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Comment on "Current budget of the atmospheric electric global circuit" by Heinz W. Kasemir

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The conventional model of the global electric circuit has been discussed in the literature for many years [*Dolezalek,* 1972; *Israel,* 1973; *Volland,* 1984; *Roble and Tzur,* 1986] and was conceptualized several decades ago in an effort to describe the electrical connection between the large potential of ionosphere relative to the Earth, the potential gradient at the surface of the Earth in areas of fair weather, and the upward flow of positive charge in the vicinity of thunderstorms. Throughout the years, measurements [*Wilson,* 1920; *Whipple and Scrase,* 1936; *Gish and Wait,* 1950; *Stergis et al.,* 1957; *Blakeslee et al.,* 1986] as well as numerical and analytical simulations [*Holzer and Saxon,* 1952; *Kasemir,* 1959; *Anderson and Freier,* 1969; *Dejnakarintra and Park,* 1974; *Hayes and Roble,* 1979; *Driscoll et al.,* 1992] have advanced our understanding of the global electric circuit. Recently, however, *Kasemir* [1994] introduced a new model for the global circuit that is claimed to be an improvement over the conventional model. This claim was based on calculations made with his new model which appeared to demonstrate the need for a fundamental change in the generally accepted model of the global circuit, despite the striking similarities between the components of his new model and the thunderstorm model introduced by *Holzer and Saxon* [1952]. However, a careful examination of Kasemir's new model reveals that computational and conceptual errors were made in its development, resulting in erroneous conclusions. It is the purpose of this paper to demonstrate the equivalence between the conventional model of the global electric circuit and Kasemir's model, and to refute Kasemir's suggestion that the conventional model of the global circuit contains some physical errors and unwarranted assumptions.

In this paper, three major issues relevant to Kasemir's new model will be addressed. The first concerns Kasemir's assertion that there are significant differences between the potentials associated with the new model and the conventional model. A recalculation of these potentials reveals that both models provide equivalent results for the potential difference between the Earth and ionosphere. The second issue to be addressed is Kasemir's assertion that discrepancies in the electric potentials associated with both models can be attributed to modeling the Earth as a sphere, instead of as a planar surface. A simple analytical comparison will demonstrate that differences in the equations for the potentials of the atmosphere derived with a spherical and a planar Earth are negligible for applications to global current flow. Finally, the third issue to be discussed is Kasemir's claim that numerous aspects of the conventional model are incorrect, including the role of the ionosphere in global current flow as well as the significance of cloud-to-ground lightning in supplying charge to the global circuit. In order to refute these misconceptions, it will be shown that these aspects related to the flow of charge in the atmosphere are accurately described by the conventional model of the global circuit.

In the latter portion of his paper, Kasemir demonstrates a difference between his new model and the conventional model of the global circuit by comparing the electric potentials of the Earth and ionosphere associated with each of the models. Listing several possible reasons for this discrepancy, Kasemir implies that the potentials calculated with his model are correct and the potentials associated with the conventional model are incorrect. In fact, however, both models provide the same potential difference between the Earth and ionosphere. The numerical differences in the potentials cited by Kasemir can be attributed to three factors: (1) An alternate choice of a reference potential, (2) an error in a calculation with his model equation, and (3) the use of a different conductivity scale height.

In the conventional model of the global circuit, the Earth is assigned a reference potential of 0 V, while the potential of the ionosphere relative to the Earth is typically taken to be 300 kV, a value that is computed from the product of the fair weather current density ($\sim 2.0\times10^{-12}$ A/m^2) and the columnar resistance of the atmosphere ($\sim 1.5\times10^{17}\Omega$). On the other hand, the analytical equation derived by Kasemir, which is the basis for his new model, was derived with a constraint that effectively assigned a zero reference potential at infinity and a large negative potential for the Earth's surface. Hence in comparing the models, we choose to discuss the potential difference between the ionosphere and the Earth, rather than discussing the individual potentials of the Earth and ionosphere associated with the models.

