

Aerodynamic Shape Sensitivity Analysis and Design Optimization of Complex Configurations Using Unstructured Grids

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Abstract

A three-dimensional unstructured grid approach to aerodynamic shape sensitivity analysis and design optimization has been developed and is extended to model geometrically complex configurations. The advantage of unstructured grids (when compared with a structured-grid approach) is their inherent ability to discretize irregularly shaped domains with greater efficiency and less effort. Hence, this approach is ideally suited for geometrically complex configurations of practical interest. In this work the nonlinear Euler equations are solved using an upwind, cell-centered, finite-volume scheme. The discrete, linearized systems which result from this scheme are solved iteratively by a preconditioned conjugate-gradient-like algorithm known as GMRES for the two-dimensional geometry and a Gauss-Seidel algorithm for the three-dimensional; similar procedures are used to solve the accompanying linear aerodynamic sensitivity equations in incremental iterative form. As shown, this particular form of the sensitivity equation makes large-scale gradient-based aerodynamic optimization possible by taking advantage of memory efficient methods to construct exact Jacobian matrix-vector products. Simple parameterization techniques are utilized for demonstrative purposes. Once the surface has been deformed, the unstructured grid is adapted by considering the mesh as a system of interconnected springs. Grid sensitivities are obtained by differentiating the surface parameterization and the grid adaptation algorithms with ADIFOR (which is an advanced automatic-differentiation software tool). To demonstrate the ability of this procedure to analyze and design complex configurations of practical interest, the sensitivity analysis and shape optimization has been performed for a two-dimensional high-lift multielement airfoil and for a three-dimensional Boeing 747-200 aircraft.

1. Introduction

With the aid of modern computers, aerodynamic design optimization procedures [1-5] have emerged which directly couple the fields of computational fluid dynamics (CFD), sensitivity analysis, and numerical optimization. These procedures have enormous potential as design tools and are therefore receiving considerable attention in the aerospace, automotive, and biomedical research communities (among others). Bottlenecks, moreover, associated with the analytic evaluation of discrete sensitivity derivatives, appear to have been addressed [3] via the use of an incremental iterative solution of the sensitivity equation [5] where memory efficient methods [6] are used to construct Jacobian matrix-vector products.

Solutions to the excessive CPU run times, to perform the design optimization, are being explored through the use of simultaneous analysis and design optimization (SAADO) [7], one-shot methods [8], and exploiting parallel computing architectures [9,10]. Another crucial hurdle, for these aerodynamic optimization procedures to become useful design tools, is their ability to analyze and design complex configurations of practical interest. Elliot and Peraire [11], with regards toward the geometrically complex domains associated with the integration of the engine into the wing design process and to the possible multipoint design of the aircraft's high lift system and cruise design, assert that this may be "the step that determines the economic viability of the vehicle".

As recently noted by Reuther et al. [2] "while flow analysis has matured to the extent that Navier-Stokes calculations are routinely carried out over very complex configurations, direct CFD based design is only just beginning to be used in the treatment of moderately

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complex three-dimensional configurations". This is primarily due to the fact that to generate a single structured grid about such a configuration is difficult, if not impossible. Thus, to handle a typical complex geometry of practical interest, some sort of domain decomposition scheme must be incorporated into the design code. For structured grid solvers, these techniques would include multiblocked, zonally patched, and overlapped (sometimes referred to as Chimera) grid algorithms. However, as the geometric flexibility of the method increases, so does the complexity of the underlying algorithm. Since the use of sensitivity analysis, to evaluate the needed gradients for a numerical optimizer, is still evolving, little work has been done toward extending these algorithms to include these domain decomposition methods. The research which has been accomplished has mostly concentrated on the use of multiblocked grids. To this end, Reuther et al. [2] have developed a multiblock-multigrid adjoint solver ("continuous" or "control theory" approach [12]) which was applied to the wing redesign of a transonic business jet. Eleshaky and Baysal [13] developed a multiblock "discrete" adjoint solver which was applied to a simple axisymmetric nozzle near a flat plate. As for the use of the more advanced domain decomposition methods (zonal and overlapped grids), and combinations of the three various types, Taylor [14,15] has differentiated an advanced flow-analysis code to perform the discrete sensitivity analysis.

An alternative, to resorting to structured grid domain decomposition methods to cope with complex configurations, is the use of unstructured grid schemes. Since triangles and tetrahedra are the simplest geometric shapes possessing area and volume, respectively, they are capable of resolving irregularly shaped domains easier and with greater efficiency. Another attribute of unstructured grids is that they may be adapted and locally enriched where needed without affecting other regions of the mesh.

