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SUB-PLATE OVERLAP CODE DOCUMENTATION

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ABSTRACT

An expansion of the plate overlap method of astrometric data reduction to a single plate has been proposed and successfully tested. Each plate is (artificially) divided into sub-plates which can then be overlapped. This reduces the area of a "plate" over which a plate model needs to accurately represent the relationship between measured coordinates and standard coordinates. Application is made to non-astrographic plates such as Schmidt plates and to wide-field astrographic plates. Indeed, the method is completely general and can be applied to any type of recording media.

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This document and its attachments contains

a User's Guide to the software

a Reference Manual for the concepts behind the software which also incorporates the High-level design document

a full set of Commented Source Code Files

with Installation Procedures

and the necessary Makefiles or equivalent

as well as Test data and procedures

within the text are included references to its use in the literature

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I. INTRODUCTION

This document outlines an astrometric method of plate reduction that was designed to attack the general problem of Schmidt plate astrometry. By "astrometric" we mean that the results are both precise and accurate. The precision is typically limited by the measurement errors; the accuracy by the size of the sub-plate. The size of the sub-plate is, in turn, determined by the density of the reference star catalog. With the successful advent of the European Space Astrometry artificial satellite HIPPARCOS, coupled with its TYCHO adjunct, precision in star catalogs of moderate density is no longer an issue. When the TYCHO Reference Catalogue is completed, which will incorporate both the TYCHO observations and those of the Astrographic Catalogue, density will no longer be an issue either.

Although the original motivation for this creation came from the poor state of the *Hubble Space Telescope* Guide Star Catalog (Taff *et al.* 1990), we have, in addition to success with wide-field Schmidt plates, already demonstrated its utility with respect to wide-field astrograph plates (Smart, Taff, and Morrison 1995). Extrapolating from these outcomes, we expect that sub-plate overlap, for which the overlapping component represents the appropriate extension of the plate overlap method (Eichhorn 1960) to a single plate, to become the standard reduction procedure for all astronomical images. The TYCHO Reference Catalogue would be the logical reference star catalog in most instances. Finally, to further exploit the new catalogs and to simplify the application of the procedure to other types of telescopes, we have made two improvements in the code that implements this process. One is described in Sub-section IIIe, the other in the Appendix. The former allows for a spherical, instead of gnomonic, projection (Dick 1991) and the latter allows for differential astronomical refraction across the plate.

II. PROBLEM STATEMENT

The promise of Schmidt plates for astrometry is their large field-of-view. The difficulty of astrometry with Schmidt plates is the pattern of systematic deformations that appear in them (see Fig. 1). Schmidt plates, because they can cover so much of the sky

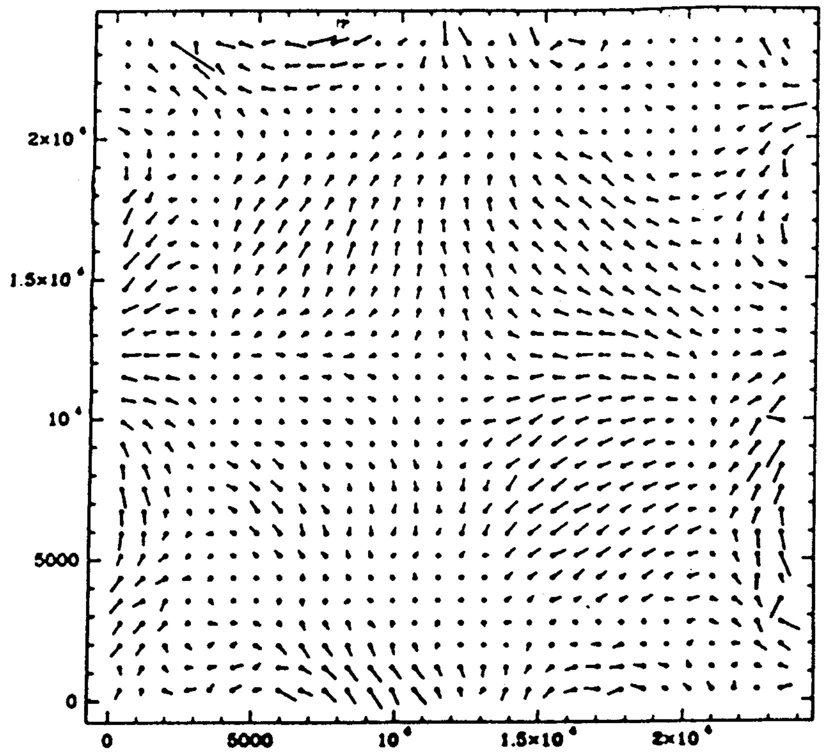

Fig. 1. Vector residual map of the typical, wide field-of-view, Schmidt plate.

relatively deeply, can contain an enormous amount of information. The possibility of tremendous numbers of precision positions for all types of astronomical objects, and all types of astrophysics, is extraordinarily appealing. This aim had not yet been achieved because there was never any satisfactory, let alone astrometric, method of reducing entire, large-scale, Schmidt plates. We review why this was so and demonstrate our solution to the problem. The software documented herein represents the only success in the global astrometric reduction of a *single* Schmidt plate. Utilizing it we have been able to reach the lower bounds set by reference catalog quality and plate measurement errors. This is also true for wide field-of-view astrographic plates.

If we may slightly abuse the concept of variance, one can represent the total variance, s^2, of a star's position, as deduced from a photographic plate, as consisting of two parts,

$$s^2 = s^2_{plate} + s^2_{ref}.$$

In this expression s_{plate} is the standard deviation in a star's position arising from all plate modelling errors and s_{ref} is the standard deviation in a star's position arising from all reference frame errors. Each of these we can in turn split into two components; for example s_{ref} can be written as

$$s^2_{ref} = \sigma^2_{ref,ran} + \sigma^2_{ref,sys}.$$

The first part on the right hand side represents the contribution of the random errors in the reference frame to s^2_{ref} as well as those resulting from our attachment to it. The second element arises from the systematic errors.

In an analogous fashion for the plate modelling components we can write

$$s^2_{plate} = \sigma^2_{plate,ran} + \sigma^2_{plate,sys}.$$

The random part contains both the plate measurement variance and the plate constant variance. The measurement component should be constant over the celestial sphere, except perhaps in the Milky Way where the act of plate measurement itself is most difficult. The systematic component varies across each plate, being much worse along a plate's borders than in its center (see Fig. 1). In the case of wide-field Schmidt plates, this is almost certainly a consequence of an insufficient plate model, the physical deformation of the plate, and the characteristics of Schmidt optics.

III. PROBLEM SOLUTION

From the above recitation it is clear that to dramatically improve the situation for Schmidt plate reduction the traditionally used plate reduction method itself must be re-constituted.

a) The Lack of a Plate Model

Ideally, the plate model is a deduction from a complete theoretical understanding of the telescope optics, the plate holder, the measuring and developing processes, and so on. In practice, a good, empirically deduced form for the plate model is acceptable. Neither exists for Schmidt plates (this *includes* recent attempts at seventh order polynomial models based on Fig. 1). A post-facto analysis of the residuals in the star positions, utilizing an independent catalog, can also be helpful in identifying neglected terms, magnitude or color effects, and so forth. Indeed, such an investigation has now shown the presence of a systematic magnitude effect (Morrison, Röser, Lasker, Smart, and Taff 1995). Correlation analysis of the residuals may also reveal significant scales of poor fitting; this did not yield a satisfactory method in this case (Lattanzi, Bucciarelli, and Taff 1994).

Why does there not exist a global model for Schmidt plates? That is, why aren't there relatively simple, low-order polynomial formulas of the form

$$\xi = \Xi(x,y,m,c;\mathbf{P}), \quad \eta = H(x,y,m,c;\mathbf{P}) \tag{1}$$

which relate the measured coordinates of a star (x,y) with apparent magnitude m and color index c to its standard coordinates (ξ,η) with an auxiliary vector of plate constants (**P**)? For one thing, if there were such formulas as the above, then we would (albeit crudely) expect that the estimated positional error Σ would behave as

$$\Sigma \sim (M/N)^{1/2}\sigma \quad \text{as} \quad N \to \infty \tag{2}$$

where M is the dimension of **P**---that is the number of plate modelling parameters, N is the number of reference stars used in the statistical fitting of the plate models, and σ is the average positional precision of a reference star. No one has ever reported such convergence in probability for Schmidt plates. Indeed, reading the literature, it is clear that all low-order polynomial expansions for Ξ and H are equally poor representations of the standard coordinate to measured coordinate mapping. In addition to the many journal articles on the subject (*e.g.* Dieckvoss 1955, 1960, Anderson 1971, Dodd 1972, Fresneau 1978, or de Vegt 1979), the exponentially larger experience with the astrometric reduction of Schmidt plates by the constructors of the *Hubble Space Telescope* Guide Star Catalog also supports our conclusion. Finally, our own substantial experiments with the northern half of the Guide Star Catalog plate collection allows us to prove the absence of a low-order polynomial plate model. But why? We shall answer this after summarizing the additional evidence to the contrary.

