Pergamon

J. Quant. Spectrosc. Radiat. Transfer Vol. 55, No. 6, pp. 837–848, 1996

0022-4073(95)00190-5
0022-4073/96 $15.00 + 0.00

NASA-CR-204545

NDB
IN-90-CR

ARE PLANETARY REGOLITH PARTICLES BACK SCATTERING? RESPONSE TO A PAPER BY M. MISHCHENKO

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(Received 22 September 1994; received for publication 11 December 1995)

Abstract—In a recent paper [*JQSRT* **52,** 95 (1994)] Mishchenko asserts that natural soil particles are strongly forward scattering, whereas particles on the surfaces of objects in the solar system have been inferred to be back scattering. Mishchenko suggests that this apparent discrepancy is an artifact caused by using an approximate light scattering model to analyse the data, and that planetary regolith particles are actually strong forward scatterers. The purpose of the present paper is to point out the errors in Mishchenko's paper and to show from both theoretical arguments and experimental data that inhomogeneous composite particles which are large compared to the wavelength of visible light, such as rock fragments and agglutinates, can be strongly back scattering and are the fundamental scatterers in media composed of them. Such particles appear to be abundant in planetary regoliths and can account for the back scattering character of the surfaces of many bodies in the solar system. If the range of phase angles covered by a data set is insufficient, serious errors in retrieving the particle scattering properties can result whether an exact or approximate scattering model is used. However, if the data set includes both large and small phase angles, approximate regolith scattering models can correctly retrieve the sign of the particle scattering asymmetry. Copyright © 1996 Elsevier Science Ltd

INTRODUCTION

Present models of light scattering by planetary regoliths or soils are based on the equation of radiative transfer.[1] This equation is strictly applicable only to continuous media, but by appropriately defining the extinction, scattering and absorption coefficients, it can also describe the behavior of radiation in particulate media in which the scatterers are separated by distances large compared to their sizes. However, in regoliths the particles are touching and there is some uncertainty as to whether or not the radiative transfer equation can be applied to regoliths and, if so, what are the fundamental scattering units in them. One way of answering, which will be used in this paper, is to assume that the radiative transfer equation is applicable, and to adopt an operational definition. The fundamental scattering unit of a regolith is defined to be the particle whose mean single scattering albedo and angular phase function would make the calculated bidirectional reflectance and polarization match the ones that would be measured if completely sampled over all angles of incidence, i, viewing, e, and phase, g. If this definition is adopted, then a new question is, what the relation is between the single scattering albedo and angular scattering function of the fundamental scattering units and those that the actual particles in the regolith would have if isolated. Most regolith models assume that they are the same, or at least similar.[1]

In a recent paper Mishchenko[2] (hereinafter referred to as Paper I) attacked this problem using structure factor theory. In the first part of his paper he showed that the scattering properties of dielectric spheres and spheroids are similar whether they are isolated or densely packed in clusters. This is an important contribution, because it tends to support one of the major assumptions underlying regolith scattering models. His analysis also shows that the intensity of light scattered in the extreme forward direction by a particle large compared to the wavelength in a cluster is smaller than if the particle is isolated. This result is also important because it demonstrates mathematically that the diffracted component of the radiance scattered by a large particle is reduced by the presence of close neighbors, as the author has previously shown using intuitive arguments.[7,8]

In the second part of Paper I, Mishchenko then goes on to assert that, "it is well known that independently scattering ice and soil particles have positive and usually large asymmetry parameters and are (strongly) forward scattering." However, some analyses[1,3–6] of photometric observations of the surfaces of bodies in the solar system have retrieved mean particle angular scattering functions that are back scattering. These analyses were done using a model[7,8] for scattering by a particulate medium in which the singly scattered component of the radiance is calculated exactly, while the multiply scattered portion is approximated by a two-stream solution to the equation of radiative transfer for isotropic scatterers. (This model will be referred to as the isotropic multiple scattering approximation, or IMSA, model.) This model yields an expression for the scattered radiance that is convenient because it is mathematically simple and analytic, but which can be quantitatively inaccurate if the particles of the medium have a high albedo, are strongly anisotropic scatterers and the angles of incidence and viewing are such that the radiance is dominated by multiply scattered light. Such approximate models are useful, even though they are inexact, because of the large observational errors inherent in absolute planetary photometric measurements; it is difficult to carry out absolute planetary photometry to much better than $\pm 10\%$.

In Paper I Mishchenko correctly points out that the result of his structure factor analysis shows that particles that are forward scattering when isolated remain forward scattering in a regolith. Because Mishchenko claims that all soil particles are inherently forward scattering, he suggests that the apparent back scattering character of planetary regolith particles is an artifact introduced by use of the IMSA model. He demonstrates how this might happen by several examples.

The purpose of this paper is to point out the errors in the second part of Paper I. It will be shown that inhomogeneous composite particles are a major class of backscattering dielectric particles that exist in nature, and that they constitute the fundamental scattering unit of media in which they occur. It will also be shown that when a particle scattering asymmetry of the wrong sign is retrieved, the error is caused by an inadequate range of phase angles in the data being analysed and not by the IMSA model.

DISCUSSION

The present discussion is not concerned with the strong, narrow, forward scattered component of light diffracted by large particles, nor with the opposition effect, the sharp peak in intensity scattered by a particulate medium in the directions toward the source of collimated radiation. Also, there is no doubt that clear, homogeneous dielectric particles are strongly forward scattering because of the light refracted and transmitted through them. Hence, particles in planetary regoliths can be expected to have a forward scattered component. Indeed, as long ago as 1963, the author showed[9] that the integral phase function of the Moon requires that the angular scattering function of particles of lunar soil have a moderate forward scattering lobe, and also pointed out that the angle at which the maximum in the polarization of moonlight occurs implies that the particles have a transmitted, forward scattered component.[8,12] The question considered here is how strong this component is in planetary regoliths relative to the back scattered radiance, whether it is extremely large and dominates the phase curve, as in Mie scattering by a homogeneous dielectric sphere, or whether its amplitude is comparable to, or even smaller than, the back scattered component.

