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## Introduction

The primary purpose of this investigation has been to study various aspects of multimode pulsations in variable white dwarfs. My collaborators have been D.E. Winget and his colleagues at the University of Texas (Austin), S.D. Kawaler at Iowa State University, J.R. Buchler at the University of Florida, and M.-J. Goupil of the Observatoire de Paris. In particular, I have concerned myself with nonlinear interactions among pulsation modes in white dwarfs (and, to some extent, in other variable stars), analysis of recent observations where such interactions are important, and preliminary work on the effects of crystallization in cool white dwarfs.

## Nonlinear Interactions

The detailed paper by Buchler, Goupil, and Hansen (On the Role of Resonances in Nonradial Pulsators, Astron. & Astrophys., 321, 159) describes some of the consequences of nonlinear interactions between modes where the frequencies of the modes are nearly commensurate; that is, where the frequencies are related by rational fractions. Examples of such modes are those split nearly evenly by rotation or chance rational coincidences. We demonstrate, in simple examples, that these interactions, or resonances, place strict constraints on the possible identifications of angular indices associated with the modes (as those indices appearing in spherical harmonics). Examples of these identifications are suggested for various observed stars. Such identifications are useful in eliminating or strengthening those suggested by other means in the analysis of pulsation spectra. We also demonstrated, by means of numerical experiments, that resonantly interacting modes may show different types of behavior ranging from sitting quietly to chaotic excursions in amplitude and phase. (One example of the latter may be the variable white dwarf GD358 which over some ten years behaved nicely as a multimode pulsator but, when last seen, showed only one strong mode.) We also remarked on the phenomenon which we call "frequency locking" whereby observed modes are in exact resonance even though the linear frequencies are only approximately in resonance. A possible example of this may be the observed equal splitting into multiplets in some stars. Splitting is used to deduce rotation properties but, with frequency locking, the deductions immediately become suspect.

## Observations of PG1351+489

In the Spring of 1995 I was PI for a "Whole Earth Telescope" (WET) observation run of the DB variable white dwarf PG1351+489. The light curve has long been known to be dominated by a signal (mode) of frequency  $f_0 \approx 2.05$  mHz (488 seconds period) but harmonics at  $nf_0$  also appear as do signals at  $3/2 f_0$ ,  $5/2 f_0$ , and so on. The new WET data, of over a week's duration using some 10 telescopes around the world, also show clear evidence of a signal at  $f_0/2$ ;, i.e., a subharmonic. If  $3/2 f_0$  and  $f_0$  are both true modes coupled by a 3:2 resonance interaction, then my work with Buchler and Goupil requires that the  $f_0$  mode is a quadrupole mode ( $\ell = 2$ ). (The WET data also show some weak evidence for rotational splitting of this mode.) At present, the data do not offer any other clear hints. However, the combination of extended coverage (WET) and theory (resonant interactions) do point the way to understanding the seismology of this star.