One of the first clues that Ξ and H do not exist as any simple functions of ξ and η is the complete lack of agreement as to their form, which terms are important, and so on in previous work. While it is true that most of the testing has been in analogy with astrographic work, and therefore relied on low-order polynomials in *x, y, m,* and *c*, after testing with hundreds of plates from a dozen Schmidt telescopes no one model or group of models has been successful. A second clue is the pattern of residuals one obtains when two overlapping Schmidt plates are separately reduced and then inter-compared. One sees swirls and flows very similar to the patterns one obtains when a reduced Schmidt plate is compared to a high density, high precision catalog as in Fig. 1. Several attempts to determine the scale length of correlation amongst the residuals were undertaken (see Taff, Lattanzi, and Bucciarelli 1990). One used the correlation analysis suggested by Fresneau (1978; significantly improved upon in Lattanzi, Bucciarelli, and Taff 1994) while the other examined the discrepancy in determining position angle as a function of star separation (this is an important concept in the operational usage of the *Hubble Space Telescope* Guide Star Catalog). The conclusions were that the scale length of the correlations were 1-2° in size. The uniformity and symmetry properties of the pattern seen in Fig. 1 is almost certainly a consequence of the bending of the flat photographic plates to fit the spherical focal surface holder of Schmidt optical systems. As with efforts to model the standard coordinate to measured coordinate relationship, we believe that simple treatments of the Schmidt optical system or of the deformations induced by Schmidt telescope plateholders are futile (Shepard 1953, Dixon 1961, Fresneau 1979, Dieckvoss and de Vegt 1966).

Summing this together, namely the absence of a theoretically compelling form for the plate model functions, the lack of empirical success in reducing real plates (independent of the Schmidt telescope), the failure to achieve statistical convergence, and the empirically determined patterns that we discovered, all points to the same, inescapable conclusion: Schmidt plate astrometry is dominated by systematic effects. Therefore, no global (*i.e.,* area > 3-4 sq. deg.) reduction of a large scale Schmidt plate will succeed. One needs to deal with small pieces of Schmidt plates and the smaller the better. Only practical considerations, having to do with reference star areal density, limit the optimum sub-area size from below. In more detail, the application of the sub-plate overlap technique is limited by the tension between small sizes for sub-plates and the reference catalog density.

b) Sub-plate Overlap

Since the fundamental difficulty of astrometry with Schmidt plates is that the reduction is dominated by systematic effects, not plate measurement error nor reference star catalog errors, the key to the successful astrometric reduction of Schmidt plates is to accept and recognize this, to abandon the search for a global plate model---especially one that is a low order polynomial derived from experience with astrographs---and to directly deal with the global pattern impressed on Schmidt plates. This pattern, see Fig. 1, is strikingly clear and must be conquered before any sort of traditional "plate modelling" can work. Sub-plates, and sub-plate overlap, were invented (Taff 1989) as a method of breaking the large, typically 6.°5 square, Schmidt plate into smaller, more tractable pieces. Since sub-plates exist in the mind and not on the glass, we can make them as small as we please. Eventually, with a high density reference star catalog, we must be able to get below the 1-2° scale of the systematic structure and model the standard coordinate (ξ,η) to measured coordinate (x,y) mapping.

By sub-plates (see Taff 1989 for the details) we mean the artificial division of an entire Schmidt plate into smaller units. Figure 2 shows an example. The raison d'etre of sub-plates is to break the embrace of the systematic effects. Clearly the size of the sub-plates must be ~ 1°. Because sub-plates contain a minimum amount of complex systematic effects, simple forms for the plate models Ξ and H in Eqs. (1) are acceptable.

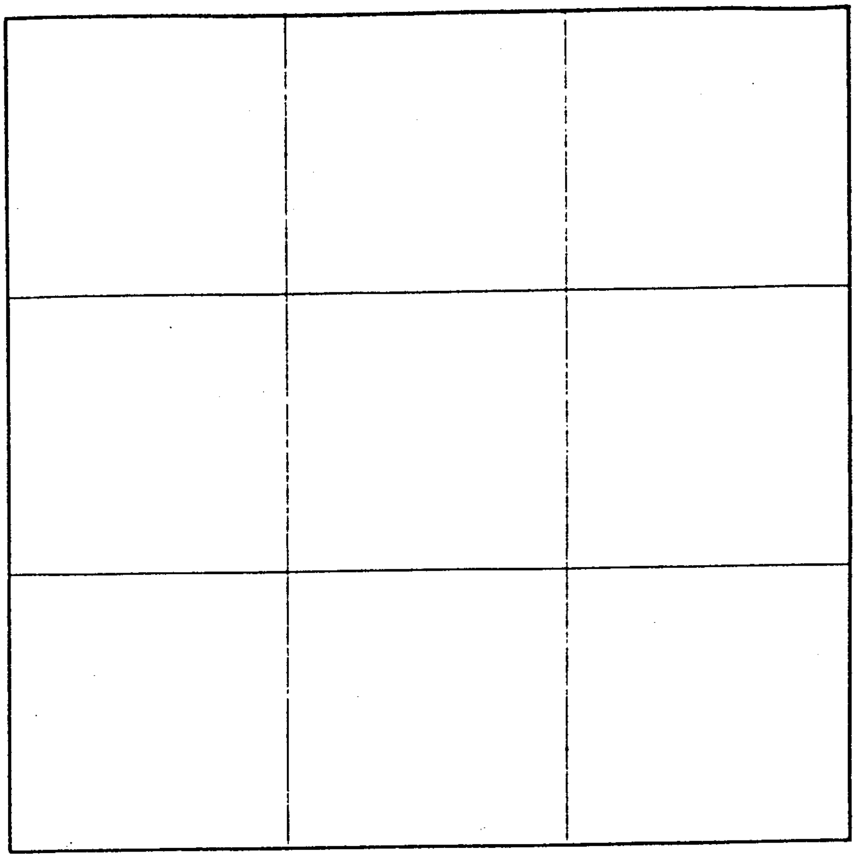

Fig. 2. Basic sub-plate array for nine square sub-plates on a square plate.

In fact, the general linear model

$$\xi = a + bx + cy, \qquad \eta = d + ex + fy \tag{3}$$

should suffice. Each sub-plate could have its own origin so the ranges of x and y can be limited. Note that here there is no longer any attempt to interpret Eqs. (3) in a physical way. They can be viewed as the leading terms in a Taylor series of the remaining systematic effects. Were we to include additional terms, either higher-order in x and y or to represent magnitude or color effects, then two different problems would arise. The inclusion of more terms means that more sub-plate constants have to be determined. As the current large-scale reference catalogs, such as the AGK3U and the CPC2, are limited to a star density of 8-10 stars per square degree, there are insufficient reference stars to support a precise evaluation of additional sub-plate constants. A corollary difficulty is that with few reference stars [$N \sim M$ in Eq. (2)], the random errors in these star's positions, magnitudes, and colors emerge as systematic errors in the value of the sub-plate constants. Only a large sample of reference stars will mitigate against this possibility.

Thus, more likely than not, the poor outcomes one obtains outside of the core of a Schmidt plate will not be successfully dealt with by any analytically simple model. The difficulty is inherent in the absolute scale of the large plate and its non-astrometric properties (especially the effects of bending the plates). Hence, the plate must not be reduced in its entirety. Each plate must be partitioned into pseudo-plates, which were called sub-plates (see Fig. 2). The purpose of this sub-division is to artificially decrease the size of the "plate" thereby allowing simplistic plate models to function because one is now underneath the scale length of the biases. Furthermore, by overlapping sub-plates some of the advantages of the plate overlap method can be brought to bear.

Though the Schmidt plate is (theoretically) infinitely divisible, once a certain size for the sub-plates is reached, no new reference stars are introduced. Thus, different sub-plates share the same reference stars leading to the same sub-plate model in Eqs. (3). As a concrete example, current catalogs coupled with the scale of Schmidt plate systematic errors would lead one to compromise on a 2/3° sub-plate size and 50% overlapping. As 2/3° x 2/3° x 9 stars/sq. deg. = 4 stars, the 50% overlapping, yields (on the average), one different reference star per sub-plate quarter. The second complication that the introduction of higher-order terms into the sub-plate model gives rise to is indeterminacy of the coefficients of the quadratic, cubic,...terms owing to insufficient exploration of x, y ranges. For two-thirds of a degree sub-plates, the relative values of $|x|$ and $|y|$ only vary over 0.006 rad.

c) Sub-plate Pattern Design

Once one conceives of sub-plates and sub-plate overlap, the next questions to be addressed are the geometrical form of the sub-plates and the nature of the overlap. Again, ideally a good theoretical model or empirical data would allow one to choose between overlapping concentric circles or overlapping pie-shaped wedges; see Figs. 3a and 3b. Lacking either theoretical or empirical demands for such possibilities, we can decide the issue on two other grounds---the corners of the plate must be fully penetrated and the sub-plates must not deal with orthogonal directions anisotropically. As the discussion in the following paragraph indicates, this leads *uniquely* to square sub-plates. The maximal square sub-plate overlapping scheme, based on traditional ideas, is shown in Fig. 4.