Lunar soil is especially relevant to this discussion because the Moon is strongly back scattering.[10] The full moon is more than 10 times brighter than the half moon (not including the opposition surge, which is about 7° in half-width). The most straightforward way of accounting for this is if the particles of lunar regolith are themselves back scattering (with a moderate forward scattering lobe, as discussed above). However, if the particles are forward scattering, some other explanation for the back scattering nature of the surface must be found. The only other way of accomplishing this is to make the surface extremely rough on a scale large compared with the mean particle size; that is, the surface is required to be covered with very deep, steep-walled depressions in which shadows cause the surface to appear to be back scattering. The latter explanation is implicit in the

Lumme-Bowell model,[1] which forces *a priori* all planetary regolith particles to be forward scattering.

However, explanations invoking roughness are inadequate because in the laboratory, where the surfaces of samples are macroscopically smooth, lunar soils are still found to be strongly back scattering. This is illustrated by Fig. 1, which shows that, when viewed at 60° from the normal, the brightness of lunar soil decreases monotonically by a factor of four as the phase angle changes from 5 to 120°. For a low albedo surface, such as lunar soil, viewed and illuminated at large angles from the normal, the scattered radiance is dominated by single scattering, which is directly proportional to the particle scattering function. If the particles were strongly forward scattering, one would expect to see some indication of this in the form of increased brightness at large phase angles.

Hence, the mean scattering function of particles of lunar regolith must have a strong back scattering component. However, Paper I denies that this is physically possible because it claims that all natural dielectric particles are strongly forward scattering. Paper I attempts to argue that the apparent back scattering character of planetary regolith particles deduced from applying the IMSA model to observations is an artifact resulting from the inexact description of multiple scattering. While such an explanation might conceivably be possible for some of the high albedo satellites in the solar system, it is clearly untenable for the Moon, whose Bond albedo is only 0.07, so that the scattering properties are strongly dominated by singly scattered radiation, which is calculated exactly in the IMSA model.

The lunar regolith consists of fragments of rocks, minerals and glass. It is mafic in composition; that is, the major minerals are anorthites, pyroxenes, olivines and ilmenite. The mean particle size is about 50 μm,[11] but a wide range of sizes occurs. The most abundant particles in mature lunar soils are agglutinates. These are particles with complex shapes and internal structures, including cracks and voids, and consist of fragments of rocks and minerals welded together by a matrix of shock-melted glass. Although agglutinitic glass is a dielectric, it is highly absorbing because it contains submicron-sized particles of metallic iron.

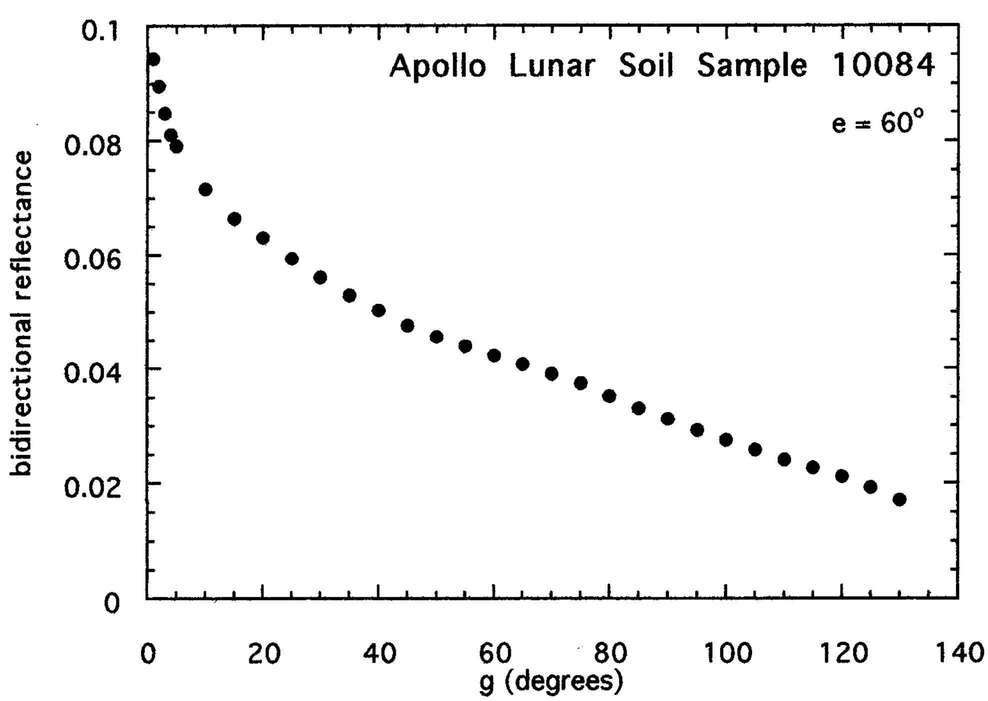

Fig. 1. Radiance (relative to a PTFE standard at e = 0 and i = 5°) of NASA Apollo lunar soil sample number 10084 measured in visible light with e = 60° as i varies in the principle plane, plotted against phase angle. The apparatus is described in Ref. 16. The sample was prepared by gently sprinkling the soil into the sample holder from a height of less than a centimeter.

Although smooth surfaced, convex, opaque particles are forward scattering, it has been demonstrated both experimentally[12] and theoretically[13] that roughening their surfaces can make them back scattering. However, the majority of the particles in lunar soil are dielectric, so that some other explanation for the back scattering nature of lunar regolith particles must be found. Since individual dielectric mineral particles are forward scattering, as Mishchenko correctly states, the only remaining candidates are the rock fragments and agglutinates. It will be demonstrated in this paper that inhomogeneous, composite particles, such as rock fragments and agglutinates, can indeed be efficient back scatterers, which explains the back scattering nature of the lunar surface and, by extrapolation, of the surfaces of other bodies in the solar system also.

