            iR = i
            If i < nPeak Then
                iR=iR + 1
```

$$
\text { Do While ( } \mathrm{iR}<\mathrm{nPeak} \text { And iFlag }(\mathrm{iR})=9 \text { ) }
$$

$$
\mathrm{i} \mathrm{R}=\mathrm{i} \mathrm{R}+1
$$

Loop
$\mathrm{iR}=\mathrm{iR}-1$
End If

```
PkLoc(kPeak) \(=\operatorname{PkLoc}(\mathrm{i})\)
iLeft0(kPeak) \(=\operatorname{iLeft}(i L)\)
iRight (kPeak) \(=\) iRight \((i \operatorname{R})\)
\(\mathrm{iL}=\mathrm{iLeft0}\) (kPeak)
\(i \mathrm{R}=\mathrm{iRight}(\mathrm{kPeak})\)
\(a L=P S D G(i L)\)
\(\mathrm{aR}=\mathrm{PSDG}(\mathrm{iR})\)
delta \(=(\mathrm{aR}-\mathrm{aL}) /(\mathrm{iR}-\mathrm{iL})\)
root \(=\mathrm{aL}+(\operatorname{PkLoc}(\mathrm{kPeak})-\mathrm{iL}) *\) delta
\(\operatorname{PkAmp}(k P e a k)=\operatorname{PSDG}(\operatorname{PkLoc}(k P e a k))-\) root
```

    End If
    Next i
For $\mathrm{i}=1$ To kPeak
iLeft(i) $=\mathrm{iLeft0(i)}$
Next i
For $\mathrm{i}=1$ To kPeak
$\mathrm{iL}=\mathrm{iLeft}(\mathrm{i})$
$\mathrm{iR}=\mathrm{iRight}(\mathrm{i})$
$a L=P S D G(i L)$
$\mathrm{aR}=\mathrm{PSDG}(\mathrm{iR})$
delta $=(a R-a L) /(i R-i L)$
For $\mathrm{j}=\mathrm{iL}$ To iR
$\operatorname{Floor}(\mathrm{j})=\mathrm{aL}+(\mathrm{j}-\mathrm{iL}) *$ delta
Next j
Nexti
For $\mathrm{i}=1$ To iLeft(1)
Floor(i) $=$ Floor(iLeft(1))

Next i

For $\mathrm{i}=\mathrm{iRight}(\mathrm{kPeak})$ To n
Floor(i) $=$ Floor $($ iRight $(k P e a k))$
Next i
PkLoc=Extract(PkLoc, kPeak, 1) 'Global for passing to VB
PkAmp=Extract(PkAmp, kPeak, 1) 'Global for passing to VB
ApplyRainFall $=$ Extract(Floor, n, 1)
'Global for passing to VB
'Function Returns Noise Floor

End Function
$\% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \%$

PART2: In Visual Basic Host side, two arrays of data, PkLoc() and PkAmp() are passed from DSP-EZ, and then Functions FindPeak and SortMax are called to find the cutoff value so to obtain the TOPO data for plotting.

```
********************************************************
, ** Procedure SortMax **
, ** **
```

```
- ** Returned Twk() is sorted PkVal() in descending order **
********************************************************
Sub SortMax (PkVal() As Single, Twk() As Single, m As Integer)
Dim i, k, mm, z As Integer
Dim T1() As Integer
Dim t2() As Single
Dim iWk() As Integer
ReDim T1(m)
ReDim t2(m)
ReDim iWk(m)
For \(\mathrm{i}=0\) To \(\mathrm{m}-1\)
iWk(i) \(=0\)
Twk(i) \(=-1000\)
Next i
\(\mathrm{mm}=\mathrm{m}-1\)
For \(\mathrm{i}=0\) To \(\mathrm{m}-1\)
\(\mathrm{z}=\mathrm{mm}\)
For \(\mathrm{k}=0\) To \(\mathrm{m}-1\)
If \(\mathrm{PkVal}(\mathrm{i})>=\operatorname{Twk}(\mathrm{k})\) Then
\(\mathrm{z}=\mathrm{k}\)
Exit For
End If
Next k
For \(\mathrm{k}=\mathrm{z}\) To m-2
\(\mathrm{T} 1(\mathrm{k}+1)=\mathrm{iWk}(\mathrm{k})\)
\(\mathrm{t} 2(\mathrm{k}+1)=\operatorname{Twk}(\mathrm{k})\)
Next \(k\)
For \(\mathrm{k}=\mathrm{z}+1\) To \(\mathrm{m}-1\)
\(\mathrm{iWk}(\mathrm{k})=\mathrm{T} 1(\mathrm{k})\)
\(\operatorname{Twk}(\mathrm{k})=\mathrm{t} 2(\mathrm{k})\)
Next k
\(\mathrm{iWk}(\mathrm{z})=\mathrm{i}\)
\(\operatorname{Twk}(\mathrm{z})=\operatorname{PkVal}(\mathrm{i})\)
Next i
```

End Sub
, $* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * ~$
, ** Function FindPeak **
1 ** **
1 ** Call : SortMax and obtain Twk() **
, ** Return: Peak Value(single) **
, $* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * ~$
1
'
Function FindPeak (A() As Single, ChoicePK As Integer, Neigh As Integer, n As Long) As Single
Dim NumPk As Integer
Dim Nex, i, j, MinPk As Integer

Dim PeakLoc() As Integer
Dim Peakamp() As Single
Dim Twk() As Single

```
ReDim PeakLoc(n)
ReDim Peakamp(n)
ReDim Twk(n)
```

```
NumPk \(=0\)
```

NumPk $=0$
For $\mathrm{i}=2$ To $\mathrm{n}-2$
For $\mathrm{i}=2$ To $\mathrm{n}-2$
If $(\mathrm{A}(\mathrm{i})>\mathrm{A}(\mathrm{i}-1))$ And $(\mathrm{A}(\mathrm{i})>\mathrm{A}(\mathrm{i}+1)$ ) Then
If $(\mathrm{A}(\mathrm{i})>\mathrm{A}(\mathrm{i}-1))$ And $(\mathrm{A}(\mathrm{i})>\mathrm{A}(\mathrm{i}+1)$ ) Then
$\mathrm{Nex}=\mathrm{i}+2$
$\mathrm{Nex}=\mathrm{i}+2$
NumPk = 1
NumPk = 1
PeakLoc(0) $=\mathrm{i}$
PeakLoc(0) $=\mathrm{i}$
Peakamp $(0)=\mathrm{A}(\mathrm{i})$
Peakamp $(0)=\mathrm{A}(\mathrm{i})$
Exit For
Exit For
End If
End If
Next i
Next i
For $\mathrm{i}=$ Nex To $\mathrm{n}-2$
For $\mathrm{i}=$ Nex To $\mathrm{n}-2$
If $\mathrm{A}(\mathrm{i})>\mathrm{A}(\mathrm{i}-1)$ And $\mathrm{A}(\mathrm{i})>\mathrm{A}(\mathrm{i}+1)$ Then
If $\mathrm{A}(\mathrm{i})>\mathrm{A}(\mathrm{i}-1)$ And $\mathrm{A}(\mathrm{i})>\mathrm{A}(\mathrm{i}+1)$ Then
If ( $\mathrm{i}<$ PeakLoc(NumPk -1 ) + Neigh) Then
If ( $\mathrm{i}<$ PeakLoc(NumPk -1 ) + Neigh) Then
PeakLoc(NumPk-1) $=\mathrm{i}$
PeakLoc(NumPk-1) $=\mathrm{i}$
Peakamp $($ NumPk - 1$)=\mathrm{A}(\mathrm{i})$
Peakamp $($ NumPk - 1$)=\mathrm{A}(\mathrm{i})$
Else
Else
NumPk $=$ NumPk +1
NumPk $=$ NumPk +1
PeakLoc(NumPk-1) = i
PeakLoc(NumPk-1) = i
Peakamp(NumPk-1) $=$ A(i)
Peakamp(NumPk-1) $=$ A(i)
End If
End If
End If
End If
Nexti
Nexti
SortMax Peakamp(), Twk(), NumPk
SortMax Peakamp(), Twk(), NumPk
If ChoicePK >= NumPk Then
If ChoicePK >= NumPk Then
$\mathrm{MinPk}=\mathrm{NumPk}$
$\mathrm{MinPk}=\mathrm{NumPk}$
Else
Else
$\mathrm{MinPk}=$ ChoicePK
$\mathrm{MinPk}=$ ChoicePK
End If
End If
FindPeak $=$ Twk $(\mathrm{MinPk}-1)$

```
FindPeak \(=\) Twk \((\mathrm{MinPk}-1)\)
```


## End Function

If CommChannel $=$ TopoChan Then
If bufferreadyfromdsp(frmDemo.DspEz1, TopoChan) Then
GetBufferFromDsp frmDemo.DspEz1, TopoChan, Databuffer(), validpts Plotbuffer plot5, Databuffer(), validpts
lblFilYmax.Caption $=$ plot5. yMax
lblFilYmin.Caption $=$ plot5. $y$ Min
End If
End If
If CommChannel $=$ LocChan Then
GetBufferFromDsp DspEz1, CommChannel, Databuffer(), validpts
For $\mathrm{i}=0$ To validpts -1
$\operatorname{PkLoc}(\mathrm{i})=$ Databuffer $(\mathrm{i})$

Next $i$
End If
If CommChannel $=\mathrm{AmpCh}$ an Then
GetBufferFromDsp DspEz1, CommChannel, Databuffer(), validpts
For $\mathrm{i}=0$ To validpts -1
PkAmp(i) = Databuffer(i)
Next i

If validpts $>20$ Then
CutOff $=$ FindPeak(Databuffer(), 20, 5, validpts)
End If
If CutOff $<5$ Then
LowVal $=5$
Else
LowVal = CutOff
End If
HighVal $=1000-$ LowVal
frmTopo!txtCutOff.Text $=\mathbf{S t r}($ CutOff $)$
For $\mathrm{i}=0$ To validpts -1
If PkAmp(i) $>=$ CutOff Then
$\mathrm{Sx}=\operatorname{PkAmp(i)} / 60$
If $S x>1$ Then
$S x=1$
End If
Ex $=$ Sx * Broad $+4 *$ PkLoc(i)
TopoData(i) = LowVal
frmTopo!Picture1.Line ( 4 * PkLoc(i), TopoData(i))-(Ex, TopoData(i))
End If
Next i
$\mathrm{r} 1 \%=$ BitBlt(ByVal frmTopo.Picture1.hDC, ByVal HDisp, ByVal 0, ByVal frmTopo.Picture1.Width

- HDisp, ByVal frmTopo.Picture1.Height - VDisp, ByVal frmTopo.hDC, ByVal frmTopo.Picture 1.Left, ByVal frmTopo.Picture1.Top + VDisp, ByVal SRCCopy)
$\mathrm{r} 1 \%=$ BitBlt(ByVal frmTopo.Picture1.hDC, ByVal frmTopo.Picture1.Left, ByVal
frmTopo.Picture 1.Height - VDisp, ByVal frmTopo.Picture1.Width, ByVal VDisp, ByVal frmTopo.hDC, ByVal 0, ByVal 0, ByVal Whiteness)
End If


### 2.2.2 Hyper-Coherence Analysis - Nonlinear correlation characteristics between integer spectral components

### 2.2.2.1 General background, algorithm and simulation example of hyper-coherence function

In machinery health monitoring, the shaft synchronous (RPM) component and its harmonics in a monitoring signal contains much subtle information about the system operating condition. Many mechanical failure modes, such as shaft misalignment, loose coupling, or rubbing, will cause waveform distortion resulting in the generation of strong harmonics. For example, the radial rubbing of a rotating shaft will cause waveform clipping. This waveform clipping will then generate strong odd synchronous harmonics which can be easily identified from its PSD. However, due to a complex operational environment, many other independent sources may coincidentally generate spectral components coincident with harmonic frequencies appearing to be synchronous harmonics. With this potential for ambiguity, a diagnostic method which can identify whether an apparent harmonic is a true harmonic is indispensable. The hyper-coherence method developed by ASRI provides such a nonlinear technique for harmonic identification. Hyper-coherence represents a hierarchy of higher joint moments between a reference spectral component and any of its harmonics. It can identify when a spectral component at a harmonic frequency is truly a harmonic or not. This is achieved by measuring a special phase difference to identify the degree of correlation between the fundamental and a harmonic.

To describe the nonlinear interaction between harmonically related spectral components in a given stationary, zero mean signal, we define the Hyper-spectrum of order $n$ by the relation
$H(n ; f 1)=E\left[X^{n}(f 1) X^{*}(n f 1)\right], \quad n=1,2,3, \ldots$
where f 1 is an arbitrary reference frequency, and nf 1 is an integer multiple of f 1 . Thus, the ascending terms in $\mathrm{H}(\mathrm{n} ; \mathrm{f} 1)$ represents a single value from the linear spectrum, bispectrum, tri-spectrum, etc., at the specific value $\mathrm{f} 1=\mathrm{f} 2=\ldots=\mathrm{fn}$.

In analogy with the ordinary coherence function, the hyper-coherence can be defined as a normalized hyper-spectrum:

$$
\begin{equation*}
\mathrm{G} 2(\mathrm{n} ; \mathrm{f} 1)=\frac{\left|\mathrm{E}\left[\mathrm{X}^{\mathrm{n}}(\mathrm{f} 1) \mathrm{X}^{*}(\mathrm{nf} 1)\right]\right|^{2}}{\mathrm{E}\left[\left|\mathrm{X}^{\mathrm{n}}(\mathrm{f} 1)\right|^{2} \mathrm{E}\left[|\mathrm{X}(\mathrm{nf} 1)|^{2}\right]\right)} \quad \mathrm{n}=1,2,3 . . \tag{2.2}
\end{equation*}
$$

The hyper-coherence function defines the nonlinear correlation between a reference frequency component in a vibration signal and its harmonics. A major benefit is determination of whether an apparent harmonic in a complex vibration signal is correlated with the fundamental or caused by extraneous noise. The technique has been frequently applied to space shuttle main engine turbopump measurements in order to discriminate between a true Sync harmonic and other independent components. A typical example is the discrimination between a true 3 N and a so-called "pseudo-3N" components.

### 2.2.2.2 Test Example for hyper-coherence Analysis

An example encountered from SSME hot firing test can best demonstrate the practical application of hyper-coherence for machinery diagnostics. Figures 2.1-a shows the PSD taken from a HPFTP accelerometer during SSME test 901-436. The peak marked "N" is corresponding to the fundamental RPM component, and the remaining major peaks are its harmonics $2 \mathrm{~N}, 3 \mathrm{~N}$ and 4 N . Engine failure occurred near the end of the test. Debris was found in a down stream coolant line during tear-down hardware inspection. Off-line diagnostic signal analysis was performed after the fact to investigate the failure. Anomaly was detected due to a high level amplitude of 3 N and this is considered to be defect signature associated with several different failure modes.

To confirm it as a defect signature, Hyper-Coherence analysis was performed. Figure $2.1-\mathrm{c}$ shows the hyper-coherence function resulting from this test. It shows the correlation between the fundamental RPM (N) and all of its harmonics including itself. High level coherence peaks indicate that $2 \mathrm{~N}, 3 \mathrm{~N}$, and 4 N are all highly correlated with the reference component $N$. Therefore, they are all true harmonics of the Sync frequency component, and the original high level 3N PSD of the test is a true defect signature.

A similar situation occurred later during another SSME engine test 901-471. Figure 2.1-b shows the PSD taken from a HPFTP accelerometer during this test with similar abnormally high 3N PSD amplitude. However, the hyper-coherence analysis was able to identify this apparent defect signature as a false-alarm. Figure 2.1-d shows the hypercoherence function resulting from this test. This time, the hyper-coherence at 3 N is much smaller. This indicates that the spectral peak at the 3 N frequency is not a true third harmonic. The signature turned out to be the so-called pseudo 3 N component which is due to some independent source in the HPFTP not related to the machinery rotational process.

It should be pointed out that even though the hyper-coherence is only defined at the harmonic frequencies of the reference frequency. However, if the frequency of a higher harmonic is greater than the nyquist frequency, it will be folded back below the nyquist frequency due to aliasing effect. In this case, the hyper-coherence can still pick correlation of this aliased harmonic despite the frequency of this aliased harmonic is no longer a integer multiple of the reference frequency. For this reason, the hyper-coherence is still computed for all frequencies other than the harmonic frequencies. However, the



Figure 2.2 HYPER-COHERENCE Hc(f), (Ref Freq= 500.00) of clipped Sine Wave
hyper-coherence below the reference frequency is not computed. This is because the phase of the reference term, $\mathrm{X}^{\mathrm{n}}$, in equation (1) becomes a small number when n becomes smaller than 1, which would appear to be a constant phase. Figure 2.2 shows a simulation example for aliased harmonics. The simulation signal is a sine wave at 500 Hz . The amplitude of the sine wave is truncated at one side in order to generated its harmonics. Figure 2.2(a) shows the PSD of the simulation signal. The reference component ( N ) along with its harmonics, $2 \mathrm{~N}, 3 \mathrm{~N}, \ldots$. upto to 10 N , are non-aliased harmonics. The peaks marked $(11 \mathrm{~N}),(12 \mathrm{~N})$, upto ( 20 N ) are aliased harmonics. The peaks marked [21N], [22N], upto [30N] are double aliased harmonics. Figure 2.2(b) shows its hyper-coherence function with 500 Hz as the reference frequency. The hypercoherence pick strong correlation for all the non-aliased harmonics ( 1 N to 10 N ), and moderate correlation for aliased (and doubled aliased) harmonics ( 11 N to 30 N ).

### 2.2.2.3 DSP Code Overview of hyper-coherence function using Data Translation's DSPEZ programming language running on C40 DSP.

The hyper-coherence algorithm has been implemented on ASRI's TI-C40 DSP, using Data Translation's DSP-EZ programming language. This section lists the complete DSP Codes of it. The DSP-EZ utilizes a series of well-defined functions to construct a signal processing application directly on DSP. Therefore, the processing algorithm/flow-chart of any particular application program can be clearly defined. This format has proved to be acceptable as conversion format..

| 1 | $* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *$ |
| :--- | :--- |
| 1 | $* * *$ Function Hyper Coherence $* * *$ |
| 1 | $* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *$ |

Function HyperCoherence( i1 As Integer,m1 As Integer)
Dim BHWnd As FloatArray
Dim WndData0 As FloatArray
Dim Tempbuff As FloatArray
Dim Fs As Single
Dim kref As Integer
Dim kmax As Integer
Dim kmin As Integer
Dim bw As Single
Dim R1 As Single
Dim R2 As Single
Dim R3 As Single
Dim R4 As Single
Dim J1 As Complex
Dim J2 As Complex
Dim J3 As Complex
$\operatorname{Dim} \mathrm{d}, \mathrm{j}, \mathrm{i}, \mathrm{m}$ As Integer
Dim dBScale As Single
Dim Power As Single



