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TECHNICAL NOTE

D-319

AN APPROXIMATE ANALYTICAL METHOD FOR STUDYING ATMOSPHERE ENTRY OF VEHICLES WITH MODULATED AERODYNAMIC FORCES

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AN APPROXIMATE ANALYTICAL METHOD FOR STUDYING ATMOSPHERE ENTRY OF VEHICLES WITH MODULATED AERODYNAMIC FORCES

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SUMMARY

The dimensionless, transformed, nonlinear differential equation developed in NASA TR R-11 for describing the approximate motion and heating during entry into planetary atmospheres for constant aerodynamic coefficients and vehicle shape has been modified to include entries during which the aerodynamic coefficients and the vehicle shape are varied. The generality of the application of the original equation to vehicles of arbitrary weight, size, and shape and to arbitrary atmospheres is retained. A closed-form solution for the motion, heating, and the variation of drag loading parameter m/C_DA has been obtained for the case of constant, maximum resultant deceleration during nonlifting entries. This solution requires certain simplifying assumptions which do not compromise the accuracy of the results.

The closed-form solution has been used to determine the variation of m/C_DA required to reduce peak decelerations and to broaden the corridor for nonlifting entry into the earth's atmosphere at escape velocity. The attendant heating penalty is also studied.

INTRODUCTION

A number of studies have been made of the motion and heating of vehicles entering planetary atmospheres. In one such study (ref. 1) an approximate analytical method for studying entry into any planetary atmosphere was developed and applied to vehicles of arbitrary constant weight, size, and shape with constant aerodynamic coefficients. In reference 2 the method of reference 1 was applied to the study of corridor depth and guidance requirements. Also presented in reference 2 is a limited discussion of the effect on corridor depth of varying (modulating) the aerodynamic coefficients and the vehicle shape during entry. Other investigators have made studies of the trajectories for modulated entries. For example, entries at circular and supercircular velocities with varying lift-drag ratios, but with constant drag, have been studied in reference 3;

entries at circular and supercircular velocities with a particular variation of drag with lift have been studied in references 4, 5, and 6; and nonlifting entries at circular velocity (and one case of entry at supercircular velocity) with varying drag have been studied in reference 7. The analyses of references 3 through 7 all employ time as the independent variable and separate sets of calculations are required for each specific vehicle and planetary atmosphere.

The purpose of the present investigation is to extend the method of reference 1 to include vehicles with variable aerodynamic coefficients and variable vehicle shapes. The general applicability of the method to vehicles of arbitrary weight, size, and shape and to arbitrary atmospheres is retained. Calculations are made to illustrate the effect of varying vehicle drag coefficient and area on the corridor depth and heating characteristics of nonlifting vehicles entering the earth's atmosphere at escape velocity. The type of variation considered is that which reduces the maximum resultant deceleration to some specified constant value during the modulation period. The variation of vehicle drag coefficient and area required to maintain a constant resultant deceleration is determined.

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NOTATION

a	resultant deceleration, ft sec^{-2}
A	reference area for drag, sq ft
B	constant defined by the deceleration at the beginning of modulation, $G_1/\sqrt{\beta r}$
C_D	drag coefficient, $2D/\rho V^2 A$
D	drag force, lb
F_p	conic perigee parameter (eq. (22)), dimensionless
g	gravitational acceleration, ft sec^{-2}
g_c	gravitational conversion constant, 32.2 ft sec^{-2}
G	deceleration in g's, a/g
L	lift force, lb
m	mass of vehicle, slugs
q_s	convective heating rate per unit area at the stagnation point, Btu ft^{-2} sec^{-1}

$\bar{q}$ dimensionless function proportional to convective heating rate (eq. (26))

Q_s total convective heat absorbed per unit area at the stagnation point, Btu ft^{-2}

$\bar{Q}$ dimensionless function proportional to total heat absorbed, (eq. (28))

r distance from planet center, ft

R radius of curvature of vehicle surface, ft

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$\bar{r}$ dimensionless radius used in the definition of F_p (See eq. (23).)

t time, sec

u tangential velocity component normal to a radius vector, ft sec^{-1}

$\bar{u}$ dimensionless velocity ratio, $u/\sqrt{gr}$

v radial velocity component, ft sec^{-1}

V resultant velocity, $\sqrt{u^2 + v^2} = u/\cos\gamma$

$\bar{V}$ dimensionless velocity ratio, $V/\sqrt{gr}$

W weight of vehicle at earth's surface, mg_c, lb

y altitude, ft

Z dimensionless function of $\bar{u}$ (eq. (3))

β atmosphere density decay parameter, ft^{-1}

γ flight-path angle relative to the local horizontal, negative for descent

Δ vehicle parameter, m/C_DA

Δy_p corridor depth between conic perigee altitudes, statute miles

ρ atmosphere density, slugs ft^{-3}

$\bar{\rho}_o$ mean value for exponential approximation to atmosphere density-altitude relation, slugs ft^{-3}

Subscripts

i	initial value
max	maximum value
mod	modulated-entry value
o	surface of a planet
ov	overshoot boundary
p	conic perigee point
un	undershoot boundary
unmod	unmodulated-entry value
1	point in the trajectory where modulation begins
2	point in the trajectory where modulation ends
I,II,III	phase of a trajectory