Paper I states that the IMSA model assumes that somehow "—particles become back scattering when they are densely packed—"in the form of "—aggregates or chains of smaller particles". This is a misrepresentation of the assumptions of that model. The only situation in which the IMSA model assumes that aggregates of smaller particles behave optically like larger particles is when the particles are small compared with the wavelength. However, it is widely accepted that this, in fact, does happen because of cooperative interactions among submicroscopic, densely packed particles. Indeed, this assumption underlies the DDA (discrete dipole approximation) method for calculating the scattering properties of a large particle by synthesizing it by an array of dipoles. What is presently uncertain is how these cooperative effects manifest themselves and what constitutes the fundamental scattering unit in a medium of submicroscopic particles. However, contrary to the statements in Paper I, the author has never assumed that this is also the case when the particles are large compared with the wavelength.

Hence, concerning the behavior of large particles, the author is (and always has been) in complete agreement with Mishchenko when he says in Paper I, "However, if such composite particles are densely packed to form a regolithic surface, the individual aggregates or chains become indistinguishable (there is no way to determine whether a grain is the last element of the previous chain or the first element of the next chain), and the light incident on the surface will "see" the scattering medium as a single superaggregate or superchain composed of the smaller grains. Thus, it is the smaller grains rather than the indistinguishable "individual" aggregates or chains that play the role of single scatterers."

The difficulty arises when Paper I attempts to apply the same reasoning to composite particles. It argues that a composite particle behaves optically just like an aggregate or chain of its constituent particles, and that it is the sub-particles that play the role of the fundamental scatterers of the medium rather than the composite particles. Paper I also states that invoking composite particles to explain back scattering "—requires an *unrealistically complicated* internal structure for regolithic and soil particles" (author's emphasis).

Now, anyone who agrees with the statement that real composite particles are "unrealistically complicated" has, quite obviously, never looked at a thin section of an agglutinate, or even an ordinary fragment of rock, under a petrographic microscope. Most natural rocks are incredibly messy things. They consist of grains of different minerals welded together in optical contact, often containing cracks, voids, inclusions, and lamellae where the composition, phase or crystalline orientation have changed. Agglutinates, being the result of violent hypervelocity impact processes, are even more messy.

The errors in the arguments in Paper I regarding composite particles can be seen by considering Fig. 2. Figure 2(A) shows schematically a medium consisting of particles large compared to the wavelength, of various compositions, arranged and oriented randomly, but touching each other at a few points in a condition of mutual support. The medium is assumed to be in a vacuum on the surface of an airless body. Mishchenko and the author are in complete agreement that the single scattering units in this medium are the individual particles, not the aggregates or chains. Figure 2(B) shows the same particles, but now welded together into rock fragments, which are composite particles in which the interior faces of the particles are nearly everywhere in optical contact with each other. Figure 2(C) also shows the same particles as Fig. 2(A), but now incorporated into agglutinates, in which one type of particle has been shock melted and injected around the other particles, enclosing them in optical contact before solidifying.

According to the reasoning in Paper I, all three media are optically identical and there should be no difference in their scattering properties. However, clearly, this cannot be the case, because

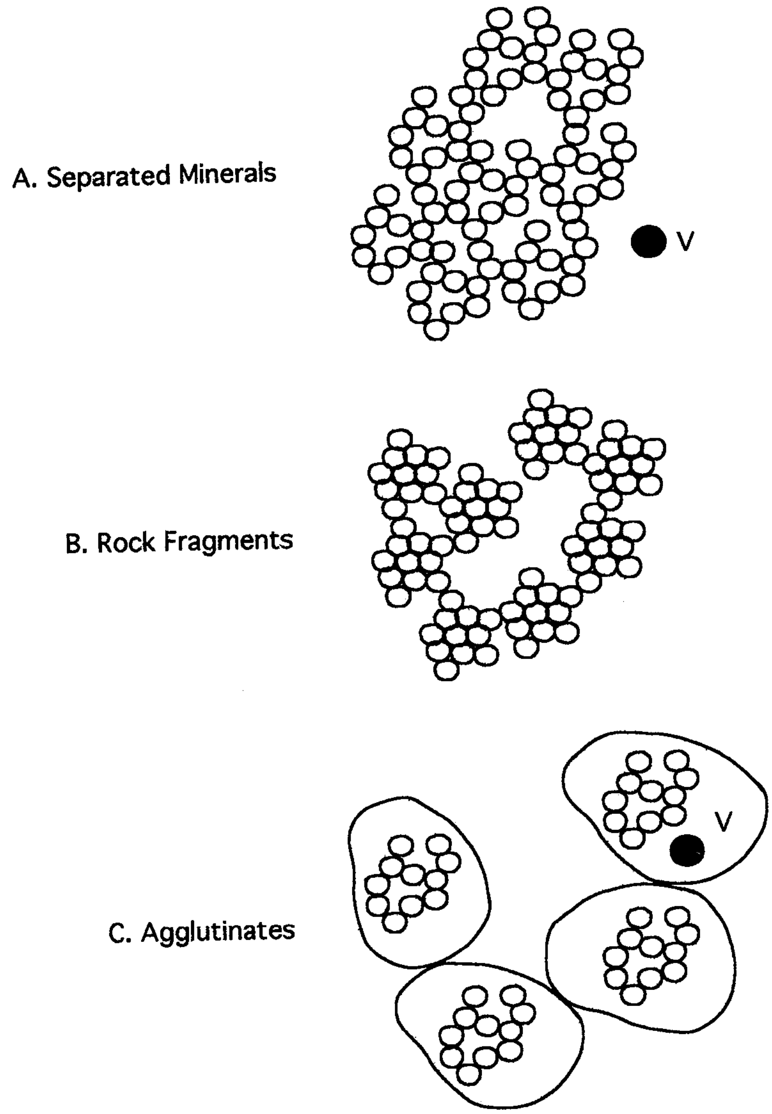

Fig. 2. Schematic diagram of portions of three media consisting of the various minerals, which are the same in all three figures. A. Medium of separate mineral particles. B. Medium of the same mineral particles as in A, but welded together into rock fragments in which the minerals are in optical contact. C. Medium of the same mineral particles as in A, but one of the mineral types has melted and flowed around the other minerals before solidifying to form agglutinates.