```
                            HAA6(d)=HAA6(d)+J1*J2
Else
                            HAA7(d)=HAA7(d)+J1*J2
End If
If il=0 Then
\(\operatorname{HBB} 0(\mathrm{~d})=\operatorname{HBB} 0(\mathrm{~d})+\operatorname{Abs}(\mathrm{J} 1) * \operatorname{Abs}(\mathrm{~J} 1)\)
ElseIf \(\mathrm{i} 1=1\) Then
\(\operatorname{HBB} 1(\mathrm{~d})=\operatorname{HBB} 1(\mathrm{~d})+\operatorname{Abs}(\mathrm{J} 1) * \operatorname{Abs}(\mathrm{~J} 1)\)
Elself i1=2 Then
\(\operatorname{HBB} 2(\mathrm{~d})=\mathrm{HBB} 2(\mathrm{~d})+\mathrm{Abs}(\mathrm{J} 1) * \operatorname{Abs}(\mathrm{~J} 1)\)
ElseIf \(\mathrm{i} 1=3\) Then
\(\operatorname{HBB} 3(\mathrm{~d})=\mathrm{HBB} 3(\mathrm{~d})+\mathrm{Abs}(\mathrm{J} 1) * \operatorname{Abs}(\mathrm{~J} 1)\)
ElseIf \(\mathrm{i} 1=4\) Then
\(\operatorname{HBB} 4(\mathrm{~d})=\mathrm{HBB} 4(\mathrm{~d})+\operatorname{Abs}(\mathrm{J} 1) * \operatorname{Abs}(\mathrm{~J} 1)\)
Elself il=5 Then
\(\operatorname{HBB5}(\mathrm{d})=\operatorname{HBB} 5(\mathrm{~d})+\operatorname{Abs}(\mathrm{J} 1) * \operatorname{Abs}(\mathrm{~J} 1)\)
Elself i1=6 Then
\(\operatorname{HBB6}(\mathrm{d})=\mathrm{HBB} 6(\mathrm{~d})+\operatorname{Abs}(\mathbf{J} 1) * \operatorname{Abs}(\mathbf{J} 1)\)
Else
\(\operatorname{HBB} 7(\mathrm{~d})=\mathrm{HBB} 7(\mathrm{~d})+\operatorname{Abs}(\mathrm{J} 1) * \operatorname{Abs}(\mathrm{~J} 1)\)
End If
\(\mathrm{R} 4=\mathrm{Abs}(\mathrm{XX}(\mathrm{d}))\)
If \(\mathrm{i} 1=0\) Then
\[
\mathrm{HCC} 0(\mathrm{~d})=\mathrm{HCC} 0(\mathrm{~d})+\mathrm{R} 4 * \mathrm{R} 4
\]
ElseIf i1=1 Then
\(\mathrm{HCC1}(\mathrm{~d})=\mathrm{HCC1}(\mathrm{~d})+\mathrm{R} 4 * \mathrm{R} 4\)
ElseIf i1=2 Then
\(\mathrm{HCC} 2(\mathrm{~d})=\mathrm{HCC} 2(\mathrm{~d})+\mathrm{R} 4 * \mathrm{R} 4\)
ElseIf i1=3 Then
\(\mathrm{HCC} 3(\mathrm{~d})=\mathrm{HCC} 3(\mathrm{~d})+\mathrm{R} 4 * \mathrm{R} 4\)
ElseIf i1=4 Then
\(\mathrm{HCC} 4(\mathrm{~d})=\mathrm{HCC} 4(\mathrm{~d})+\mathrm{R} 4 * \mathrm{R} 4\)
ElseIf \(\mathrm{i} 1=5\) Then
\(\mathrm{HCC} 5(\mathrm{~d})=\mathrm{HCC} 5(\mathrm{~d})+\mathrm{R} 4 * \mathrm{R} 4\)
ElseIf i1=6 Then
HCC6(d) \(=\mathrm{HCC} 6(\mathrm{~d})+\mathrm{R} 4 * \mathrm{R} 4\)
Else
\[
\mathrm{HCC} 7(\mathrm{~d})=\mathrm{HCC} 7(\mathrm{~d})+\mathrm{R} 4 * \mathrm{R} 4
\]
End If
Next d
tempbuff=AD.Scan(buffsize)
If ( \(\mathrm{ml}=\) NumOfAvg) Then
For \(\mathrm{d}=0\) To kmin- 1
If \(\mathrm{i} 1=0\) Then
\(\operatorname{HBCO}(\mathrm{d})=0.001\)
ElseIf il=1 Then
\(\mathrm{HBCl}(\mathrm{d})=0.001\)
ElseIf i1=2 Then
\(\mathrm{HBC} 2(\mathrm{~d})=0.001\)
ElseIf il=3 Then
\(\mathrm{HBC} 3(\mathrm{~d})=0.001\)
ElseIf i \(1=4\) Then
\(\mathrm{HBC} 4(\mathrm{~d})=0.001\)
```