The geometrical issue of sub-plate form can be dealt with fairly quickly given the two aforementioned constraints. Remembering our High School geometry, if we start with the simplest (non-degenerate) polygons, that is triangles of non-zero area, then full penetration of the plate corners means we must use right angle triangles. We can divide all right angle triangles into two mutually exclusive and exhaustive sets; isosceles right angle triangles and non-isosceles right angle triangles. Utilization of non-isosceles right angle triangles results, in effect, in using non-equilateral rectangles. Even if the overall pattern of rectangles was isotropic, within a rectangle there would be anisotropy. Therefore, this option is precluded by one of our initial constraints.

Isosceles right angle triangles leads to squares. Squares satisfy both constraints. No regular polygon with more than four sides can fully penetrate the corners (being equilateral and regular also means being equiangular and there are an insufficient number of right angles in any higher numbered-sided figure). Finally, general convex shapes will neither fill the corners nor be isotropic.

Figure 4 is for illustrative purposes, in that the choice of nine sub-plates is arbitrary. It does, nonetheless, demonstrate two general features of this approach. The interior of each plate is now overlapped four times and the border is overlapped twice except for the four corners. When there are nine principal sub-plates, or nine sub-plates in the basic pattern, 44% of the plate is four-times overlapped, 44% is doubly overlapped, and 11% is not overlapped at all. When there are 16 fundamental sub-plates per plate (*i.e.,* that is 16 in the basic configuration---the first pattern laid down), the percentages are 53%, 38%, and 9%. For 25 sub-plates in the basic pattern, the percentages are altered to 64%, 32%, and 4%. Finally, the smallness of the area occupied by each sub-plate eliminates the need for complex sub-plate models allegedly based on an analysis of the optical system, and so forth

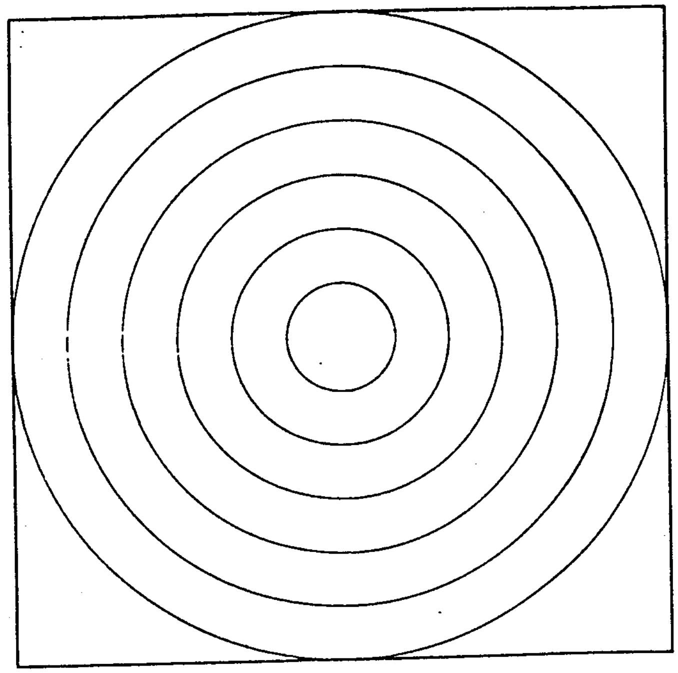

Fig. 3a. Concentric circle sub-plate overlap. Useful if the plate model is azimuthal.

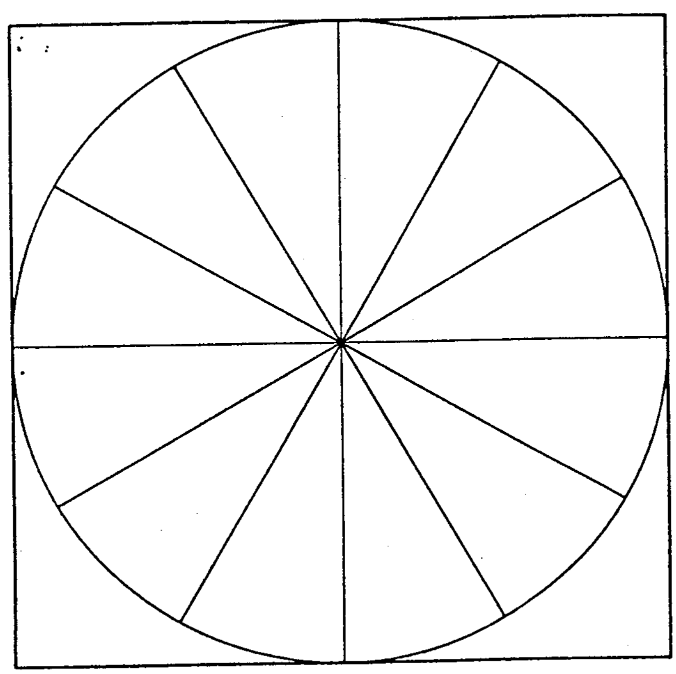

Fig. 3b. Pie-shaped wedge sub-plate overlap.

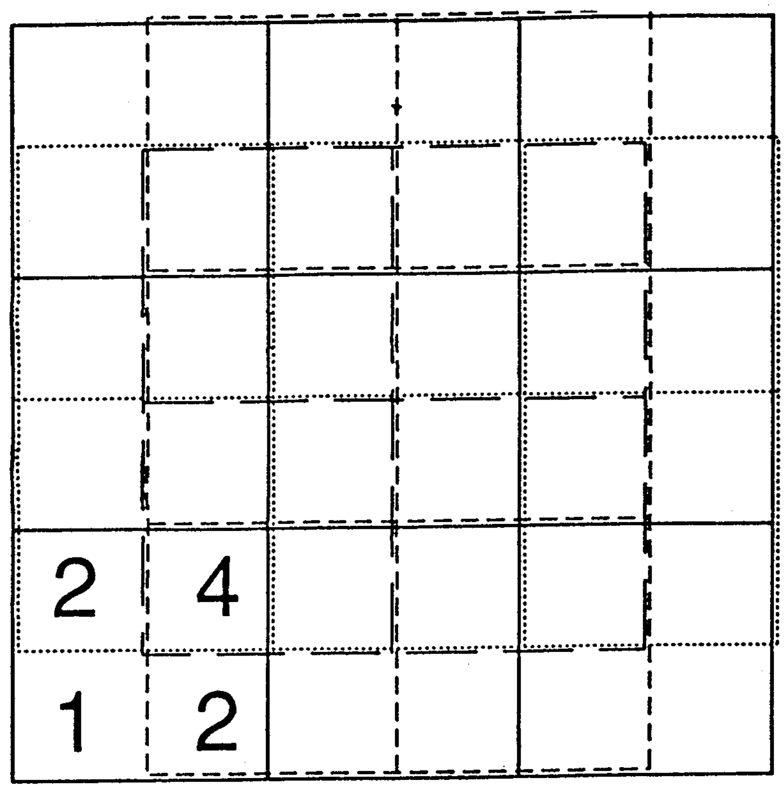

Fig. 4. Sparse maximal sub-plate overlapping (see the numerical values). Only the corners are not overlapped. Sub-plate borders are slightly displaced for clarity.

In general, a square photographic plate of linear dimension D can hold $(D/d)^2$ square sub-plates in the basic pattern. Altogether there will be $(2D/d - 1)^2$ sub-plates in the pattern of Fig. 4. Of these the four quarter sub-plates in the corners are not overlapped, $8(D/d - 1)$ quarter sub-plates of the perimeter sub-plates will be doubly overlapped, and the remaining $4(D/d - 1)^2$ interior quarter sub-plates will be four times overlapped.

The overlapping pattern shown in Fig. 4 is quite unimaginative. It rests on combining the standard edge-in-center-line and corner-in-center patterns used for overlapping plates on the sky. As the sub-plates are artificial---existing only in our minds and the computer's memory, and the plane infinitely divisible---we can choose to overlap as densely as we please. Figure 5 shows a plate with four principal sub-plates overlapped by moving each new pattern in one-third steps relative to the previous one instead of only half-way between sub-plates. Note that the diagonal possibilities are included too. Now, only the outer ninths of the corner sub-plates are not overlapped at all. There are still $(D/d)^2$ square sub-plates in the basic pattern but $(3D/d - 2)^2$ overall of which $(3D/d - 4)^2$ are nine times overlapped (there are four central "arrays"; an induction is possible).