in Fig. 2(A) the optical environment of all of the particles is a vacuum, but in Fig. 2(B) and 2(C) it consists of adjacent particles. Hence, the change in the refractive index as a ray of light enters or exits a sub-particle will be smaller in the media depicted in Fig. 2(B) or 2(C) than in 2(A). Since the albedo and angular scattering pattern of a particle is strongly influenced by the contrast in refractive index across its surfaces, the scattering patterns of the particles and, hence, of the medium are altered.

An extreme example of the incorrectness of claiming that all of the media in Fig. 2 are optically identical is illustrated by the particle labeled "V". This represents a small volume of empty space in 2(A) that has been incorporated into the agglutinate in 2(C) in the form of a bubble or void. Surely, no one could argue that a void surrounded by empty space scatters light like a void surrounded by a dielectric.

Light entering one of the dielectric sub-particles in a composite particle will be more weakly internally reflected from the forward surface of the sub-particle than if the particle were in vacuum because of the decreased contrast in refractive index caused by the proximity of the adjacent particle; that is, the dielectric sub-particles become more forward scattering. When light is incident on an opaque sub-particle, such as magnetite, which is a common mineral on the surface of the Earth and in meteorites, or ilmenite, which is common on the Moon, less light is reflected and more is absorbed. The light that is forward scattered by the dielectric sub-particles has a high probability of encountering an opaque sub-particle or absorbing matrix material and being absorbed before exiting from the composite particle, so that little escapes to vacuum from the forward surface. It is only the light scattered to the sides and rear by the sub-particles near the back-facing surface of the composite particle that readily escapes. The net result is that the media of Fig. 2(B) and 2(C) consist of larger particles than the medium of 2(A) with lower single scattering albedos and with scattering functions that have decreased forward scattering lobes and increased back scattering lobes.

The measured scattering properties[12] of a composite particle are illustrated in Fig. 3. This figure shows that the effect of adding progressively more internal scatterers to a clear spherical particle is to change the phase function from forward scattering to isotropic to back scattering. The particle with a high density of internal scatterers has been constructed entirely of elements that are strongly forward scattering and, yet, is back scattering.

Paper I argues that the inhomogeneities and sub-particles inside a composite particle are the primary scattering units because there is no way of distinguishing a sub-particle in one composite particle from a similar sub-particle inside an adjacent composite, so that the sub-particles form continuous chains. To the contrary, chains or aggregates of small particles in optical contact inside a larger particle, such as depicted schematically in Fig. 2(B) or 2(C), are readily distinguishable from similar aggregates inside another large particle by at least two criteria. (1) A light wave can propagate from any point inside a given composite particle to any other point by a variety of paths of finite cross sectional area without going through vacuum, whereas to reach a point in another composite particle it must either travel through vacuum or through one of the support points of infinitesimal area. (2) Two particles in a chain may be separated without altering their internal

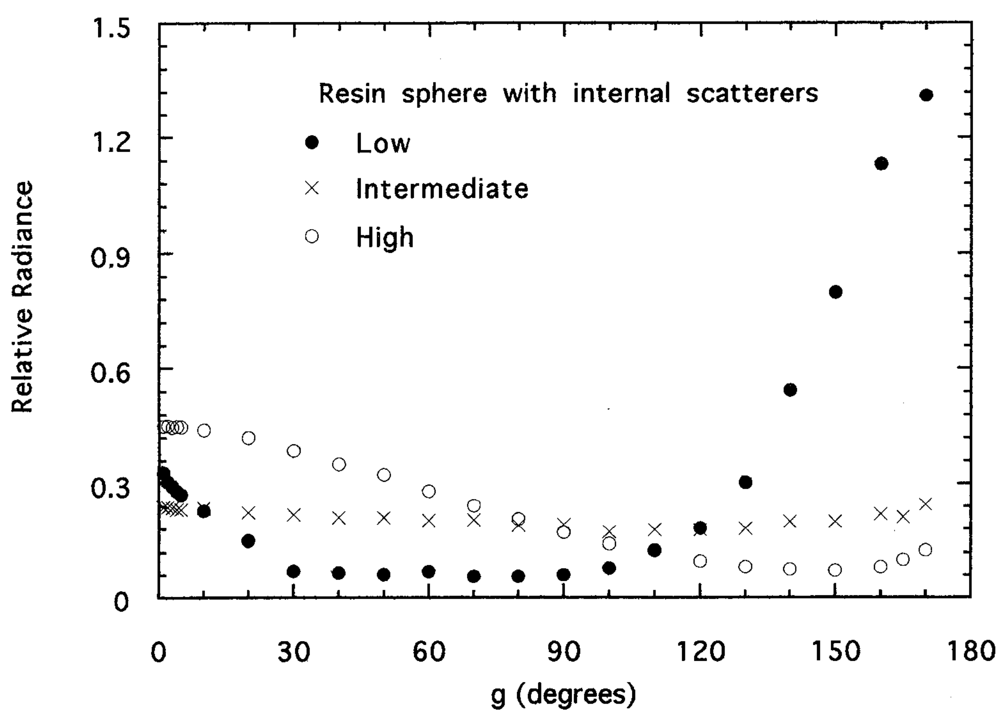

Fig. 3. Angular phase functions of composite spherical particles approximately 1 cm in diameter consisting of a resin matrix to which has been added varying amounts of TiO_2 particles about 300 nm in size (adapted from McGuire and Hapke[12]). The terms "high", "medium" and "low" refer to the relative number of TiO_2 particles in the resin.