As for unstructured grid approaches to aerodynamic design optimization, Beux and Dervieux [16] performed spatially first-order accurate sensitivity analysis and optimization of a two-dimensional nozzle using a continuous adjoint method to derive the optimality conditions, but a discrete approach for computer implementation. Newman, Taylor, and Burgreen [17] subsequently developed a two-dimensional, and later a three-dimensional [3], second-order spatially accurate discrete sensitivity analysis approach which has been used to perform the design optimization of airfoils and transport wings in transonic flow. Elliot and Peraire [11] have also developed an unstructured discrete sensitivity analysis approach which was used to match target pressure distributions for a two-element airfoil, a 3D infinite wing, and a wing-body configuration. Subsequently, Elliot and Peraire [4] have applied their algorithm to perform the inverse pressure design of a business jet wing immersed in transonic flow. An equally impressive use of unstructured grid approaches, for the design of geometrically complex devices, has been performed by Burgreen and Antaki [18]. In Ref.[18], CFD-based design optimization methods are

used to improve the thrombogenic performance of an axial flow blood pump. The research of Burgreen and Antaki [18], furthermore, represents the expansion of traditional aerodynamic design optimization procedures into the biomedical field to aid in artificial heart design. More recently, Anderson and Venkatakrishnan [19] have developed an unstructured grid approach to sensitivity analysis which truly utilizes a continuous adjoint approach. Moreover, in Ref.[19], limitations of the continuous adjoint approach are discussed and a hybrid continuous-discrete approach, which addresses some of these deficiencies, is developed.

In this work the current unstructured grid approach to aerodynamic design optimization [3,17] is demonstrated on non-trivial, complex configurations. Presented herein is a discussion of the algorithms used to solve the nonlinear fluid and the linear sensitivity equations, to parameterize the design surfaces, and to perform the unstructured grid adaptation. Special considerations that arise from the use of unstructured meshes, as well as the use of memory efficient Jacobian matrix-vector product methods which make large-scale optimization possible, are discussed. To demonstrate this procedure, the aerodynamic shape sensitivity analysis and design optimization of a high-lift multielement airfoil and for a complete Boeing 747-200 aircraft is performed. Accuracy is assessed by comparing the analytically obtained sensitivity derivatives with central finite-differences.

2. Aerodynamic Design Optimization Problem

Aerodynamic design optimization is simply a constrained minimization problem which attempts to reduce an objective or cost function $F(Q, X, \beta_k)$ subject to constraints $C_j(Q, X, \beta_k)$. Here, Q is the aerodynamic state vector, X is the computational mesh, and β_k is the vector of design variables which control the shape of the configuration.

A procedure to accomplish this minimization is obtained by linearizing the above constrained problem and then solving the resulting set of equations. For a gradient-based optimization method, such as the Method of Feasible Directions [20] used in the present work, frequent evaluations of the objective function and constraints as well as sensitivity gradient information are required. The sensitivity gradients of the objective function, ∇F , and the constraints, ∇C_j , are commonly referred to as the sensitivity derivatives. These sensitivity derivatives may be simply evaluated by finite differences; however, this approach is not only computationally expensive, it has been found at times to produce highly inaccurate gradient approximations. The preferable approach is to obtain the sensitivity derivatives quasi-analytically via

$$\nabla F = \left(\frac{\partial F}{\partial Q} \right)^T \frac{dQ}{d\beta_k} + \frac{\partial F}{\partial X} \frac{dX}{d\beta_k} + \frac{\partial F}{\partial \beta_k} \quad (1a)$$

$$\nabla C_j = \left(\frac{\partial C_j}{\partial Q} \right)^T \frac{dQ}{d\beta_k} + \frac{\partial C_j}{\partial X} \frac{dX}{d\beta_k} + \frac{\partial C_j}{\partial \beta_k} \quad (1b)$$

To compute the sensitivity derivatives in Eqs.(1a,b), the sensitivity of the state vector $dQ/d\beta_k$ is needed. This, consequently, results in the difficulty of solving an extremely large system of linear equations. The methods used in the present work to obtain the state vector, and the sensitivity of the state vector, will be discussed in sections to follow.

3. Fundamental Equations

Nonlinear Aerodynamic Analysis

The nonlinear fluid model considered in this work will be the three-dimensional Euler equations. These equations represent the conservation of mass, momentum and energy for an inviscid compressible flow. Applying the backward Euler time-integration scheme to the unsteady term and linearizing the right hand side of the semi-discrete approximation yields

$$\left[\frac{V_i}{\Delta t_i} I + \frac{\partial R_i}{\partial Q} \right] \Delta Q = -R_i \quad (2)$$

where R_i represents the steady-state residual

$$R_i = \oint_{\partial \Omega} \bar{E} \cdot \hat{n} dA = \sum_{j=\kappa(i)} E_{i,j} \quad (3)$$

with the face area through which the flux passes contained within E .

In the present work, the inviscid flux vector, E , and the Jacobian, $\partial R_i / \partial Q$, are evaluated using the flux vector splitting technique of Van Leer [21]. The Jacobian matrix may then be expressed in terms of the Van Leer fluxes as

$$\frac{\partial R_i}{\partial Q} = \sum_{j=\kappa(i)} \left(\frac{\partial E_{i,j}^+}{\partial Q_{f,j}} \frac{\partial Q_{f,j}^-}{\partial Q} + \frac{\partial E_{i,j}^-}{\partial Q_{f,j}} \frac{\partial Q_{f,j}^+}{\partial Q} \right) \quad (4)$$

where the flux Jacobians are evaluated with variables interpolated to the j cell faces, and $\partial Q_{f,j}^\pm / \partial Q$ represents the cell center contribution from the interpolation or reconstruction. When higher-order spatial accuracy is desired in Eq.(4), the form of the Jacobian becomes extremely complicated and the computational stencil very large. This is due to the upwind biased interpolation scheme that must be used for unstructured grids. The order of accuracy of the aerodynamic analysis, however, is determined from the evaluation of the residual vector, and a first-order Jacobian has been found sufficient to converge the implicit scheme. Since the left hand side operator is not an exact linearization of the residual, the ability to achieve quadratic convergence is now lost. It should be noted that an inexact linearization is not permitted for the aerodynamic sensitivity equation. This is because the underlying equations are linear and no approximations to

the higher-order Jacobian matrix are allowed.. This will be discussed in a subsequent section.