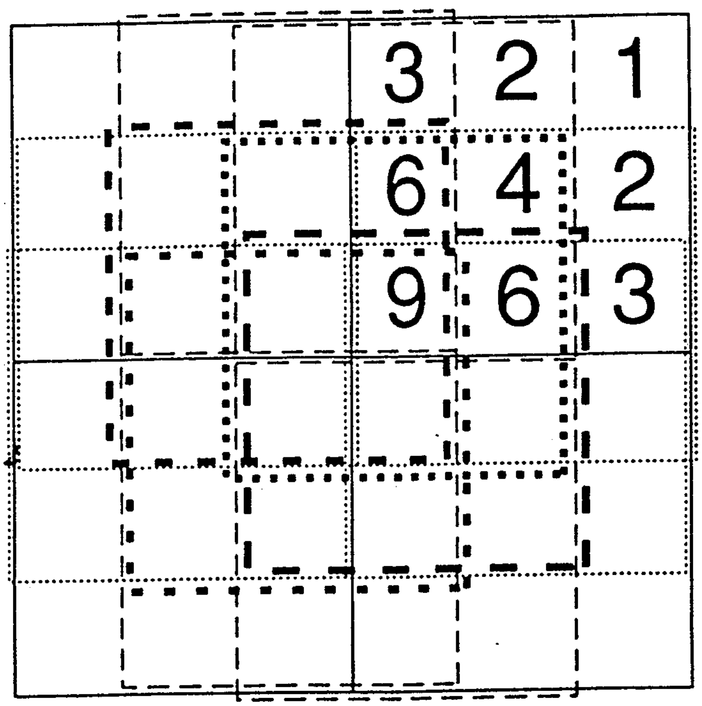

Fig. 5. Overlapping by one-third step sizes instead of half-step sizes as in Fig. 4. Again the numbers indicate the degree of overlapping.

As these kinds of divisions are made finer two problems arise. One is that no new reference stars are added nor old ones subtracted as we move infinitesimally across the plate; even with a very high star density such as the proposed TYCHO Reference Catalogue (~ 25 stars/sq. deg.). The other difficulty is that there is a degeneracy inherent in this technique since there is only a **single** measurement for each star. Hence, the incorporation of traditional overlapping constraints can exercise considerable unwanted influence. Thus, they are not used but the lack of repetitive observations is a limitation on the utility of more highly overlapping. Finally, we have not yet discussed how to combine the many different values of a single star's coordinates, each one obtained from a different sub-plate. We deal with the latter topic in Sub-section d below.

Before the completion of the TYCHO Reference Catalogue, which would be dramatically strengthened by the inclusion of the Guide Star Catalog positions, the number of sub-plates per plate would have been determined by the local star density in whichever of the AGK3U (northern hemisphere) or the CPC2 (southern hemisphere) is being used as the reference catalog. In general, a 1.°5 x 1.°5 sub-plate will contain 10-20 reference stars. Another advantage of smaller sub-plates is the diminished influence the border of the plate will have on its central region, thus minimizing the chances of having much better precision at the edges for uniformly worse precision than was previously available in the core of each plate. (That the border can influence the center at all is a consequence of the overlapping aspect we have implemented.)

d) The Minimum Variance Unbiased Estimator for Sub-plates

In this Sub-section we address the formulation of the minimum variance unbiased estimator for overlapping sub-plate combinations. This problem requires either an assumption regarding the degree of sharing of reference stars on different sub-plates or a detailed counting thereof so that the degree of correlation among the different positions deduced for the star can be evaluated. The existence of these correlations, especially for the higher levels of overlapping, make the problem both cumbersome to develop and long to work out. For instance, with 33% spacing between the different sub-plate patterns there are nine positions (and hence nine weights) to deal with and 36 (8×9 /2) different non-zero terms in the covariance matrix. Also, the usual techniques of partial differentiation may be insufficient to find the correct answer. The reasons for this are that there are two very different constraints on the hypothesized weights $\{w_n\}$ for the different stellar positions we are entertaining. First they must sum to unity. Admittedly this constraint can be analytically dealt with---either with a Lagrange multiplier or by algebraic elimination. Second, and more important, each weight is subject to the constraint $w_n \in [0,1]$ and this is much more difficult to enforce. Thus, we have no *a priori* assurance that even if we formulate the minimum variance unbiased estimator problem for overlapping sub-plate combinations as a standard one in partial differentiation that we will arrive at an acceptable solution by equating all the appropriate first partial derivatives equal to zero. Finally, we have already performed substantial numerical simulations of parts of this problem using data from over 800 real Schmidt plates (see Taff, Lattanzi, and Bucciarelli 1990). Thus, we have considerable confidence that equal weights, the choice empirically favored, was the correct one.

However, the usage of equal weights represents just as strong an assumption (or assertion) as that required to advance any other weighting scheme. Therefore, we have explicitly formulated and solved the minimum variance unbiased estimator problem for two of the overlapping sub-plate combinations most commonly used. Thus, we can now be fully confident of our statistical underpinnings with regard the choice of the equal weighting solution. To facilitate this we make one (compound) supposition; that the reference star catalog is *locally* homogeneous. By this we mean first that, over the area spanned by the larger photographic plate on which the sub-plates are being constructed, the stars are, to zero'th order, uniformly distributed across the plate. The second sense this assumption has is that over the much smaller area spanned by a sub-plate, and all those which overlap it, the reference stars have equal error qualities, to first order. This statement, in turn, has two meanings also. It means both that the precision and the accuracy of the reference star catalog is homogeneous on this smaller scale-length and that the measuring machine (and whence the measurement error) is uniform on the scale of a sub-plate dimension. This composite assumption allows us to both explicitly compute the covariance matrix for all the overlapping sub-plates **and** to make a reasonable surmise regarding the mean errors of any coordinate deduced via the sub-plate algorithm.

Because there is some subtlety in these suppositions, we expand a little on them below. Using these assumptions, we then deal with the 50% overlapping problem first. In this instance we fully and explicitly demonstrate that the minimum variance unbiased estimator problem for overlapping sub-plate combinations is the one with equal weights. We also present the outcome of a Monte Carlo simulation of the *nearly* equal weights situation to illustrate what the resulting distribution of weights would be without these assumptions. Next, having analytically solved the equal weights problem we can use it to illustrate some geometrical symmetries that the general problem possesses. In particular, we will reveal how the formulation, and hence the solution, to the 33% minimum variance unbiased estimator overlapping problem can be significantly simplified. As we have never had access to a reference catalog dense enough to support 25% overlapping, we stop here.

Now to more fully explain what the assumptions mean and why we adopt them. The hypothesis that the areal distribution of the reference star catalog is uniform is mainly one of convenience and true to at least zero'th order. It allows us to transform the details of the reference star correlation evaluation problem into one of evaluating the degree of the geometrical overlaps. Using this assumption, for a uniform distribution of reference stars, the extent of the overlap uniquely fixes the numerical value of the correlation coefficient. The hypothesis that the reference star positions have roughly equal precision---locally---is also one of analytical convenience; below we display the penultimate step in the solution to the 50% overlapping problem without making this assertion. Taking this statement to be true greatly simplifies our ability to penetrate the algebra and plays an equal role with the celestial sphere surface density uniformity assumption in leading to a conclusion that equal weights are the ones to use to obtain the minimum variance unbiased estimator. However, these two assumptions are logically independent and no more than is required to obtain an answer. Finally, using Eqs. (6) below, we perform a Monte Carlo simulation to demonstrate the effects of relaxing the constant mean error hypothesis; see Fig. 6.

For further elucidation, consider the predicted standard coordinate for a star after the sub-plate solutions have been found. With *a, b,* and *c* the estimates for the three parameters in the xi coordinate plate model, and *x* and *y* the values of the measured coordinates for this star, we write, as is customary, repeating a part of Eq. (3)

$$\xi = a + bx + cy. \tag{3}$$

Thus, the predicted variance of ξ, σ_ξ, has two different types of terms; one set owing to the variances of the plate model parameters (and their covariances amongst themselves) and one set owing to the measurement errors in x and y (and their covariances amongst themselves). There is no correlation between the measurement errors and the formal mathematical statistical error estimates for the plate constants. It is this two-component aspect of σ_ξ which makes the second hypothesis made above itself compound.