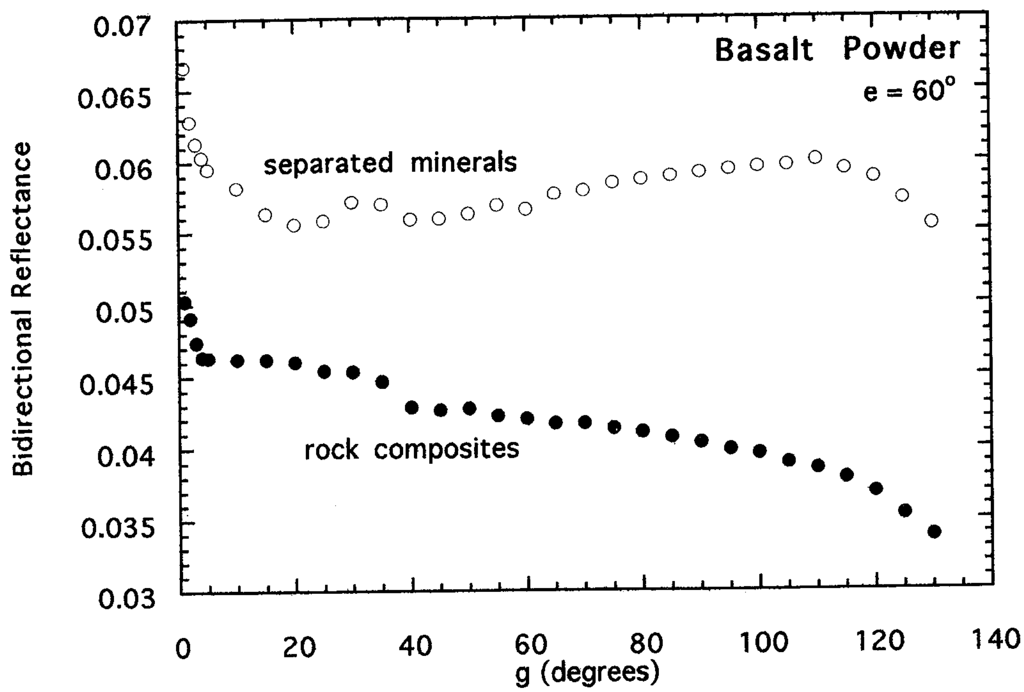

Fig. 4. Radiance (relative to a PTFE standard at e = 0 and i = 5°) of ground basalt powder measured in visible light with e = 60° as i varies in the principle plane, plotted against phase angle; open circles: 0.1 mm minerals; closed circles: 1 mm rock fragments consisting of the same minerals, but welded together into composite particles. The apparatus is described in Ref. 16.

refractive indices relative to their immediate surroundings; however, if a composite particle is broken up into its constituent sub-particles, their relative refractive indices will be drastically altered. These are exactly the same criteria that one would use for the medium depicted in Fig. 2(A) to distinguish the interior of one particle from the interior of another. Hence, the composite particles, rather than the sub-particles, are the fundamental scattering units of the media of Fig. 2(B) and 2(C).

Theoretical arguments are well and good, but science ultimately rests on experiment. Therefore, the predictions of these opposing models were tested experimentally. A fine-grained basalt, consisting of grains of the minerals anorthite, pyroxene and magnetite approx. 75–100 μm in size welded together, was ground in a corundum mortar and pestle and wet-sieved to remove any finer clinging particles to a particle size range of 1000–1325 μm. Half of this sample was further ground and wet-sieved to a size range of 74–105 μm. After drying, the porosities of the two samples were measured and were found to be similar, $0.43 \pm .01$ for the coarse rock fragments and $0.48 \pm .02$ for the finer mineral separates. The powders were then gently poured into sample holders to form optically thick layers that were smooth to the eye. These were placed under a stereoscopic microscope and their three-dimensional surface structures were examined. On the scale of the particle size, both powders had virtually identical appearances and were equally rough, except that the coarse powder consisted of welded, muticrystalline, composite rock particles, while the fine powder consisted of separate minerals of the same size as those making up the coarse particles.

The bidirectional reflectances of the powders at different angles were measured by a goniometric photometer in unpolarized visible light. The results are shown in Fig. 4, where the bidirectional reflectances of the powders are plotted against phase angle. If the arguments in Paper I were correct there should be no difference between the intensity of light scattered by the two samples, since the mineral grains that make up the rock fragments should be the basic scatterers in both samples. Clearly, this is not the case. The coarse powder consisting of the rock composites has a lower reflectance, smaller forward scattered lobe and larger back scattered lobe than the fine powder consisting of the separate individual mineral grains. The difference in albedo is obvious even from simple visual examination of the samples in their jars. These differences are exactly as expected if the composites, and not the sub-particles, are the basic scatterers in the coarse powder. Thus, experiment has demonstrated unequivocally that (1) the fundamental scattering units of a medium consisting of composite particles have properties that are similar to those expected of composite particles rather than of the constituent sub-particles, and (2) natural composite particles can be back scattering.

Next, the question of whether or not the apparent back scattering nature of regolith particles is an artifact of the IMSA model will be considered. Figures 10 and 11 of Paper I present the results of a rigorous numerical calculation in which it is shown that a medium of particles with a single scattering albedo $w = 1.00$ and with a strongly forward scattering phase function has a bidirectional reflectance that is similar, over a range of phase angles between 0 and 90°, to the reflectance, calculated using the IMSA model, of a medium of back scattering particles. Paper I cites this as proof that the IMSA model will retrieve the wrong sign of the particle scattering asymmetry.