A higher-order upwind scheme is obtained by expanding the cell-centered solution to the left and right of each cell interface using a Taylor series expansion [22]. This expansion may be expressed as

$$Q_f^\pm = Q + \nabla Q \cdot \Delta \bar{r} \quad (5)$$

where the solution gradient, ∇Q , at the center of the cell is found using the geometric invariant features of tetrahedra. The expression for the solution gradient at the cell center may be obtained from application of Green's theorem as

$$\nabla Q \cdot \Delta \bar{r} = \frac{1}{4} \left[\frac{1}{3} (Q_{n1} + Q_{n2} + Q_{n3}) - Q_{n4} \right] \quad (6)$$

where Q_{n1} , Q_{n2} , Q_{n3} are the primitive variables at the three nodes that constitute the face through which the flux passes, $\Delta \bar{r}$ is the distance from the centroid of the tetrahedron to the center of that face, and Q_{n4} are the same variables at the fourth node of the tetrahedron.

The data at the nodes are obtained from the cell centered solution by using either an inverse distance or a pseudo-Laplacian weighting procedure [23]. Both procedures, described in Ref.[24], attempts a multidimensional weighted averaging of the form

$$Q_n = \left(\sum_{i=1}^N w_i Q_i \right) / \left(\sum_{i=1}^N w_i \right) \quad (7)$$

where w_i is the computed weighting factor from the desired node, n , to the surrounding N cell centers.

Aerodynamic Shape Sensitivity Analysis

As noted in a previous section, to determine the needed sensitivity derivatives, the sensitivity of the state vector $dQ/d\beta_k$ is required. To obtain this, the discrete residual vector (for a steady-state solution) may be recast as

$$R(Q(\beta_k), X(\beta_k), \beta_k) = 0 \quad (8)$$

where the explicit and implicit dependencies of the residual on the state vector, the computational mesh, and the design variables are asserted.

At this point, one of two discrete formulations may be used to determine the sensitivity derivatives. These formulations are referred to as the direct differentiation method and the adjoint variable method. For reasons which will be summarized in the conclusions, the direct approach is used in the current work. For a more detailed discussion of both methods, and their associated boundary conditions, the reader is referred to Ref.[25].

In this formulation Eq.(8) is directly differentiated with respect to the vector of design variables and rearranged to produce the following linear equation

$$\frac{\partial R}{\partial Q} \frac{dQ}{d\beta_k} = - \left(\frac{\partial R}{\partial \beta_k} + \frac{\partial R}{\partial X} \frac{dX}{d\beta_k} \right) \quad (9)$$

where $\partial R/\partial Q$ and $\partial R/\partial X$ are the Jacobian matrices evaluated at a converged flow solution, and $dX/d\beta_k$ is the grid sensitivity term. It should be noted that the task of constructing exactly or analytically all of the required Jacobians and derivatives by hand, and then building the software for evaluating these terms can be extremely complex. This problem is exemplified by the inclusion of even the most elementary turbulence model (for viscous flow) or use of a sophisticated grid generation package for adapting (or regenerating) the computational mesh to the latest design. A promising possible solution to this problem, however, has been found in the use of a technique known as automatic differentiation (AD). AD involves the application of a precompiler software tool called ADIFOR (Automatic Differentiation of FORtran, Ref.[26]). This software has been utilized, with much success, to obtain complex derivatives from advanced CFD and grid generation codes for use within aerodynamic design optimization procedures [14,15,27-29].

In the present work, the Jacobians $\partial R/\partial Q$, $\partial R/\partial X$ as well as all derivatives (except for the grid sensitivity term) are constructed by hand. This is due to the fact that an inviscid fluid model is assumed, with the inviscid fluxes being constructed via the flux vector splitting scheme of Van Leer (a scheme which is continuously differentiable and well documented). ADIFOR, on the other hand, is used on the unstructured grid adaptation algorithm to provide the required grid sensitivity terms. Details of this algorithm and the evaluation of grid sensitivities will be discussed in a later section.

The solution of Eq.(9) above poses the difficulty of solving an extremely large linear system for each design variable. Solving these systems, however, is made more tractable when the above equations are recast into what has been termed the incremental iterative form [5,14,29,30] as follows

$$\bar{A} \Delta^n \left(\frac{dQ}{d\beta_k} \right) = - \left(\frac{\partial R}{\partial \beta_k} + \frac{\partial R}{\partial X} \frac{dX}{d\beta_k} + \frac{\partial R}{\partial Q} \left(\frac{dQ}{d\beta_k} \right)^n \right) \quad (10a)$$

$$\left(\frac{dQ}{d\beta_k} \right)^{n+1} = \left(\frac{dQ}{d\beta_k} \right)^n + \Delta^n \left(\frac{dQ}{d\beta_k} \right) \quad (10b)$$

where \bar{A} may now be any convenient approximation to the higher-order Jacobian which converges the linear system. This is because the equations are now cast in *delta* form, with the physics contained in the right-hand-side vector. It has been found that the first-order Jacobian works well for use in the coefficient matrix of Eq.(10a), and is therefore used in the present work. A more detailed and thorough discussion of this incremental iterative technique may be found in the above cited literature.