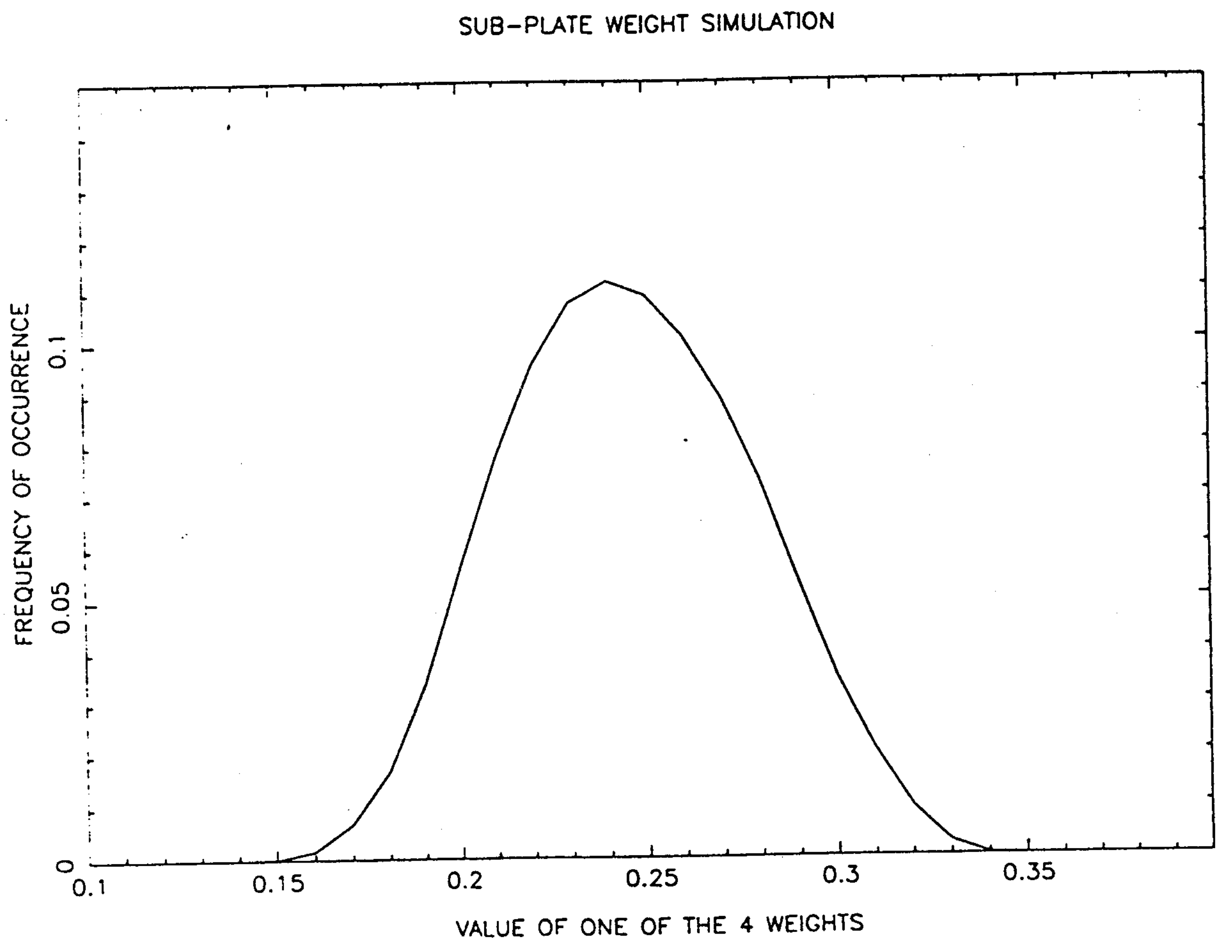

Fig. 6. A Monte Carlo simulation of the nearly equal variances of the values of relative weights obtained from Eqs. (6). Note that the mean and the mode are not co-incident.

Now, the measurement errors have principally to do with the local nature of the photographic emulsion (charge coupled device chip), the local nature of the celestial sphere (in case of rapidly varying background owing to a nebulosity say), and the local nature of the measuring screw systematic errors. Any other important factors will also be local in nature, so we argue that this piece of the second meaning to the master assumption is weak.

The diagonal part of the variances of the plate model parameters looks like

$$\sigma_a^2 + \sigma_b^2 x^2 + \sigma_c^2 y^2.$$

The variances of the plate model parameters depend directly on the number of the reference stars, their distribution on the sub-plate, and their precision. By hypothesis all these are constant across a plate. The relevant amplitude of these variances is controlled by the ranges of x and y and these are minimal. To see why, remember that for the plate model solution of each sub-plate we could have translated all the axes to the center of every sub-plate so that Eq. (3) would have read instead

$$\xi - \xi_0 = a + b(x - x_0) + c(y - y_0). \tag{4}$$

In this form the range of $|x - x_0|$ or $|y - y_0|$ is clearly limited to half the size of a sub-plate. With 1° or even 2°-sized sub-plates this is at most 0.017 radians. Thus, the potential for the placement of a reference star on a sub-plate to have any effect on the standard coordinate mean error is minimal too. Hence, we feel that the second aspect of the second hypothesis can be easily accepted.

Let us now consider the origin of the four contributors to the ultimate 50% overlapping sub-plate produced position for a star lying in the lower left quarter of the basic sub-plate. Imagine the entire pattern shifted; first up, second to the right, third diagonally up and to the left, fourth diagonally up and to the right, and fifth diagonally down and to the right. Clearly, the result of any of these motions is to produce a zero overlap with the lower left quarter of the underlying sub-plate. Alternatively, consider the entire pattern shifted as follows; first down, second to the left, and third diagonally down and to the left. This exhausts all the independent motions and, just as clearly as above, the consequence of these actions is to produce a partial overlap with the lower left quarter of the fundamental sub-plate. In the first two cases of the latter set of movements the geometrical overlap with the original sub-plate is one-half while for the third case of the latter set of maneuvers the geometrical overlap is one-quarter. Using the assumption that the reference stars are uniformly distributed across the photographic plate allows us to assert that these territorial overlap fractions are precisely the sought-for correlation coefficients between the positions for a star lying in the lower left quarter of the basic sub-plate and each of these shifts of the design. The three shifts of the pattern which have non-zero correlation coefficients and the original sub-plate contribute the four positions we need to average together. Finally, we require the correlation coefficients among the second set of moves just discussed. Once again, the uniform distribution of reference stars assumption allows us to use their areal overlap to compute them. Between the left and down shifts it is one-quarter while between either of these and the diagonally down and left maneuver it is one-half.

Our estimate for the overlapping sub-plate determined xi standard coordinate for a star lying in the lower left quarter of the fundamental sub-plate is a linear combination of the four contributors, viz.

$$\Xi = \Sum w_n \xi_n. \tag{5}$$

The weights w_n are subject to the constraints discussed above; namely that they sum to unity and that they are non-negative (which together implies that they are individually bounded above by unity). Because Ξ is a linear combination of the individual values, operating on the expression with the expectation operator of the random errors merely reproduces Ξ (because this is also the expected value of each of the ξ_n). The treatment for the other standard coordinate proceeds in an entirely analogous fashion.

The variance of Ξ is computed by applying the propagation of error formula to Eq. (5). This introduces not only the weights $\{w_n\}$ and the correlation coefficients but the individual mean errors of each ξ_n. Before using the uniformity of the reference star distribution on the plate to assert that these are all equal because all sub-plates are *statistically* equal---with regard to the elimination of systematic errors in the plate as a whole, with regard to the errors of measurement, and with regard to the placement and quality of the reference stars in this small area of the celestial sphere---let us examine the complete resulting expression;

$$\sigma_{\Xi}^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + w_3^2\sigma_3^2 + (1 - w_1 - w_2 - w_3)^2\sigma_4^2$$
$$+ w_1w_2\sigma_1\sigma_2 + w_1w_3\sigma_1\sigma_3 + w_1(1 - w_1 - w_2 - w_3)\sigma_1\sigma_4/2$$
$$+ w_2w_3\sigma_2\sigma_3/2 + w_2(1 - w_1 - w_2 - w_3)\sigma_2\sigma_4 + w_3(1 - w_1 - w_2 - w_3)\sigma_3\sigma_4,$$

where we have explicitly substituted $1 - w_1 - w_2 - w_3$ for w_4 (and #4 corresponds to the diagonally shifted pattern in our scheme with #1 referring to the original sub-plate). If we evaluate the first partial derivatives of σ_{Ξ}^2 with respect to w_1, w_2, and w_3 set them equal to zero, then we obtain the following triplet of equations;

$$w_1(2\sigma_1^2 - \sigma_1\sigma_4 + 2\sigma_4^2) + w_2(\sigma_1\sigma_2 - \sigma_1\sigma_4/2 - \sigma_2\sigma_4 + 2\sigma_4^2) + w_3(\sigma_1\sigma_3 - \sigma_1\sigma_4/2$$
$$- \sigma_3\sigma_4 + 2\sigma_4^2) = 2\sigma_4^2 - \sigma_1\sigma_4/2,$$

$$w_1(2\sigma_4^2 + \sigma_1\sigma_2 - \sigma_1\sigma_4/2 - \sigma_2\sigma_4) + w_2(2\sigma_2^2 + 2\sigma_4^2 - 2\sigma_2\sigma_4) + w_3(2\sigma_4^2 +$$
$$\sigma_2\sigma_3/2 - \sigma_2\sigma_4 - \sigma_3\sigma_4) = 2\sigma_4^2 - \sigma_2\sigma_4, \qquad (6)$$

$$w_1(2\sigma_4^2 + \sigma_1\sigma_3 - \sigma_1\sigma_4/2 - \sigma_3\sigma_4) + w_2(2\sigma_4^2 + \sigma_2\sigma_3/2 - \sigma_2\sigma_4 - \sigma_3\sigma_4) + w_3(2\sigma_3^2$$
$$+ 2\sigma_4^2 - 2\sigma_3\sigma_4) = 2\sigma_4^2 - \sigma_3\sigma_4.$$

Obviously, obtaining the solution of this system of three inhomogeneous linear equations in three unknowns is trivial but that does not mean that it is analytically transparent from the point of view of the departures of σ_1, σ_2, σ_3, and σ_4 from equality (say all equal to σ). To aid in understanding the numerical consequences of non-equal variances among the contributing sub-plate positions to the overall weighting scheme, we have performed a Monte Carlo simulation of the values obtained from Eqs. (6) when σ_1, σ_2, σ_3, and σ_4 vary from strict equality. A typical outcome is shown in Fig. 6. This is a consequence of uniformly sampling the interval $[\sigma - \Delta\sigma, \sigma + \Delta\sigma]$ with $\Delta\sigma$ equal to 10% of the value of σ. Further decreasing the range sharpens the distribution around the mode while enlarging the spread about the mean does the opposite. In addition, when the range of possibilities for the weights increases, the probability that the correct solution to the minimum variance problem can be obtained by the partial differentiation method decreases. Hence, there was something to our concerns regarding the enforcement of the non-linear constraints on the weights. Despite the asymmetric appearance of the figure---the mean is <u>not</u> equal to the mode---the average value of all the (acceptable) weights is precisely 1/4.

