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# The Effect of Temperature Dependent Material Nonlinearities on the Response of Piezoelectric Composite Plates

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## THE **EFFECT OF** TEMPERATURE **DEPENDENT MATERIAL NONLINEARITIES ON THE** RESPONSE **OF PIEZOELECTRIC COMPOSITE PLATES**

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#### **ABSTRACT**

Previously developed analytical formulations for piezoelectric composite plates are extended to account for the nonlinear effects of temperature on material properties. The temperature dependence of the composite and piezoelectric properties are represented at the material level through the thermopiezoelectric constitutive equations. In addition to capturing thermal effects from temperature dependent material properties, this formulation also accounts for thermal effects arising from: (1) coefficient of thermal expansion mismatch between the various composite and piezoelectric plies and (2) pyroelectric effects on the piezoelectric material. The constitutive equations are incorporated into a layerwise laminate theory to provide a unified representation of the coupled mechanical, electrical, and thermal behavior of smart structures. Corresponding finite element equations are derived and implemented for a bilinear plate element with the inherent capability to model both the active and sensory response of piezoelectric composite laminates. Numerical studies are conducted on a simply supported composite plate with attached piezoceramic patches under thermal gradients to investigate the nonlinear effects of material property temperature dependence on the displacements, sensory voltages, active voltages required to minimize thermal deflections, and the resultant stress states.

#### **NOMENCLATURE**



- b : width of plate h
- D : electric displacement K
- d : piezoelectric charge constant M
- E : electric field p
- F : mechanical force vector Q
- f : body force per unit volume q
- : thermal force **vector**
- : thickness of plate
- : stiffness matrix
- : mass matrix
- : pyroelectric constant
- : electric charge vector
- : surface electric charge

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#### **Subscripts**

- $(i, i)$  :  $\partial(j)$  /  $\partial u_i$
- uu : structural component
- u $\phi$ , $\phi$ u : piezoelectric component
- $\phi$  : dielectric component

#### **Superscripts**



- a : active component
- E : constant voltage condition
- s : sensory component
- T : constant temperature condition
- $\sigma$ : constant stress condition
- *0* : through the thickness shape functions

# **INTRODUCTION**

The development of structural components consisting of piezoelectric materials offers great potential for improving the performance of advanced aerospace vehicles. By taking advantage of the direct and converse piezoelectric effects, as well as the pyroelectric phenomena, piezoelectric composite materials can be utilized in a variety of shape, vibration, and noise control applications. These potential performance advantages have resulted in extensive development of both analytical and experimental methods to characterize the behavior of piezoelectric materials. Detailed overviews of the current state of piezoelectric material technology have been reported by Crawley (1994) and Rao and Sunar (1994).

Although the typical application of most aerospace structures will involve operations in extreme temperature environments, only limited research has been performed to study the implications of thermal effects on the active and sensory response of piezoelectric composite materials. In general, there are three distinct physical mechanisms which will influence the thermal response of piezoelectric composite structures: (1) induction of thermal stresses due to coefficient of thermal expansion (CTE) mismatch between the various composite and piezoelectric layers, (2) pyroelectric effects on the piezoelectric element, and (3) the temperature dependence of the composite and piezoelectric material properties. Previously reported analytical models incorporating thermal effects have investigated only the first two thermal physical mechanisms (i.e. CTE mismatch and pyroelectric phenomena). Mindlin (1974) initiated analytical studies of thermopiezoelectricity by formulating two-dimensional plate equations. Laminated thermopiezoelectric plate and thin shell theories were reported by Tauchert (1992) and Tzou and Howard (1994), respectively. Subsequent developments in thermopiezoelectric finite elements were performed by Rao and Sunar (1993), Jonnalagadda et al. (1994), Tzou and Ye (1994), and Chandrashekara and Kolli (1995). Many of these previous approaches utilize various single layer theories, which have been shown by Robbins and Reddy (1991, 1993) to have limitations in the analysis of both thick laminates and laminates

containing strong inhomogeneities. In order to remedy these limitations, layerwise thermopiezoelectric finite element approaches have been developed by Lee and Saravanos (1995, 1996a, 1996b).

This paper extends previously developed layerwise finite element formulations [Lee and Saravanos (1995, 1996a, 1996b)] to also account for the nonlinear temperature dependence of material properties. The updated mechanics provides a comprehensive thermal analysis capability for modeling smart structures, which captures the coupled mechanical, electrical, and thermal response of piezoelectric laminates at the material level through the thermopiezoelectric constitutive equations. A layerwise laminate theory is implemented with the displacements, electric potential, and temperature modeled as variable fields through the thickness to provide more accurate analysis of piezoelectric composite laminates. A corresponding finite element formulation is implemented for a bilinear plate element. Numerical studies are conducted to determine the nonlinear effects of incorporating material property temperature dependence on the displacements, sensory voltages, active voltages required to minimize the thermal deflections, and stresses of piezoelectric composite plates subject to thermal gradients.