Two particularly attractive features of the incremental iterative strategy are that (i) a more diagonally dominant

matrix may be used to drive the solution of the linear systems (as opposed to the sometimes ill-conditioned higher-order Jacobian), and (ii) the higher-order Jacobian now resides on the right-hand-side of the equations and may be dealt with in an explicit manner. When in this form, only the k -vectors resulting from the matrix-vector product of $(\partial R/\partial Q) \cdot (dQ/d\beta_k)$ are of concern. Hence, CPU time and memory efficient methods for constructing the exact matrix-vector product can be utilized. To this end, Barth and Linton [6] have developed a new technique which permits the construction of the higher-order Jacobian-vector product using slightly less memory than that which would be required to evaluate the first-order Jacobian-vector product. This is accomplished by avoiding the need to assemble the full Jacobian prior to multiplication. With the details omitted (and the reader directed to Ref.[6] for the proof and further explanation of this method), this technique, applied to the desired matrix-vector product in Eq.(10a), may be symbolically written (using the notation of Eq.(4)) as

$$\frac{\partial R_i}{\partial Q} \frac{dQ}{d\beta_k} = \sum_{j=\kappa(i)} \left(\frac{\partial E_{i,j}^+}{\partial Q_{f,j}^-} \left(\frac{dQ}{d\beta_k} \right)_{f,j}^- + \frac{\partial E_{i,j}^-}{\partial Q_{f,j}^+} \left(\frac{dQ}{d\beta_k} \right)_{f,j}^+ \right)$$

where $(dQ/d\beta_k)_{f,j}^-$ and $(dQ/d\beta_k)_{f,j}^+$ are the vectors reconstructed from $dQ/d\beta_k$ using the same scheme employed in the CFD analysis. The resulting vector, from this matrix-vector product, is then scattered to the adjacent cells in the same manner as used for the nonlinear flow-residual calculation.

It should be noted that this method only requires the storage of the 5x5 flux Jacobians, and the reconstructed vectors, at the cell faces. Since this product is to be used in the sensitivity analysis, the memory which was utilized to compute the flux Jacobians for the first-order Jacobian, and that used to reconstruct the CFD state vector, may be reused. Thus, the spatially first- and higher-order accurate sensitivity analysis may be performed with *virtually* the same memory as the CFD analysis. The only additional memory is due to the storage of the grid and metric sensitivity terms and the derivative $\partial R/\partial \beta_k$ or $(\partial R/\partial X) \cdot (dX/d\beta_k)$. (Note that for geometric design variables $\partial R/\partial \beta_k$ is zero, and for non-geometric design variables $(\partial R/\partial X) \cdot (dX/d\beta_k)$ is zero). Another attribute of this method is that the matrix-vector product computation only requires a fraction of the CPU time originally needed to assemble the full higher-order Jacobian; hence, the benefits are two-fold. The use of Barth and Linton's technique within the incremental iterative method has significant ramifications in that it makes large-scale optimizations of practical three-dimensional configurations possible.

4. Solution Methodology

The linear systems resulting from the nonlinear aerodynamic analysis, as well as those from the

aerodynamic shape sensitivity analysis, can be expressed in the simple form $Ax=b$. Within the optimization process, it is evident that the aerodynamic analysis not only consumes more CPU time (than the shape sensitivity analysis) to converge the nonlinear systems, it also is needed more frequently. Thus, solution algorithms which have high convergence rates are imperative when multiple analyses are to be performed. To this end, it has been found that a fully implicit iterative solver, utilizing preconditioned GMRES techniques developed by Saad and Schultz [31], often out perform conventional explicit as well as classical iterative solvers [32-34]. In previous work [3,17], all linear systems resulting from the solution of Eq.(2) and Eq.(10a) for the aerodynamic analysis and shape sensitivity analysis, respectively, utilized GMRES. However, the selection of the preconditioner used in conjunction with GMRES essentially governs the performance and memory required by this algorithm. Computations performed in Refs.[3,17] used an incomplete LU factorization (ILU(0)) as a left preconditioner for Eq.(2) and as a right preconditioner for Eq.(10a). In the current work, the two-dimensional computations about the multielement airfoil use GMRES, but due to the memory requirements associated with this algorithm, it was not possible to utilize it for the Boeing 747-200 configuration. Hence, a Gauss-Seidel iterative method was used to solve the fluid and sensitivity equations for this geometry.