Using his new model equation, Kasemir calculated the potential of the Earth to be −43.6 kV and the ionospheric potential at 100 km above the ground to be $-4.23\,\mu$V, yielding a potential difference between the ionosphere and Earth of 43.6 kV. This potential difference is almost an order of magnitude smaller than the 300 kV potential difference associated with the conventional model. However, an error was made in the calculations of the Earth and ionospheric potentials. With Kasemir's equation for the global potential function,

$$\Phi_3^{(\text{global})} = \frac{I_g}{4\pi a \sigma r^2} \tag{1}$$

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the fair weather potential can be recalculated as function of radial distance from the center of the Earth, r. By substituting in the same input parameters used by Kasemir into the above equation (i.e., k is the radius of the Earth=6.378×10^6 m, i_g is the fair weather current density=-2.0×10^{-12} A/m^2, I_g is the global fair weather current=$i_g \times (4\pi k^2)=-1022$ A, a is 1/(scale height)=ln(10)/10,000 m, and σ is conductivity of the atmosphere=$2\times10^{-14}\ e^{a(r-k)}$ S/m), the corrected potentials for the Earth ($r=k$) and ionosphere ($r=k+100$ km) were found to be −434 kV and −43.4 μm , respectively. The remaining discrepancy (143 kV), however, vanishes when the potential difference of the conventional model is recalculated using the same conductivity scale height implemented by Kasemir (i.e., $1/a$ = 4343 m), thereby adjusting the columnar resistance of the atmosphere. Using this conductivity scale height, the columnar resistance of the atmosphere, R_{columnar}, between the surface of the Earth and 100 km in altitude can be calculated as follows:

$$R_{columnar} = \int_{k}^{k+100\,\text{km}} \frac{1}{\sigma_o e^{a(r-k)}}\, dr$$

$$= \frac{-1}{a} \frac{1}{2\times10^{-14} e^{a(r-k)}} \Bigg|_{r=k}^{r=k+100\,\text{km}} \tag{2}$$

$$= 2.17\times10^{17}\,\Omega.$$

By changing the columnar resistance of the conventional model from $1.5\times10^{17}\ \Omega$ to $2.17\times10^{17}\ \Omega$ and multiplying by the fair weather current density of 2.0×10^{-12} A/m^2, the potential difference between the ionosphere and Earth also changes from 300 kV to 434 kV. Hence the ionosphere to Earth potential differences associated with the new model and the conventional model are the same for an identical atmospheric columnar resistance, as shown in Table 1.

Kasemir wrongly attributed the differences he found between his model and the conventional model to his use of a spherical coordinate system that allowed the Earth to be modeled as a sphere, rather than as a planar surface. However, the equation derived by *Holzer and Saxon* [1952] for the electric potential of the atmosphere above a planar Earth is only slightly different from the equation associated with Kasemir's new model, and, except for an arbitrary offset potential, the differences between Kasemir's new model and Holzer and Saxon's model are negligible for applications related to global current flow. Despite the difference in coordinate systems, the similarity of the results obtained with the two models is expected. This is made apparent by the fact that a sphere with an infinite radius is equivalent to a planar surface and that the radius of curvature of the Earth appears infinite in comparison to the altitudes and dipole dimensions associated with a typical thundercloud.

By combining a spherical coordinate system with the method of images, Kasemir derived a mathematical expression that described the quasi-static value of the electric potential of the atmosphere in terms of a current source above the sphere (I), its image below the surface of the sphere (I_b), and a "global current" source at the center of the sphere (I_g), as shown in Figure 1a. This made it possible for Kasemir to express the net electric potential above the Earth, $\Phi^{(\text{total})}$ as the superposition of three potentials, or simply

$$\Phi^{(\text{total})} = \Phi_1^{(\text{source})} + \Phi_2^{(\text{image})} + \Phi_3^{(\text{global})} \tag{3}$$

