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CHINKS IN SOLAR DYNAMO THEORY:

TURBULENT DIFFUSION, DYNAMO WAVES AND MAGNETIC HELICITY

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Chinks in Solar Dynamo Theory: Final Report

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ABSTRACT

In this first year of our investigation we explored the role of compressibility and stratification in the dissipation of magnetic fields. The predictions of Mean Field Electrodynamics have been questioned because of the strong feedback of small scale magnetic structure on the velocity fields. In 2-D, this nonlinear feedback results in a lengthening of the turbulent decay time. In 3-D alpha-quenching is predicted. Previous studies assumed a homogeneous fluid. This first year we present recent results from 2-D compressible MHD decay simulations in a highly stratified atmosphere that more closely resembles to solar convection zone. We have applied for NCCS T3E time to assist in the performance of our 3-D calculations.

1. Introduction

The goal of this project is to explore three key questions in solar dynamo theory: (1) In 2-D turbulent convection, how does the decay time of an imposed magnetic field vary as the field strength increases? (2) In a 3-D turbulent, compressible fluid, can a large scale, oscillatory field be maintained by a combination of small scale helical turbulence and a localized large scale shear, when feedback of the magnetic field on the momentum equation is included? (3) How much magnetic helicity is generated by a dynamo in a highly conducting fluid? How is the magnetic helicity distributed? When realistic boundary conditions are applied, what is the relative helicity in the "coronal field"?

During the first year of this project we have made substantial progress on the first problem and have started work on the second. We have modified the existing compressible MHD code to include a random forcing by thermal blobs (see §2 below). This replaces the conventional forcing in phase space used in homogeneous turbulence models. The parameters of interest in these simulations are γ, the ratio of specific heats, Δ the dimensionless temperature difference across the layer, σ the dimensionless viscosity, ζ the dimensionless resistivity, and Q the Chandrasekhar number.

2. Equations and numerical methods

Fully-compressible magnetohydrodynamics can be described by the nondimensionalized equations

$$\frac{\partial \rho}{\partial t} = -\mathbf{u} \cdot \nabla \rho - \rho \nabla \cdot \mathbf{u}, \tag{1}$$

$$\frac{\partial \mathbf{u}}{\partial t} = -\mathbf{u}\cdot\nabla\mathbf{u} - \frac{1}{\rho}\nabla p + \frac{\Delta\gamma}{\gamma-1}\hat{\mathbf{z}} + R_e^{-1}\nabla\cdot\sigma - \frac{M_a^2}{\rho}(\nabla^2 A)\nabla A, \tag{2}$$

$$\frac{\partial A}{\partial t} = -\mathbf{u}\cdot\nabla A + R_m^{-1}\nabla^2 A \tag{3}$$

$$T = T(x,z,t) \tag{4}$$

together with boundary conditions at the sides, $x = 0$, λ, and the top and bottom, $z = 0$, 1, equation of state $p = \rho T$ and prescription of the temperature T. Here the density ρ, velocity $\mathbf{u}$, temperature T and magnetic flux function A (with the magnetic field $\mathbf{B} = -\nabla A$) have been nondimensionalized using the depth of the annulus d and the temperature T_1 and density ρ_1 at its top and the mean magnetic field B. Time is measured in units of the isothermal sound speed transit time at the top $d/\sqrt{R_* T_1}$ where R_* is the gas constant. The resulting parameters γ, Δ, R_e, R_m and M_a denote, respectively, the ratio of the specific heats of the fluid, the dimensionless temperature difference across the height, the viscous and magnetic Reynolds numbers of the fluid and the dimensionless Alfvén speed at the upper boundary. λ denotes the horizontal aspect ratio of the computational domain relative to the height. The components of the viscous stress tensor σ is defined by

$$\sigma_{ij} = \rho\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3}\delta_{ij}(\nabla\cdot\mathbf{u})\right) \quad \text{with} \quad i = x,z; \quad j = x,z; \quad \text{and} \quad l = x,z \tag{5}$$

In the absence of motion and current, equations (1)-(4) permit a static solution with

$$T_s = 1 + \Delta z, \quad \rho_s = T_s^m, \quad P_s = T_s^{m+1}, \quad A_s = \sin(\pi z)/\pi \tag{6}$$

where $m = 1/(\gamma - 1)$.