Let us examine this example critically to see where the difficulty lies. The geometry assumed in the example had the angle of viewing $e = 0°$ while the angle of incidence i varied between 0 and 90°. Now, at these angles the brightness distribution is extremely insensitive to particle scattering function when the single scattering albedo is high. This is illustrated by Fig. 5(A), which shows the radiance, calculated rigorously,[14] scattered from two media, one consisting of particles with single scattering albedo $w = 1.00$ and particle scattering functions $p(g) = 1 - \cos g$ (strongly forward scattering), and the other with $w = 1.00$ and $p(g) = 1 + \cos g$ (strongly back scattering), where g is the phase angle. Note that the curves are identical. Hence, at the geometry of the example in Paper I one can say virtually nothing about the particle phase function by model fitting. If the curve for $p(g) = 1 + \cos g$ represented data measured on an actual medium of strong back scatterers, a theoretical curve for a medium with scatterers having a strongly forward scattering phase function $p(g) = 1 - \cos g$ could be fitted to it exactly. Thus, for this example, serious retrieval errors will result even with an exact scattering model, and it is specious to blame the retrieval of an incorrect particle phase function on an inexact approximation for multiple scattering.

However, Fig. 5(B) shows the radiances scattered from the same media as in Fig. 5(A), but viewed at $e = 60°$. Now the scattering functions are clearly different. The maxima of the radiances are displaced toward or away from the direction to the detector, depending on whether the particles are back or forward scattering. The surface brightness is nonuniform in such a way that it is obvious that the particles are forward scattering, and if the IMSA model were fitted to the curves in these figures, the correct sign of the asymmetry would, in fact, be retrieved. For instance, an analyst might begin by attempting to fit an isotropically scattering particle phase function to the curve for $p(g) = 1 - \cos g$. But this solution would be too high at small phase angles and too small at large phases, which would require a forward scattering particle function to improve the fit.

This is also true in the examples given in Figs 13–15 of Paper I, where the brightnesses of media of forward scattering particles are calculated over a large range of phase angles and compared with the IMSA model. At large phase angles the IMSA model underestimates the brightnesses of media of forward scattering particles, so that the model particles would have to be made more forward scattering, not erroneously back scattering, as Paper I claims. To be sure, because the multiple scattering is not treated exactly in the IMSA model, the asymmetry factor may have the incorrect value, or it may not even be possible to retrieve a unique value, but its sign will not be wrong.

The importance of a large range of phase angles in the data set is also illustrated by Fig. 4, where it is obvious that the fine mineral grains have a forward scattering lobe, but that the coarse powder is back scattering. It is readily apparent in Fig. 1, where no amount of fiddling with the multiply scattered contribution could make a strongly forward scattering particle appear to be as back scattering as those in the lunar soil. (Of course, one could make the *ad hoc* hypothesis that a forward scattering lobe lies entirely outside the range of phase angles where data was taken, $g = 0$–$130°$. However, even then the particle would have to posses a large back scattering lobe at smaller phase angles.)

Another example cited in Paper I as proving that the IMSA model retrieves the wrong sign of the asymmetry is provided by the planet Venus. Inserting isotropic scatterers into the IMSA model provides a good fit[16] to the brightness profile across the disk of Venus at a phase angle of 23.3°.

Since the Venus cloud particles are known to be forward scattering spheres, Paper I cites this as proof that the IMSA model retrieves incorrect particle scattering functions. However, the exact solution[14] for a medium of isotropic scatterers at the same angles also is an excellent fit[17] to this

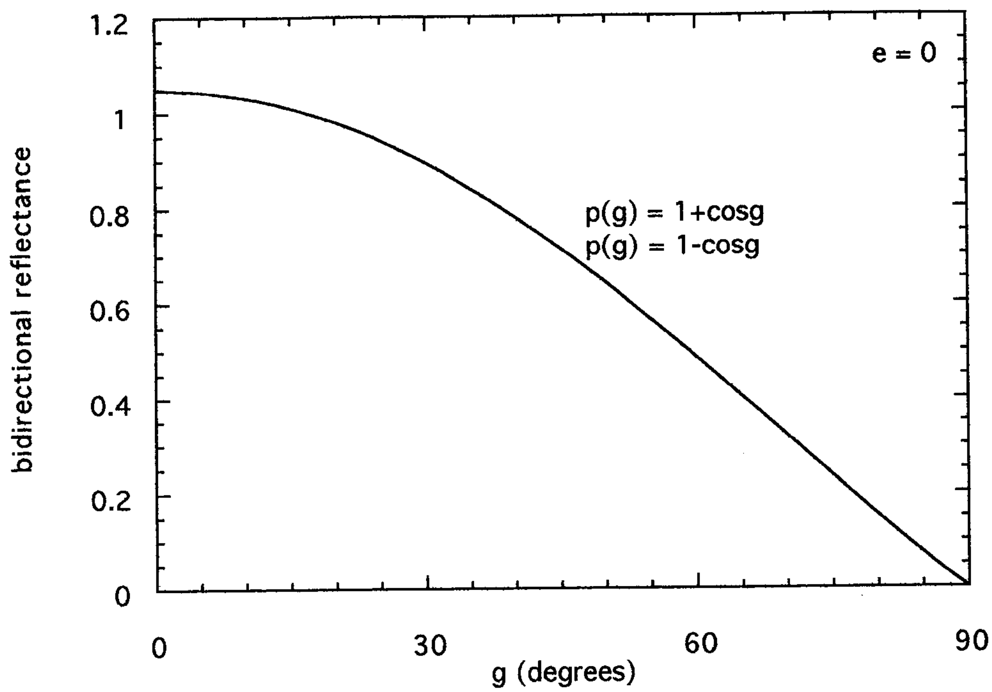

Fig. 5(A). Radiances (calculated from Chandrasekhar[14]) of two media with single scattering albedo w = 1.00, one composed of particles having the scattering function p(g) = 1 + cosg and the other having the function p(g) = 1 − cosg, with e = 0 as i varies, plotted against phase angle.