To proceed further, we now factor the common mean error of ξ_n out from the propagation of error formula and deal with the ratio of (the variance of Ξ) to (the variance of ξ). Call this value *r*. After performing some straightforward algebraic reductions, *r* can be written as

$$r(w_1,w_2,w_3) = w_1/2 + w_1^2/2 - w_1(w_2 + w_3)/2 - 3w_2w_3/2 + w_2 + w_3 + (1 - w_1 - w_2 - w_3)^2 = r(w_1,w_3,w_2).$$

Computing the first partial derivatives of *r* with respect to the three independent weights in its argument list, setting them equal to zero, and solving the resulting system of equations yields an even value of 1/4 for each of the four weights.

Also observe from the geometry and from the discussion of shifting the patterns presented above, that *r* is a symmetric function of the left (#2) and down (#3) shifts. Thus, before solving the extremum problem for *r* we can exploit this 90° rotational invariance and set $w_2 = w_3$. Dealing with the simplified form thus obtained, this too (of course) yields 1/4 for the solution for each weight. This new form for *r* is

$$r = 1 + 3w_1^2/2 + 5w_2^2/2 - 3w_1/2 - 2w_2 + 3w_1w_2.$$

Lastly, we can enforce the 180° rotational symmetry of the situation on the analytical formulation and demand that w_1 and w_2 be equal. Having made all the weights equal, the normalization constraint now forces them to be one-quarter.

The end consequence is a value for r of $(3/4)^2$ or a relative weight of the overlapped position of 3/4. The naive expectation would have been that the four different estimates for the position would have yielded a one-half reduction of the individual weight [that is the one obtained from a simple square root of the number of degrees of freedom argument as in Eq. (2)]. Clearly the correlations have reduced the benefit in the formal estimate of the reduction in the random errors.

We now outline how to treat the 33% overlapping problem in a similar fashion as well as how to significantly simplify it using what we have learned about the 50% overlapping problem. Consider the case where our interest is directed toward the central sub-plate ninth. Then, of all the possible shifts of the primary sub-plate that could be made, let us concentrate on only those which will contribute one of the eight other values of the xi coordinate to the mean position of a star. Moving the pattern down by 1/3, up by 1/3, to the left by 1/3, or to the right by 1/3 clearly still covers 2/3 of the underlying sub-plate and the value of their correlation coefficients is 2/3. (However, moving the pattern in any of these fashions by 2/3 covers none of the primary sub-plate.) Next consider moving the pattern diagonally up and to the left by 1/3, up and to the right by 1/3, down and to the right by 1/3, or down and to the left by 1/3. These too cover equal amounts of the chief sub-plate; this amount is 4/9 and once again the assumption of the uniformity of the reference star distribution allows us to identify this value with the appropriate correlation coefficients. Next, we come to shifting the configuration diagonally by 2/3 (in all four possible ways) instead of just 1/3. This yields no non-zero overlaps.

In a similar fashion we can build up the remaining terms in the covariance matrix; more interesting is how we can use the rotational and translational symmetries to reduce what appears to be a nine-dimensional problem to a two-dimensional one. Translational symmetry implies that we can group the up and down shifts together as well as the right and left movements. Furthermore, rotational symmetry allows us to group all these together. (In other words, the sub-plate ninths on the perimeter of the basic sub-plate which are not in the corners can all be treated as equals.) Pursuing the use of the rotational symmetries further, for so far we have only used a 90° rotation, we can argue that all the sub-plate ninths in the corners can, in fact, be treated as equals; adjacent corners based on a 90° rotation, diagonally opposite ones based on a 180° rotation. Finally, using the normalization constraint simplifies the function we need to minimize yet again thereby rendering this problem tractable (in the equal variance case). Skipping the details of the mathematics, wherein these symmetries are reflected as indicated above, the numerical result is uniform weight of one-ninth.

The ultimate value for r is now $(2/3)^2$ implying a relative weight of the overlapped position of 2/3. In this instance the simplistic expectation would have been that the nine contributing values for the position would have yielded a one-third reduction in the weight. Once again we can clearly see the effects of the correlations in reducing the benefit in the formal estimate of the reduction in the random errors.

In our (very limited) experience with enforcing the traditional plate overlap constraints on sub-plates, the preponderance of the constraint equations relative to the equations of condition has severe repercussions; namely the propagation of the same sub-plate model over the entire plate. (Part of this result is a consequence of the very high weight we gave to the constraint equations relative to the normal equations.) This situation is the reverse of the traditional one. Hence, while we kept sub-plates and overlapped them, the ξ or η value for a star is the (weighted) average of all its ξ or η values. Now the purpose of the overlapping is to reduce the errors in the sub-plate constants induced by the errors in the reference star positions. One disadvantage of not enforcing an overlapping constraint is that individual sub-plates are free to "float" against the background of the Schmidt plate's borders. A suggestion to control small shifts or rotations of the sub-plates is presented in Sub-section f. We have not numerically tested this procedure because the improvement brought about by such a scheme is minimal in the presence of non-negligible measurement error, big sub-plates, and poor reference catalogs.

e) Sub-plate Alternatives

Sub-plate overlap is not the best method for reducing a set of wide field-of-view Schmidt plates from the same instrument. Another technique, which uses residual maps for an *ex post facto* correction is usually superior. We refer to it as the "mask" technique.

Although Dodd (1972) experimented with a version of this, my colleagues and I have fully developed and tested it (Taff, Lattanzi, and Bucciarelli 1990). However, it is inapplicable to the astrometric reduction of a single plate for it requires an integration of tens, if not hundreds, of plates to be statistically significant. See also the reseau experiments of Fresneau (1978) and Warnock and Usher (1980).

Even though the gnomonic projection (Taff 1981, 1991) is inappropriate for a Schmidt optical system---because it preserves tangents of angles instead of the angles themselves---it is a useful first approximation. Moreover, as it is the mapping for astrographic and long focus astrometry, most workers have used it and therefore are intimately familiar with it. Hence, as with all other Schmidt plate astrometrists, we used to use it too. The software delivered herein has been upgraded to utilize the spherical coordinate projection too (Dick 1991).

f) Bonding The Sub-plates

As each sub-plate is separately reduced, with its own reference stars, there is the possibility that the sub-plate model may tear the sub-plate away from its moorings on the plate. Not only translation and rotation are possible, so is magnification or de-magnification. Only a sub-plate-to-sub-plate bonding can control sub-plate movements and keep the whole structure rigid. How to do so? We propose using the AGK3R/SRS (or perhaps better the IRS) stars.

Since one is using the sub-plate technique one is necessarily utilizing a large-scale photographic star catalog as the reference catalog. This means that low density, high precision catalogs, such as the AGK3R, SRS, or perhaps CAMC catalogs, are not contributing to the sub-plate modelling. There are, typically, ~ 42 AGK3R or SRS stars per 6.°5 x 6.°5 Schmidt plate and they will be present on many different sub-plates. Because the precision in these catalogs is better than that of the large- scale reference catalogs, we can use the independent information in their star-to-star spacing to control the shift, orientation, and scale changes in the sub-plates.

Specifically, if i and j refer to two AGK3R/SRS stars on a plate, then the great circle distance between them D_{ij} is given by

$$\cos D_{ij} = \cos(\alpha_i - \alpha_j)\cos\delta_i \cos\delta_j + \sin\delta_i \sin\delta_j \tag{7a}$$

where α and δ are right ascension and declination. When re-written in terms of standard coordinates the outcome is

$$\cos D_{ij} = \{f^2 + \eta_i\eta_j + \xi_i\xi_j\}/[(f^2 + \eta_i^2 + \xi_i^2)(f^2 + \eta_j^2 + \xi_j^2)]^{1/2} \tag{7b}$$

where f is the telescope's focal length. The customary convention is to measure ξ and η in radians in which case f is equal to unity.

When the standard coordinates are interpreted as the average values of these variables over all sub-plates then we can re-write the least squares problem for the sub-plate models, which is to separately minimize R_ξ and R_η,

$$R_\xi = \Sigma\,[\xi_{nm} - (a_n + b_n x_{nm} + c_n y_{nm})]^2, \quad R_\eta = \Sigma\,[\eta_{nm} - (d_n + e_n x_{nm} + f_n y_{nm})]^2$$

as to minimize R,

$$R = R_\xi + R_\eta + R_c.$$

The constraint part of R, R_c, has the form

$$R_c = \tfrac{1}{2}\,\Sigma\,[F(\Xi_i,H_i,\Xi_j,H_j) - \cos D_{ij}]^2$$

with F defined by

$$F = (1 + H_iH_j + \Xi_i\Xi_j)/[(1 + H^2_i + \Xi^2_i)(1 + H^2_j + \Xi^2_j)]^{1/2}.$$

Ξ is given by Eq. (5), with an analogous expression for H, and D_{ij} is a **number** from Eq. (7a) with catalog values of α_i, δ_i, α_j, and δ_j. [An idea, vaguely of this form, was put forward by Stephenson (1974)].