where $\Phi_1^{(\text{source})}$, $\Phi_2^{(\text{image})}$, and $\Phi_3^{(\text{global})}$ are the potentials corresponding to the source, image, and global currents, respectively. Kasemir noted, and we concur, that the potentials associated with the source and image terms ($\Phi_1^{(\text{source})}$ and $\Phi_2^{(\text{image})}$) derived with a spherical Earth are effectively equivalent to the source and image potentials computed using a planar Earth for locations in the local environment of the source. However, Kasemir came to the erroneous conclusion that the global potential function, given by equation (1), was the distinguishing element that made the net electric potential derived with a spherical Earth different from the net electric potential obtained with the planar Earth.

As depicted in Figure 1b, the electric potential above a planar Earth can be computed as a superposition of terms created by the source current, the image current, and the potential assigned to the planar surface. Using a planar Earth to derive the equations for the net electric potential of the atmosphere, *Holzer and Saxon* [1952] demonstrated that the equivalent global potential above a planar surface can be written as

$$\Phi_{(\text{planar})}^{(\text{global})} = V_\infty + \left(V_{\text{Earth}} - V_\infty\right)e^{-az} \tag{4}$$

where z is the altitude above the Earth, V_{Earth} is the potential assigned to the Earth, and V_∞ is the potential associated with a location far from the source current at $z=\infty$. It should be noted that Holzer and Saxon arbitrarily assigned the Earth a reference potential of zero (they could just as easily have taken $V_\infty=0$).

Table 1. A Comparison of the Ionosphere to Earth Potential Differences Computed for Kasemir's New Model, the Conventional Model With a Commonly Used Columnar Resistance, and the Conventional Model With the Columnar Resistance Used by Kasemir

Global Circuit Model	Location of Zero Reference Potential	Potential Difference Between the Ionosphere (~100 km) and Earth
Kasemir's new model (R_{columnar}= $2.17\times10^{17}\ \Omega$)	infinity	−43.4 μV − (−434 kV) = 434 kV
Conventional model (R_{columnar}= $1.50\times10^{17}\ \Omega$)	Earth	300 kV − 0 kV = 300 kV
Conventional model (R_{columnar}= $2.17\times10^{17}\ \Omega$)	Earth	434 kV − 0 kV = 434 kV

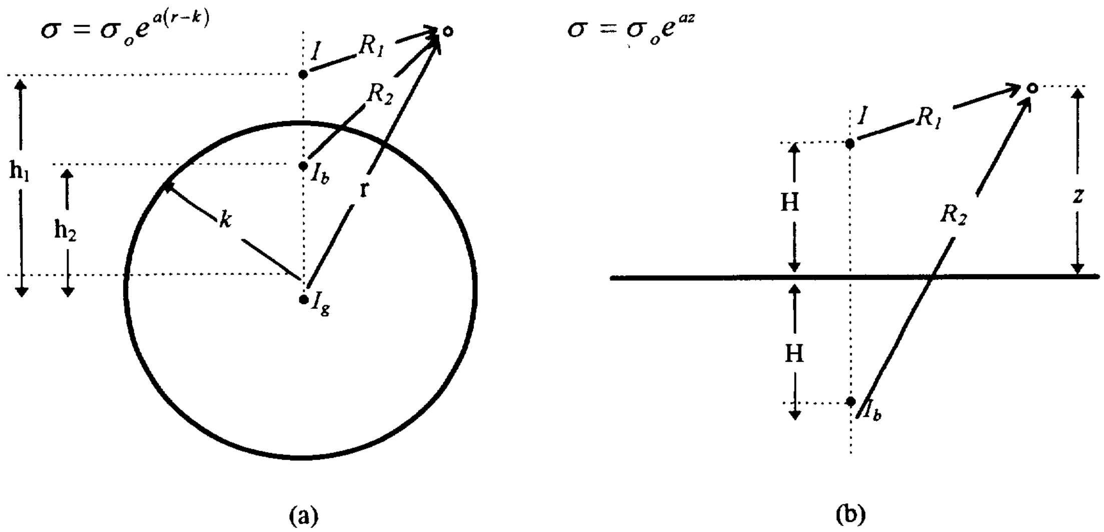

Figure 1. A pictorial representation of the geometry used to model the global electric circuit when the Earth is described as a (a) sphere and a (b) planar surface.