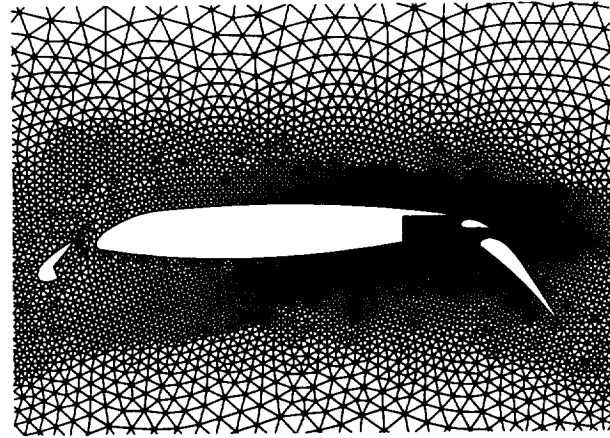
As a final note, it has been observed that the *ordering* of the cells in a grid has an affect on the rate of convergence of iterative solvers [35]. With this in mind, many researchers [34,36,37] currently working with unstructured grids (which usually have a random ordering) have adopted renumbering algorithms such as Cuthill-McKee (CM) [38] or reverse-CM [39]. These algorithms attempt to reorder a given mesh such that the bandwidth of the coefficient matrix is minimized. In the present work, reordering is accomplished, during the initial preprocessing of the grid, using the Gibbs-Poole-Stockmeyer [40] algorithm.

5. Design Surface and Grid Representation

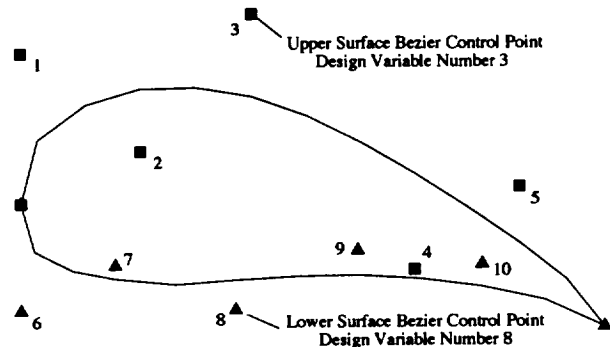
A key aspect in any design optimization procedure is *how* the design surface and computational mesh are to be represented. This selection will ultimately determine (i) the type and number of design variables used, (ii) the grid adaptation or regeneration method, and (iii) the means through which grid sensitivities are calculated. In the following sections, the techniques used in the present work for (i), (ii), and (iii) will be discussed.

Design Surface Parameterization

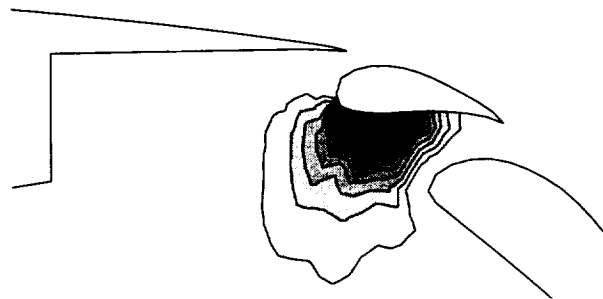
Once an aerodynamic shape optimization code has been developed and verified, only the design surface parameterization routines change from application to application. The particular parameterization technique utilized depends on the geometry being studied and the design problem formulation. An excellent example of a sophisticated wing parameterization method capable of



(a) Unstructured mesh (7614 nodes and 14919 cells).



(b) Design surface parameterization for the vane.



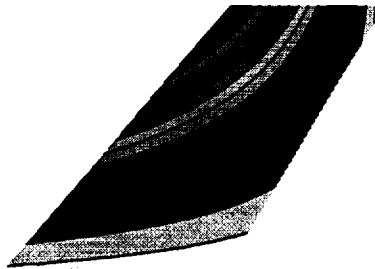
(c) Grid sensitivity $dy/d\beta_7$.

Fig. 1: Mesh, vane parameterization, and grid sensitivity for the multielement airfoil.

modeling wing-section (airfoil) definitions, taper distribution, sweep, span and spanwise bending, global angle-of-attack, and twist schedule was developed in Ref.[41], and discussed at length in Ref.[1]. In the current work, however, simpler parameterization techniques have been used for demonstrative purposes and to keep the number of design variables to a minimum for the Boeing 747-200 configuration.



(a) Unstructured mesh (63828 nodes and 352547 cells).



(b) Grid sensitivity $dz/d\beta_1$.

Fig. 2: Surface mesh and grid sensitivity for the Boeing 747-200 configuration.

For the high-lift multielement airfoil, the upper and lower surfaces of the vane were parameterized with separate Bezier curves. Details of this type of parameterization for the design of airfoils may be found in Refs.[17,32]. The design variables are the vertical locations of the 10 interior Bezier control points. The unstructured mesh, and this parameterization, are shown in Fig. 1a and 1b, respectively. As for the Boeing 747-200 geometry, the dihedral and twist schedule along the wing, outboard of the outermost engine nacelle, was parameterized with cubic polynomials. At the point where the engine strut meets the wing, point and slope continuity are enforced. Thus, only the coefficients of the quadratic and cubic terms are free and are therefore chosen as the design variables. This constitutes 2 design variables for the dihedral and 2 for the twist. The unstructured surface mesh for the Boeing 747-200 configuration is depicted in Fig. 2a, and is derived from the model tested in the NASA Ames 11 foot Transonic Pressure Tunnel (Test AR0502). It should be noted that the twist distribution of this geometry is not representative of the normal shape of the production wing.