```
ElseIf il=5 Then
                    HBC5(d)=0.001
ElseIf il=6 Then
            HBC6(d)=0.001
ElseIf i1=7 Then
            HBC7(d)=0.001
        End If
        Next d
            For d=kmin To kmax-1
            If il=0 Then
            HBC0(d)=Abs(HAA0(d)) * Abs(HAA0(d))/(HBB0(d)*HCC0(d))
            ElseIf il=1 Then
            HBC1(d)=Abs(HAA1(d)) * Abs(HAA1(d)) /(HBB1(d)*HCC1(d))
            ElseIf il=2 Then
            HBC2(d)=Abs(HAA2(d)) * Abs(HAA2(d)) /(HBB2(d)*HCC2(d))
            ElseIf il=3 Then
            HBC3(d)=Abs(HAA3(d))* Abs(HAA3(d))/(HBB3(d)*HCC3(d))
            ElseIf i1=4 Then
            HBC4(d)=Abs(HAA4(d)) * Abs(HAA4(d)) /(HBB4(d)*HCC4(d))
            ElseIf i1=5 Then
            HBC5(d)=Abs(HAA5(d)) * Abs(HAA5(d)) /(HBB5(d)*HCC5(d))
            ElseIf il=6 Then
            HBC6(d)=Abs(HAA6(d))* Abs(HAA6(d))/(HBB6(d)*HCC6(d))
            Else
            HBC7(d)=Abs(HAA7(d)) * Abs(HAA7(d)) /(HBB7(d)*HCC7(d))
                        End If
            Next d
    tempbuff=AD.Scan(buffsize)
    For d=0 To Nmax-1
            If i1=0 Then
            HPSD0(d)=10*log10(HPSD0(d)/NumOfAvg)+ dBScale
            ElseIf i1=1 Then
            HPSD1(d)=10*log10(HPSD1(d)/NumOfAvg)+ dBScale
            ElseIf il=2 Then
            HPSD2(d)=10*log10(HPSD2(d)/NumOfAvg)+ dBScale
            ElseIf il=3 Then
            HPSD3(d)=10*log10(HPSD3(d)/NumOfAvg)+ dBScale
            ElseIf il=4 Then
            HPSD4(d)=10* log10(HPSD4(d)/NumOfAvg)+ dBScale
            ElseIf i1=5 Then
            HPSD5(d)=10*log10(HPSD5(d)/NumOfAvg)+ dBScale
            ElseIf i1=6 Then
            HPSD6(d)=10*log10(HPSD6(d)/NumOfAvg)+ dBScale
            Else
            HPSD7(d)=10*log10(HPSD7(d)/NumOfAvg)+dBScale
            End If
            Next d
    End If
End Function
```


### 2.2.3 Bi-Spectral Analysis -Nonlinear correlation characteristics between three spectral components with frequency sum and difference

The dynamic response of a rotating machine can exhibit frequency sum or difference phenomena. This type of nonlinear phenomena is primarily due to amplitude modulation between various rotational components and fluid/structure interactions. Amplitude modulation can be generated by at least six phenomena. Enrich and Eshleman have published works using analytical techniques to explain how these modulations are physically generated. The characteristics of such nonlinear interactions are usually reflected in a response waveform which can be identified by using the bi-spectral technique.

However, due to the lack of phase relationship information among different frequency components, conventional linear spectral analysis is not able to describe or identify such nonlinear phenomena. In linear spectral analysis, the time series of any fluctuating physical quantity is regarded as the superposition of statistically uncorrelated waves and can be described, in part, by its power spectrum which shows the power distribution in the frequency domain. However, if nonlinear interactions occur, some coherent phase relationship exists among different frequency components. As a result, the information content of harmonics and sidebands for machinery fault diagnostics is usually not fully exploited.

### 2.2.3.1 General background, algorithm and simulation example of bi-coherence function

As the power spectral density (PSD) function is a second moment statistic of a random signal, the auto-bi-spectrum (ABS) represents the third joint moment among three different waves at frequencies $w_{1}, w_{2}$ and the sum frequency $w_{1}+w_{2}$, and can be estimated by:

$$
\begin{equation*}
B_{X X X}\left(\omega_{1}, \omega_{2}\right)=E\left[X\left(\omega_{1}\right) X\left(\omega_{2}\right) X^{*}\left(\omega_{1}+\omega_{2}\right)\right] \tag{3.1}
\end{equation*}
$$

Where $X(\omega)$ is the Fourier transform of $x(t)$. The auto bi-coherence $(A B C)$, a normalized bi-spectrum, is defined as:

$$
\begin{equation*}
b_{X X X}^{2}\left(\omega_{1}, \omega_{2}\right)=\frac{\left|B_{X X X}\left(\omega_{1}, \omega_{2}\right)\right|^{2}}{E\left[\left|X\left(\omega_{1}\right) X\left(\omega_{2}\right)\right|^{2}\right] E\left[\left|X\left(\omega_{1}+\omega_{2}\right)\right|^{2}\right]} \tag{3.2}
\end{equation*}
$$

The discrete time formulation for bi-coherence function is:

$$
\begin{equation*}
\hat{b}_{X X X}^{2}\left(\omega_{1}, \omega_{2}\right)=\frac{\left.1 \sum_{i=1}^{N A}\left[X_{i}\left(\omega_{1}\right) X_{i}\left(\omega_{2}\right) X_{i}^{*}\left(\omega_{1}+\omega_{2}\right)\right]\right|^{2}}{\left\{\sum_{i=1}^{N A}\left[\left|X_{i}\left(\omega_{1}\right) X_{i}\left(\omega_{2}\right)\right|^{2}\right]\right\}\left\{\sum_{i=1}^{N A}\left[\left|X_{i}^{*}\left(\omega_{1}+\omega_{2}\right)\right|^{2}\right]\right\}} \tag{3.3}
\end{equation*}
$$

Where $\quad \mathrm{X}_{\mathrm{i}}(\omega)$ is the FFT of the i -th block ensemble average data
NA is the total number of ensemble average blocks
By using Schwarz' inequality, it can be shown that the bi-coherence is always bounded by zero and unity. Bi -coherence $\mathrm{b}_{\mathrm{xxx}}\left(\omega_{1}, \omega_{2}\right)$ is a function of two frequencies, $\omega_{1}$ and $\omega_{2}$. Therefore, a three dimensional plot would be required in order to display a complete bi-coherence function. To minimize the computation requirement associated with the complete 3D bi-coherence function, a special bi-coherence function $\mathrm{b}_{\mathrm{XXX}}\left(\omega_{\mathrm{r}}, \omega\right)$ is frequently utilized for machinery diagnostic application. One of the bi-frequency arguments, $\omega_{1}$, can be fixed at some particular reference frequency of interest, ( e.g. $\omega_{1}=$ $\omega_{\mathrm{r}}=$ bearing characteristic frequency or Sync frequency, etc.) while the other frequency argument, $\omega_{2}=\omega$, sweeps through the entire analysis frequency range. As will be discussed next, there exists a certain limit for the second frequency argument, $\omega_{2}=\omega$, which is function of the reference frequency $\omega_{\mathrm{r}}$ and the sampling frequency $\omega_{\mathrm{S}}$.

The maximum frequency available for a discrete time data is the nyquist frequency, $\omega_{\mathrm{n}}=$ $\omega_{\mathrm{s}} / 2$, which is defined as half of its sampling frequency. Therefore, there exists a region over which the ABS is defined in the bi-frequency domain $\left(\omega_{1}, \omega_{2}\right)$ : First of all, both $\omega 1$ \& $\omega 2$ must be limited by $\pm \omega_{\mathrm{n}}$, which is corresponding to a square region in the bifrequency domain. In addition, the sum frequency $\omega 1+\omega 2$ of the third (implicit) spectral component in the ABS function must also be limited by $\pm \omega_{\mathrm{n}}$, which is corresponding to the region between the $2-45$ degree lines (line D-D \& E-E in figure 3.1) in the bifrequency domain. As a result, the ABS is only defined over the hexagon region as shown in figure 1. Furthermore, due to the symmetric property of the ABS, this hexagon region cab be further reduced. Referring to the definition of ABS in equation 3.1, bispectrum is the cumulant average of the 3 spectral components:
$B\left(\omega_{1}, \omega_{2}\right)=E\left[X\left(\omega_{1}\right) X\left(\omega_{2}\right) X\left(-\omega_{1}-\omega_{2}\right)\right] . \quad\left\{\right.$ Note: $\left.X^{*}\left(\omega_{1}+\omega_{2}\right)=X\left(-\omega_{1}-\omega_{2}\right)\right\}$

Since the order of the spectral sequence is irrelevant, a total of 6 different permutations of these 3 spectral components exist which would lead to 6 identity among ABS:
permutations:
$\left[\omega_{1}, \omega_{2} ;-\omega_{1}-\omega_{2}\right]\left[\omega_{2},-\omega_{1}-\omega_{2} ; \omega_{1},\right]\left[-\omega_{1}-\omega_{2}, \omega_{1} ; \omega_{2}\right]$
$\left[\omega_{2}, \omega_{1} ;-\omega_{1}-\omega_{2}\right]\left[\omega_{1},-\omega_{1}-\omega_{2} ; \omega_{2}\right]\left[-\omega_{1}-\omega_{2}, \omega_{2} ; \omega_{1}\right]$
Identities:

$$
\begin{aligned}
\operatorname{Bxxx}\left(\omega_{1}, \omega_{2}\right) & =\operatorname{Bxxx}\left(\omega_{2},-\omega_{1}-\omega_{2}\right) \\
& =\operatorname{Bxxx}\left(\omega_{2}, \omega_{1}\right)
\end{aligned} \quad=\operatorname{Bxxx}\left(-\omega_{1}-\omega_{2}, \omega_{1}\right) \quad\left\{\begin{array}{l}
\left.1,-\omega_{1}-\omega_{2}\right)=\operatorname{Bxxx}\left(-\omega_{1}-\omega_{2}, \omega_{2}\right)
\end{array}\right.
$$

Furthermore, the complex conjugate of the ABS provides additional 6 different permutations and 6 more identities as:
$B^{*}\left(\omega_{1}, \omega_{2}\right)=E\left[X^{*}\left(\omega_{1}\right) X^{*}\left(\omega_{2}\right) X^{*}\left(-\omega_{1}-\omega_{2}\right)\right]=\mathrm{E}\left[X\left(-\omega_{1}\right) X\left(-\omega_{2}\right) X\left(\omega_{1}+\omega_{2}\right)\right]$.
permutations:

$$
\begin{aligned}
& {\left[-\omega_{1},-\omega_{2} ; \omega_{1}+\omega_{2}\right]\left[-\omega_{2}, \omega_{1}+\omega_{2} ;-\omega_{1},\right]\left[\omega_{1}+\omega_{2},-\omega_{1} ;-\omega_{2}\right]} \\
& {\left[-\omega_{2},-\omega_{1} ; \omega_{1}+\omega_{2}\right]\left[-\omega_{1}, \omega_{1}+\omega_{2} ;-\omega_{2},\right]\left[\omega_{1}+\omega_{2},-\omega_{2} ;-\omega_{1}\right]}
\end{aligned}
$$

Identities:

$$
\begin{aligned}
\operatorname{Bxxx}\left(\omega_{1}, \omega_{2}\right) & =B^{*} \operatorname{xxx}\left(-\omega_{1},-\omega_{2}\right) \\
=B^{*} \operatorname{xxx}\left(-\omega_{2},-\omega_{1}\right) & =B^{*} \operatorname{xxx}\left(-\omega_{2}, \omega_{1}+\omega_{2}\right)=B^{*} \operatorname{xxx}\left(\omega_{1}+\omega_{2},-\omega_{1}+\omega_{2}\right)=B^{*} \operatorname{xxx}\left(\omega_{1}+\omega_{2},-\omega_{2}\right)
\end{aligned}
$$

This would lead to the following 3 symmetric properties of ABS :

1. Symmetric with respect to $45^{\circ}$ line A-A
2. Conjugate Symmetric with respect to $-45^{\circ}$ line $\mathrm{B}-\mathrm{B}$
3. Symmetric with respect to $-30^{\circ}$ line C-C: $\omega_{1}=-2 \omega_{2}$


Figure 3.1 Symmetric Property of Auto Bi-Coherence Function
If the ABS in the triangular region in the 1st quadrant (the shaded triangle in figure 1 ) is known, then the ABS in the entire hexagon region can be obtained based on these 3 symmetric properties, . As a result, this triangular region is the only region over which the computation of ABS is required. However, for the special bi-coherence function $\mathrm{b}\left(\omega_{\mathrm{r}}, \omega\right)$ discussed above, the frequency argument, $\omega$, should sweep through the entire first quadrant from 0 upto the -45 degree boundary line (line D-D). The parameter "fmax" in the computer codes to be discussed below is refereed to this boundary line.

### 2.2.3.2 Test Example of bi-coherence function

The bi-coherence algorithm implemented on ASRI's TI-C40 DSP has been preliminary tested utilizing a carefully-design test signal containing both nonlinearly-correlated wave and linearly-independent wave at the same frequency so that, with a known degree of nonlinear correlation in the test signal, the resulting bi-coherence value can be evaluated for its accuracy. In addition, Mr. Bapty of Vanderbilt requested ASRI to provide a set of simulation data with correct bi-coherence results so that he can test the bi-coherence program on the MPP system for verification purpose. Therefore, a simple simulation example has been designed which generates three time histories $\mathrm{x} 1(\mathrm{t}), \mathrm{x} 2(\mathrm{t})$ and $\mathrm{x} 3(\mathrm{t})$ as follows:

$$
\begin{aligned}
X_{1}(t) & =\cos \left(\omega_{a} t+\Phi_{1}\right)+\cos \left(\omega_{b} t+\Phi_{2}\right)+\cos \left[\left(\omega_{a}+\omega_{b}\right) t+\left(\Phi_{1}+\Phi_{2}\right)\right]+N_{1}(t) \\
X_{2}(t) & =\cos \left(\omega_{a} t+\Phi_{1}\right)+\cos \left(\omega_{b} t+\Phi_{2}\right)+\cos \left[\left(\omega_{a}+\omega_{b}\right) t+\Phi_{3}\right]+N_{2}(t) \\
X_{3}(t) & =\cos \left(\omega_{a} t+\Phi_{1}\right)+\cos \left(\omega_{b} t+\Phi_{2}\right)+0.5^{*} \cos \left[\left(\omega_{a}+\omega_{b}\right) t+\left(\Phi_{1}+\Phi_{2}\right)\right] \\
& +0.5^{*} \cos \left[\left(\omega_{a}+\omega_{b}\right) t+\Phi_{3}\right]+N_{3}(t)
\end{aligned}
$$

where $\phi 1, \phi 2$ and $\phi 3$ are independent random phase; $N 1(t), N 2(t)$ and $N 3(t)$ are independent Gaussian White Noise, and the sum frequency $\omega_{C}(2700 \mathrm{~Hz})=\omega_{a}$ $(1000 \mathrm{~Hz})+\omega_{\mathrm{b}}(1700 \mathrm{~Hz})$, is the frequency of the third sine wave. Each one of these time series is composed of three sine waves at frequency $\omega_{\mathrm{a}}, \omega_{\mathrm{b}}$ and $\omega_{\mathrm{C}}$. The first two waves at $\omega_{\mathrm{a}}$ and $\omega_{\mathrm{b}}$ are identical for each of the time histories. But the third wave at $\omega$ c is quite different. For $\mathrm{x} 1(\mathrm{t})$, the phase at $\omega_{\mathrm{C}}$ is equal to the sum of the phases at $\omega_{\mathrm{a}}$ and $\omega_{\mathrm{b}}$, which represents perfect phase coupling. For $\mathrm{x} 2(\mathrm{t})$, the random phase at $\omega_{\mathrm{C}}$ indicates total independence among these three sine waves. For $\mathrm{x} 3(\mathrm{t}), 50 \%$ of the component at $\omega_{\mathrm{C}}$ is phase coupled with $\omega_{\mathrm{a}}$ and $\omega_{\mathrm{b}}$, and the remaining $50 \%$ with random phase independent of $\omega_{\mathrm{a}}$ and $\omega_{\mathrm{b}}$.

Since all these three signal have the same energy distribution in frequency domain, therefore, they all have identical PSD (Power spectral density) as shown in figures 3.2-a, $3.2-\mathrm{b}$ and $3.2-\mathrm{c}$ for $\mathrm{x} 1(\mathrm{t})$, $\mathrm{x} 2(\mathrm{t})$ and $\mathrm{x} 3(\mathrm{t})$ respectively. Figures $3.3-\mathrm{a}, 3.3-\mathrm{b}$, and $3.3-\mathrm{c}$ show their corresponding auto-bicoherence function with the first frequency argument $\omega 1$ fixed at 1000 Hz . The X -axis in figure 3.3 represents the second frequency argument $\omega 2$ of the bi-coherence function. In figure 3.3-a, there is a strong peak at bi-frequency ( $\omega$ $\left.\mathrm{a}=1000 \mathrm{~Hz}, \omega_{\mathrm{b}}=1700 \mathrm{~Hz}\right)$ with amplitude close to one. This indicates the components at $\omega_{\mathrm{a}}, \omega_{\mathrm{b}}$ and $\omega_{\mathrm{c}}$ in $\mathrm{x} 1(\mathrm{t})$ are highly correlated due to strong phase coupling. For $\mathrm{x} 2(\mathrm{t})$, since the phases at these three frequencies are independent random phases, no significant bicoherence peaks are apparent in figure 3.3-b. For $\mathrm{x} 3(\mathrm{t})$, the amplitude at bi-frequency $\left(\omega_{\mathrm{a}}=1000 \mathrm{~Hz}, \omega_{\mathrm{b}}=1700 \mathrm{~Hz}\right)$ in figure 3.3-c, is only 0.729 . The corresponding bicoherence square $\left(0.729^{2}=0.53\right)$ indicates that only $50 \%$ of the power at $\omega_{c}$ is correlated with $\omega_{\mathrm{a}}$ and $\omega_{\mathrm{b}}$, and the remainder is totally independent. Therefore, the bi-coherence analysis completely distinguishes these three time histories even though they have identical PSDs.



### 2.2.3.3 DSP Code Overview of bi-coherence function using Data Translation's DSPEZ programming language running on C40 DSP

The bi-coherence algorithm has been implemented on ASRI's TI-C40 DSP, using Data Translation's DSP-EZ programming language. This section lists the complete DSP Codes of $i t$.

Complete codes of bi-coherence function $b\left(\omega_{r}, \omega\right)$ on C40 DSP using Data Translation's DSP-EZ programming language

```
'+--------------------------
```

'| Bicoherence.DSP |
'+---------------------------

Const PERCENTchan $=0$
Const Col5chan=1
Const TimeChan $1=1$
Const FFTChan $1=2$
Const BPTimeChan1=3
Const BPFFTChan $1=4$
Const ABSChan $1=5$
Const FILTchan $=6$
Const FILTERPOINTS $=0$
Const CUTOFFLower $=1$
Const CUTOFFUpper $=2$
Const FILTER_SIZE $=201$
Const BUFFSIZEE $=2048$
Const FFT_SIZE $=2048$
'+-----------+
'| Arrays |
'+------------+
Dim ADbuff As FloatArray
Dim BPFilter As FloatArray
Dim TimeData As FloatArray
Dim FFTdata As ComplexArray
Dim ImpulseResponse As FloatArray
Dim XX As ComplexArray 'Bicoherence calculation
Dim PSD As FloatArray
Dim AA As ComplexArray
Dim BB As FloatArray
Dim CC As FloatArray
Dim BC As FloatArray
'+---------------+
'| Variables |
'+----------------+

Dim Frequency As Single
Dim FMax As Single

```
        Dim dbScale As Single
        Dim MaxFreq As Single
        Dim MinFreq As Single
        Dim Kmax As Single
        Dim NumOfAvg As Single
        ************************
        **** Function Main ****
        ***********************
Function Main()
'+------------------
'| Declaration |
'+-----------------+
Dim cmdbuff As FloatArray
Dim Utilization As FloatArray
Dim DAdata As FloatArray
Dim BHWnd As FloatArray
Dim WndData As FloatArray
Dim MagData As FloatArray
Dim ZZ As ComplexArray
Dim YY As ComplexArray
Dim XX1 As ComplexArray
Dim ADREADY As Integer
Dim SampleTime As Single
Dim RetrieveTime As Single
Dim Parameterchecked As Integer
Dim i, j, k, kount,m,n, ecode, Length, start, \(\mathrm{t}, \mathrm{d}, \mathrm{p}, \mathrm{DSize}\) As Integer
Dim bw, ref, nmax, Fs As Single
Dim R1, R2, R3, R4, kref As Single
Dim J1 As Complex
Dim J2 As Complex
Dim J3 As Complex
```