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Superscripts

'	differentiation with respect to $\bar{u}$

ANALYSIS

In reference 1 the two-component equations of motion for entry into planetary atmospheres

$$\left.\begin{aligned} -\frac{d^2y}{dt^2} = -\frac{dv}{dt} &= g - \frac{u^2}{r} - \frac{L}{m}\cos\gamma + \frac{D}{m}\sin\gamma \\ \frac{du}{dt} + \frac{uv}{r} &= -\frac{D}{m}\cos\gamma\left(1 + \frac{L}{D}\tan\gamma\right) \end{aligned}\right\} \tag{1}$$

are transformed into a single, ordinary, nonlinear, differential equation with the aid of certain simplifying assumptions and by the introduction of the dimensionless independent variable

$$\bar{u} = u/\sqrt{gr} \tag{2}$$

and the dimensionless dependent variable Z, defined by

$$Z = \frac{\bar{\rho}_o \sqrt{\frac{r}{\beta}}}{2\left(\frac{m}{C_D A}\right)} \bar{u} e^{-\beta y} \tag{3}$$

The resulting differential equation for the Z function is

$$\bar{u}Z'' - \left(Z' - \frac{Z}{\bar{u}}\right) - \frac{1 - \bar{u}^2}{\bar{u}Z} \cos^4\gamma + \sqrt{\beta r}\, \frac{L}{D} \cos^3\gamma = 0 \tag{4}$$

A447 where $\cos \gamma = \sqrt{1 - \sin^2\gamma}$ can be expressed in terms of Z and Z' through the equation for the flight-path angle

$$\sqrt{\beta r} \sin \gamma = Z' - \frac{Z}{\bar{u}} \tag{5}$$

In the analysis in reference 1 the terms $m/C_D A$ and L/D are constant throughout the entry trajectory.

When the terms $m/C_D A$ and L/D are allowed to vary during entry (are modulated), the differential equation for the Z function is, as derived in appendix A, given by

$$\bar{u}Z'' - \left(Z' - \frac{Z}{\bar{u}}\right) + \left\{\bar{u}Z\left[\frac{\Delta''}{\Delta} - \left(\frac{\Delta'}{\Delta}\right)^2\right] + \bar{u}Z'\left(\frac{\Delta'}{\Delta}\right)\right\}$$

$$- \frac{1 - \bar{u}^2}{\bar{u}Z} \cos^4\gamma + \sqrt{\beta r}\, \frac{L}{D} \cos^3\gamma = 0 \tag{6}$$

where

$$\Delta \equiv m/C_D A$$

and

$$\Delta' = \frac{d\Delta}{d\bar{u}} \equiv \frac{d(m/C_D A)}{d\bar{u}} \tag{7}$$

The attendant expression for the flight-path angle can be given by

$$\sqrt{\beta r} \sin \gamma = Z' - \frac{Z}{\bar{u}} + Z\left(\frac{\Delta'}{\Delta}\right) \tag{8}$$

Thus, the differential equation for the Z function during modulation contains the same terms as the equation developed in reference 1 for unmodulated entries plus the three additional terms in the curly brackets (cf. eqs. (4) and (6)). Equation (6) is applicable to both lifting and nonlifting entries. However, as a result of the assumptions made in the derivation, equation (6) is limited to shallow lifting entries for which $|L/D \tan \gamma| \ll 1$ (see appendix A) and is applicable to nonlifting entries for which the angles of descent are large as well as small.

Equation (6) can be reduced to a differential equation in which $\bar{u}$ and Z are, respectively, the independent and dependent variables if a drag polar and the type or purpose of the modulation are specified. In this manner it is possible to express L/D, Δ, Δ', and Δ'' in terms of Z, Z', Z'', and $\bar{u}$.[1] Solution of the resulting equation for $Z(\bar{u})$ permits A
the calculation of many quantities which describe the modulated portion 4
of the trajectory; for example, resultant deceleration, flight-path angle, 4
range, time, density, altitude, dynamic pressure, Reynolds number, con- 7
vective heating rate, and total heat absorbed at the stagnation point. Detailed expressions for these quantities can be found in reference 1.

Nonlifting Entries Modulated for Specified Maximum Deceleration

As a specific application of the present analysis, modulated nonlifting entries will be studied. In this study m/C_DA will be varied in such a manner that the maximum resultant deceleration does not exceed some specified value G_1. The modulated trajectory consists of three distinct phases which can be described with the aid of sketch (a). The deceleration time histories of an unmodulated and a modulated entry are shown for a given entry angle (γ_i) and entry velocity ($\bar{u}_i$). During phase I the vehicle enters the atmosphere at a given entry angle and velocity ($\bar{u}_i$) and maintains a constant m/C_DA; the deceleration increases monotonically until the specified value of the deceleration G_1 is attained at velocity $\bar{u}_1$. At this velocity, phase II begins with the modulation of m/C_DA in such a manner that the deceleration is held constant until the velocity is reduced to the value $\bar{u}_2$. This velocity is determined by specifying that termination of the modulation will allow the resultant deceleration to decrease monotonically as the vehicle completes its entry. Phase III is this final phase of the entry.

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[1]For example, in an analysis to reduce the maximum resultant deceleration G_{max}, the equation for G given in reference 1 is used to relate G, L/D, Z, and $\bar{u}$; that is, for shallow entries $G = \sqrt{\beta r}\,\bar{u}Z\sqrt{1 + (L/D)^2}$.