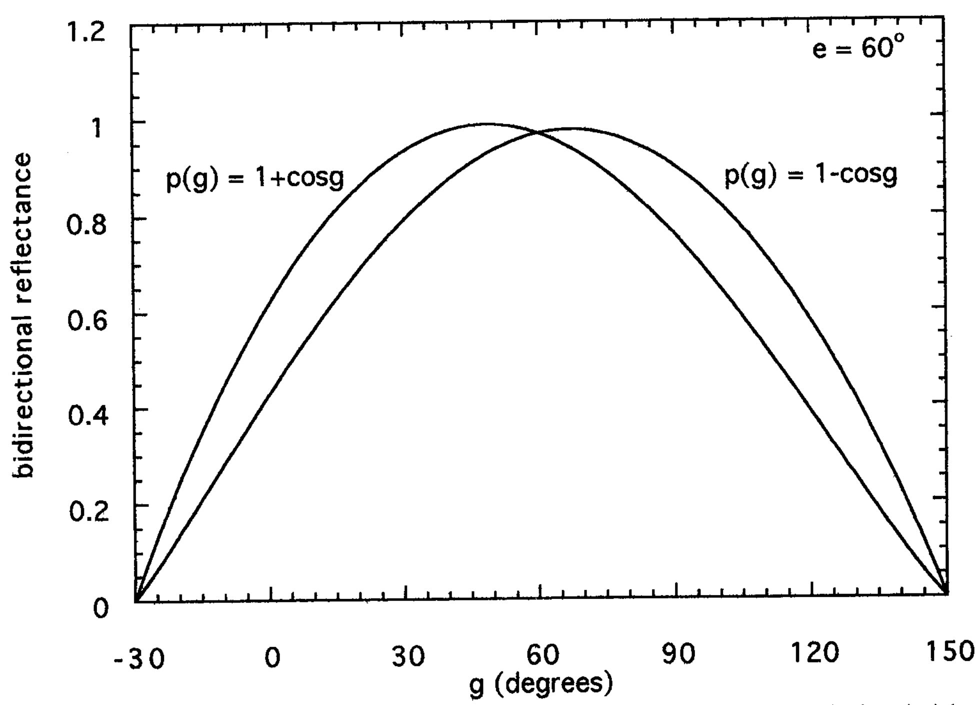

Fig. 5(B). Radiances of the same two media as in Fig. 5(A), except with e = 60° as i varies in the principle plane, plotted against phase angle.

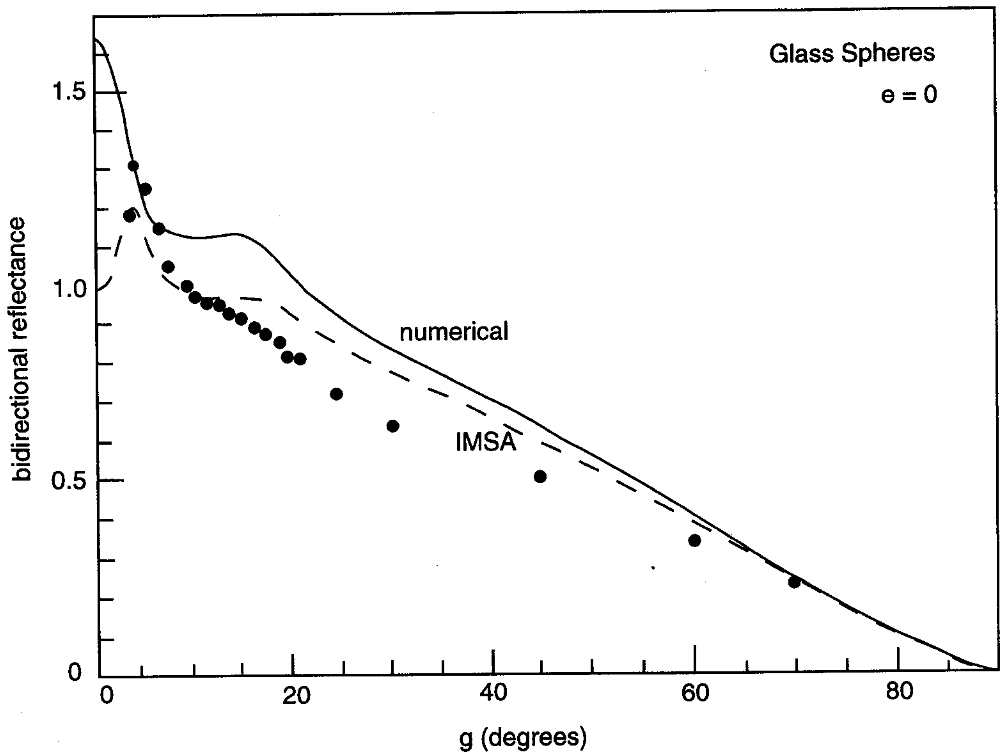

Fig. 6. Radiance of a medium of glass spheres (relative to a $BaSO_4$ standard at e = 0 and i = 30°), with e = 0 as i varies in the principle plane, plotted against phase angle; (●) measured data; (—) numerical calculation using a doubling computer program; (---) approximate IMSA model. (Adapted from Goguen[16].)

data set. So is a rigorous numerical solution[18] for a cloud of forward scattering spheres. Hence, this example is totally irrelevent. All it demonstrates is the obvious: that a particle phase function cannot be retrieved from data taken at a single phase angle, even if one is using an exact scattering model. In fact, all of the examples[16] cited in Paper I as putative proof that the IMSA model gives incorrect results fall into this category of limited range of phase angles, so that these criticisms have little merit.