```
'| get the initial A/D frequency |
```

'| get the initial A/D frequency |
'+---------------------------------------------
'+---------------------------------------------
'kount = 0
'kount = 0
Do While Not Host.GetDataReady(CTRLchan)
Do While Not Host.GetDataReady(CTRLchan)
Loop
Loop
cmdbuff = Host.GetBuffer(CTRLchan)
cmdbuff = Host.GetBuffer(CTRLchan)
Frequency = cmdbuff(0)
Frequency = cmdbuff(0)
'+------------------------------------
'+------------------------------------
'I Define A/D parameters |
'I Define A/D parameters |
'+-------------------------------------
'+-------------------------------------
AD.Frequency = Frequency
AD.Frequency = Frequency
AD.ClockSource = DTDSP_ICLK
AD.ClockSource = DTDSP_ICLK
AD.TriggerSource = DTDSP
AD.TriggerSource = DTDSP
AD.NumChannels =1
AD.NumChannels =1
AD.AcqMode = DTDSP_WRAP
AD.AcqMode = DTDSP_WRAP
AD.InternalChannelBufferSize = BUFFSIZE*2
AD.InternalChannelBufferSize = BUFFSIZE*2
'+------------------------------------
'+------------------------------------
II Define D/A parameters I
II Define D/A parameters I
+----------------------------------
+----------------------------------
'DA.Frequency = Frequency
'DA.Frequency = Frequency
'DA.ClockSource = DTDSP_ICLK

```
'DA.ClockSource = DTDSP_ICLK
```

```
'DA.TriggerSource = DTDSP_ITRG
'DA.NumChannels = 1
DA.AcqMode = DTDSP_WRAP
'DA.InternalChannelBufferSize = BUFFSIZE*2
'+-----------------------------------------------
'I initialize global array to all zeros |
'+---------------------------------------------
ImpulseResponse(BUFFSIZE/2) = FFT_SIZE
Do While Not Host.GetDataReady(CTRLchan)
Loop
'+---------------------
'| initialize filter |
P+---------------
'+--------------------------------------
'| generate Window Data |
'+-------------------------------------
BHWnd = GenWindow(BLACKMAN HARRIS_61,BUFFSIZE)
dBScale = -20* log10(10*FFT_SIZE) +6.0
SampleTime = 1.0/AD.Frequency * FFT_SIZE
RetrieveTime = 0
Utilization = GenLine(0,0,1)
'+-----------------------------
II start acquiring data |
'+----------------------------
AD.ChannelType=DTDSP_DI
AD.Filter=AD.Frequency /2
AD.Start()
1 *****************
l * Main looping *
l}*********************
'+--------------------------------------------------------
'I acquire parameters from buffer
Fs=Frequency/2
bw=Fs/BUFFSIZE
nmax=BUFFSIZE/2
ref=500
Start=0
Length = BUFFSIZE/2
NumOfAVG=10
ParameterChecked = 1
XX=GenComplexCos(0,0,10000,FFT_SIZE/2+1)
Do While ParameterChecked = 1
    If Host.GetDataReady(Col5chan) Then
        cmdbuff=Host.GetBuffer(Col5chan)
        NumOfAvg=cmdbuff(0)
        ref=cmdbuff(1)
    End If
    PSD=GenLine(0,0,FFT_SIZE/2+1)
    kref=ref/bw+0.5
    kmax=nmax-kref
```

AA=GenComplexCos(0,0,10000,Kmax+1)
AA $=$ AA * 0
BB=GenLine(0,0,Kmax+1)
CC=GenLine(0,0,Kmax+1)
BC=GenLine(0,0,FFT_SIZE/2+1)
For m = 1 To NumOfAVG
TimeStart()
ADbuff = AD.Scan (BUFFSIZE)
RetrieveTime = TimeRead()
TimeStop()

For $\mathrm{d}=0$ To kmax
$\mathrm{BC}(\mathrm{d})=\operatorname{Abs}(\mathrm{AA}(\mathrm{d})) * \operatorname{Abs}(\mathrm{AA}(\mathrm{d})) /(\mathrm{BB}(\mathrm{d}) * \mathrm{CC}(\mathrm{d}))$
Next d
For $\mathrm{d}=\mathrm{Kmax}+1$ To Nmax
$B C(d)=0$
Next d
If Host.PutDataReady(BPTimechan1) Then
Host.PutBuffer(BPTimechan1)=BC
If Host.PutDataReady(PERCENTchan) Then
Host.PutBuffer(PERCENTchan) $=$ Utilization
End If
End If
For $\mathrm{d}=0$ To nmax $\operatorname{PSD}(\mathrm{d})=10^{*} \log 10(\operatorname{PSD}(\mathrm{~d}) / \mathrm{NumOfAvg})+\mathrm{dBScale}$
Next d
If( Host.PutDataReady(ABSchan1)) Then
Host.PutBuffer(ABSchan1) = PSD
If Host.PutDataReady(PERCENTchan) Then
Host.PutBuffer(PERCENTchan) $=$ Utilization
End If
End If
'+---------------------------------------------
'I Upgrade filter if necessary
'+----------------------------------------------1
If (Host.GetDataReady(CTRLchan) And (AD.DataReady(0) < BUFFSIZE))
Then
ProcessControl()
End If
Loop 'End of Do While loop
End Function
*********************************
**** Function ProcessControl $* * * *$
*********************************
Function ProcessControl()
Dim ctrlbuff As FloatArray
Dim FilterDataTemp As FloatArray
Dim FFTDataTemp As ComplexArray
Dim ready As Integer
Dim lowercut, uppercut As Single
Dim length As Integer
ready = TRUE
Do While ready
ctrlbuff = Host.GetBuffer(CTRLchan)
ready $=$ Host.GetDataReady(CTRLchan)
Loop
'+------------------------------------
| | generate a band pass filter object |
'+-----------------------------------
lowercut $=$ ctrlbuff(CUTOFFLower)
uppercut =ctrlbuff(CUTOFFUpper)
length $=\operatorname{ctrlbuff}($ FILTERPOINTS $)$
BPFilter $=$ InitBandPassFilter(lowercut,uppercut,Frequency,length)
FilterDataTemp $=$ Filter(BPFilter,ImpulseResponse)

FFTDataTemp $=\mathrm{fft}($ FilterDataTemp)
FilterDataTemp $=\mathrm{dB}$ (FFTDataTemp)
FilterDataTemp $=$ FilterDataTemp $-20 * \log 10($ FFT_SIZE $)+6.0$
'+----------------------------------------1
'| Send data to host for filter plotting |
'+---------------------------------------1
Host.PutBuffer(FILTchan) = FilterDataTemp
End Function


Function ApplyFFT(SomeData As FloatArray) As FloatArray
Dim Temp 1 As FloatArray
Dim Temp2 As ComplexArray
Dim BHWnd As FloatArray
BHWnd=GenWindow(BLACKMAN_HARRIS_61,BUFFSIZE)
Templ=Window(BHWnd, SomeData)
Temp2=fft(Temp1)
ApplyFFT $=\mathrm{dB}($ Temp2 $)+\mathrm{dBScale}$
End Function
$* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *$
$* * \quad$ Function CopyVector
$* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *$$\quad * *$

Function CopyVector(XX As ComplexArray, SomeVal As Integer) As ComplexArray Dim Temp As ComplexArray
Dim i As Integer
Temp $=$ GenComplexCos $(0,0,10000, \mathrm{Kmax}+1)$
For $\mathrm{i}=0$ To kmax
Temp $(\mathrm{i})=\mathrm{XX}($ SomeVal +i$)$
Next i
CopyVector $=$ Temp
End Function

```
** Function Conjugate **
*********************************
```

Function Conjugate( SomeData As ComplexArray) As ComplexArray
Dim Temp As ComplexArray
Dim i As Integer
Dim r1, r2 As Single
Temp $=$ GenComplexCos $(0,0,10000, \mathrm{Kmax}+1)$
Temp $=$ SomeData
For $\mathrm{i}=0$ To Kmax
$\mathrm{r} 1=-\operatorname{Imag}(\operatorname{Temp}(\mathrm{i}))$
$\mathrm{r} 2=\operatorname{Real}(\operatorname{Temp}(\mathrm{i}))$
$\operatorname{Temp}(\mathrm{i})=\operatorname{MakeComplex}(\mathrm{r} 2, \mathrm{r} 1)$
Next i
Conjugate $=$ Temp
End Function

### 2.2.4 Tri-Spectral Analysis - higher order coherence estimations among four spectral components and modulation sideband

### 2.2.4.1 General background, algorithm and simulation example of tri-coherence function

The auto-tri-coherence function has been implemented on a TI-C40 DSP, and has been converted to MSFC's MPP system. The tri-spectral analysis can identify a different type of correlation when four different spectral components are cubically correlated. A special case of it application is to identify whether an apparent PSD sideband is due to modulation or simply three independent waves. In machinery diagnostics, many failure modes will cause amplitude modulation between an envelope signal and its carrier signal, and, as a result, will generate a pair of sidebands around a center frequency component. Therefore, such sideband phenomenon is commonly treated as an off-nominal signature. The computer code for the tri-coherence program is currently being implemented and tested on ASRI's C40/PC computer system. In addition, a simulation example to be used by Vanderbilt for the verification of the tri-coherence program in the MPP system has been designed, and will be provided to Vanderbilt. The complete codes in a conversion format from ASRI's DSP system to Vanderbilt MGA Model Based Programming environment will be transferred to Vanderbilt in the next reporting period. The technical discussion in this report provides some background information to aide Vanderbilt's programmers in implementing and testing the tri-coherence program.

The tri-spectrum represents a fourth joint moment among four different waves. For a stationary random time series $\mathrm{x}(\mathrm{t})$, its fourth order cumulant spectrum, the tri-spectrum, is defined as :

$$
\begin{align*}
T_{X X X X}\left(\omega_{1}, \omega_{2}, \omega_{3}\right)= & E\left[X\left(\omega_{1}\right) X\left(\omega_{2}\right) X\left(\omega_{3}\right) X^{*}\left(\omega_{1}+\omega_{2}+\omega_{3}\right)\right] \\
& -E\left[X\left(\omega_{1}\right) X\left(\omega_{2}\right)\right] E\left[X\left(\omega_{3}\right) X^{*}\left(\omega_{1}+\omega_{2}+\omega_{3}\right)\right]  \tag{4.1}\\
& -E\left[X\left(\omega_{1}\right) X\left(\omega_{3}\right)\right] E\left[X\left(\omega_{2}\right) X^{*}\left(\omega_{1}+\omega_{2}+\omega_{3}\right)\right] \\
& -E\left[X\left(\omega_{2}\right) X\left(\omega_{3}\right)\right] E\left[X\left(\omega_{1}\right) X^{*}\left(\omega_{1}+\omega_{2}+\omega_{3}\right)\right]
\end{align*}
$$

Where $X(\omega)$ is the Fourier transform of $x(t)$. And the tri-coherence is a normalized trispectrum, normalized by the power of the four spectral components at $w 1, w 2, w 3$ and w1+w2+w3.

In general, the tri-spectrum $\mathrm{T}_{\mathrm{Xxxx}}\left(\mathrm{w}_{1}, \mathrm{w}_{2}, \mathrm{w}_{3}\right)$ is a function of three frequency arguments. However, a special tricoherence function $\mathrm{T}_{\mathrm{X}}(\mathrm{w})$ defined in equation 2 can be used to identify the correlation within a PSD sideband pattern by fixing the first and second frequency arguments, w1 \& w2, at the frequencies of two consecutive components within a cluster of sidebands spaced by frequency $\Delta$. This special tricoherence function can be used to identify whether an apparent PSD sideband structure is due to modulation signal or simply three totally independent components.

$$
\begin{align*}
T_{X}\left(\omega \mid \omega_{s 1}, \omega_{s 2}\right)= & T_{X X X X}\left(-\omega_{s 1}, \omega_{s 2}, \omega\right)=E\left[X^{*}\left(\omega_{s 1}\right) X\left(\omega_{s 2}\right) X(\omega) X^{*}\left(-\omega_{s 1}+\omega_{s 2}+\omega\right)\right] \\
& -E\left[X^{*}\left(\omega_{s 1}\right) X\left(\omega_{s 2}\right)\right] E\left[X(\omega) X^{*}\left(-\omega_{s 1}+\omega_{s 2}+\omega\right)\right] \tag{4.2}
\end{align*}
$$

Where $\quad \omega_{\mathrm{S} 1}=$ Frequency of a Sideband component

$$
\omega_{\mathrm{s} 2}=\text { Frequency of the Next Sideband Component (i.e. } \omega_{\mathrm{s} 2}=\omega_{\mathrm{s} 1}+\Delta \text { ) }
$$

The auto tri-coherence, $\mathrm{t}_{\mathrm{X}}(\mathrm{w})$, a normalized tri-spectrum, is defined as:

$$
\begin{equation*}
t_{X}^{2}\left(\omega \mid \omega_{s 1}, \omega_{s 2}\right)==\frac{\left|T_{X}\left(\omega \mid \omega_{s 1}, \omega_{s 2}\right)\right|^{2}}{E\left[\left|X\left(\omega_{s 1}\right) X\left(\omega_{s 2}\right) X(\omega)\right|^{2}\right] E\left[\left|X^{*}\left(\omega_{s 1}+\omega_{s 2}+\omega\right)\right|^{2}\right]} \tag{4.3}
\end{equation*}
$$

The discrete time formulation for the special tri-coherence function is:

$$
\begin{equation*}
\hat{t}_{X}^{2}\left(\omega \mid \omega_{s 1}, \omega_{s 2}\right)=\frac{\left|\sum_{i=1}^{N A}\left[X_{i}^{*}\left(\omega_{s 1}\right) X_{i}\left(\omega_{s 2}\right) X_{i}(\omega) X_{i}^{*}\left(-\omega_{s 1}+\omega_{s 2}+\omega\right)\right]-\left[X_{i}^{*}\left(\omega_{s 1}\right) X_{i}(\omega)\right]\left[X_{i}\left(\omega_{s 2}\right) X_{i}^{*}\left(-\omega_{s 1}+\omega_{s 2}+\omega\right)\right]\right|^{2}}{\left.\left.\left.\left\{\sum_{i=1}^{N A} \| X_{i}^{*}\left(\omega_{s 1}\right) X_{i}\left(\omega_{s 2}\right) X_{i}(\omega)\right)^{2}\right]\right\}\left(\sum_{i=1}^{N A}\left|X_{i}^{*}\left(-\omega_{s 1}+\omega_{s 2}+\omega\right)\right|^{2}\right]\right\}} \tag{4.4}
\end{equation*}
$$

Where $\quad X_{i}(w)$ is the FFT of the $i$-th block ensemble average data NA is the total number of ensemble average blocks
$\mathrm{w}_{\mathrm{s} 1}=$ Frequency of a Sideband component
$w_{s} 2=$ Frequency of the Next Sideband Component (i.e. $w_{s} 2=w_{s} 1+D$ )
Due to the existence of nonlinear phase correlation among the participating sideband components, this special tri-coherence function should show a strong coherence peaks at these sideband frequencies except at $\omega_{\mathrm{s}}$ as well as the last component within the sideband sequence. Conversely, due to a lack of phase correlation, no coherence peak will be identified for a sequence of independent (uncorrelated) spectral peaks. As a result, this special tri-coherence function can discriminate between a modulation sideband signal and independent sideband signal.

It should be pointed out that, the purpose of the second numerator term in equation (4.4):

$$
\begin{equation*}
\mathrm{I} \sum\left[X_{i}^{*}\left(\omega_{s 1}\right) X_{i}(\omega)\right]\left[X_{i}\left(\omega_{s 2}\right) X_{i}^{*}\left(-\omega_{s 1}+\omega_{s 2}+\omega\right)\right] \mathrm{I}^{2} \tag{4.5}
\end{equation*}
$$

is to force $\hat{t}_{X}^{2}\left(\omega \mid \omega_{s 1}, \omega_{s 2}\right)=0$ when $\mathrm{w}=\mathrm{w}_{\mathrm{s} 1}$, since its tri-spectrum at this frequency

$$
\begin{aligned}
T_{X}\left(\omega_{s 1} \mid \omega_{s 1}, \omega_{s 2}\right) & =T_{X X X X}\left(-\omega_{s 1}, \omega_{s 2}, \omega_{s 1}\right)=E\left[X^{*}\left(\omega_{s 1}\right) X\left(\omega_{s 2}\right) X\left(\omega_{s 1}\right) X^{*}\left(\omega_{s 2}\right)\right] \\
& =E\left[\left|X^{*}\left(\omega_{s 1}\right)\right|^{2}\left|X\left(\omega_{s 2}\right)\right|^{2}\right]
\end{aligned}
$$

The frequency argument, $\omega$, of the special tri-coherence function, $\mathrm{t}_{\mathrm{X}}(\mathrm{w})$, is defined over a frequency range from 0 to a maximum frequency $\omega_{\max }$. This limit frequency for $\omega$ is a function of the reference frequency $\omega_{\mathrm{S} 1}, \omega_{\mathrm{s} 2}$ and the sampling frequency, since $\mathrm{t}_{\mathrm{X}}(\mathrm{w})$ is only defined over the triangular region as shown in figure 4.1. Therefore the frequency argument, $\omega$, should sweep through the entire first quadrant from 0 upto the -45 degree boundary line which corresponding to the maximum frequency $\omega_{\max }$.


Figure 4.1 Frequency range for the special Auto Tri-Coherence Function

### 2.2.4.2 Simulation Example for Tri-Coherence Analysis Program

A simulation example has been designed and generated to demonstrate the sideband identification capability of the tri-coherence function. The data of this simulation example will also be provide to Vanderbilt for testing and verification of the tricoherence program in the MPP system. The modulation sideband signal $x(t)$ is composed of an envelop signal at frequency $\omega_{\mathrm{e}}$ and drifting phase $\psi_{\mathrm{e}}{ }^{(\mathrm{t})}$, and a carrier signal at frequency $\omega_{\mathrm{c}}$ \& phase $\psi_{\mathrm{c}}(\mathrm{t})$. A small amount of Gaussian White Noise is added to the signal. The sampling frequency is $10,240 \mathrm{~Hz}$. The modulation (multiplication) between the envelop and the carrier signal will generate a PSD side-band structure with left, center and right side-band components at frequencies $\omega_{\mathrm{C}}-\omega_{\mathrm{e}}, \omega_{\mathrm{C}}$, and $\omega_{\mathrm{C}}+\omega_{\mathrm{e}}$ respectively. Notice that, not only the frequency becomes sum and difference, but also their phase become sum and difference. Such phase correlation can be identified by using the special tri-coherence. The independent sideband signal $y(t)$ is composed of three independent sine waves at frequency $\omega_{\mathrm{C}}-\omega_{\mathrm{e}}, \omega_{\mathrm{C}}, \omega_{\mathrm{C}}+\omega_{\mathrm{e}}$, and phase $\psi_{\mathrm{L}}(\mathrm{t}), \psi_{\mathrm{C}}(\mathrm{t}) \psi_{\mathrm{R}}(\mathrm{t})$, respectively. The PSD of this independent sideband signal will have an apparent side-band structure with left, center and right side-band components at frequencies $\omega_{\mathrm{c}}-\omega_{\mathrm{e}}, \omega_{\mathrm{c}}$, and $\omega_{\mathrm{C}}+\omega_{\mathrm{e}}$ respectively. However, the phases among all these three waves within $y(t)$ are totally independent of each other.




$x(t)=\left\{10+2 * \cos \left[\omega_{e} t+\psi_{e}(t)\right]\right\} \cos \left[\omega_{c} t+\psi_{c}(t)\right]+G W N(t)$
$y(t)=10^{*}\left\{\cos \left[\left(\omega_{c}-\omega_{e}\right) t+\psi_{L}(t)\right]+\cos \left[\omega_{c} t+\psi_{c}(t)\right]+\cos \left[\left(\omega_{c}+\omega_{e}\right) t+\psi_{R}(t)\right]\right\}+G W N(t)$
Where $\quad \omega_{\mathrm{e}}=600 \mathrm{~Hz}$
$\omega_{\mathrm{C}}=3000 \mathrm{~Hz}$
Figure 4.2 and 4.3 shows the time histories and PSDs of $x(t)$ and $y(t)$. An apparent sideband structure at $2400 \mathrm{~Hz}, 3000 \mathrm{~Hz}$, and 3600 Hz is shown in both PSDs which are almost identical to each other. From the time history and PSD, it is difficult to discriminate between the independent and modulation sideband signal. This is because their difference is hidden in the phase coupling relationship among different frequencies. Figure 4.4 shows the special tri-coherence function for $y(t)$ and $x(t)$ with the first and the second frequency arguments fixed at the negative left side-band $(-2400 \mathrm{~Hz})$ and the center frequency $(3000 \mathrm{~Hz})$. The frequency of the tri-coherence is defined over the 0 to 4520 Hz (i.e. $5120-600 \mathrm{~Hz}$ ). For the modulation sideband signal $x(t)$, the tri-coherence indeed identify a strong coherence at the center frequency, 3000 Hz , which indicates that it is a true modulation generated side-band signal. While for the independent sideband signal $y(t)$, no significant correlation is identified since the apparent side-band in $y(t)$ is due to three totally independent waves. Therefore, the tri-coherence analysis successfully discriminate between these 2 sideband signal which all have identical PSD.

### 2.2.4.3 DSP Code Overview of tri-coherence function using Data Translation's DSPEZ programming language running on C40 DSP