#### **GOVERNING EQUATIONS**

This section outlines the governing equations required to develop the layerwise finite element formulation for thermopiezoelectric composite structures. The mechanical response is governed by the equations of motion

$$
\sigma_{ij,j} + f_i = \rho \ddot{u}_i \tag{1}
$$

while the electrical response is described by the electrostatic equation

$$
D_{i,i} = 0 \tag{2}
$$

where  $i, j = 1, 2, 3$ . The constitutive equations for a thermopiezoelectric material, with temperature dependent properties, employing standard contracted notation are

$$
S_{\alpha} = s_{\alpha\beta}^{E,T}(T) \sigma_{\beta} + d_{\alpha m}^{T}(T) E_{m} + \alpha_{\alpha}^{E,T}(T) \theta
$$
 (3)

$$
D_m = d_{m\alpha}^T(T) \sigma_\alpha + \epsilon_{mk}^{\sigma,T}(T) E_k + p_m^{\sigma,T}(T) \theta
$$
 (4)

where  $\alpha$ ,  $\beta = 1$ ,...,6 and k, m = 1, 2, 3. The small deformation strain-displacement relations are

$$
S_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i})
$$
 (5)

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and the electric field vector is related to the electric potential by

$$
E_i = -\Phi_{i}
$$
 (6)

Through use of the divergence theorem and neglecting body forces, Eqs. (1) and (2) can be expressed in an equivalent variational form as

$$
\int_{V} (\rho \ddot{u}_{i} \delta u_{i} + \sigma_{ij} \delta S_{ij} - D_{i} \delta E_{i}) dV = \int_{\Gamma_{t}} t_{i} \delta u_{i} d\Gamma + \int_{\Gamma_{p}} q \delta \phi d\Gamma
$$
\n(7)

where V represents the volume of both the composite and piezoelectric materials,  $\Gamma_t$  is the bounding surface on which surface tractions are applied, and  $\Gamma_{p}$  represents the piezoelectric material surface to which electrical charges are applied.

By incorporating Eqs. (3)-(6) and introducing both the layerwise and finite element approximations into Eq. (7), the following discretized finite element equations can be obtained in a compact form with the electrical potential partitioned into active and sensory components

$$
\begin{bmatrix}\n[M_{uu}] & 0 \\
0 & 0\n\end{bmatrix}\n\begin{bmatrix}\n\{\ddot{U}\}\n\end{bmatrix} + \begin{bmatrix}\n[K_{uu}] & [K_{u\phi}^{ss}]\n\end{bmatrix}\n\begin{bmatrix}\n\{U\}\n\end{bmatrix} = \n\begin{bmatrix}\n\{F\} + \{F_{th}\} - [K_{u\phi}^{sa}]\{\Phi^a\}\n\end{bmatrix}
$$
\n(8)

The mass, stiffness, external force, and thermal force submatrices are functions of the various material properties, interpolation functions, and plate geometric parameters such that

$$
M_{uu} = M_{uu}(\rho, \psi, R, a, b, h)
$$
  
\n
$$
K_{uu} = K_{uu}(C, \psi, R, a, b, h)
$$
  
\n
$$
K_{u\phi} = K_{u\phi}(e, \psi, R, a, b, h)
$$
  
\n
$$
K_{\phi\phi} = K_{\phi\phi}(\epsilon, \psi, R, a, b, h)
$$
  
\n
$$
F_{th} = F_{th}(\lambda, \psi, R, a, b, h)
$$
  
\n
$$
Q_{th} = Q_{th}(p, \psi, R, a, b, h)
$$
 (9)

More detailed **descriptions** of the layerwise theory and finite element formulation, along with the specific forms of the submatrices can be found in Lee and Saravanos (1996b).

#### **TEMPERATURE DEPENDENT MATERIAL PROPERTIES**

The various elastic, dielectric, piezoelectric, and pyroelectric properties of piezoelectric materials are influenced differently by temperature variations depending on the composition and manufacturing technique. Piezoelectric materials also possess a characteristic limiting temperature, called the Curie temperature, beyond which the material loses its piezoelectric properties. Thus, in typical applications the operating temperatures must be considerably lower than the Curie temperature. In this study, temperature dependent material properties of a PZT-5A piezoceramic were obtained from manufacturer's data, while published properties of an AS4/3501-6 carbon/epoxy composite were acquired from Daniel and Ishai (1994).