Unstructured Grid Adaptation

The mesh movement strategy adopted considers the mesh as a system of interconnecting springs. This system is constructed by representing each edge of each tetrahedron by a tension spring. In the current method, the stiffness of the springs is assumed inversely proportional to the length of its edge. Then, for each mesh point, the external forces due to the connecting boundary springs are summed and resolved into Cartesian components. The result is a set of linear systems which may be solved for the displacements of each node using several Jacobi iterations. For further details on this method the reader is directed to the literature [42-44]. Reference [44] has the added advantage of edge (2D; face in 3D) swapping based on the Delaunay criterion which greatly improves the performance of the method. This technique, however, is not currently implemented in the present grid adaptation algorithm, but is deemed a vital improvement.

Grid Sensitivities

Efficient and accurate evaluation of grid sensitivities is an extremely important and vital aspect in any design optimization procedure (which uses discrete sensitivity analysis). The technique used to obtain the grid sensitivities from the unstructured grid adaptation procedure results from the direct application of ADIFOR. Here, the subroutines which define the design variables β_k , and the subroutines which perform the unstructured grid adaptation to produce the mesh $X(\beta_k)$, are differentiated using ADIFOR. The result is an additional set of subroutines which, upon compilation and execution, will return the grid sensitivities, $dX/d\beta_k$.

To verify that these sensitivities were indeed correct for both geometries, the design variables were perturbed, the grid adapted, and the grid sensitivities calculated via ADIFOR generated subroutines. These sensitivities were then compared with those obtained using finite-difference. Quantitatively, ADIFOR generated grid sensitivities matched finite-difference to approximately 8 significant digits. A qualitative representation of computed grid sensitivities are depicted in Fig. 1c and Fig. 2b for the multielement airfoil and Boeing 747-200 configurations, respectively. Figure 1c illustrates the sensitivity of the internal mesh to one of the design variables (Bezier control points) on the lower surface of the vane. Figure 2b represents the sensitivity of the surface grid to a twist design variable (specifically the coefficient of the quadratic term in the spanwise twist schedule).

6. Results and Discussion

In the present work, aerodynamic shape sensitivity analysis and design optimization for two sample complex configurations are examined: a two-dimensional high-lift multielement airfoil and a three-dimensional Boeing 747-200 aircraft. The flow conditions for the multielement airfoil calculation were a subsonic Mach number of 0.20 at 16.02 degrees angle-of-attack. For the Boeing 747-

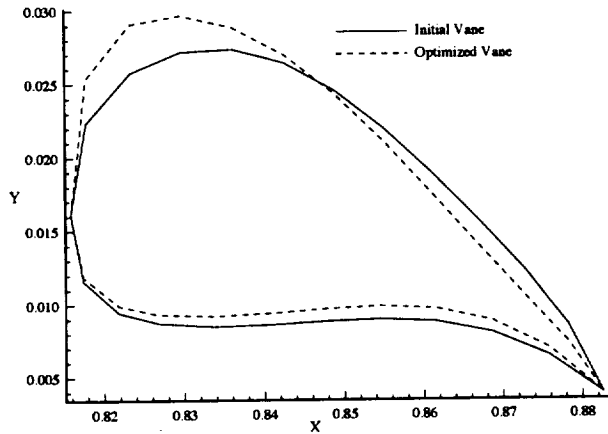


Fig. 3: Initial and optimized vane shapes.

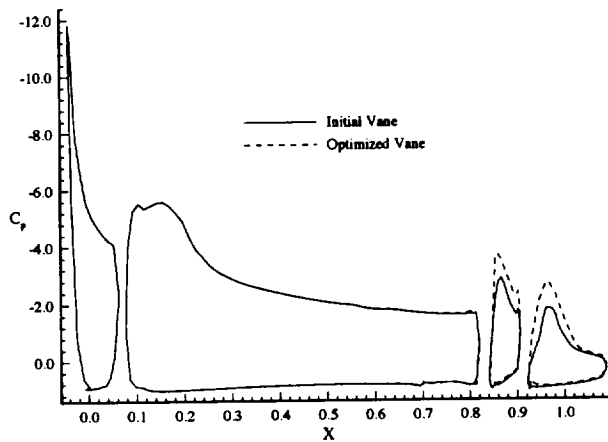


Fig. 4: Pressure coefficient distributions on the multielement airfoil.

200, a transonic Mach number of 0.84 was chosen with a freestream angle-of-attack of 2.73 degrees. To verify the accuracy of present discrete sensitivity analysis approach, the sensitivity derivatives of lift-to-drag ratio, with respect to the geometric design variables previously discussed, are computed and compared with finite-differences. It should be noted that the work associated with computing sensitivity derivatives via the direct differentiation method does not scale with the number of output functions. Hence, for the computations shown, any number of output function sensitivities (i.e., for lift, drag, pitching moments...etc.) can be computed with little effort. To evaluate the corresponding finite-difference derivatives, however, requires two analysis runs per design variable for central differences. Thus, due to the expense of finite-difference derivatives, only those derivative comparisons with respect to one of the geometric design variables were performed.