```
'+------------------------
| TricCoherence.DSP |
'+-------------------------
                                    ****************************************************
                                    **** DSP Basic Program Global Declaration ****
                                    ***************************************************
    Option Explicit
'+----------------
| Constants |
'+------------------
    Const CTRLchan =0
    Const Col5chan=1
    Const PERCENTchan = 0
                Const TimeChan1 = 1
    Const FFTChan1 =2
    Const BPTimeChan1=3
    Const BPFFTChan1=4
    Const ABSChan1=5
        Const TimeChan2 = 6
    Const FFTChan2 =7
    Const BPTimeChan2=8
```

| Const BPFFTChan $2=9$ |
| :---: |
| Const ABSChan2 $=10$ |
| Const FILTchan $=11$ |
| Const RealChan $=12$ |
| Const FILTERPOINTS $=0$ |
| Const CUTOFFLower $=1$ |
| Const CUTOFFUpper $=2$ |
| Const FILTER_SIZE =201 |
| Const BUFFSIZE $=2048$ |
| Const FFT_SIZE $=2048$ |
| '+----------+ |
| '\| Arrays | |
| Dim ADbuff As FloatArray |
| Dim BPFilter0 As FloatArray |
| Dim BPFilter 1 As FloatArray |
| Dim TimeData As FloatArray |
| Dim FFTdata As ComplexArray |
| Dim ImpulseResponse As FloatArray |
| Dim XX As ComplexArray ' Tricoherence calculation |
| Dim PSD As FloatArray |
| Dim AA As ComplexArray |
| Dim BB As FloatArray |
| Dim CC As FloatArray |
| Dim BC As FloatArray |
| '+---------------+ |
| '\| Variables | |
| -------+ |
| Dim Frequency As Single |
| Dim FMax As Single |
| Dim dbScale As Single |
| Dim MaxFreq As Single |
| Dim MinFreq As Single |
| Dim NumOfAvg As Single |
| ********************** |
| **** Function Main **** |
| ************************ |
| Function Main() |
| '+------------------- |
| 'I Declaration I |
| ------------------+ |
| Dim cmdbuff As FloatArray |
| 'Dim Utilization As FloatArray |
| 'Dim LossRatio As FloatArray |
| Dim DAdata As FloatArray |
| 'Dim BHWnd As FloatArray |
| 'Dim WndData As FloatArray |
| Dim MagData As FloatArray |
| Dim ADREADY As Integer |
| 'Dim SampleTime As Single |
| 'Dim RetrieveTime As Single |
| Dim Parameterchecked As Integer |
| Dim i, j, k, kount,m,n, ecode, Length, start,t ,d,p,DSize As Integer |
| Dim ref1, ref2 As Single |

```
'\----------------------------------
'+---------------------------------------------
'kount = 0
Do While Not Host.GetDataReady(CTRLchan)
Loop
cmdbuff = Host.GetBuffer(CTRLchan)
Frequency = cmdbuff(0)
'+---------------------------------
'I Define A/D parameters I
'+-----------------------------------
AD.Frequency = Frequency
AD.ClockSource = DTDSP_ICLK
AD.TriggerSource = DTDSP_ITRG
AD.NumChannels =2
AD.AcqMode = DTDSP_WRAP
AD.InternalChannelBufferSize = BUFFSIZE*2
'+-------------------------------------
'I Define D/A parameters |
'+-------------------------------------
'DA.Frequency = Frequency
'DA.ClockSource = DTDSP_ICLK
'DA.TriggerSource = DTDSP_ITRG
'DA.NumChannels = 1
'DA.AcqMode = DTDSP_WRAP
'DA.InternalChannelBufferSize = BUFFSIZE*2
'+----------------------------------------------------
'I initialize global array to all zeros |
'+-------------------------------------------------
ImpulseResponse = GenLine(0,0,BUFFSIZE)
ImpulseResponse(BUFFSIZE/2) = FFT_SIZE
Do While Not Host.GetDataReady(CTRLchan)
Loop
'+----------------------
I initialize filter |
'+----------------------
ProcessControl()
'l---------------------------
'+-------------------------------------
'BHWnd = GenWindow(BLACKMAN_HARRIS_61,BUFFSIZE)
dBScale = -20* log10(10*FFT_SIZE) + 6.0
'SampleTime = 1.0/AD.Frequency * FFT_SIZE
'RetrieveTime = 0
'Utilization = GenLine(0,0,1)
'LossRatio = GenLine(0,0,1)
'+------------------------------
II start acquiring data |
'+-----------------------------
AD.ChannelType=DTDSP_DI
AD.Filter=AD.Frequency / 2
AD.Start()
```

```
* Main looping *
****水水****水水米**
ref1=1000
ref2=3000
' ref1 = first reference frequency specified by user
' ref2 = second reference frequency specified by user
Start =0
Length = BUFFSIZE/2
NumOfAVG=10
ParameterChecked = 1
Do While ParameterChecked = 1
If Host.GetDataReady(Col5chan) Then
cmdbuff=Host.GetBuffer(Col5chan)
NumOfAvg=cmdbuff(0)
ref1=cmdbuff(1)
ref2=cmdbuff(2)
End If
' +--------------------------------------------------------------
'I Calculate tricoherence
' +--------------------------------------------------------------
TriCoherence(ref1, ref2)
'+--------------------------------------
I Upgrade filter if necessary
'+------------------------------------------
If (Host.GetDataReady(CTRLchan) And (AD.DataReady(0) < BUFFSIZE))
Then
                            ProcessControl()
End If
Loop 'End of Do While loop
End Function
                                    *******************水水***************
                                    **** Function ProcessControl ****
***********************************
Function ProcessControl（）
Dim ctrlbuff As FloatArray
Dim FilterDataTemp As FloatArray
Dim FFTDataTemp As ComplexArray
Dim ready As Integer
Dim lowercut，uppercut As Single
Dim length As Integer
ready \(=\) TRUE
Do While ready
ctrlbuff \(=\) Host．GetBuffer（CTRLchan）
ready \(=\) Host．GetDataReady（CTRLchan）
Loop
＇＋－－－－－－－－－－－－－－－－－－－－－－－－－－－－－－－－－－－－－
＇I generate a band pass filter object｜
＇＋－－－－－－－－－－－－－－－－－－－－－－－－－－－－－－－－－－－＋
lowercut \(=\) ctrlbuff（CUTOFFLower）
uppercut＝ctrlbuff（CUTOFFUpper）
length \(=\) ctrlbuff（FILTERPOINTS）
BPFilter0＝InitBandPassFilter（lowercut，uppercut，Frequency，length）
BPFilter 1 ＝InitBandPassFilter（lowercut，uppercut，Frequency，length）
FilterDataTemp \(=\) Filter（BPFilter0，ImpulseResponse）
```

FFTDataTemp $=\mathrm{fft}($ FilterDataTemp)
FilterDataTemp $=\mathrm{dB}$ (FFTDataTemp)
FilterDataTemp $=$ FilterDataTemp $-20 * \log 10($ FFT_SIZE $)+6.0$
'+-----------------------------------------
'I Send data to host for filter plotting |
'+---------------------------------------1
Host.PutBuffer(FILTchan) = FilterDataTemp End Function


Function ApplyFFT(SomeData As FloatArray) As FloatArray
Dim Temp1 As FloatArray
Dim Temp2 As ComplexArray
Dim BHWnd As FloatArray
BHWnd=GenWindow(BLACKMAN_HARRIS_61,BUFFSIZE)
Temp1=Window(BHWnd, SomeData)
Temp2=fft(Temp1)
ApplyFFT=dB(Temp2)+dBScale
End Function

```
*********************************************
** Function TriCoherence() **
**********************************************
```

Function TriCoherence( ref11 As Single, ref22 As Single)
Dim ProcessTime As Single
Dim SampleTime As Single
Dim RetrieveTime As Single
Dim LossRatio As FloatArray
Dim BHWnd As FloatArray
Dim WndData As FloatArray
Dim Utilization As FloatArray
Dim kref, kref1, kref2, kmax As Single
Dim bw, nmax, Fs As Single
Dim R1, R2, R3, R4 As Single
Dim JJ1 As Complex
Dim JJ2 As Complex
Dim J1 As Complex
Dim J2 As Complex
Dim J3 As Complex
$\operatorname{Dim} \mathrm{d}, \mathrm{j}, \mathrm{i}, \mathrm{m}$ As Integer