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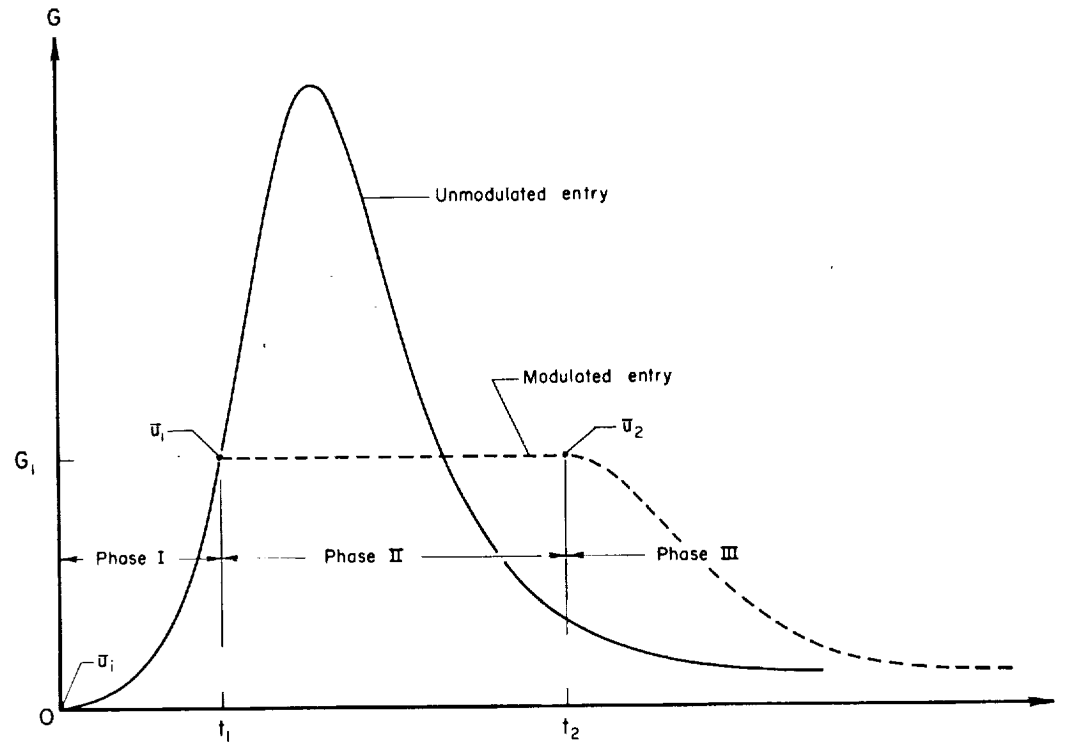

Sketch (a)

The present problem is reduced to the determination of the solution during the modulated portion of the trajectory (phase II) since the solutions for the unmodulated portions of the trajectory (phases I and III) are indicated in reference 1. There are three basic steps necessary to the solution for phase II. First, a relationship between Z, $\bar{u}$, γ, G, and m/C_DA must be obtained in order to determine the required variation in m/C_DA. Secondly, the termination of the modulation period must be determined (i.e., the velocity $\bar{u}_2$ must be determined); and finally, the solution for phase II must be matched with the solutions for phases I and III. The first step is accomplished by means of the equation from reference 1 for the resultant deceleration:

$$G = \frac{\sqrt{\beta r}\ \bar{u}Z}{\cos^2\gamma}$$

During the modulation period G is constant and equal to G_1; hence

$$G_1 = \frac{\sqrt{\beta r}\ \bar{u}Z}{\cos^2\gamma} \tag{9}$$

By combining equation (6) for $L/D = 0$ with equations (8) and (9) one can obtain a differential equation for the Z function in terms of βr, $\bar{u}$, and G_1, but the mathematical treatment is tedious and the resulting equation is difficult to program for numerical solutions. This situation can be circumvented, however, if only shallow entries are considered, so that during phase II the flight-path angle is sufficiently small that $\cos^2\gamma \approx 1$.[2] Thus, from equation (9), the resultant deceleration during the modulation period for shallow entries is given by

$$G_1 = \sqrt{\beta r}\,\bar{u}Z \tag{10}$$

From equation (10) the Z function is defined during phase II as

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$$Z = B\,\frac{1}{\bar{u}} \tag{11}$$

where

$$B = \frac{G_1}{\sqrt{\beta r}} \tag{12}$$

By substitution of equation (11) and its derivatives into equation (6) ($L/D = 0$), the differential equation which describes the variation in m/C_DA required to maintain a constant resultant deceleration is obtained as

$$\frac{\Delta''}{\Delta} - \left(\frac{\Delta'}{\Delta}\right)^2 - \frac{1}{\bar{u}}\left(\frac{\Delta'}{\Delta}\right) + \frac{4}{\bar{u}^2} - \frac{1 - \bar{u}^2}{B^2}\cos^4\gamma = 0 \tag{13}$$

and the related expression for the flight-path angle is

$$\sin\gamma = \frac{B}{\sqrt{\beta r}}\left[\frac{1}{\bar{u}}\left(\frac{\Delta'}{\Delta}\right) - \frac{2}{\bar{u}^2}\right] \tag{14}$$

As a result of the multiplying factor $\cos^4\gamma$ of the last term in equation (13), solutions for equation (13) must be obtained by numerical methods.

If during phase II the flight-path angle is further assumed to be small enough that $\cos^4\gamma \approx 1$, a closed-form solution for Δ can be

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[2]Although the inaccuracies due to assuming $\cos^2\gamma \approx 1$ were not investigated, they are believed to be negligible for the entry angles considered in this report.

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derived. The resulting expression for the variation with velocity of the m/C_DA required to maintain constant resultant deceleration is

$$\frac{\Delta}{\Delta_1} = \exp\left\{2\left[1 + C\left(\frac{\bar{u}}{\bar{u}_1}\right)^2\right] \ln\left(\frac{\bar{u}}{\bar{u}_1}\right) + E\left[1 - \left(\frac{\bar{u}}{\bar{u}_1}\right)^2\right] + \frac{1}{2} C\bar{u}_1^2\left[1 - \left(\frac{\bar{u}}{\bar{u}_1}\right)^4\right]\right\} \tag{15}$$

where

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$$\left.\begin{aligned} C &= \frac{\bar{u}_1^2}{4B^2} \\ E &= C(1 - 2G_1 \sin\gamma_1 - \bar{u}_1^2) \end{aligned}\right\} \tag{16}$$

The details of the derivation of equation (15) are presented in appendix B. The expression for the flight-path angle obtained with the closed-form solution is

$$\sin\gamma = \sin\gamma_1 + \frac{1}{G_1}\left\{\ln\left(\frac{\bar{u}}{\bar{u}_1}\right) + \frac{\bar{u}_1^2}{2}\left[1 - \left(\frac{\bar{u}}{\bar{u}_1}\right)^2\right]\right\} \tag{17}$$

The accuracy of the closed-form solution relative to a numerical solution for equation (13) will be discussed later.