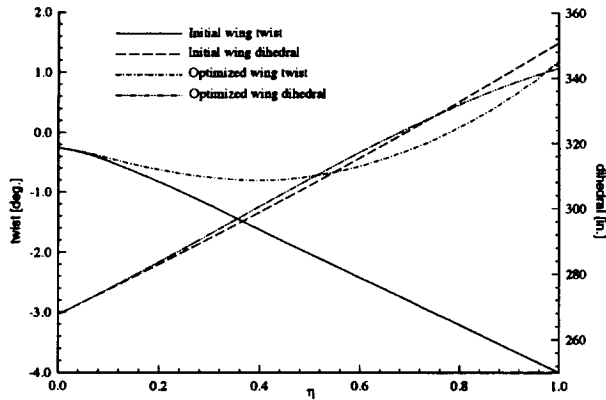
For the multielement airfoil, the geometric design variable selected was the same Bezier control point used

above in Fig. 1c for the grid sensitivity illustration. For this configuration the analytically obtained derivative $dC_L/d\beta_7$ was computed to be 0.210754 as compared to the central finite-difference value, 0.210733. The geometric design variable selected for the Boeing 747-200 was, once again, the one presented above in the grid sensitivity demonstration of Fig. 2b; namely, the coefficient of the quadratic term in the spanwise twist schedule. Here, the computed derivative $d(L/D)/d\beta_1$ yielded 1.3231, and the finite-difference calculation 1.3229. As should be expected with consistent, discrete sensitivity analysis, computed derivatives match finite-difference to approximately 4 significant digits for both geometries.

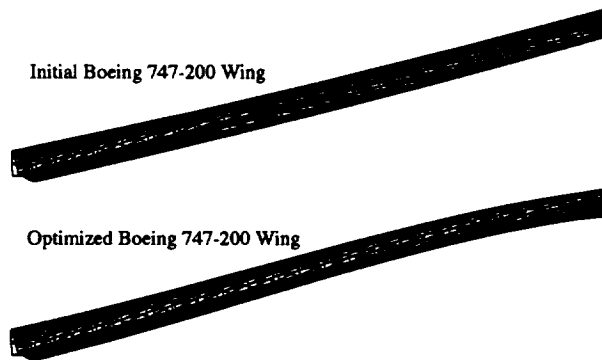
Before presenting the design results, some brief words about the importance of design problem formulation need to be asserted. Sensitivity analysis is merely an extra level of computation that provides additional information to the designer. When the sensitivity analysis routines are coupled with the fluid solver, a mesh movement strategy, and a numerical optimizer, a functional design tool is produced. The eventual designs created with this tool will be only as good as the formulated design problem. If improperly formulated, designs can be produced that will violate constraints such as those needed for manufacturability or structural feasibility, or produce a design that has superb performance at one operating condition, but is unacceptable off the design point. Thus, experience is required in formulating meaningful design problems.

For the multielement airfoil case, it is recognized that the goal of a flap system is to maintain the highest possible lift-to-drag ratio at the maximum lift coefficient [45]. With this in mind, the problem formulation consisted of maximizing the lift coefficient. Note, it is not asserted that inviscid flow analysis and sensitivity analysis are capable of modeling the physics or properly designing such a configuration (especially at high angle-of-attack situations such as take off and landing), but rather that an unstructured grid method easily discretizes the domain and that it is possible to carry out this type of design study. Furthermore, a geometric constraint has been placed on the thickness of the trailing edge to keep it from becoming excessively thick or thin.

Results of this design study are summarized in Table 1. This optimization required 7 design cycles with 83 CFD analyses along the line searches and a total run time of a little over 40.5 min. on a CRAY Y/MP. The objective function was increased by 6.2%. Figure 3 depicts the initial and final optimized vane produced by this shape optimization procedure. Illustrated in Fig. 4 are the corresponding pressure coefficient distributions about the multielement airfoil. It should be noted that the horizontal location of the vane and flap have been altered so that their C_p distributions may be easily distinguished. As seen, the pressure distributions about the leading edge slat and the main airfoil remain roughly unchanged, but those on the vane and flap have been greatly altered. Another



(a) Initial and optimized twist and dihedral distributions.



(b) Upstream view of the surface mesh on the outboard wing stations.

Fig. 5: Optimization results for the Boeing 747-200 wing redesign.

interesting design problem, which may have produced more dramatic improvements, could have been the design optimization of the shape, location, and orientation of the leading edge slat.

For the Boeing 747-200 aircraft, the design problem was formulated to maximize the lift-to-drag ratio. Once again, constraints that the lift coefficient at the final design be greater than the initial value, and that the drag coefficient be reduced for the optimized shape, have been incorporated. The results of this optimization are also shown in Table 1. It should be noted that since this design study was for purely demonstrative purposes, the optimization was halted after 3 design cycles and not allowed to continue until an isolated local minimum was found. Observe that the objective function has been improved by 2.7%, but at the cost of 23.4 CRAY Y/MP hours. This represents about 6 converged CFD analyses. As noted in the introduction, however, techniques are now being studied to reduce these excessive CPU times. Nevertheless, the initial and final twist and dihedral schedules are shown in Fig. 5a. Note that a positive twist angle is defined herein as leading edge up. As seen, this optimized wing has a greater twist at the tip station and an

altered dihedral distribution. The initial and optimized surface meshes are viewed from an upstream vantage point in Fig 5b to show the dihedral distributions of each wing. Surface pressure contours for both the initial and final designs are illustrated in Fig. 6 and 7 for the upper and lower surfaces, respectively. It can be observed from close inspection that the upper surface of the optimized wing has a greater region of lower pressure than the initial wing and that the lower surface has a slightly higher pressure. Once again, a design problem which possibly could have produced more significant increases in the lift-to-drag ratio would have been to perform the shape optimization of the wing airfoil sections. However, this current procedure has demonstrated the ease with which an unstructured grid approach to aerodynamic shape sensitivity analysis and optimization may be used to analyze and design geometrically complex configurations.