The temperature variation of the piezoelectric and composite material properties are implemented into the constitutive equations Eqs. (3)-(4) in a piecewise linear fashion. The variation of PZT-5A properties  $(d_{31}, \epsilon_{33}, \alpha_{11},$  and  $p_3)$  are shown in Figure 1, while the temperature dependence of the AS4/3501-6 moduli and  $\alpha_{22}$  are depicted in Figure 2. All properties are nondimensionalized with their respective room temperature values shown in Table 1. Figures 1 illustrates the nonlinear variation of the various piezoceramic properties with temperature. The greatest nonlinearity occurs for  $p_3$ , while the other properties exhibit a more gradual variation. For purposes of this study, all material properties not illustrated in Figures 1 and 2 are assumed to remain constant with temperature.

#### **APPLICATIONS**

This section presents results of numerical studies to demonstrate the effects of incorporating temperature dependent material properties. The problem examined consists of a 37.2 cm x 22.8 cm  $x$  0.75 mm  $[0/±45]$ , AS4/3501-6 carbon/epoxy plate with discrete PZT-5A piezoceramic patches attached on both the top and bottom surfaces. There are fifteen 6.0 cm x 6.0 cm x 0.13 mm piezoceramic patches uniformly attached to each surface as shown in Figure 3, which also depicts the finite element discretization for this problem. A total of eight discrete layers are used through the thickness of the plate (one layer for each ply of the composite and piezoceramic). The plate is simply supported along the two edges parallel to the y-axis and free on the two edges parallel to the x-axis. Linear thermal gradients of different values are applied through-the-thickness of the plate with the bottom surface (z/h = -0.5) fixed at 20 $^{\circ}$ C for all cases and the upper surface (z/h = 0.5) varying from 45°C to 170°C. The values of the thermal gradient are selected to fall within the temperature range of the AS4/3501-6 properties (Fig. 2). The objective of the numerical study is to contrast the effects of incorporating temperature dependent material properties with corresponding results when material properties are assumed to remain temperature independent at the room temperature values shown in Table 1.

**Thermal** Deflections. A combined active/sensory configuration of the piezoceramic patches is used to study the thermally induced bending deflections and the corresponding sensory voltages in the plate. In this configuration, the piezoceramic patches on the upper surface of the plate are configured as sensors with free electric potentials, while 0 Volts are applied to all the patches on the bottom surface. All piezoceramic patches are grounded on the surface in contact with the carbon/epoxy plate. The resulting thermally induced deflection at the center of the plate is shown in Figure 4 for different applied gradients. The deflections are nondimensionalized with respect to the total laminate thickness  $(h=1.01 \text{ mm})$ . The figure shows that when material properties are assumed to be temperature independent, a linear variation in deflection is predicted. In contrast, a nonlinear variation in the deflection is observed when temperature dependence is incorporated into the analysis. Greater discrepancies between the two cases are observed as larger thermal gradients are applied.

Sensory Voltages. The corresponding sensory voltages generated by the central piezoceramic patch on the upper surface of the plate is shown in Figure 5. As in the case of the deflections, the assumption of constant material properties leads to a linear variation of the sensory voltages, while nonlinearities are introduced when temperature dependent material properties are modeled. Closer agreement is observed between the two cases than for the deflections. In actual smart structures applications, the measured sensory voltages would be used as input into a control algorithm to generate the necessary applied voltages to the actuators to minimize the thermal deflections.

Thermal Shape Control. A fully active configuration of the piezoceramic patches is used to study thermal shape control applications. In this configuration, all the piezoceramic patches are used as actuators to minimize the initial bending deformation. The same active voltages are applied to the inner surface of all the piezoceramic patches (i.e. the surface in contact with the carbon/epoxy plate), while the outer surface is grounded. The initial bending deformation to be minimized is shown in Figure 6 for the case in which the plate is subjected to a 100°C gradient with 0 Volts applied to the actuators. This deformation can be gradually eliminated through the application of increasing active voltages to the piezoceramic actuators. Figures 7 and 8 illustrate the reduction in bending achieved by applying active voltages of 72 V and 145 V, respectively. These results demonstrate the capability for thermal shape control using piezoceramic actuators.