The velocity $\bar{u}_2$ at which the modulation is terminated is determined in the following manner. For a given set of entry conditions and a specified deceleration limit, the constants B, C, and E are fixed, and it is a simple matter to determine from equation (15) that m/C_DA increases to a maximum value and then decreases. Continuation of the modulation after m/C_DA has reached the maximum would sustain a constant deceleration above that which would result if the modulation were terminated. Consequently, the value of the velocity $\bar{u}_2$ at which phase II is terminated is determined as that value for which Δ/Δ_1 is a maximum. As noted in appendix B the value of $\bar{u}_2$ is obtained as that value of the velocity which satisfies the relation

$$\frac{1}{B^2} \ln\left(\frac{\bar{u}}{\bar{u}_1}\right) - 2C\left(\frac{\bar{u}}{\bar{u}_1}\right)^2 + \frac{2}{\bar{u}_1^2}\frac{1}{\left(\frac{\bar{u}}{\bar{u}_1}\right)^2} + \left(2C + \frac{\sqrt{\beta r}}{B}\sin\gamma_1\right) = 0 \tag{18}$$

It now remains to match the solution for phase II with the solutions for the unmodulated portions of the trajectory, phases I and III. This is done by matching the altitude and the flight-path angle at the velocities for the beginning and end of modulation ($\bar{u}_1$ and $\bar{u}_2$, respectively). The quantities Z_1, or Δ_1, and γ_1 are known at velocity $\bar{u}_1$ from the first phase of the trajectory (obtained as indicated in ref. 1); hence, with the aid of equation (14) the solutions for phases I and II are matched by satisfying the conditions

$$\left.\begin{aligned} \Delta_{II_i} &= \Delta_1 \\ \left(\frac{\Delta'}{\Delta}\right)_{II_i} &= \frac{\sqrt{\beta r}}{B}\,\bar{u}_1 \sin\gamma_1 + \frac{2}{\bar{u}_1} \end{aligned}\right\} \tag{19}$$

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where the subscript II_i denotes initial values for phase II. The Z function and Δ are known at the end of phase II (eqs. (11) and (15)) and $\Delta'/\Delta = 0$ at velocity $\bar{u}_2$ (Δ/Δ_1 is a maximum at $\bar{u} = \bar{u}_2$); hence, with the aid of equations (5) and (14) the solutions for phases II and III are matched by satisfying the conditions

$$\left.\begin{aligned} Z_{III_i} &= B\,\frac{1}{\bar{u}_2} \\ Z'_{III_i} &= -B\,\frac{1}{\bar{u}_2^{\,2}} \end{aligned}\right\} \tag{20}$$

which, together with the velocity $\bar{u}_2$, constitute the initial conditions from which the trajectory characteristics of phase III are calculated as indicated in reference 1. Within the basic approximations noted earlier, the closed-form solution is applicable to any vehicle making modulated, nonlifting, shallow entries into any exponential atmosphere from any approach velocity.

Accuracy of the Closed-Form Solution

The inaccuracies due to assuming $\cos^4\gamma \approx 1$ in obtaining the closed-form solution for phase II may be judged from a comparison of some trajectory parameters for which the calculations for phase II were obtained from the closed-form solution and from the numerical solution of equation (13). The time history of the velocity $\bar{u}$, the flight-path angle γ, and W/C_DA[3] are shown in figure 1 for a typical modulated,

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[3]Since the remainder of the report is concerned only with entries into the earth's atmosphere, the parameter W/C_DA is subsequently used.

10g-limited, nonlifting entry into the earth's atmosphere at escape velocity ($\bar{V}_i = 1.4$; $\gamma_i = -8^\circ$). Excellent agreement is exhibited between the solid curves representing the trajectory characteristics as computed from use of the numerical solution of equation (13) and the dashed curves representing the corresponding trajectory characteristics as computed from use of the closed-form solution (eq. (15)).

RESULTS AND DISCUSSION

The closed-form solution has been employed to study the effect of varying W/C_DA on nonlifting shallow entries into the earth's atmosphere
A at escape velocity. In what follows, the effect of modulation on the
4 time history of the resultant deceleration, flight-path angle, W/C_DA, and
4 velocity is discussed first. The effect of modulation on the corridor
7 depth and heating during entry is then discussed.

Trajectory Parameters

Typical time histories of the resultant deceleration, flight-path angle, W/C_DA, and velocity for nonlifting vehicles ($L/D = 0$) are presented in figure 2 for unmodulated and modulated entries into the earth's atmosphere at escape velocity ($\bar{V}_i = 1.4$). The particular curves shown are for an entry angle $\gamma_i = -6^\circ$. For the modulated entry, W/C_DA was varied to maintain a peak deceleration of 10g during phase II (see fig. 2(a)). It will be noted in figure 2(b) that the flight-path curvature during phase II of the modulated entry is considerably less than that for the unmodulated entry during the corresponding time interval. The reduced deceleration of the modulated entry results primarily from the reduced flight-path curvature. Since W/C_DA is presented as a fraction of the value at the beginning of modulation, the variation in W/C_DA shown in figure 2(c) can be applied to vehicles of any weight, size, and shape. The relative areas under the velocity time-history curves (fig. 2(d)) are indicative of the greater range for the modulated entry.