7. Conclusions

A three-dimensional unstructured grid approach to aerodynamic shape sensitivity analysis and optimization has been demonstrated on geometrically complex configurations of practical interest. It was shown that shortcomings of discrete sensitivity analysis can be bypassed by the use of an efficient Jacobian matrix-vector product technique within the incremental iterative form of the sensitivity equation and through the use of automatic differentiation to obtain grid sensitivities. This approach was demonstrated through the shape optimization of the vane in a two-dimensional multielement airfoil configuration and by the twist and dihedral schedule on the outboard stations of the wing on a Boeing 747-200 aircraft. The complexity of the geometries studied herein illustrate the advantages of using unstructured grids for aerodynamic shape optimization.

In the current work the direct differentiation approach, as opposed to the adjoint variable approach, was used to perform the discrete sensitivity analysis. As is well known, for design problem formulations in which the number of design variables exceed the number of constraints plus one (for the objective function), the adjoint method is the preferred approach. However, when the reciprocal is true and there are more constraints than design variables, as in the case of multidisciplinary optimization, the direct approach is more attractive. Since the ultimate goal of the present work is the development of a multidisciplinary analysis and optimization procedure (using discrete sensitivity analysis), the direct differentiation method was adopted. Moreover, the current algorithm has been extended to incorporate an existing finite-element code to perform the aeroelastic analysis [46,47], and work is currently underway to perform the discrete aeroelastic sensitivity analysis. Results are forthcoming.

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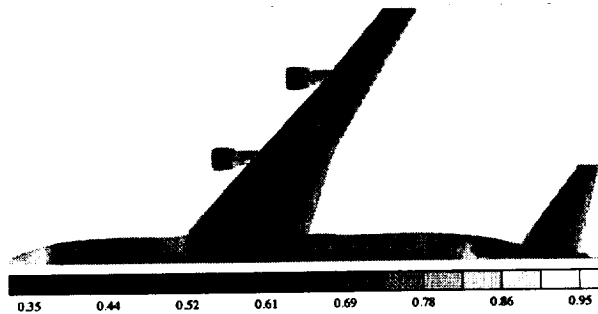
Table 1: Summary of optimization results.

	Initial Objective	Final Objective	Function Evaluations	Gradient Evaluations	Memory [†] [MW]	CPU Y/MP [‡] [hr.]
MEA	3.792	4.028	82	7	2.67/4.18	0.062/0.675
B747	14.136	14.522	24	3*	63.3/104.1	3.7/23.4

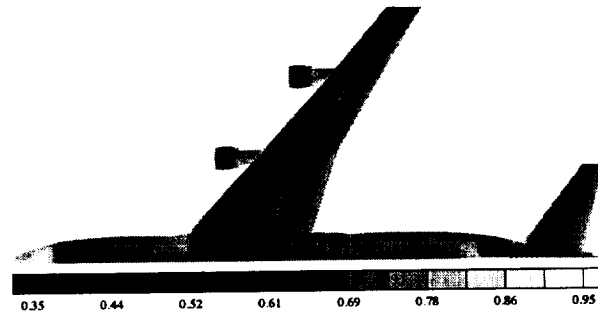
* Stopped after third design cycle.

† Memory for CFD analysis/memory for sensitivity analysis.

‡ CPU time for converged CFD analysis/total optimization run time.

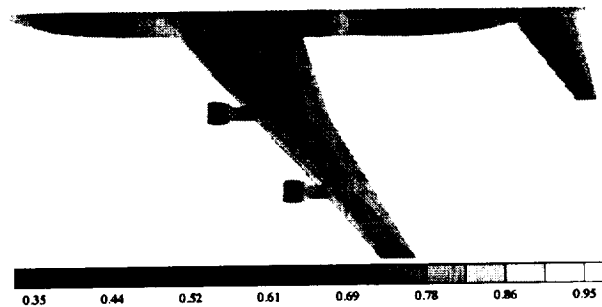


(a) Pressure contours on the initial geometry.

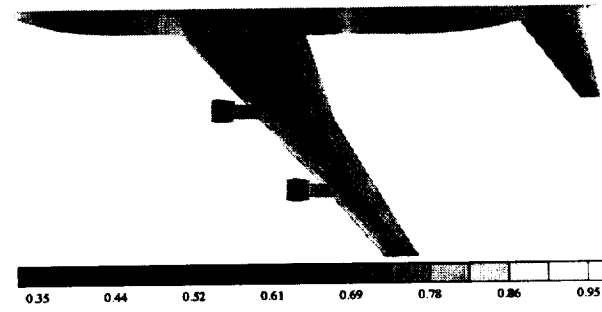


(b) Pressure contours on the optimized geometry.

Fig. 6: Upper surface pressure contours for the initial and optimized B747-200 configuration.



(a) Pressure contours on the initial geometry.



(b) Pressure contours on the optimized geometry.

Fig. 7: Lower surface pressure contours for the initial and optimized B747-200 configuration.

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