At first glance, the global potential functions given by equations (1) and (4) appear to be totally different. However, we will now establish that these equations are equivalent when the boundary conditions are the same. In order to demonstrate this equivalence, a relationship must be obtained between the global current, I_g, and the potential difference between the Earth and a point distant from the Earth. This can be accomplished by computing the value of $V_{Earth} - V_\infty$ due to the fair weather electric field, which can be summarized by the following equation:

$$V_{Earth} - V_\infty = -\int_{r=\infty}^{r=k} E\,dr = -\int_{r=\infty}^{r=k} \frac{i_g}{\sigma} dr$$
$$= -i_g \int_{r=\infty}^{r=k} \frac{1}{\sigma_o e^{a(r-k)}} dr = \frac{i_g}{a\sigma_o} \tag{5}$$

where i_g is the global current density associated with the fair weather current. Since the global current density, i_g, is related to the total global current, I_g, by the surface area of the Earth, the above potential difference can be expressed as

$$V_{Earth} - V_\infty = \frac{I_g}{4\pi k^2 a\sigma_o} . \tag{6}$$

Substituting this relationship back into equation (4) gives

$$\Phi^{(global)}_{(planar)} = V_\infty + \frac{I_g}{4\pi a\sigma_o e^{az} k^2} = V_\infty + \frac{I_g}{4\pi a\sigma k^2} . \tag{7}$$

For an observation location near the Earth (i.e., $r/k \cong 1$), the above equation for the electric potential is equivalent to equation (1) when V_∞ is zero. Therefore equations (1) and (7) are equivalent when the same boundary potentials are used for the Earth (V_{Earth}) and for a distance far from the Earth (V_∞), which demonstrates the fact that there is really no appreciable difference between the electric potentials of the atmosphere when the Earth is modeled as a sphere as opposed to a planar surface. Moreover, atmospheric electric potentials computed with Kasemir's model are different from the potentials commonly associated with the conventional model of the global circuit due to a different choice in a reference potential. (Note that Kasemir included a constraint equation in his model that forced the net sum of the source current, the image current, and the global current to zero ($I + I_b + I_g = 0$). This constraint equation, in effect, becomes a boundary condition that forces the net electric potential to a value of zero at a distance far from the source, image, and global currents ($\Phi^{total} = 0$ at $r = \infty$)).

At this point, we have established that the differences between the atmospheric potentials associated with Kasemir's new model and the conventional model are negligible. In addition, it can be shown that other errors Kasemir attributes to the conventional model are a result of his misinterpretation of the details of the global circuit. In his paper, Kasemir incorrectly concludes that many concepts contained within the conventional model are inaccurate, including the amount of charge on the Earth and in the atmosphere, the role of the ionosphere in the global circuit, and the significance of cloud-to-ground lightning in global current flow. As a result, we discuss these important details of the global circuit in the following paragraphs in order to demonstrate the validity of the concepts inherent in the conventional model of the global circuit.

First, Kasemir believed that a conducting sphere with a potential of zero also should have a net charge of zero, and as a result, he erroneously concluded that the conventional model could not account for the Earth's electrical charge. In fact, the conventional model offers no such restriction, and an estimate can be obtained for the average amount of charge that resides on the surface of the Earth. Assuming that an electric field equivalent to a fair weather field of −100 V/m is uniformly distributed across the surface of the Earth, Gauss's law reveals that the Earth holds approximately 450,000 C of negative charge. The true amount of charge held by the Earth is less than this value, since this simple computation does not include the large positive electric fields found underneath thunderstorms. Not surprisingly, however, it can also be demonstrated with Gauss's law that the atmosphere also holds an amount of positive charge equal to the negative charge held by the Earth, with most of this positive charge located within a few kilometers of the Earth's surface. As a result of these simple calculations, it is important to recognize that very little of the charge circulated in the global electric circuit is held in the ionosphere.