Corridor Depth

In reference 2 the corridor depth in statute miles Δy_p for successful entry into an exponential earth atmosphere was shown to be

$$\Delta y_p = 10 \log_{10} \frac{\left(F_p \frac{W}{C_DA}\right)_{un}}{\left(F_p \frac{W}{C_DA}\right)_{ov}} \qquad (21)$$

where the subscripts un and ov, respectively, refer to the undershoot and overshoot boundaries of the corridor and F_p is the conic perigee parameter given by

$$F_p = \frac{Z_i}{\bar{u}_i} \frac{e^{\beta r_p(\bar{r} - 1)}}{\sqrt{\bar{r}}} \tag{22}$$

for

$$\bar{r} = \frac{r_i}{r_p} = \frac{1 + \sqrt{1 - \bar{V}_i^2(2 - \bar{V}_i^2)\cos^2\gamma_i}}{\bar{V}_i^2\cos^2\gamma_i} \tag{23}$$

For the case of a nonlifting vehicle for which the initial W/C_DA is the same for both boundaries

$$\Delta y_p = 10 \log_{10} \frac{F_{p_{un}}}{F_{p_{ov}}} \tag{24}$$

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and for a given entry altitude and vehicle (Z_i), and a given entry velocity $(\bar{u}_i = \bar{V}_i \cos \gamma_i)$ the corridor depth depends only upon the allowable entry angle (see eqs. (22), (23), and (24)). Thus the increased corridor depths subsequently presented result from lowering the undershoot boundary by using modulation which permits deceleration-limited entries at steeper angles than is possible without modulation.

Corridor depths and the corresponding entry angles attainable by modulating W/C_DA during shallow nonlifting entries into the earth's atmosphere at escape velocity are shown in figure 3 as a function of the ratio of W/C_DA at the end of modulation to W/C_DA at the beginning of modulation. Results are shown for deceleration limits of 5, 10, and 15g. It can be seen that for a given deceleration limit, the larger the available change in W/C_DA, the broader is the corridor into which entry may be made. For example, the 7-mile depth of the 10g-limited corridor for an unmodulated entry is increased to 30 miles for a modulated entry during which W/C_DA at the end and beginning of modulation changes by a factor of 21. It is interesting to note from reference 2 that in order to use lift to attain a 10g-limited, 30-mile corridor without modulation, the entering vehicle must have an L/D of ±0.3. It is also shown in reference 2 that without modulation the minimum peak deceleration for nonlifting entry into the earth's atmosphere at escape velocity is 6.6g, which occurs for $\gamma_i = -4^\circ$. Consequently, no calculations for a 5g corridor have been made for entry angles less than -4°.

Heating

For the entries considered herein, peak heating occurs during the high-altitude portion of the trajectory where the density and attendant Reynolds numbers are low; consequently, the following discussion of heating during entry will cover only those results obtained when a laminar boundary layer is considered.

Convective heating rate at the stagnation point.- The convective heating rate per unit area at the stagnation point during entry into the earth's atmosphere, as given in reference 1, is

$$q_s = \frac{590}{\sqrt{g_c}}\sqrt{\frac{W}{C_DAR}}\,\frac{\bar{q}}{\cos^3\gamma} \tag{25}$$

where

$$\bar{q} = \left\{\begin{array}{l} \bar{u}^{5/2}\,Z^{1/2} \text{ for phases I and III} \\ \\ B^{1/2}\,\bar{u}^2 \text{ for phase II} \end{array}\right\} \tag{26}$$

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Equation (25) has been used to calculate the stagnation-point heat-transfer rate for modulated and unmodulated entries which have the same maximum deceleration. Results for the maximum heating rate are shown in figure 4 for deceleration limits of 10 and 15g. In making these calculations it was assumed that the radius of curvature R of the vehicle surface at the stagnation point was identical and constant for both the modulated and unmodulated entries. It was also assumed that the initial W/C_DA was the same for the modulated and the unmodulated entries. It should be noted that the data presented in figure 4 can be used to calculate the ratio of stagnation-point heating rates for the same modulated and unmodulated entries for which $R_{mod} \neq R_{unmod}$, provided R remains constant during entry. This is done by multiplying the ratio of heating rates shown in figure 4 by the square root of the ratio of the radius of curvature of the vehicle making the unmodulated entry to that of the vehicle making the modulated entry. If R varies during entry, the stagnation-point heating rate will depend upon the variation of R with time or velocity (see eqs. (25) and (26)).

The heating-rate penalty incurred by the use of modulation (to increase corridor depth) can be seen in figure 5 which shows the ratio of maximum heating rates for modulated and unmodulated entries as a function of corridor depth. For example, in the case cited previously, in which the depth of a 10g-limited corridor is increased from 7 to 30 miles, the maximum heating rate for the modulated entry which provides the 30-mile corridor is 90 percent greater than that for the unmodulated entry which provides the 7-mile corridor.

Total convective heat absorbed at the stagnation point.- The total convective heat absorbed per unit area at the stagnation point during a modulated entry into the earth's atmosphere can be written (see ref. 1)

$$Q_s = \frac{15,900}{\sqrt{g_c}}\int\sqrt{\frac{W}{C_DAR}}\left(\frac{d\bar{Q}}{d\bar{u}}\right)d\bar{u} \tag{27}$$

where

$$\frac{d\bar{Q}}{d\bar{u}} = \left\{\begin{array}{l} \dfrac{\bar{u}^{3/2}}{Z^{1/2}\cos^2\gamma} \text{ for phases I and III} \\ \\ \dfrac{\bar{u}^2}{B^{1/2}} \text{ for phase II} \end{array}\right\} \tag{28}$$

Equation (27) has been used to calculate the total heat absorbed per unit area at the vehicle stagnation point for modulated and unmodulated entries which have the same deceleration limit. The results are shown in figure 6 for deceleration limits of 10 and 15g. In the calculations the same assumptions were made with regard to vehicle shape and W/C_DA that were made in calculating the heating-rate data of figure 4 (i.e., $R_{mod} = R_{unmod}$ and $(W/C_DA)_1 = (W/C_DA)_{unmod}$). For vehicles which satisfy these assumptions, the penalty in total heat absorbed at the stagnation point can be seen in figure 6. As in the case of the heat-transfer rate, the data of figure 6 may be similarly used to calculate the total heat absorbed at the stagnation point for different vehicles when $R_{mod} = \text{constant} \neq R_{unmod}$; otherwise $R(\bar{u})$ must be available for use in equation (27).