```
'I generate Window Data I
'+-----------------------------------
BHWnd = GenWindow(BLACKMAN_HARRIS_61,BUFFSIZE)
SampleTime = 1.0/AD.Frequency * FFT_SIZE
RetrieveTime =0
Utilization = GenLine(0,0,1)
LossRatio = GenLine(0,0,1)
'+------------------------------------------------------
'I acquire parameters from buffer
Fs=Frequency/2
bw=Fs/BUFFSIZE
nmax=BUFFSIZE/2
```

```
XX=GenComplexCos(0,0,10000,FFT_SIZE/2+1)
kref1=ref11/bw+0.5
kref2=ref22/bw+0.5
kref=kref2-kref1
kmax=nmax-kref
ProcessTime = 0
j=AD.NumChannels - 1
For i =0 To j
"'+-------------------------------
' +-------------------------------------------------
            AA = GenComplexCos(0,0,10000,FFT_SIZE/2+1)
        AA = AA * 0
        BB = GenLine(0,0,FFT_SIZE/2+1)
        CC = GenLine(0,0,FFT_SIZE/2+1)
        BC = GenLine(0,0,FFT_SIZE/2+1)
        For m=1 To NumOfAV}\overline{G
        TimeStart()
        ADbuff = AD.Scan (BUFFSIZE)
        RetrieveTime = TimeRead()
'+------------------------------------
'I Extract data from buffer
'+---------------------------------------
    TimeData = Extract(ADbuff, BUFFSIZE,i*BUFFSIZE)
'+--------------------------------------------------------
'I calculate average of % utilization
'+---------------------------------------------------
                            Utilization(0) =.6* Utilization(0)+.4*((SampleTime-
RetrieveTime)/SampleTime)*100)
                        TimeStart()
'+---------------------------------------------------------------------
I Put FFT data to host for plotting
    WndData=Window(BHWnd, TimeData)
    FFTData=fft(WndData)
'+------------------------------
'+------------------------------------------------------------------------
    XX=FFTData
    JJ1=XX(kref1)
    JJ2=XX(kref2)
    R1= -Imag(JJ1)
    R2= Real(JJ1)
    J2=MakeComplex( R2, R1) 'conjg (JJ1)
    J1=J2*JJ2
    For d=0 To nmax
            R1=Abs(XX(d))
            PSD(d)= PSD(d)+R1*R1
    Next d
    PSD = PSD + Abs(XX)*Abs(XX)
    For d=0 To kmax
```



### 2.2.5 Envelope Detection Method - Demodulation of envelope signal using Hilbert Transform

### 2.2.5.1 General background, algorithm and simulation example of high frequency envelope analysis

The envelope detection method for bearing fault detection is based on the observation that the bearing characteristic frequency may modulate structural or sensor resonant frequencies excited by defect impacts. This impact will produce transient ringing pulses at the impact repetition rate (the modulating frequency). The harmonics of such impact excitation extend well into the high frequency region ( $10 \mathrm{KHz}-100 \mathrm{KHz}$ ) and excite the structural or the sensor resonant frequencies. The resonant response of this motion is detected and refereed to as the carrier frequency. What is of interest however is not the carrier frequency, rather the modulating frequency of the carrier (impact rate). Use of an amplitude demodulation algorithm such as the Hilbert Transform will retrieve the modulating frequency in the resulting envelope signal, and as a result, indicate the faulty component.

Consider a vibration signal due to periodic impact on a structure with resonant frequency $\mathrm{w}_{\mathrm{c}}$. Assuming that, each impact will ring the structure at a dominant frequency $\mathrm{w}_{\mathrm{c}}$, and the waveform will repeat itself with a repetition rate equal to the periodic impact rate. In the time domain, this signal can be modeled as an envelope signal e(t) multiplied by a carrier signal as:
$x(t)=e(t) \cos \left[w_{c} t+f(t)\right]$
The envelope represents the periodic impact with a dominant frequency $\mathrm{w}_{\mathrm{e}}$, and the carrier represent the structural response at natural frequency wc. In the frequency domain, its PSD will have a side-band structure with left, center and right sideband. Notice that, it is not sufficient to determine the envelope signal just from this measured signal alone. This signal $x(t)$ can be treated as the real part of a complex signal whose imaginary part $y(t)$ is equal to the envelope $e(t)$ multiplied by this sine wave with identical phase drifting. If such imaginary part signal can be found, then the envelop signal is just equal to the amplitude of the complex signal. It can be shown that, if the frequency of the envelop signal is much smaller then the carrier frequency $w_{c}$, then this imaginary part signal $y(t)$ is just equal to the Hilbert transform of the real part signal $x(t)$, which is equivalent to perform a 90 degree phase shift of $x(t)$.
$\mathrm{y}(\mathrm{t})=\mathrm{HT}[\mathrm{x}(\mathrm{t})]=\mathrm{x}(\mathrm{t}) * \mathrm{~h}(\mathrm{t})=\operatorname{IFT}[\mathrm{X}(\mathrm{w}) \mathrm{H}(\mathrm{w})]$
Where : $\quad H(w)=$ Transfer Function of 90 degree phase shiftor $=-j \operatorname{sgn}(w)$
HT = Hilbert Transform
IFT = Inverse Fourier Transform

Therefore, in the time domain, $y(t)$ is equal to the convolution of $x(t)$ and the impulse response function (IRF) of the $\mathrm{HT} h(\mathrm{t})$. In the frequency domain, $\mathrm{Y}(\mathrm{w})$ is simply equal to $\mathrm{X}(\mathrm{w}) \mathrm{H}(\mathrm{w})$. The transfer function $\mathrm{H}(\mathrm{w})$ will have unity amplitude with 90 degree phase at positive frequency and -90 degree phase at negative frequency. Therefore, the imaginary signal can be estimated by taking the FFT of the real part signal $x(t)$, multiplied by the transfer function $\mathrm{H}(\mathrm{w})$ of HT, and then followed by an IFFT to generate the imaginary part signal $y(t)$. The envelop signal then can be recovered through the amplitude of $x(t) \& y(t)$.

### 2.2.5.2. Algorithm/Code of Envelope Analysis Program

Figure 5.1 shows the signal processing algorithm for high frequency envelope analysis using Hilbert transform technique. The input measurement signal $x(t)$ after bandpass filtered can be modeled as a low frequency envelope signal e(t) multiplied by a high frequency carrier signal with a center frequency wc and phase drifting $f(t)$.. The purpose here is to recover the low frequency envelop signal $\mathrm{e}(\mathrm{t})$ from the high frequency measurement signal. The major function of the Hilbert Transform within this algorithm is to find its corresponding imaginary part signal from the real part measurement signal. Since the amplitude of the transfer function of HT is one across the entire frequency, and the phase of the transfer function of HT is -90 degree at all positive frequency and 90 degree phase at all negative frequency. Therefore, the desired imaginary part signal $y(t)$ is simply equal to the HT of the real part signal $\mathrm{x}(\mathrm{t})$. This is equivalent to performing a 90 degree phase shifting of $x(t)$. The HT operation can be effectively implemented in the frequency domain, as shown in the algorithm, and the envelop signal is obtained from the amplitude of the complex analytical signal $z(t)$.

Figure 5.2 shows an example of the envelope analysis. The raw time history shown in figure $5.2(\mathrm{a})$ is taken from a dynamic pressure measurement during an ATD HPOP Inducer test at NASA/MSFC's ITL (Inducer Test Leg) water flow test facility. The "bursting" waveform in the raw time history is generated due to cavitation-generated bubble collapsing process. Such cavitation phenomenon can be detected by performing spectral analysis on the envelope signal of the time history instead of the time history itself. Figure 5.2(b) shows the resulting envelope signal which "envelopes" the original time history. The periodicities of the bursting waveform in the original time history can be detected from the PSD of the envelope signal. Figure 5.3(a) shows the RAW PSD of the original signal which has been bandpass filtered from 6 KHz to 10 KHz . Figure 5.3 (b) shows the resulting ENVELOPE PSD, where strong 4 N component ( 280 Hz ) along with its harmonics are recovered as an indication of full-blade (4-blade) cavitation condition.


- $\left\{\begin{array}{l}\text { Measured Bandpass Signal } x(t)=e(t) \cos \left[\begin{array}{ll}\left.\omega_{c} t+\phi(t)\right] \\ 90^{\circ} & \text { Phase Shifted Signal } \\ y & (t)=e(t) \sin [ \\ \left.\omega_{c} t+\phi(t)\right]\end{array}\right.\end{array}\right.$
$\Rightarrow$ Analytical Signal $z(t)=x(t)+j y(t)=e(t) \exp j\left[\omega_{c} t+\phi(t)\right]$
- $\mathbf{H}(\omega)=$ Transfer Function of $90^{\circ}$ Phase Shiftor $=-\mathrm{j} \operatorname{sgn}(\omega)$


Figure 5.1 Envelope Analysis Algorithm using Hilbert Transform




Figure 5.3 (a) RAW PSD (b) ENVELOPE PSD of ITL test data

### 2.2.5.3 DSP Code Overview of envelope analysis using Data Translation's DSP-EZ programming language running on C40 DSP

The complete codes for the envelope analysis program running on C40 DSP using Data Translation's DSP-EZ programming language is listed below.


| '+-----------------+ |
| :---: |
| + |
| Dim cmdbuff As FloatArray |
| Dim DAdata As FloatArray |
| Dim Channel As FloatArray |
| Dim FWArray As FloatArray |
| Dim TFWArray As FloatArray |
| Dim SFW As FloatArray |
| Dim Sum As FloatArray |
| Dim T As FloatArray |
| Dim SCount As Integer |
| Dim NumOfAVG As Integer |
| Dim AVG2 As Integer |
| Dim ADREADY As Integer |
| Dim NumPSDChd As Integer |
| Dim i, j, k, n, Length, start As Integer |
| Dim bw, ref, Fs As Single |
| FFT_SIZE $=2048$ |
| BUFFSIZE $=2048$ |
| nmax $=$ FFT_SIZE/ $2+1$ |
| Start=0 |
| Length=nmax |
| NumOfAvg=1 |
| AVG2 $=1$ |
| Decimation $=1$ |
| Channel $=$ GenLine $(0,0,16)$ |
|  |
| 'I get the initial A/D frequency I |
|  |
| Do While Not Host.GetDataReady(CTRLchan) |
| Loop cmdbuff $=$ Host.GetBuffer(CTRLchan) |
| Frequency $=$ cmdbuff(0) |
| FFT_SIZE $=$ cmdbuff $(1)$ |
| If Host.GetDataReady(UserChan) Then cmdbuff=Host.GetBuffer(UserChan) <br> NumPSDChd $=\operatorname{cmdbuff}(0)$ <br> For $\mathrm{i}=1$ To NumPSDChd Channel(i-1) $=$ cmdbuff(i) <br> Next i |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |
|  |
|  |
|  |
|  |
| AD.TriggerSource $=$ DTDSP_ITRGAD. NumChannels $=8$ |
|  |  |

```
AD.AcqMode = DTDSP_NORMAL
AD.InternalChannelBufferSize = BUFFSIZE * 4
'+----------------------------------
II Define D/A parameters I
'+-------------------------------------
DA.Frequency = Frequency
'DA.ClockSource = DTDSP_ICLK
'DA.TriggerSource = DTDSP_ITRG
'DA.NumChannels =1
'DA.AcqMode = DTDSP_Normal
'DA.InternalChannelBufferSize = BUFFSIZE*3
'+-----------------------------------------------------
'| initialize global array to all zeros |
'+---------------------------------------------------
ImpulseResponse = GenLine(0,0,BUFFSIZE)
ImpulseResponse(BUFFSIZE/2)= FFT_SIZE
Do While Not Host.GetDataReady(CTRLchan)
Loop
'+-----------------------
I Initialize filter |
'+-------------------+
I start acquiring data I
'+-------------------------------
AD.ChannelType=DTDSP_DI
AD.Filter=AD.Frequency / 2
AD.Start()
: }\begin{array}{l}{***************}\\{,}\\{*M, Main looping*}\\{***************}
'+---------------------------------------------------------
'| acquire parameters from buffer
'+----------------------------------------------------------
Do While 1
If Host.GetDataReady(Userchan) Then cmdbuff=Host.GetBuffer(Userchan)
                                    NumOfAVG=cmdbuff(0)
                                    Start=cmdbuff(1)
                                    Length = cmdbuff(2)
                                    Decimation = cmdbuff(3)
                    Sum = Sum *0
                    SCount = 0
                    End If
                        +-------------------------------------
        | Extract data from buffer
        +-------------------------------------
    Sum = Sum * 0
    SFW = SFW * 0
    If NumOfAVG < Decimation Then
        AVG2 = 1
            Else
                AVG2 = NumOfAVG/Decimation
            End If
```

For $\mathrm{j}=1$ To AVG2
For $\mathrm{i}=1$ To Decimation
ADbuff $=$ AD.Scan(FFT_SIZE $)$
TimeData $=$ Extract(Adbuff, FFT_SIZE, Channel(0) *
FFT_SIZE)
FilterData $=$ Filter(BPFilter0, TimeData $)$
FilterData $=$ HilbertTransform(FilterData)
TFWArray $=$ ApplyDecimation(Channel(0), FilterData )
If $\mathrm{i}=1$ Then
FWArray = TFWArray
Else
FWArray $=$ FWArray \& TFWArray
End If
PSD = ApplyFFT(TimeData)
Sum $=$ Sum + PSD
Next i
$\mathrm{SFW}=\mathrm{SFW}+\mathrm{Abs}(\mathrm{FWArray})$
Next j
If NumOfAVG + Decimation $<4$ Then
T = AD.Scan(FFT_SIZE)
End If
If Host.PutDataReady(Channel(0)) Then
Host.PutBuffer $($ Channel $(0))=$ Sum/NumOfAVG
End If
If Host.PutDataReady $($ Channel $(0)+8)$ Then
Host.PutBuffer(Channel $(0)+8)=$ ApplyFFT(SFW/AVG2)
End If
If (Host.GetDataReady(CTRLchan) And (AD.DataReady(0) <
FFT_SIZE)) Then
ProcessControl()
End If
Loop 'End of Do While loop
End Function
$1 \quad * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * ~$
1 $\quad * * * *$ Function ProcessControl ${ }^{* * * *}$
, $\quad * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *$
Function ProcessControl()
Dim ctrlbuff As FloatArray
Dim FilterDataTemp As FloatArray
Dim FFTDataTemp As ComplexArray
Dim ready As Integer
Dim lowercut, uppercut As Single
Dim length As Integer
ready $=$ TRUE
Do While ready
ctrlbuff = Host.GetBuffer(CTRLchan)
ready $=$ Host.GetDataReady(CTRLchan)
Loop
'I generate a band pass filter object |


Function ApplyFFT(SomeData As FloatArray) As FloatArray
Dim Temp1 As FloatArray
Dim Temp2 As ComplexArray
Dim BHWnd As FloatArray
Dim dBScale As Single