Another issue raised by Kasemir was the role of the ionosphere in global current flow [*Kasemir*, 1994, page 10,707, point 3, section 5]. He incorrectly asserts that the conventional model requires an insulating ionosphere, instead of a conducting ionosphere, in order for charge flowing upward from a thundercloud to change its upward direction of flow and distribute itself horizontally over the ionosphere. When examining the flow of charge in the global circuit, the ionosphere plays a significant role by allowing the current supplied by thunderstorms to spread horizontally and return to Earth in the form of fair weather current. Without properly considering the forces involved, one might come to an incorrect conclusion that a highly conductive ionosphere would cause the upward flowing current supplied by thunderstorms to continue to flow upward beyond the ionosphere. The positive charge that flows upward toward the ionosphere is repelled by the positive charge in the electrified cloud due to Coulombic forces; and similarly, these same forces cause this positive charge in the atmosphere to migrate toward the negatively charged Earth. The highly conductive ionosphere simply provides a convenient path for the positive charge flowing upward from thunderstorms to travel horizontally away from the storms and return to the negatively charged Earth as fair weather current.

Still another important topic discussed by Kasemir is the role of the thunderstorm in supplying current to the global electric circuit. The relationship between regional thunderstorm activity and fluctuations in the fair weather current density has long been recognized, suggesting that thunderstorms serve as one of the major contributors to the global circuit. Additionally, electrical measurements obtained over thunderstorms have clearly demonstrated that thunderstorms are capable of producing positive upward currents that range from 0.1 to over 6 A with an average of about 0.7 A [*Gish and Wait*, 1950; *Stergis et al.*, 1957; *Blakeslee et al.*, 1989]. Using this information, it is easy to see that the estimated 1800 thunderstorms around the globe can collectively supply the 1000 A needed to fulfill the current budget of the global circuit and maintain a potential difference between the Earth and ionosphere.

Despite this evidence, Kasemir concluded through calculations made with his new model that thunderstorms can be responsible for only a small fraction of the global current budget. In addition, he suggested that the current supplied to the global circuit by cloud-to-ground lightning is almost negligible. Perhaps the most perplexing aspect of Kasemir's results is that the equations he used to compute current contributions to the global circuit are almost identical to the equations derived by *Holzer and Saxon* [1952], yet Holzer and Saxon came to the conclusion that cloud-to-ground lightning does supply current to the global circuit. Assuming that a generator current supplies positive charge to the upper charge center of a thunderstorm and negative charge to the lower charge center, Holzer and Saxon suggested that cloud-to-ground lightning lowers negative charge from the lower charge center to ground, causing a temporary charge imbalance in the thundercloud, thus increasing the amount of positive charge supplied to the global circuit. This concept was later verified both analytically and numerically by *Driscoll et al.* [1992] through a time-averaged analysis of the currents in the vicinity of a thunderstorm. From this analysis, it was shown that negative cloud-to-ground lightning adds to the upward flow of positive current above a thunderstorm, thus increasing the thunderstorm's contribution to the global electric circuit. A careful examination of Kasemir's model reveals that when he calculated the global current contribution made by cloud-to-ground lightning, he included the instantaneous electrostatic removal of the charge from the atmosphere, but he did not include the electrodynamic effects associated with supplying charge to the location in the atmosphere where the charge was removed from. As a result, Kasemir's conclusion that cloud-to-ground lightning only minimally contributes to the global circuit is erroneous.

In conclusion, a thorough examination of Kasemir's new model for the global electric circuit reveals that errors were made in his calculations as well as in his conclusions derived from these calculations. We show that there are only minor differences between the models of the global circuit when the Earth is described by a planar surface as opposed to a sphere. Finally, we refute Kasemir's assertion that numerous aspects of the conventional model are incorrect.

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