NASA/CR- -1998- 208200

IN-27-CR
151416

Heat Conduction in Ceramic Coatings: Relationship between Microstructure and Effective Thermal Conductivity

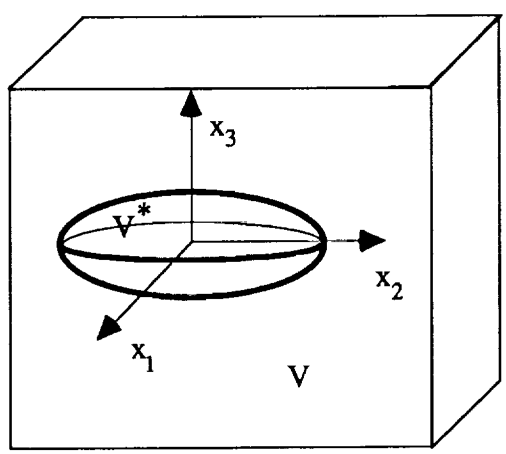
Technical report for Task 2 (Second Year)
(period of performance: February 7, 1998 - May 6, 1998)

Contract Number: NAS3-97002

Analysis of the effective thermal conductivity of ceramic coatings and its relation to the microstructure continued. Results (obtained in Task 1) for the three-dimensional problem of heat conduction in a solid containing an inclusion (or, in particular, cavity - thermal insulator) of the ellipsoidal shape, were further advanced in the following two directions:

- closed form expressions of $\boldsymbol{H}$ tensor have been derived for special cases of ellipsoidal cavity geometry: spheroid, crack-like spheroidal cavity and needle shaped spheroidal cavity;
- these results for one cavity have been incorporated to contrast heat energy potential for a solid with many spheroidal cavities (in the approximation of non-interacting defects).

This problem constitutes a basic building block for further analyses, since the ellipsoidal shape covers a variety of practically important pore geometries.



The problem is formulated as the determination of the change in thermal conductivity due to inclusion. Namely:

$$\Delta \boldsymbol{Q} = \boldsymbol{H} \cdot \boldsymbol{G} \qquad (1)$$

where ΔQ is the heat flux change per reference volume V, $\boldsymbol{G}$ is the far-field temperature gradient and second rank tensor $\boldsymbol{H}$ is a function of the inclusion shape and the inclusion conductivity.

Mathematical considerations based on the analysis of the ellipsoidal shapes in the framework of Eshelby-type theory and on utilization of Green's function for the heat conduction problem in an unbounded medium, show that $\boldsymbol{H}$ has the following form:

$$\boldsymbol{H} = \frac{V^*}{V}(k_* - k_0)(A_1\boldsymbol{ll} + A_2\boldsymbol{mm} + A_3\boldsymbol{nn}) \tag{2}$$

where $V^* = \frac{4\pi}{3}a_1a_2a_3$ is the volume of the ellipsoidal inclusion with semi-axes a_1, a_2, a_3 with unit vectors $\boldsymbol{l}, \boldsymbol{m}, \boldsymbol{n}$; k_* and k_0 are conductivities of the inclusion and of the matrix, correspondingly. Coefficients A_1, A_2, A_3 are given in terms of elliptic integrals (see Report for Task 1).

In the case when the inclusion is a *spheroid* $(a_1 = a_2 \equiv a)$, tensor $\boldsymbol{H}$ takes the form

$$\boldsymbol{H} = \frac{V^*}{V}(k_* - k_0)\left\{\left[1 + \frac{k_* - k_0}{k_0} f_0(\gamma)\right]^{-1}(\boldsymbol{I} - \boldsymbol{nn}) + \left[1 + \frac{k_* - k_0}{k_0}(1 - 2f_0(\gamma))\right]^{-1}\boldsymbol{nn}\right\} \tag{3}$$

or, in components:

$$H_{ij} = \frac{V^*}{V}(k_* - k_0)\left\{\left[1 + \frac{k_* - k_0}{k_0} f_0(\gamma)\right]^{-1}(\delta_{ij} - n_in_j) + \left[1 + \frac{k_* - k_0}{k_0}(1 - 2f_0(\gamma))\right]^{-1} n_in_j\right\} \tag{4}$$

where it is denoted:

$\gamma = a/a_3$ - aspect ratio of the spheroidal inclusion,

$\boldsymbol{n} = n_1\boldsymbol{e}_1 + n_2\boldsymbol{e}_2 + n_3\boldsymbol{e}_3$ - unit vector along the axis of symmetry of spheroid,

$\boldsymbol{I} = \boldsymbol{e}_1\boldsymbol{e}_1 + \boldsymbol{e}_2\boldsymbol{e}_2 + \boldsymbol{e}_3\boldsymbol{e}_3$ - unit second rank tensor,

$f_0(\gamma) = \dfrac{1 - g(\gamma)}{2(1 - \gamma^2)}$, $\quad g(\gamma) = \dfrac{\gamma^2}{\sqrt{\gamma^2 - 1}}\arctan\sqrt{\gamma^2 - 1}$ (for oblate shape, $\gamma > 1$),

$g(\gamma) = \dfrac{\gamma^2}{2\sqrt{1 - \gamma^2}}\ln\dfrac{1 + \sqrt{1 - \gamma^2}}{1 - \sqrt{1 - \gamma^2}}$ (for prolate shape, $\gamma < 1$).