**Figure** 9 **depicts** the **active** voltages applied to **all** the patches to **minimize** the **deflection at** the center **of** the **plate for** different **applied** gradients. Once **again, a nonlinear variation is predicted by** the **temperature dependent material property case, while** the **constant material property case shows a linear variation** that **increasingly deviates as larger** thermal gradients are **applied.** The **increasing** discrepancy **in** the **active voltages** required **to** minimize the **initial deformation** between the **two cases corresponds to** the **same trend observed for** the **deflection at the center of** the **plate (Fig.** 4). **Since the** temperature **dependent case results in significantly larger deflections for** higher temperatures, **this in part justifies why larger active voltages** are **necessary to control the deflections.**

**Thermal** Stresses. The stress **fields** associated with the plate in a fully active configuration are presented in this section. A continuous piezoceramic layer is used instead of discrete patches to reduce the need for a highly refined mesh in this study. The resulting normal stress  $(\sigma_{xx})$  near the center of the plate (x/a=0.5, y/b=0.4, z/h=0.8) for different applied gradients is shown in Figure 10 for applied active voltages of 0 and 100 Volts. The stresses are nondimensionalized using a product of the corresponding longitudinal PZT-5A modulus,  $E_{11}$ , and the total laminate thickness, h. As in the previous cases, the assumption of constant material properties leads to a linear variation of the stresses with temperature, while nonlinearities are introduced when the temperature dependence in material properties is incorporated. Once again, the two cases show larger discrepancies as larger

thermal gradients are applied. The application of an active voltage of 100 V produces a corresponding reduction in normal stress from the 0 V case. The variation of the stress through-thethickness near the center of the plate is illustrated in Figure 11 for the case in which the plate is subjected to a 150°C gradient with 100 V applied to the actuators. The results indicate that incorporating temperature dependent material properties produce generally higher stress levels in all plies of the plate.

One of the major advantages of using a layerwise approach is to improve the accuracy of out-ofplane shear stress predictions. Figure 12 shows the variation of the out-of-plane shear stress  $(\sigma_{xz})$ near the free edge of the plate  $(x/a=0.8, y/b=0.5, z/h=0.9)$  for different applied gradients with active voltages of 0 and 100 V. The stresses are nondimensionalized using a product of the corresponding PZT-5A shear modulus,  $G_{13}$ , and the total laminate thickness, h. Once again, the use of temperature dependent material properties introduces a nonlinear variation in the stress with temperature, which increasingly deviates from the constant material case as higher applied gradients are applied. Application of active voltages of 100 V produces a decrease in the shear stress from the 0 V case. The variation of the stress through the thickness near the free edge of the plate with an applied gradient of  $150^{\circ}$ C and  $100$  V is shown in Figure 13. Closer agreement between the constant material property and temperature dependent material property case is observed for the shear stress.

#### **SUMMARY**

Layerwise mechanics were extended to account for the temperature dependence of the composite and piezoelectric material properties, which were incorporated as piecewise linear functions of temperature into the thermopiezoelectric constitutive relations. This formulation provides a unified capability to comprehensively model the coupled mechanical, electrical, and thermal behavior of active and sensory piezoelectric composite laminates. A finite element formulation was developed and implemented for a bilinear plate element. Numerical studies were conducted on a simply supported carbon/epoxy plate with attached piezoceramic patches subjected to thermal gradients. Results indicate that significant nonlinearities in the variation of displacement, sensory voltages, active voltages to minimize the centerline deflections, and stresses are introduced when temperature dependent material properties are incorporated into the analysis. This indicates the importance of including temperature dependent material properties in the analysis of thermal piezoelectric composite structures. Future experimental work is required in connection with the analytical developments to characterize the thermal behavior of piezoelectric composite structures.

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### **TABLE** 1: **Material properties of piezoceramic (PZT-5A) and carbon/epoxy (AS4/3501-6) composite**

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 $\lambda$ 



Figure 1.--Temperature dependence of PZT-5A properties.  $(d_{31}$  and  $\varepsilon_{33}$  data obtained from American Piezo Ceramics, Inc., Mackeyville, PA) ( $\alpha_{11}$  and  $p_3$ data obtained from Morgan Matroc, Inc., Bedford, OH).



Figure 2.-Temperature dependence of AS4/3501-6-properties.



Figure 3.-Geometry and finite element mesh of AS4/3501-6 carbon/epoxy plate with attached PZT-5A piezoceramic patches.



Figure 4.-Comparison of deflections at the center of the plate for different applied thermal gradients.



Figure 5.- Comparison of sensory voltages at the center of the plate for different applied thermal gradients.



Figure 6.-Induced bending deflection under 100 °C gradient with actuators grounded.



**Figure 7.--Active shape control: 72 V applied to actuators.**



**Figure 8.reActive shape control: 145 V applied to actuators.**

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**Figure 9.\_Comparison of active voltages to minimize centerline deflections for different applied thermal gradients.**















Figure 13.-Through the thickness variation of  $\sigma_{\text{XZ}}$  under 150 °C gradient with 100 V applied to actuators.



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