```
dBScale }=-20*\operatorname{log}10(10*FFT_SIZE)+6.
BHWnd=GenWindow(BLACKMAN_HARRIS_61,FFT_SIZE)
Temp1=Window(BHWnd, SomeData)
Temp2=fft(Temp1)
ApplyFFT=dB(Temp2)+dBScale
\begin{tabular}{ll}
\(* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *\) \\
& \(* * \quad\) Function Conjugate
\end{tabular}
```

End Function

Function Conjugate( A As Complex) As Complex
Dim r1, r2 As Single
rl $=\operatorname{Real}(\mathrm{A})$
$\mathrm{r} 2=-\operatorname{Imag}(\mathrm{A})$
Conjugate $=$ MakeComplex (r1, r2)
End Function


Function ApplyDecimation(Channel_ID As Integer, InArray As FloatArray ) As

## FloatArray

Dim TArray As FloatArray
Dim PArray As FloatArray
Dim i, Size As Integer


### 2.2.6 GHC - Generalized Hyper-Coherence Analysis

### 2.2.6.1 General background, algorithm and simulation example of Generalized hyper-coherence function

In a rotor system, the fundamental shaft rotational component can be treated as a driving source which drives some other rotational mechanisms and generates new spectral components at different frequencies such as cage frequency or outer ball pass frequency associated with bearing defect. These two different frequency components are correlated to each other in a particular nonlinear fashion due to its kinematic geometry. Since the frequency ratio of these two components is arbitrary, the nonlinear higher order cumulant spectral analysis, such as bi-spectrum and tri-spectrum would not be able to identify such correlation. The Generalized hyper-coherence (GHC) can identify their correlation by correlating their instantaneous frequency signal since the rate of change of their instantaneous phases should be synchronize with each other. Consider a gear box example with non-integer gear ratio R. Assuming that the first gear with M1 teeth is driving the second gear with M2 teeth. The angular displacements (phases) at frequencies N 1 and N 2 will be proportional to each other, and these two spectral components are correlated in such a way that their phase variation are synchronous to each other. Assuming perfect gear mesh with no broken or worn gear tooth. If gear 1 goes through one cycle of rotation with angular displacement $a(t)$, then gear 2 will finish $R$ cycle of rotation with angular displacement $\mathrm{R}^{*} \mathrm{a}(\mathrm{t})$. In other word, their angular displacement will be proportional to each other by a factor of $R$. As a result, certain coherent phase relationship would exist between the spectral components at frequencies N 1 and N 2 . However, in general the gear ratio R (M1/M2) would be an non-integer number. Therefore, such phase coherence become very difficult to identify due to the ambiguity introduced by the Riemann surface phase wrapping in the phase estimation. Based on this observations, the GHC was developed to identify such phase coherence by correlating the rate of change of their phases, which is also called the instantaneous frequency. By taking the rate of change of phase, the ambiguous phase term will be eliminated, and their phase correlation will now be reflected in the frequency domain as frequency synchronization.

If a vibration signal is treated as an FM signal with different spectral components at different center (carrier) frequencies. Assuming that, there is some intelligence being frequency modulated in the vibration signal as the instantaneous frequency at these different carrier frequency. In order to recover the intelligence, we need to demodulate the FM signal to estimate its instantaneous frequency signal. A narrow band random process can be modeled as a sine wave with slowly varying amplitude $A(t)$ and phase $p(t)$ :

$$
\begin{equation*}
x(t)=A(t) \cos \left[\omega_{C} t+\varphi(t)\right] \tag{6.1}
\end{equation*}
$$

The instantaneous frequency $\mathrm{Wi}(\mathrm{t})$ is defined by:

$$
\begin{equation*}
\omega_{\mathrm{i}}(\mathrm{t})=\frac{\mathrm{d} \varphi(\mathrm{t})}{\mathrm{dt}} \tag{6.2}
\end{equation*}
$$



Figure 6.1 Frequency Demodulation Using Synchronous Receiver
Figure 6.1 shows the block diagram of an FM demodulator using the Synchronous Receiver method. The input signal may contain several spectral components (FM signal) at different carrier frequencies. Here, the 90 degree phase shifting is performed through the multiplication of a locally generated sinusoid and cosine waves ( synchronous receiver) at the desired carrier frequency wc. This will effectively shift the spectral component from the carrier frequency wc to both zero frequency and two times of that frequency. A low-pass filter is then used to remove any high frequency component. The output of the low-pass filters represent the slowly varying amplitude and phase (i.e. $\mathrm{A}(\mathrm{t})$ $\exp [j \varphi(t)])$. The instantaneous frequency signal wi(t) then can be obtained by,

$$
\begin{equation*}
w_{i}(t)=\dot{\phi}(t)=\frac{u(t) \dot{v}(t)-\dot{u}(t) v(t)}{u^{2}(t)+v^{2}(t)} \tag{6.3}
\end{equation*}
$$

Where $\quad \begin{aligned} u(t) & =A(t) \cos [\varphi(t)] \\ v(t) & =A(t) \sin [\varphi(t)]\end{aligned}$
For the purpose of identifying frequency synchronization for machinery diagnostics. Just imagine that the vibration signal is composed of several FM signal modulated at different carrier frequency. Each component is then passing through an FM-demodulator to generate a series of new random signal representing the instantaneous frequency signal at each carrier frequency. A linear correlator is then used to correlate among these instantaneous frequency signal with a specified reference carrier frequency. Their correlations are then summarized in the frequency domain to generate the GHC, which indicates the degree of frequency lock-in between the reference component and any other spectral component at arbitrary frequency. Figure 6.2 shows the flow diagram of the GHC program.

GHC Menu: NDEC=GHC Decimation Cycle
for each GHC tracking component, Request 3 parameters:
(1) Fc (2) LPBW (3) Filter Order=Order

- Fs= sampling frequency in Hz
- NN=block size (or buffersize)
- $\mathrm{Fc}=$ Center frequency in Hz
- LPBW = low pass bandwidth in Hz
- Define Pinc=2p*Fc*NN/Fs
- Define $\mathrm{MM}=\mathrm{NN} / \mathrm{NDEC}$ (Note: NN/NDEC must be integer)


## Program start (or re-start):

- initial phase by setting phs=0
- initial zl $(0)=z 2(0)=0$
- greate two filter object filter1 \& filter2 with identical filter setting:
lower cutoff frequency $=0$
higher cutoff frequency $=$ LPBW
filter order $=$ order

1. get one block of data raw( t$)$ array
2. generate $(\cos +\mathrm{j} \sin )$ array using GENCOSCOMPLEX with:

Phase: phs $=$ phs + Pinc
frequency $=\mathrm{Fc}$
Sampling frequency $=F s$
Arraysize $=$ NN
break complex $(\cos +j \sin )$ array into $x x() \& y y()$ array for real \& imaginery pary
3. generate $\quad x x=2 *^{\text {raw }}{ }^{*} x x$
$y y=-2 *{ }^{*} w^{*} y y$
4. use filter 1 to perform low-pass filter of xx : $\mathrm{xx}=$ lowpass filter 1 of xx use filter2 to perform low-pass filteri of $y y$ : $\quad y y=l o w p a s s$ filter2 of $y y$
5. average/decomation:

Array z1 ( 1 to MM ) $=\mathrm{xx}(1$ to NN$)$ with NDEC point average and decimation Array z2 (1 to MM) $=\mathrm{yy}(1$ to NN) with NDEC point average and decimation
6. Calculate Instantaneous frequency Array IFQ() using equation (3)

$$
w_{i}(t)=\dot{\phi}(t)=\frac{u(t) \dot{v}(t)-\dot{u}(t) v(t)}{u^{2}(t)+v^{2}(t)}
$$

Note for New Phase update: T= time period of one clock
1st block $=\cos \left(\omega_{\mathrm{C}} \mathrm{t}+\mathrm{f}_{0}\right)$
2nd block $=\cos \left[\omega_{c} t+\left(w_{c} T+f_{0}\right)\right]$
$\Rightarrow$ phase increment $=\omega_{\mathrm{C}} \mathrm{T}=2 \mathrm{p}^{*} \mathrm{Fc}^{*} \mathrm{NN} / \mathrm{Fs}$
Figure 6.2 GHC Program Flow Diagram

### 2.2.6.2. Simulation Example for GHC Analysis Program

A set of simulation data has been generated which will be used by both ASRI and Vanderbilt to test and evaluate the performance of the GHC program in the MPP system. The simulation signal is composed of three spectral components:

1. Reference component
2. Correlated Component
3. Uncorrelated Component

The first component represents the Synchronous frequency component at 1000 Hz . The second one represents a Sync-related component at 2600 Hz whose instantaneous phase/frequency is synchronous to the reference component. The third component represents an independent component at 3800 Hz with an independent instantaneous phase/frequency variation. The sampling frequency of the simulation is 10240 Hz . A total amount of 40 seconds of data was generated. Figure 6.3 shows the PSD of the simulation signal with 25 average of 4096 -point FFT block. Its is composed of three spectral components marked by "R", "C", and "U", which represent the Reference, Correlated and Un-correlated component. Figure 6.4(a), (b) and (c) show the demodulated instantaneous frequency signal with carrier frequencies set at " R ", " C ", and "U" respectively. Strong frequency correlation or synchronization can be clearly visualized between the reference and correlation component, while no correlation between the reference and uncorrelated component.

## SSME Test Example for Generalized Hyper-Coherence Analysis

An example encountered from SSME hot firing test can best demonstrate the practical application of hyper-coherence for machinery diagnostics. Figures 6.5 shows the PSD taken from a HPFTP accelerometer during SSME test $901-471$. The peak marked " N " is corresponding to the fundamental RPM component, and the remaining major peaks are its apparent harmonics $2 \mathrm{~N}, 3 \mathrm{~N}$ and 4 N . Notice that, the PSD shows an abnormally high 3 N PSD amplitude. The hyper-coherence analysis as discussed in the last month progress report was able to identify this spectral peak at the 3 N frequency is not a true third harmonic. The signature turned out to be the so-called pseudo 3 N component which is due to some independent source in the HPFTP not related to the machinery rotational process. Figure 6.6 shows the instantaneous frequency signal for (a) N , (b) 4 N , and (c) 3 N components. The IF signal of 4 N component is highly correlated with N , while the IF signal of 3 N is only weakly correlated with N .

The complete software codes in a conversion format from ASRI's DSP system to Vanderbilt MGA Model Based Programming environment will be developed and tested in the next reporting period. Due to the schedule slippage of Vanderbilt tasks in converting the other programs (e.g. Tri-coherence, bi-coherence, hyper-coherence, etc.) to the MPP system, ASRI has requested a no cost extension of this contract until 31 August 1996 to the Contracting Officer (COR). No other major problem which would impact the contract cost has been encountered.

### 2.2.6.3 DSP Code Overview of GHC function using Data Translation's DSP-EZ programming language running on C40 DSP.


(a)

Reference

(b)

Correlated

(C)

Uncorrelated


Figure $6.4 \quad \operatorname{cor}(1,2)=1.00 \operatorname{cor}(1,3)=-.09 \operatorname{cor}(2,3)=-.09$




```
    Dim FFT_SIZE As Integer
    Dim nmax As Integer
    Dim MM As Integer 'For GHC
    Dim NDEC As Integer
    Dim RCFactor As Single
    Dim Coeff As Single
    Dim const1 As Single
    ***********************
    **** Function Main ****
    *水*********************
'Function Main()
'+-------------------+
' Declaration |
'+-------------------
Dim NumBuffers As Integer
Dim NumOfAVG As Integer
Dim ADREADY As Integer
Dim NumPSDChd As Integer
Dim i, j, k, n, Length, start As Integer
Dim bw As Single
Dim Decimation As Integer
Dim DecLength As Integer
Dim DecStart As Integer
Dim PSDChan As Integer
Dim Phs0, Phs1, Phs2,Phs3 As Single
Dim Interval As Single
Dim cmdbuff As FloatArray
Dim cmdbuffer As FloatArray
Dim DAdata As FloatArray
Dim Channel As FloatArray
Dim S0 As FloatArray
Dim AA As FloatArray
Dim BB As FloatArray
Dim CC As FloatArray
Dim DD As FloatArray
Dim G0 As FloatArray
Dim G1 As FloatArray
Dim G2 As FloatArray
Dim G3 As FloatArray
FFT_SIZE \(=2048\)
BUFFSIZE \(=2048\)
nmax \(=\) FFT_SIZE \(/ 2+1\)
\(\mathrm{Fc}=\operatorname{GenLine}(0,0,4)\)
LPBW=GenLine \((0,0,4)\)
Order \(=\) GenLine \((0,0,4)\)
\(\mathrm{G} 0=\operatorname{GenLine}(0,0,3)\)
\(\mathrm{G} 1=\operatorname{GenLine}(0,0,3)\)
\(\mathrm{G} 2=\operatorname{GenLine}(0,0,3)\)
\(\mathrm{G} 3=\operatorname{GenLine}(0,0,3)\)
cmdbuffer=GenLine \((0,0,19)\)
'+------------------------------------------1+
'I get the initial \(\mathrm{A} / \mathrm{D}\) frequency \(\mid\)
```



```
InitFilters(LPBW, Order)
constl = Frequency / (2*PI*NDEC)
Interval =2 * PI * FFT_SIZE/Frequency
Do While 1
If Host.GetDataReady(GHchan) Then
    cmdbuffer=Host.GetBuffer(GHchan)
    For j=0 To 3
        Fc(j)=cmdbuffer(1+j*4)
        LPBW(j)=cmdbuffer(2+j*4)
        Order(j)=cmdbuffer(3+j*4)
    Next j
    NDEC=cmdbuffer(16)
    NumBuffers = cmdbuffer(17)
    RCFactor = cmdbuffer(18)
    MM = FFT_SIZE/NDEC
    Coeff = exp(- PI * RCFactor)
    const1 = Frequency / (2*PI*NDEC)
InitFilters(LPBW, Order)
'Host.DebugPrint(" Coeff=" & Str(Coeff))
End If
```

$\mathrm{AA}=\mathrm{GenLine}(0,0,0)$ 'initialized for concatenation
$\mathrm{BB}=\operatorname{GenLine}(0,0,0)$
$\mathrm{CC}=\operatorname{GenLine}(0,0,0)$
$\mathrm{DD}=\operatorname{GenLine}(0,0,0)$
+-----------------------------------------
I Extract data from buffer
+-----------------------------------------
$\mathrm{S} 0=\mathrm{S} 0 * 0$
For $\mathrm{k}=1$ To NumBuffers
'For $\mathrm{j}=1$ To NumOfAvg
TimeData $=$ Host.GetBuffer(DiskChan)
PSD = ApplyFFT(TimeData)
$\mathrm{S} 0=\mathrm{S} 0+\mathrm{PSD}$
'Next j
Host.PutBuffer(PSDChan-1) = PSD 'S0/NumOfAvg

For i=0 To 3 If $i=0$ Then

Phs0 $=$ Phs0 + Interval* $\mathrm{Fc}(\mathrm{i})$
U0 $=$ GenComplexCos(phs0,Fc(i),Frequency,FFT_SIZE)
G0 $=\mathrm{GHC}(\mathrm{G0}, \mathrm{U} 0$, Filter0X, Filter0Y)
$\mathrm{AA}=\mathrm{AA} \& \mathrm{IFQ}$
ElseIf $\mathrm{i}=1$ Then
Phs1 = Phs1 + Interval * Fc(i)
$\mathrm{U} 1=$ GenComplexCos(phs 1,Fc(i),Frequency,FFT_SIZE)
G1=GHC(G1, U1, Filter1X, Filter1Y)
$\mathrm{BB}=\mathrm{BB} \& \mathrm{IFQ}$
ElseIf $\mathrm{i}=2$ Then