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CONCLUDING REMARKS

The differential equation developed for entries during which the aerodynamic coefficients and vehicle shape vary applies to vehicles of arbitrary weight, size, and shape and to arbitrary atmospheres. The closed-form solution for the motion, heating, and variation of W/C_DA was obtained by making certain approximations which do not compromise the accuracy of the results. It was used to calculate trajectory parameters of nonlifting entries into the earth's atmosphere at escape velocity. As an example of the results obtained, the 7-mile 10g-limited corridor for an unmodulated entry can be increased to 30 miles for a modulated entry with a W/C_DA change of 21. For this same increase in corridor depth, however, the maximum, laminar, convective heating rate and the total heat absorbed at the stagnation point are increased by factors of 1.9 and 1.8, respectively. These heating results were calculated for the case in which the radii of curvature were constant and equal during the modulated and unmodulated entries, and the initial values of W/C_DA were identical.

Ames Research Center
National Aeronautics and Space Administration
Moffett Field, Calif, Aug. 12, 1960

APPENDIX A

DIFFERENTIAL EQUATION OF THE Z FUNCTION FOR MODULATED ENTRIES

The development of the differential equation of the Z function for modulated, two-dimensional (no lateral forces) entries into spherically symmetric, exponential, planetary atmospheres closely parallels that presented in reference 1 for unmodulated entries. Deviations from the analysis of reference 1, it will be seen, occur only when velocity derivatives are obtained, since, in the present analysis, the aerodynamic forces and vehicle shape (and weight) are considered to depend on velocity.

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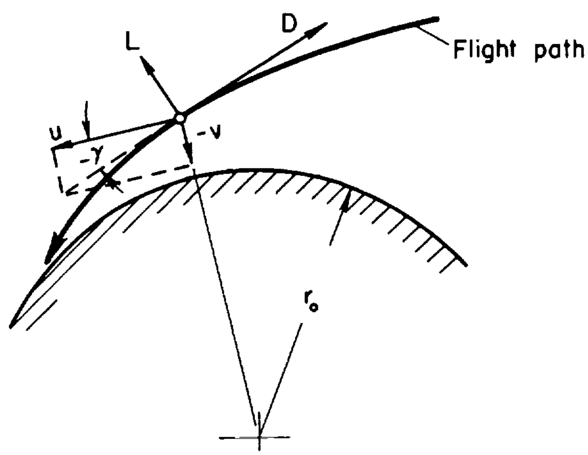
By use of a polar coordinate system with velocity components, aerodynamic forces, and flight-path angle defined as in sketch (b), the



Sketch (b)

pair of motion equations in the radial and tangential directions, respectively, can be shown to be

$$-\frac{d^2y}{dt^2} = -\frac{dv}{dt} = g - \frac{u^2}{r} - \frac{L}{m}\cos\gamma + \frac{D}{m}\sin\gamma \tag{A1}$$

$$\frac{du}{dt} + \frac{uv}{r} = -\frac{D}{m}\cos\gamma\left(1 + \frac{L}{D}\tan\gamma\right) \tag{A2}$$

By utilizing the same approximations used in reference 1 so that uv/r can be neglected compared to du/dt (i.e., $|dr/r| \ll |du/u|$), by introducing the drag coefficient $D = (1/2)C_D\rho V^2 A$ and the exponential atmosphere $(\rho = \bar{\rho}_o e^{-\beta y})$, by noting that $V = u/\cos\gamma$, and by introducing the independent variable

$$\bar{u} \equiv \frac{u}{\sqrt{gr}} \tag{A3}$$

so that

$$\frac{d\bar{u}}{dt} = \frac{1}{\sqrt{gr}}\frac{du}{dt} \tag{A4}$$

one can write equations (A1) and (A2)

$$-\frac{1}{g}\frac{d^2y}{dt^2} = -\frac{1}{g}\frac{dv}{dt} = 1 - \bar{u}^2 + \frac{r\bar{\rho}_o e^{-\beta y}\bar{u}^2}{2\left(\frac{m}{C_D A}\right)\cos^2\gamma}\left(\sin\gamma - \frac{L}{D}\cos\gamma\right) \tag{A5}$$

$$\frac{d\bar{u}}{dt} = -\sqrt{\beta g}\,\frac{\sqrt{\frac{r}{\beta}}\;\bar{\rho}_o e^{-\beta y}\bar{u}^2}{2\left(\frac{m}{C_D A}\right)\cos\gamma}\left(1 + \frac{L}{D}\tan\gamma\right) \tag{A6}$$

In order to reduce the pair of motion equations (A5) and (A6) to a single equation, the dependent variable Z defined by

$$Z = \frac{1}{2}\bar{\rho}_o\sqrt{\frac{r}{\beta}}\;\frac{e^{-\beta y}\bar{u}}{\frac{m}{C_D A}} \tag{A7}$$

is introduced. With the aid of equation (A7) equations (A5) and (A6) are more conveniently written

$$-\frac{1}{g}\frac{dv}{dt} = 1 - \bar{u}^2 + \frac{\sqrt{\beta r}\;\bar{u}Z}{\cos^2\gamma}\left(\sin\gamma - \frac{L}{D}\cos\gamma\right) \tag{A8}$$

$$\frac{d\bar{u}}{dt} = -\sqrt{\beta g}\,\frac{\bar{u}Z}{\cos\gamma}\left(1 + \frac{L}{D}\tan\gamma\right) \tag{A9}$$

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A second equation for dv/dt is developed in terms of u, Z, γ, and m/C_DA by the use of equations (A7), (A9), and the following expression for the radial component of velocity (see sketch (b))

$$\frac{dy}{dt} = v = \sqrt{gr}\;\bar{u}\tan\gamma \tag{A10}$$

The resulting equation is then equated to equation (A8) to obtain the desired single equation in terms of the foregoing variables.