• In the case of *spheroidal cavity* (insulator, $k_* = 0$) tensor $\boldsymbol{H}$ is as follows:

$$\boldsymbol{H} = -\frac{V^*}{V}k_0\left\{\frac{1}{1+f_0(\gamma)}(\boldsymbol{I}-\boldsymbol{nn})+\frac{1}{2f_0(\gamma)}\boldsymbol{nn}\right\} \tag{5}$$

• In the case of *thin spheroidal cavity* ($\gamma >> 1$):

$$\boldsymbol{H} = -\frac{V^*}{V}k_0\left\{\frac{1}{1+\pi/(4\gamma)}(\boldsymbol{I}-\boldsymbol{nn})+\frac{2\gamma}{\pi}\boldsymbol{nn}\right\} \tag{6}$$

• In the limit of a *circular crack*:

$$\boldsymbol{H} = -\frac{8a^3}{3V}k_0\boldsymbol{nn} \tag{7}$$

• In the case of *needle-shaped spheroidal cavity* ($\gamma << 1$):

$$\boldsymbol{H} = -\frac{V^*}{V}k_0\left\{\left[1-\frac{1}{2}\left(1+\gamma^2-\gamma^2\ln\frac{2}{\gamma}\right)\right]^{-1}(\boldsymbol{I}-\boldsymbol{nn})+\left(1+\gamma^2-\gamma^2\ln\frac{2}{\gamma}\right)\boldsymbol{nn}\right\} \tag{8}$$

In the approximation of non-interacting cavities (each cavity experiences the influence of the same far-field temperature gradient $\boldsymbol{G}$ unperturbed by the presence of other cavities), the *heat energy potential* $\Delta\Omega$ for a solid with many cavities is obtained as follows (in terms of derived tensors $\boldsymbol{H}^{(i)}$ characterizing $i-$th cavity):

$$\Delta\Omega = \frac{1}{2}\boldsymbol{G}\cdot\left[\sum_i \boldsymbol{H}^{(i)}\right]\cdot\boldsymbol{G} \tag{9}$$

For example, in the case of *spherical* cavities we have:

$$\sum_i \boldsymbol{H}^{(i)} = -\frac{3}{2}k_0\boldsymbol{I}\left[\frac{1}{V}\sum_i V^{*(i)}\right] = -\frac{3}{2}pk_0\boldsymbol{I} \tag{10}$$

$$\Delta\Omega = -\frac{3}{4}pk_0\boldsymbol{G}\cdot\boldsymbol{G} = -\frac{3}{4}pk_0\left(G_1^2+G_2^2+G_3^2\right) \tag{10a}$$

where parameter p is the conventional porosity.

In the case of *circular cracks* we have:

$$\sum_i \boldsymbol{H}^{(i)} = -\frac{8}{3} k_0 \left[\frac{1}{V} \sum_i \left(a^3 \boldsymbol{nn} \right)^{(i)} \right] = -\frac{8}{3} k_0 \boldsymbol{\alpha} \tag{11}$$

$$\Delta\Omega = -\frac{4}{3} k_0 \, \boldsymbol{G} \cdot \boldsymbol{\alpha} \cdot \boldsymbol{G} \tag{11a}$$

where $\boldsymbol{\alpha}$ is the second rank crack density tensor (well known in problems of effective *elastic* properties of cracked media).

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*

2. REPORT DATE
May 12, 1998

3. REPORT TYPE AND DATES COVERED
Task 2 Report 2/7/98 - 5/6/98

4. TITLE AND SUBTITLE
CONTRACT NAS3-97002
"Heat Conduction in Ceramic Coatings: Relationship between Microstructure and Effective Thermal Conductivity"

5. FUNDING NUMBERS
WU-

6. AUTHOR(S)
Prof. Mark Kachanov

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Tufts University, Grants and Contracts Administration
Attn: Theodore M. Liszczak
Packard Hall, Room 103
Medford MA 02155

8. PERFORMING ORGANIZATION REPORT NUMBER
E-

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
National Aeronautics and Space Administration
Lewis Research Center
Cleveland, Ohio 44135-3191

10. SPONSORING/MONITORING AGENCY REPORT NUMBER
NASA CR-

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Unclassified - Unlimited
Subject Category

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

Analysis of the effective thermal conductivity of ceramic coatings and its relation to the microstructure continued. Results (obtained in Task 1) for the three-dimensional problem of heat conduction in a solid containing an inclusion (or, in particular, cavity - thermal insulator) of the ellipsoidal shape, were further advanced in the following two directions:

- closed form expressions of H tensor have been derived for special cases of ellipsoidal cavity geometry: spheroid, crack-like spheroidal cavity and needle shaped spheroidal cavity;
- these results for one cavity have been incorporated to construct heat energy potential for a solid with many spheroidal cavities (in the approximation of non-interacting defects).

This problem constitutes a basic building block for further analyses.

14. SUBJECT TERMS

15. NUMBER OF PAGES 5

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	

NSN 7540-01-280-5500

Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std. Z39-18
298-102