The second issue concerns additional information. For instance, a pair of stars specifies not only a distance apart but an angle at a defined vertex. Position angle is a natural choice for this. Because we have an all-sky mentality, position angle per se we feel is unsuitable. A device that treats all plates the same, and all pairs of stars roughly equally, is the angle between the stars subtended at the plate's tangential point. If we call this A_{ij}, then from spherical trigonometry

$$\cos A_{ij} = (\eta_i\eta_j + \xi_i\xi_j)/[(f^2 + \eta_i^2 + \xi_i^2)(f^2 + \eta_j^2 + \xi_j^2)]^{1/2}$$

(or because the projection is gnomonic). Clearly there is very little additional information in A_{ij} over that contained in Eq. (7b). Moreover, were the AGK3R/SRS data exact, then these angles would be superfluous (as would most of the terms in R_c).

To see this point more clearly, imagine drawing the information content in rigorous values of D_{ij} (Fig. 7). Fixing on star number one first, D_{1n}, n = 2, 3, 4, 5 specifies a set of concentric circles. D_{2n}, for n = 3, 4, 5,.... picks out pairs of possible intersections. Beyond D_{3n}, n = 4, 5 the entire problem is determined to within a "reflection". Just as neither can D_{4n}, D_{5n},...., resolve the parity issue, nor can they further refine the places of intersection. Clearly the A_{ij} values are both irrelevant and superfluous too.

IV. SUMMARY

When we began our work on Schmidt plate astrometric reduction we confronted a problem that was dominated by systematic errors instead of random errors as is usually assumed to be the case in astronomical photographic plate reduction. Hence, the techniques of classical astrometry had to be abandoned in order to make progress. However, as can be seen from the concept of overlapping sub-plates, there is a continuity with the best of historical work. We have extended this by recognizing that we are constrained in only having one measurement of each star and not limited in how far we sub-divide a plate and how densely we overlap the different patterns. Our success extends beyond the newer versions of the *Hubble Space Telescope* Guide Star Catalog.

It includes an updated version of the AGK3 using Guide Star Catalog plates (Bucciarelli, Daou, Lattanzi, and Taff 1992), to wide-field astrograph plates (Smart, Taff, and Morrison 1995), to the discovery of additional, magnitude-dependent systematic errors in deep Schmidt plates (Morrison, Röser, Lasker, Smart, and Taff, 1996) which only became visible after the primary Schmidt plate astrometric issues had been successfully dealt with. We see no reason why this technique should not be the standard method for "plate" reduction in all cases.

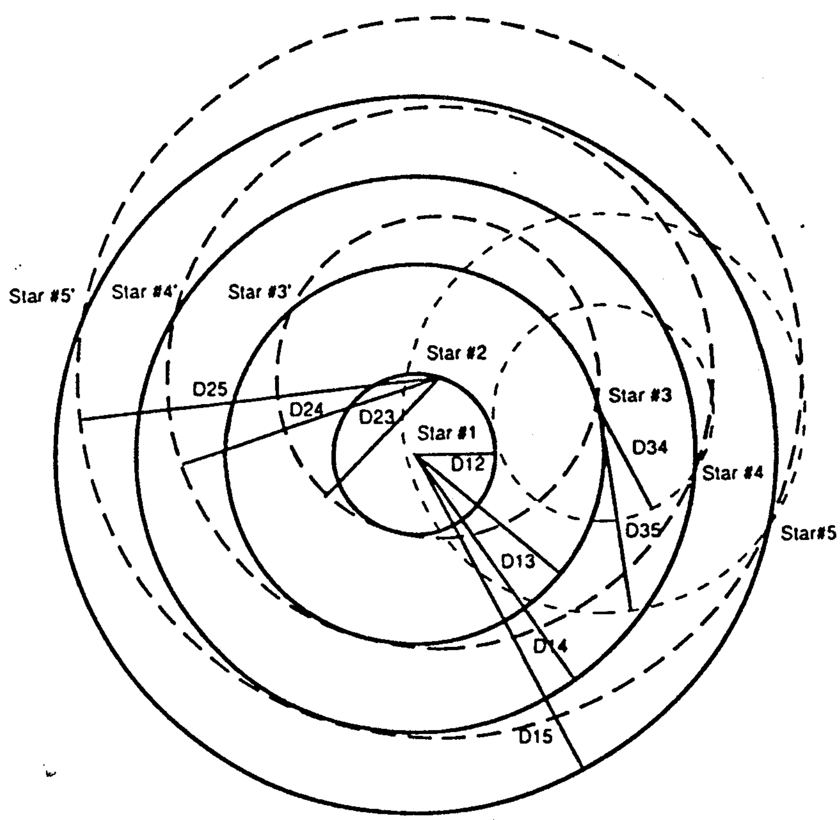

Fig. 7. Series of circles of constant angular distance demonstrating that with five objects only a left/right parity issue remains unresolved.

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Appendix: *SUBPLATE* User's Guide.

The *SUBPLATE* program can be run on any VMS or UNIX machine. If you do not have the software, you can get a copy from the Space Telescope Science Institute anonymous account 'ftp.stsci.edu' in the directory software/subplate. Files and information needed to run *SUBPLATE*:

The *SUBPLATE* program requires the following information:

- Reference Catalog with (x,y) and (ra,dec) for an object on a plate. Additional information, useful but not required is: object magnitude, the random errors in the right ascension and declination (sigma_ra and sigma_dec), and a column with a flag (0,1) to count or reject an object for the subplate's solution.
- Observatory position and atmospheric parameters for the astronomical refraction correction.
- A comparison catalog to check the subplate solutions or a catalog with standard coordinate (x,y) position to obtain the corresponding to equatorial coordinates (ra,dec).

Inputs to the Subplate program are done through Fortran Namelist files, consisting of a block of information that begins with a '$' in column 2 and the name of the block; the block ends with '$end' also starting in column 2.

The program needs three ASCII files:

1) *SUBPLATE* Input File.
2) Reference Catalog Description. This is a Namelist file with block name '$columns'. A template file called reference.nml is supplied for you to copy and edit. For an example, see below.
3) Comparison Catalog Description. This is a Namelist file with block name '$columns'. A template file called comparison.nml is supplied for you to copy and edit. For an example, see below.

Running the program:

To run the program just type: subplate The program will ask you for the

SUBPLATE Input File name.

1) *SUBPLATE* Input File. This is a Namelist file the user needs to enter after the program asks for it. It contains the 'nm_subplate' and the 'plate_info' Namelist blocks. The Namelist entry consists of a key word name, and an equal sign ('=') and its value. The data type for each key word needs to be precise; *i.e.*,

String key word type: Enclose the value in single quotes.

Integer key word: An integer value.

Floating point: A decimal number is required. *e.g.* 0.123e10, 0.11 (not .11), 123.0 (not 123).

The '$nm_subplate' name list has the following key words:

ressftr: New name to contain residual information from SUBPLATE (string type).

statsftr: New name to contain statistical information from SUBPLATE (string type).

ref_file: Name of the file containing the reference star catalog (string type).

xy_to_rd: Do you want RA and DEC from x,y in the 'cmc_file'? (yes, no) (string type).

cmc_file: Name of the file containing the comparison star catalog or a program file with (x,y) information (string type).

plotc: New name for plotting information on the comparison star catalog (string type).

plotr: New name for plotting information on the reference star catalog (string type).

refc_desc: Filename containing namelist with the reference star catalog description (string type).

comc_desc: Filename containing namelist with the comparison star catalog description (string type).

dat_file: New name with miscellaneous output information (string type).

minstar_sp: Minimum number of stars per sub-plate (default value is 6) (integer type).

min_subp: Minimum number of sub-plates (default value is 4) (integer type).

The 'ibyt' value is zero per default, but it can be reset in the 'columns' namelist. For an explanation of what is the 'ibyt' value see sub-section 2 below.

ibythi: Any 'ibyt' value greater that this is rejected (def is MAXINT) (integer type).
ibytlo: Any 'ibyt' value less than this is rejected (default is -MAXINT) (integer type).
idebug: Print (ra,dec,x,y) For ref catalog: 1; for Comparison; 2, both; 3 (integer type).

The user can edit the value of the different fields without changing the key word since these are variables for the running program.