We assume that the motions are generated by randomly placed temperature sources which share a common amplitude and scale. These sources are defined using cubic B-splines where the temperature for a single source is

$$\delta T(x,y,t;x_o,z_o,t_o,\chi,\tau) = \mathbf{B_s}\left(\frac{x-x_o}{\chi}\right)\mathbf{B_s}\left(\frac{z-z_o}{\chi}\right)\mathbf{B_s}\left(\frac{t-t_o}{\tau}\right) \tag{7}$$

Here (x_o, z_o, t_o) is the position and time of the temperature peak and χ and τ are the peak's half width and lifetime. The continuous function $\mathbf{B_s}(x)$ and its first and second derivatives vanish for $|x| > 1$ are continuous everywhere. It has the functional form of

$$\mathbf{B_s}(x) = \begin{cases} 2(x+1)^3, & \text{for } -1 < x < -0.5; \\ -6x^2(x+1)+1, & \text{for } -0.5 < x < 0; \\ 6x^2(x-1)+1, & \text{for } 0 < x < 0.5; \\ 2(1-x)^3, & \text{for } 0.5 < x < 1; \\ 0, & \text{for } |x| > 1. \end{cases} \tag{8}$$

The full temperature field has the form

$$T(x,z,t) = T_s(z)\Big(1 + \alpha\Sigma_1^N \delta T(x,y,t;x_i,z_i,t_i,\chi,\tau)\Big) \tag{9}$$

where N, α, χ and τ are specified for each simulation. The location of each fluctuation (x_i, z_i, t_i) is chosen randomly in such a way that N sources are present at any time. Each source exists at a fixed location for its lifetime $t_i - \tau/2 < t < t_i + \tau/2$. At the end of this interval, it is replaced by a new source at a new random position (x_i, z_i) with a new time center randomly chosen in the range $t + \tau/2 < t_i < t + 3\tau/2$

We assert that the viscous stress and vertical magnetic field vanish on the horizontal boundaries thus requiring the following for the vertical and radial velocity components and magnetic flux function:

$$\frac{\partial u}{\partial z} = 0, \quad w = 0, \quad \frac{\partial A}{\partial z} = 0, \quad \text{at} \quad z = 0, 1 \tag{10}$$

The vertical boundaries are taken to be periodic. In addition to these boundary conditions, we must satisfy the natural boundary condition on the pressure which arises through the combination of equation (2) and the boundary conditions (10). This condition states the the total normal force on the bounding surfaces is zero, and thus that the pressure gradient there must balance all other normal forces.

The system of equations (1) through (4) are solved using a sixth-order compact difference scheme with fourth-order time accuracy (see Hurlburt 1997 *J. Sci. Comp.* for details). For a 192×192 computational grid, each solution requires approximately ten seconds per step on an 180Mhz SGI R5000 Indy, with about 270 steps required to advance the solution one time unit.

3. Solutions

We have begun by establishing the decay rate for essentially kinematic fields. Because we have introduced a non-standard forcing, it is important to understand the reference fields before proceeding with the parameter studies. Figures 1 & 2 show two of the reference solutions: figure 1 is a weak field case (plasma $\beta = 1000$) and figure 2 is a stronger field case (plasma $\beta = 100$). For both solutions $R_e = 12,500$, $R_m = 50,000$, $\Delta T = 0.7$, $\gamma = 1.4$, and $\theta = 0.5$. Time traces of the magnetic energy are shown in figures 3. The straight solid line shows the Ohmic decay rate for the actual diffusivity.

Figure 3 shows an example of the decay of the magnetic energy in a stratified layer for four values of M_a^2 for $R_e = 11$, $R_m = 22$, $\gamma = 1.6666$, $\Delta = 5$, $\alpha = 0.7$ and $\chi = \tau = 4$. Kinematic solutions (dot-dash) with $M_a = 1.44 \times 10^{-3}$ are amplified by a factor of 10 and then decay rapidly. As the field strength increases over the range $M_a = 0.10$, 0.18 and 0.32 (dash, dot and solid respectively), the field strength is amplified less and decay at the slower ohmic rate for longer intervals. The straight solid curve represents the ohmic diffusion rate.

We presented preliminary solutions at the the Solar Physics Division Meeting in Montana. A paper on the 2-D results is in progress.