Paper I also mentions snow and ice. The measured phase curves of high albedo terrestrial frost and snow deposits[15] are bright at large phase angles, which shows that the particle phase functions are forward scattering. When quantitative analyses[6] of this data was carried out using the IMSA model, the retrieved asymmetry parameters were indeed found to be forward scattering, contrary to what would be expected from the assertions in Paper I. Hence, there is no valid reason to doubt that the IMSA model can correctly retrieve the sign of the asymmetry if the range of phase angles covered by the data set is sufficiently large.

Particle scattering parameters can be reliably retrieved only if the data includes observations over a wide range of phase angles, and this requirement is just as necessary for exact scattering models as for approximate ones. If the light scattering curve of a surface or the phase function of a body is known over only a small range of phase angles, reliable inferences cannot be made about the nature of the particle scattering function outside this range. This was the case for the satellites of the outer solar system before the Voyager missions and is still true for most of the asteroids. It is also the case in many laboratory measurements. Mishchenko is correct when he states in Paper I that the problem of retrieving particle scattering parameters from a data set of limited range in phase angle is ill-conditioned. For convenience, particle phase functions retrieved from data sets taken only at small phase angles might be described by a simple function or single asymmetry parameter that makes it appear that the particles are back scattering, but such parameters, clearly, should be regarded as provisional. If they are incorrect, this is the result of incomplete data, and would occur just as readily from using a rigorous numerical scattering model as from an approximate one. There is no justification for implying, as Paper I does, that if data over a wide range of phase angles were available, the IMSA model would retrieve a back scattering particle function from a medium of forward scatterers.

Moreover, even when the range of phase angles is large, unique values of the scattering parameters may sometimes be difficult to retrieve, as has been emphasized by the author and his colleagues.[19] Blind fitting of model parameter values to data by computer retrieval programs without careful additional analysis can lead to erroneous results, even with an exact scattering model. Both laboratory measurements and astronomical observations of light scattered by surfaces are difficult and unreliable for phase angles beyond about 150°. The scattering functions of regolith particles outside this range cannot be determined observationally, and it is rank speculation to insist that they are strongly forward scattering.

Finally, Paper I criticizes the IMSA model on the grounds that, "—the Hapke theory does not even contain such a crucial physical parameter as refractive index—." This statement is misleading. No model based on the equation of radiative transfer, including the numerical models of Paper I, explicitly contains the refractive index. The free parameters in the radiative transfer equation are the single scattering albedo and angular scattering function, and it is these that implicitly contain the refractive index. For instance, if the particles of the medium are spheres, the refractive index enters into the calculation of the particle parameters by Mie theory. If they are not spheres, then other particle models, which also contain the refractive index, must be used. This is just as true for the IMSA model as for the models of Paper I. Hence, like the other criticisms of the IMSA model in Paper I, this criticism is spurious.

Mishchenko correctly points out that present regolith scattering models have not been subjected to extensive verification, such as comparing the predictions of the models against measured radiances scattered from a particulate medium of known scattering properties. The only test of this type was carried out by Goguen,[20] who measured the bidirectional reflectance of a medium of spheres of known size and refractive index. Goguen then calculated the single scattering albedo and angular scattering function of the spheres using Mie theory and inserted these into both the IMSA model and a rigorous doubling computer model. Neither regolith model had any adjustable parameters. The results of Goguen's experiment are shown in Fig. 6. Note that the measured values are not duplicated exactly by either model, for reasons that are unclear. However, the IMSA model is a better match to the actual data than the numerical calculation. Thus, contrary to the implications of Paper I, the IMSA model, which is so computationally simple that the calculations of the scattered radiance may be done on a hand calculator, is, evidently, not greatly inferior to a rigorous numerical calculation that required 30 hours of CPU time on a SUN SPARC work station.

CONCLUDING REMARKS

Both theoretical arguments and experimental data show that back scattering dielectric particles are physically possible. Examples are rock fragments and agglutinates, which appear to be major constituents of many of the regoliths in the solar system. When they are abundant in a regolith they constitute its fundamental scattering unit. Although we do not know what the particles in icy regoliths are like, there is no reason for supposing that an impact-generated medium of ice, rock and carbonaceous material would consist only of simple, clear homogeneous particles that are strongly forward scattering. Hence, a back scattering icy regolith is physically plausible.

All of the examples given in Paper I that purported to show that the IMSA model retrieves the incorrect sign of the particle scattering asymmetry, in fact, demonstrate that the errors are caused by inadequate range of phase angle in the data set, and that under these circumstances an exact model is just as likely to give wrong answers as the IMSA model. The character of the particle scattering function of a medium can be correctly retrieved using the convenient IMSA scattering model if the analysis is done with care and if data is available over a sufficiently wide range of phase angles. Thus, even though some of the conclusions of Paper I were incorrect, this paper has served a useful purpose, because it underscores the necessity of having a data set with an adequate range in phase angles in order to correctly retrieve the particle phase function. Based on the author's experience, observations should extend *at least* over a range of phase angles between about 20 and 120°.

The phase functions of particles that make up the regoliths of bodies in the solar system contain information about the origin of the particles and the processes that occur in the regoliths. To assume that the phase function is known *a priori*, rather than to attempt to retrieve it, is to discard potentially important information. It appears that the surfaces of many of the planets and satellites in the solar system contain back scattering particles; it will be interesting to discover why this is so.

Acknowledgements—The author thanks D. DiMucci and H. Waller for valuable assistance. This research was supported by a grant from the Planetary Geology and Geophysics program of the National Aeronautics and Space Administration.

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