Before the second equation for dv/dt is developed it is convenient to introduce the notation

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$$\Delta \equiv \frac{m}{C_DA} \tag{A11}$$

so that

$$\Delta' \equiv \frac{d\Delta}{d\bar{u}} = \frac{d\left(\frac{m}{C_DA}\right)}{d\bar{u}} \tag{A12}$$

Thus, equation (A7) is rewritten

$$Z = \frac{1}{2}\,\bar{\rho}_o\sqrt{\frac{r}{\beta}}\,\frac{e^{-\beta y}\bar{u}}{\Delta} \tag{A13}$$

Proceeding now with the development, differentiation of equation (A10) yields

$$\frac{1}{g}\frac{dv}{dt} = \sqrt{\frac{r}{g}}\,\frac{d(\bar{u}\tan\gamma)}{dt} = \sqrt{\frac{r}{g}}\,\frac{d}{d\bar{u}}\left(\bar{u}\,\frac{\sin\gamma}{\cos\gamma}\right)\frac{d\bar{u}}{dt} \tag{A14}$$

Upon substituting equation (A9) in equation (A14) and noting that $\frac{d}{d\bar{u}} = \frac{d}{d\gamma}\frac{d\gamma}{d\bar{u}}$, we obtain

$$-\frac{1}{g}\frac{dv}{dt} = \frac{\bar{u}Z}{\cos^2\gamma}\left(\sqrt{\beta r}\,\sin\gamma + \frac{\sqrt{\beta r}\,\bar{u}}{\cos\gamma}\frac{d\gamma}{d\bar{u}}\right)\left(1 + \frac{L}{D}\tan\gamma\right) \tag{A15}$$

To provide the desired form of the second expression for dv/dt (eq. (A15)), the flight-path curvature $d\gamma/d\bar{u}$ is expressed in terms of $\bar{u}$, Z, γ, and Δ in the following manner. By differentiating $Z/\bar{u}$ from equation (A13) $(Z' \equiv dZ/d\bar{u})$ and keeping in mind the approximation $|dr/r| \ll |d\bar{u}/\bar{u}|$

$$\frac{Z'}{\bar{u}} - \frac{Z}{\bar{u}^2} = -\frac{1}{2}\,\bar{\rho}_o \sqrt{\frac{r}{\beta}}\,\frac{e^{-\beta y}}{\Delta}\left(\beta\,\frac{dy}{d\bar{u}} + \frac{\Delta'}{\Delta}\right)$$

$$= -\frac{Z}{\bar{u}}\left(\beta\,\frac{dy}{dt}\,\frac{dt}{d\bar{u}} + \frac{\Delta'}{\Delta}\right) \tag{A16}$$

Use of equations (A9) and (A10) in equation (A16) yields an expression for the flight-path angle

$$\sqrt{\beta r}\,\sin\gamma = \left(Z' - \frac{Z}{\bar{u}} + Z\,\frac{\Delta'}{\Delta}\right)\left(1 + \frac{L}{D}\tan\gamma\right) \tag{A17}$$

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An expression for the flight-path curvature $d\gamma/d\bar{u}$, obtained by differentiating equation (A17), is

$$\left(1 + \frac{L}{D}\tan\gamma\right)\bar{u}\,\frac{d\gamma}{d\bar{u}}$$

$$= \frac{\bar{u}}{\sqrt{\beta r}\,\cos\gamma}\left[\frac{\left(1 + \frac{L}{D}\tan\gamma\right)^2 \frac{d}{d\bar{u}}\left(Z' - \frac{Z}{\bar{u}} + Z\,\frac{\Delta'}{\Delta}\right) + \sqrt{\beta r}\left(\frac{L}{D}\right)'\frac{1 - \cos^2\gamma}{\cos\gamma}}{1 - \left(\frac{L}{D}\right)\frac{\tan\gamma}{\left(1 + \frac{L}{D}\tan\gamma\right)\cos^2\gamma}}\right] \tag{A18}$$

By assuming that $L/D\tan\gamma$ can be neglected compared to unity ($|L/D\tan\gamma| \ll 1$) and restricting the analysis for lifting vehicles to shallow entries so that during the modulation period $\cos^2\gamma \approx 1$, we can reduce equation (A18) to

$$\bar{u}\,\frac{d\gamma}{d\bar{u}} = \frac{\bar{u}}{\sqrt{\beta r}\,\cos\gamma}\,\frac{d}{d\bar{u}}\left(Z' - \frac{Z}{\bar{u}} + Z\,\frac{\Delta'}{\Delta}\right)$$

$$= \frac{1}{\sqrt{\beta r}\,\cos\gamma}\left\{\bar{u}Z'' - \left(Z' - \frac{Z}{\bar{u}}\right) + \bar{u}Z\left[\frac{\Delta''}{\Delta} - \left(\frac{\Delta'}{\Delta}\right)^2\right] + \bar{u}Z'\left(\frac{\Delta'}{\Delta}\right)\right\} \tag{A19}$$