<u>Example:</u> Namelist Block. This is what *SUBPLATE* will read. [Edit the values of each of the key words, notice that all the key words that are character variables need to be enclosed in single quotes. Do Not change any of the names to the left of the equals (=) sign, since these names are what the program expects.]

```
$nm_subplate
ressftr  = '/machine/data1/ress.dat'
statsftr = '/machine/data1/stat.dat'
ref_file = '/machine/data1/00J0.ppm'
xy_to_rd = 'yes'
cmc_file = '/machine/data1/00J0.cmc'
plotc    = '/machine/data1/plotc.dat'
plotr    = '/machine/data1/plotr.dat'
refc_desc= '/machine/data1/columns.nml'
comc_desc= '/machine/data1/xy_cat.nml'
dat_file = '/machine/data1/00j0.dat1'
minstars_sp = 6
min_subp = 4
ibythi   = 1
ibytlo   = 1
```

idebug = 2
$end

The '$plate_information' name list has the following key words:

plate_name: Plate name.
epoch: Plate epoch in units of yyyy.dddd (*e.g.* 1967.345).
center_ra_hr: Hours of Right Ascension of the tangential point (integer type).
center_ra_min: Minutes of Right Ascension of the tangential point (integer type).
center_ra_sec: Seconds of Right Ascension of the tangential point (floating type).
center_dec_deg: Degrees of Declination of the tangential point (integer type).
center_dec_min: Minutes of Declination of the tangential point (if negative Dec put -mm) (integer type).
center_dec_sec: Seconds of Declination of the tangential point (if negative Dec put -ss) (floating type).
plate_scale: Plate scale in arc_sec/mm (floating type).
row_size: Number of pixels per row (integer type).
pixel_size_x: Pixel size in x (microns/pixel) (floating type).
pixel_size_y: Pixel size in y (microns/pixel) (floating type).
ddmglim: Magnitude limit for a reference star (floating type).
iunitw: Weight of the observation equations (values allowed are 0 and 1, recommended is 1) (integer type).
median: Median calculation (0 is no, 1 is yes), recommended value is 1 (integer type).
maxiter: Maximum number of iterations (integer type).
refraction: Correct for astronomical refraction? (yes or no) (string type).
obs_lat: Observatory latitude (decimal degrees) (floating type).
obs_long: Observatory longitude (decimal degrees) (east +, west -) (floating type).
dewpoint: Dewpoint at exposure time (Celsius) (nominal value: 4.0) (floating type).
temperature: Temperature at exposure time (Celsius) (nominal value: 9.31) (floating

type).

pressure: Barometric pressure at exposure time (mm) (nominal value: 751.51) (floating type).

altitude: Observatory elevation (meters) (floating type).

exposure_year: Year of the exposure (*e.g.* 1986.0) (floating type).

exposure_month: Month of the exposure (1-12) (integer type).

exposure_day: UT day of the exposure (integer type).

exposure_hour: UT hour of the middle of the exposure (integer type).

exposure_min: UT minute of the middle of the exposure (integer type).

<u>Example:</u>

```
$plate_information
plate_name     = '00J0'
epoch          = 1982.302
center_ra_hr   = 14
center_ra_min  = 54
center_ra_sec  = 58.066
center_dec_deg = 11
center_dec_min = 24
center_dec_sec = 47.00
plate_scale    = 67.2
row_size       = 14000
pixel_size_x   = 25.28445
pixel_size_y   = 25.28445
ddmaglim       = 8.0
iunitw         = 1
median         = 1
maxiter        = 1
refraction     = 'yes'
obs_long       = -116.8633
```

```
obs_lat        = 33.3567
dewpoint        = 4.0
temperature    = 9.31
pressure        = 618.0
altitude        = 1706.0
exposure_year  = 1982
exposure_month = 4
exposure_day   = 20
exposure_hour  = 8
exposure_min   = 27
$end
```

2) Reference Catalog Description.

The input catalogs to the *SUBPLATE* program can be any text catalog in columnar form with lines of up to 300 characters each. The minimum information required by the program is the (ra,dec) and (x,y) position for each reference object in the catalog.

This is the namelist to describe the input catalog columns to the *SUBPLATE* program. Each of the columns below can be describe with up to 6 parameters which are:

1. col_name(i): The value of this parameter is any of the names indicated below inside single quote.
2. col_start(i): The start of the field counted from the left side of any input catalog line (character count from 1 to 300).
3. col_format(i): The fortran format that specified the field data type. The format width 'w' in 'Fw.d' should be wide enough to occupy the whole field.
4. col_units(i): Unit name for the column. See explanation below.
5. col_scale(i): The scale factor to applied to the numeric field for the purpose of changing units.
6. col_offset(i): The offset term to add to the numeric field. The formula applied is:

new_value = (columns_value + col_offset) * col_scale

1. col_name: The names should be in lower case only.

ident: Any index (character string only). You can choose any column from your catalog as index.

obj_ra_hr: Object coordinates at catalog epoch RA (hours). If you have hours and decimal, describe only this.

obj_ra_min: Object coordinates at catalog epoch RA (minutes of time).

obj_ra_sec: Object coordinates at catalog epoch RA (seconds of time).

dec_sign: Sign of the declination (character*1). This is for cases where you have a separate column for the declination sign.

obj_dec_deg: Object coordinate at catalog epoch DEC (degrees).

obj_dec_min: Object coordinate at catalog epoch DEC (minutes of arc).

obj_dec_sec: Object coordinate at catalog epoch DEC (seconds of arc).

obj_mag: Object magnitude.

sigra: Standard deviation in RA on the great circle at the plate epoch (sec of arc).

sigdec: Standard deviation in DEC at the plate epoch (sec of arc).

mura: Proper motion in RA (arc sec per year).

mudec: Proper motion in DEC (arc sec per year).

sigmura: Proper motion error in RA.

sigmudec: Proper motion error in DEC.

epoch_ra: Star mean epoch in RA (integer year).

epoch_dec: Star mean epoch in DEC (integer year).

obj_x: Plate measurements in pixels in the X direction.

obj_y: Plate measurements in pixels in the Y direction.

ibyt: Integer flag to discard object from operation (a value not equal to zero means 'do not account' in the solution). If you only have one column for the RA, then you need to write the col_name 'obj_ra_hr' and the units can be either 'radians' or 'hours'; you do not need to specify the fields for minutes or seconds.

Units: The *SUBPLATE* program expects the input catalog values in certain units. For this you are allow specific values for the unit field that you should put in the 'col_units(i)'. The following is the list of possible values:

2. col_name: units value (lower case only)

obj_ra_hr: 'hours', 'radians'. If your RA column is in units of hours put 'hours' if the are in units of radians, put 'radians', otherwise, specify a col_offset or a col_scale: to convert to this units.
obj_ra_min: 'minutes', 'radians'. Same comment applies.
obj_ra_sec: 'seconds', 'radians'. Same comment applies.
obj_dec_deg: 'degrees', 'radians'. Your units in the DEC column should be 'degrees' or 'radians', if not, then specify col_offset value or a col_scale value to convert to this units.
obj_ra_min: 'minutes', 'radians'. Same comment applies.
obj_ra_sec: 'seconds', 'radians'. Same comment applies.
sig_mu_ra: 'u/year' (u: arc seconds)
sig_mu_dec: 'u/year' (u: arc seconds)
epoch_ra: 'yyyy' (This means that your 'epoch_ra' column is in units of integral year, *e.g.* 1994)
epoch_dec: 'yyyy' Same, comment applies.
obj_x: 'pixels'.
obj_y: 'pixels'.

Example: This example consists of 2 lines to form a ruler (1-80) and one line of catalog information, from here the $columns namelist information is drawn.

```
12345678901234567890123456789012345678901234567890123456789012345678901234567890
123456789         1         2         3         4         5         6         7         1.  -781438.
1.   1.   0.57988566201E+01   -0.13714641511E+01   0.15744E-05   0.44897E-06
-0.19163603276E+03 -0.17538509888E+03 0.40000E-02 0.40000E-02 0.8200000E+01
```

```
$columns
col_name(1)   = 'ident'
col_start(1)  = 6
col_format(1) = 'a8'
col_name(2)   = 'obj_ra_hr'
col_start(2)  = 22
col_format(2) = 'd17.11'
col_units(2)  = 'radians'
col_name(3)   = 'obj_dec_deg'
col_start(3)  = 40
col_format(3) = 'd18.11'
col_units(3)  = 'radians'
col_name(4)   = 'sigra'
col_start(4)  = 58
col_format(4) = 'e12.5'
col_name(5)   = 'sigdec'
col_start(5)  = 70
col_format(5) = 'e12.5'
col_name(6)   = 'obj_x'
col_start(6)  = 83
col_format(6) = 'd18.11'
col_units(6)  = 'pixels'
col_scale(6)  = 4.8481368110954E-4
col_offset(6) = 199.
col_name(7)   = 'obj_x'
col_start(7)  = 102
col_format(7) = 'd18.11'
col_units(7)  = 'pixels'
col_scale(7)  = 4.8481368110954E-4
col_offset(7) = 199.
```

```
col_name(18)   = 'magnitude'
col_start(18)  = 146
col_format(18) = 'e14.7'
$end
```

3) Comparison Catalog Description. If you want to get an (ra,dec) position from a series of (x,y) positions, then you only need to describe the x,y columns and probably an index one for your input file. The column description is identical as for the Reference Catalog Description.

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An expansion of the plate overlap method of astrometric data reduction to a single plate has been proposed and successfully tested. Each plate is (artificially) divided into sub-plates which can then be overlapped. This reduces the area of a "plate" over which a plate model needs to accurately represent the relationship between measured coordinates and standard coordinates. Application is made to non-astrographic plates. Indeed, the method is completely general and can be applied to any type of recording media.

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