```
    Phs2 = Phs2 +Interval * Fc(i)
    U2 = GenComplexCos(phs2,Fc(i),Frequency,FFT_SIZE)
    G2=GHC(G2, U2, Filter2X, Filter2Y)
    CC=CC & IFQ
    Else
        Phs3 = Phs3 +Interval * Fc(i)
        U3 = GenComplexCos(phs3,Fc(i),Frequency,FFT_SIZE)
        G3 =GHC(G3, U3, Filter3X, Filter3Y)
        DD = DD & IFQ
        End If
    Next i
Next k
If Host.PutDataReady(10) Then
    Host.PutBuffer(10)=AA
End If
If Host.PutDataReady(11) Then
    Host.PutBuffer(11)=BB
End If
If Host.PutDataReady(12) Then
    Host.PutBuffer(12)=CC
End If
If Host.PutDataReady(13) Then
    Host.PutBuffer(13)=DD
End If
If (Host.GetDataReady(CTRLchan) And (AD.DataReady(0)< FFT_SIZE)) Then
            ProcessControl()
End If
Loop 'End of Do While loop
End Function
'
i ************************************
b **** Function ProcessControl ****
| *********************************
Function ProcessControl()
Dim ctrlbuff As FloatArray
Dim FilterDataTemp As FloatArray
Dim FFTDataTemp As ComplexArray
Dim ready As Integer
Dim lowercut, uppercut As Single
Dim length As Integer
ready = TRUE
Do While ready
    ctrlbuff = Host.GetBuffer(CTRLchan)
    ready = Host.GetDataReady(CTRLchan)
Loop
'+--------------------------------------------------
'I generate a band pass filter object |
'+--------------------------------------------------------
```

lowercut $=$ ctrlbuff(CUTOFFLower)
uppercut $=$ ctrlbuff(CUTOFFUpper)
length $=$ ctrlbuff(FILTERPOINTS)
BPFilter0 = InitBandPassFilter(lowercut,uppercut,Frequency,length)
BPFilter1 = InitBandPassFilter(lowercut,uppercut,Frequency,length)
FilterDataTemp $=$ Filter(BPFilter0,ImpulseResponse)
FFTDataTemp $=\mathrm{fft}($ FilterDataTemp)
FilterDataTemp $=\mathrm{dB}$ (FFTDataTemp)
FilterDataTemp $=$ FilterDataTemp $-20 * \log 10($ FFT_SIZE $)+6.0$
'+-------------------------------------+
'| Send data to host for filter plotting |
'+----------------------------------------1
Host.PutBuffer(FILTchan) = FilterDataTemp
End Function

End Function


Function ApplyFFT(SomeData As FloatArray) As FloatArray
Dim Temp1 As FloatArray
Dim Temp2 As ComplexArray
Dim BHWnd As FloatArray
Dim dBScale As Single
dBScale $=-20 * \log 10\left(10^{*}\right.$ FFT_SIZE $)+6.0$
BHWnd=GenWindow(BLACKMAN_HARRIS_61,FFT_SIZE)
Temp1=Window(BHWnd, SomeData)
Temp2=fft(Temp1)
ApplyFFT=dB(Temp2)+dBScale
End Function

## ********************************

** Function Conjugate **
********************************

Function Conjugate( A As Complex) As Complex

```
Dim r1, r2 As Single
    r1 = Real(A)
    r2 = -Imag(A)
    Conjugate = MakeComplex(r1, r2)
End Function
Function ApplyCPWBD(AA As FloatArray ) As FloatArray
Dim TData As FloatArray 'Function parameter by value only
Dim FData As ComplexArray
Dim PhData As FloatArray
Dim AmpData As FloatArray
Dim IM As FloatArray
Dim R As FloatArray
Dim DC As Single
TData \(=\mathrm{AA}\)
FData=fft(TData)
PhData \(=\) Phase \((\) FData \()\)
AmpData \(=\) Abs(FData)
AmpData \(=\operatorname{GenLine}(0,1,1024)\)
\(\mathrm{IM}=\) AmpData* \(\sin (\) PhData \()\)
\(\mathrm{R}=\) AmpData * \(\cos (\mathrm{PhData})\)
TData \(=\operatorname{irfft}(\) MakeComplex \((\mathrm{R}, \mathrm{IM}))\)
TData \(=\) Abs(TData)
DC = Sum(TData) / FFT_SIZE
TData \(=\) TData- DC
ApplyCPWBD \(=\) TData
End Function
```

```
****************************************************
** Function ApplyCPWBDD*****************************************************)
```

$* * \quad$ Function ApplyReduction
$* *$ Return T4 $4(0,1,2)=$ RMS MS,Skew, kurt
$* * \mathrm{~T} 4(0)=$ RMS, T4(1) $=$ Skew, T4(2) $=$ Kurt
$* * * * * * * * * * * * * * * * * * * * * * * * * * * * *$

Function ApplyReduction( AA As FloatArray, n As Integer) As FloatArray Dim S1, S2, S3, S4, Skew, Kurt As Integer
Dim T1 As FloatArray
Dim T2 As FloatArray
Dim T3 As FloatArray
Dim T4 As FloatArray
Dim RR As FloatArray
$R R=\operatorname{GenLine}(0,0,6)$
$\mathrm{S} 1=\operatorname{Sum}(\mathrm{AA}) / \mathrm{n}$
$\mathrm{T} 1=\mathrm{AA}-\mathrm{S} 1$
$\mathrm{T} 2=\mathrm{T} 1 * \mathrm{~T} 1$
$\mathrm{T} 3=\mathrm{T} 2 * \mathrm{~T} 1$
$\mathrm{T} 4=\mathrm{T} 2 * \mathrm{~T} 2$
$\mathrm{S} 2=\operatorname{Sum}(\mathrm{T} 2) / \mathrm{n}$
$\mathrm{S} 3=\operatorname{Sum}(\mathrm{T} 3) / \mathrm{n}$
S4 $=\operatorname{Sum}(T 4) / n$
$\mathrm{RR}(0)=\mathrm{S} 2 \wedge .5$
If $\mathrm{S} 2>0$ Or $\mathrm{S} 2<=-18$ Then

$$
R R(1)=S 3 /(S 2 \wedge 1.5)
$$

$R R(2)=S 4 /(S 2 \wedge 2)$
Else
$R R(1)=0$
$R R(2)=0$
End If
$R R(3)=\operatorname{Max}(A A)$
$R R(4)=\operatorname{Min}(A A)$
$R R(5)=S 1$
ApplyReduction $=R R$
End Function
$\begin{array}{ll}1 \\ i & * * * * * * * * * * * * * * * * * * * * \\ & * * \text { Function atn2 } * * \\ & * * * * * * * * * * * * * * * * * *\end{array}$
Function atn2(xx As Single, yy As Single) As Single Dim phi As Single
phi $=\operatorname{atn}(y y / x x)$
If (phi $>0$ And $\mathrm{xx}<0$ ) Or ( $\mathrm{phi}<0$ And $\mathrm{xx}<0$ ) Then

$$
\operatorname{atn} 2=\mathrm{phi}+\mathrm{PI}
$$

Else

$$
\operatorname{atn} 2=\mathrm{phi}
$$

End If
End Function

| ** | Function |  |
| :---: | :---: | :---: |
|  | Function GHC |  |
| *** | Global: Coeff, const1,MM, IFQ | *** |

Function GHC(GL As FloatArray, UU As ComplexArray, FX As FloatArray, FY As FloatArray ) As FloatArray
Dim i, j,k As Integer
Dim ZZx As FloatArray
Dim ZZy As FloatArray
Dim Tx As FloatArray
Dim Ty As FloatArray
Dim env As FloatArray
Dim ps As FloatArray
Dim TIFQ As FloatArray
Dim TG As FloatArray
IFQ $=$ GenLine $(0,0, \mathrm{MM})$
$\mathrm{ZZx}=\operatorname{GenLine}(0,0, \mathrm{MM})$
ZZy $=$ GenLine ( $0,0, \mathrm{MM}$ )
env $=$ GenLine $(0,0, \mathrm{MM})$
$\mathrm{ps}=$ GenLine $(0,0, \mathrm{MM})$
TIFQ $=\operatorname{GenLine}(0,0, \mathrm{MM})$
$\mathrm{TG}=\operatorname{GenLine}(0,0,3)$ 'hold values of changed GL array
Tx $=2^{*}$ TimeData $* \operatorname{Real}(\mathrm{UU})$
Ty $=-2$ * TimeData * Imag(UU)
$\mathrm{Tx}=\mathrm{Filter}(\mathrm{FX}, \mathrm{Tx})$
Ty $=\operatorname{Filter}(\mathrm{FY}, \mathrm{Ty})$
$\mathrm{j}=1$
For $\mathrm{i}=0$ To FFT_SIZE -1
$\mathrm{k}=\mathrm{j} * \mathrm{NDEC}$
If ( $\mathrm{i}<\mathrm{k}$ ) Then

$$
\mathrm{ZZx}(\mathrm{j}-1)=\mathrm{ZZx}(\mathrm{j}-1)+\mathrm{Tx}(\mathrm{i})
$$

Else

$$
\mathrm{ZZy}(\mathrm{j}-1)=\mathrm{ZZy}(\mathrm{j}-1)+\mathrm{Ty}(\mathrm{i})
$$

$\mathrm{j}=\mathrm{j}+1$
End If
Next i
ZZx = ZZx / NDEC
ZZy = ZZy / NDEC
For i =0 To MM-1

$$
\operatorname{env}(\mathrm{i})=\left(\mathrm{ZZx}(\mathrm{i})^{\wedge} 2+\mathrm{ZZy}(\mathrm{i})^{\wedge} 2\right)^{\wedge} 0.5
$$

$\mathrm{ps}(\mathrm{i})=\operatorname{atn} 2(\mathrm{ZZx}(\mathrm{i}), \mathrm{ZZy}(\mathrm{i}))$
$\mathrm{ZZx}(\mathrm{i})=\cos (\mathrm{ps}(\mathrm{i}))$
$\mathrm{ZZy}(\mathrm{i})=\sin (\mathrm{ps}(\mathrm{i}))$
Next i
$\mathrm{TG}(0)=\mathrm{ZZx}(\mathrm{MM}-1)$
TG(1) $=\mathrm{ZZy}(\mathrm{MM}-1)$
$\operatorname{TIFQ}(0)=\operatorname{const}^{*}\left(\mathrm{ZZx}(0) *(\mathrm{ZZy}(0)-\mathrm{GL}(1))-(\mathrm{ZZx}(0)-\mathrm{GL}(0))^{* Z Z y}(0)\right)$
For $\mathrm{i}=1$ To MM-1

$$
\operatorname{TIFQ}(\mathrm{i})=\operatorname{const} 1^{*}(\mathrm{ZZx}(\mathrm{i}) *(\mathrm{ZZy}(\mathrm{i})-\mathrm{ZZy}(\mathrm{i}-1))-(\mathrm{ZZx}(\mathrm{i})-\mathrm{ZZx}(\mathrm{i}-1)) * Z Z y(\mathrm{i}))
$$

Next i
$\mathrm{IFQ}(0)=(1-$ Coeff $) * \mathrm{TIFQ}(0)+$ Coeff $* \mathrm{GL}(2)$
For $\mathrm{i}=1$ To MM-1
$\mathrm{IFQ}(\mathrm{i})=(1-$ Coeff $) * T I F Q(\mathrm{i})+$ Coeff $*$ IFQ(i-1)
Next i
$\mathrm{TG}(2)=\mathrm{IFQ}(\mathrm{MM}-1)$
$\mathrm{GHC}=\mathrm{TG}$
End Function

**********************
'Function RCLowPassFilter( A As Single, XX As FloatArray) As FloatArray
'Dim TT As FloatArray
'Static $\mathrm{Pa}, \mathrm{Pb}, \mathrm{Pc}, \mathrm{Pd}, \mathrm{PP}$ As Single
'Dim i As Integer

- $\quad \mathrm{TT}=\operatorname{GenLine}(0,0, \mathrm{MM})$
- $\quad \mathrm{TT}(0)=(1-\mathrm{A}) * \mathrm{XX}(0)+\mathrm{A} * \mathrm{~Pa}$
- For $\mathrm{i}=1$ To MM-1
, $\quad \mathrm{TT}(\mathrm{i})=(1-\mathrm{A}) * \mathrm{XX}(\mathrm{i})+\mathrm{A} * \mathrm{TT}(\mathrm{i}-1)$
- Nexti
- $\quad \mathrm{Pa}=\mathrm{TT}(\mathrm{MM}-1)$
- RCLowPassFilter = TT
'
'End Function


### 3.0 Conclusion

This Final Report documents and summarizes the results of the software conversion effort. The six advanced nonlinear diagnostic DSP algorithms discussed in Section 2, have been converted to a format consistent with that required for integration into the Vanderbilt Multigraph Architecture (MGA) Model Based Programming environment. Textual/graphical descriptions of analysis algorithm structures along with their signal flow charts have been established. Simulation examples with various pertinent application scenarios were accomplished for demonstration and performance evaluation. This effort allows the real-time execution of these algorithms using the MSFC MPP Prototype System. In this report, ASRI has provided the complete computer source code, including all FORTRAN/C++ Utilities, and all other utilities/supporting software libraries that are required for operation.

Consultation/coordination on the details of these algorithms with both MSFC dynamic analysts and Vanderbilt University's Measurement and Computing Group has also been provided by ASRI. This activity include face-to-face meetings to provide discussions/briefings of the details of these algorithms with both MSFC dynamic analysts and Vanderbilt University's Measurement and Computing Group

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