Finally, upon substituting equation (A19) in equation (A15)($L/D\tan\gamma \ll 1$) and equating the result to equation (A8) the original pair of motion equations (A1) and (A2) are transformed into a single differential equation for modulated entries into exponential planetary atmospheres

$$\bar{u}Z'' - \left(Z' - \frac{Z}{\bar{u}}\right) + \bar{u}Z\left[\frac{\Delta''}{\Delta} - \left(\frac{\Delta'}{\Delta}\right)^2\right] + \bar{u}Z'\left(\frac{\Delta'}{\Delta}\right)$$

$$- \frac{1 - \bar{u}^2}{\bar{u}Z}\cos^4\gamma + \sqrt{\beta r}\,\frac{L}{D}\cos^3\gamma = 0 \qquad \text{(A20)}$$

In this equation, $\cos\gamma = \sqrt{1 - \sin^2\gamma}$ can be expressed in terms of Z, Z', Δ, and Δ' through equation (A17) ($L/D \tan\gamma \ll 1$).

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APPENDIX B

CLOSED-FORM SOLUTIONS FOR m/C_DA, $\bar{u}_2$, AND γ FOR MODULATED NONLIFTING ENTRIES

The variation in m/C_DA with velocity required to maintain a constant resultant deceleration is defined by equation (13). Upon substitution of the dependent variable

$$p = \frac{\Delta'}{\Delta} = \frac{1}{\Delta}\frac{d\Delta}{d\bar{u}} \equiv \frac{1}{m/C_DA}\frac{d(m/C_DA)}{d\bar{u}} \tag{B1}$$

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and assuming that $\cos^4\gamma \approx 1$, equation (13) reduces to

$$p' - \frac{1}{\bar{u}}p = \frac{1 - \bar{u}^2}{B^2} - \frac{4}{\bar{u}^2} \tag{B2}$$

The corresponding expression for the flight-path angle is (see eqs. (B1) and (14))

$$\sqrt{\beta r}\,\sin\gamma = \frac{B}{\bar{u}}\left(p - \frac{2}{\bar{u}}\right) \tag{B3}$$

Integration of equation (B2) yields ($1/\bar{u}$ is an integrating factor)

$$\frac{1}{\bar{u}}p = \frac{1}{B^2}\ln\bar{u} - \frac{\bar{u}^2}{2B^2} + \frac{2}{\bar{u}^2} + \text{constant} \tag{B4}$$

The integration constant can be evaluated in terms of the velocity and the flight-path angle at the beginning of modulation (i.e., from eq. (B3), at $\bar{u} = \bar{u}_1$, $p_1 = (\bar{u}_1/B)\sqrt{\beta r}\,\sin\gamma_1 + (2/\bar{u}_1)$). In this manner

$$\frac{\Delta'}{\Delta} = p = \frac{\bar{u}}{B^2}\ln\left(\frac{\bar{u}}{\bar{u}_1}\right) + \frac{\bar{u}_1^3}{2B^2}\left(\frac{\bar{u}}{\bar{u}_1}\right)\left[1 - \left(\frac{\bar{u}}{\bar{u}_1}\right)^2\right] + \frac{2}{\bar{u}} + \frac{\sqrt{\beta r}}{B}\,\bar{u}\,\sin\gamma_1 \tag{B5}$$

Proceeding by integration of equation (B1)

$$\Delta = \frac{m}{C_DA} = \exp\left(\int p\,d\bar{u} + \text{constant}\right) \tag{B6}$$

Equation (B5) is readily integrable and the integration constant can be evaluated in terms of the velocity and m/C_DA at the beginning of modulation ($\bar{u}_1$ and Δ_1, respectively). Thus, the closed-form solution for the variation of m/C_DA during a nonlifting modulated entry is

$$\frac{\Delta}{\Delta_1} = \frac{m/C_DA}{(m/C_DA)_1} = \exp\left\{2\left[1 + C\left(\frac{\bar{u}}{\bar{u}_1}\right)^2\right] ln\left(\frac{\bar{u}}{\bar{u}_1}\right) + E\left[1 - \left(\frac{\bar{u}}{\bar{u}_1}\right)^2\right]\right.$$

$$\left. + \frac{1}{2} C\bar{u}_1^2\left[1 - \left(\frac{\bar{u}}{\bar{u}_1}\right)^4\right]\right\} \qquad \text{(B7)}$$

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where

$$\left.\begin{aligned} C &= \frac{\bar{u}_1^2}{4B^2} \\ E &= C(1 - 2B\sqrt{\beta r}\sin\gamma_1 - \bar{u}_1^2 \end{aligned}\right\} \qquad \text{(B8)}$$

and B is given by equation (12).

The velocity $\bar{u}_2$ at which the modulation should be terminated has been shown to be that value at which m/C_DA reaches a maximum, that is, the value of $\bar{u}$ for which the derivative of equation (B6) (eq. (B5)) is zero, which for convenience is written

$$\frac{1}{B^2} ln\left(\frac{\bar{u}}{\bar{u}_1}\right) - 2C\left(\frac{\bar{u}}{\bar{u}_1}\right)^2 + \frac{2}{\bar{u}_1^2}\frac{1}{\left(\frac{\bar{u}}{\bar{u}_1}\right)^2} + \left(2C + \frac{\sqrt{\beta r}}{B}\sin\gamma_1\right) = 0 \qquad \text{(B9)}$$

Trial and error solutions of equation (B9) yield the velocity $\bar{u}_2$. The expression for the flight-path angle during the modulation period is obtained from substitution of equation (B5) into equation (B3)

$$\sin\gamma = \sin\gamma_1 + \frac{1}{G_1}\left\{ln\left(\frac{\bar{u}}{\bar{u}_1}\right) + \frac{\bar{u}_1^2}{2}\left[1 - \left(\frac{\bar{u}}{\bar{u}_1}\right)^2\right]\right\} \qquad \text{(B10)}$$

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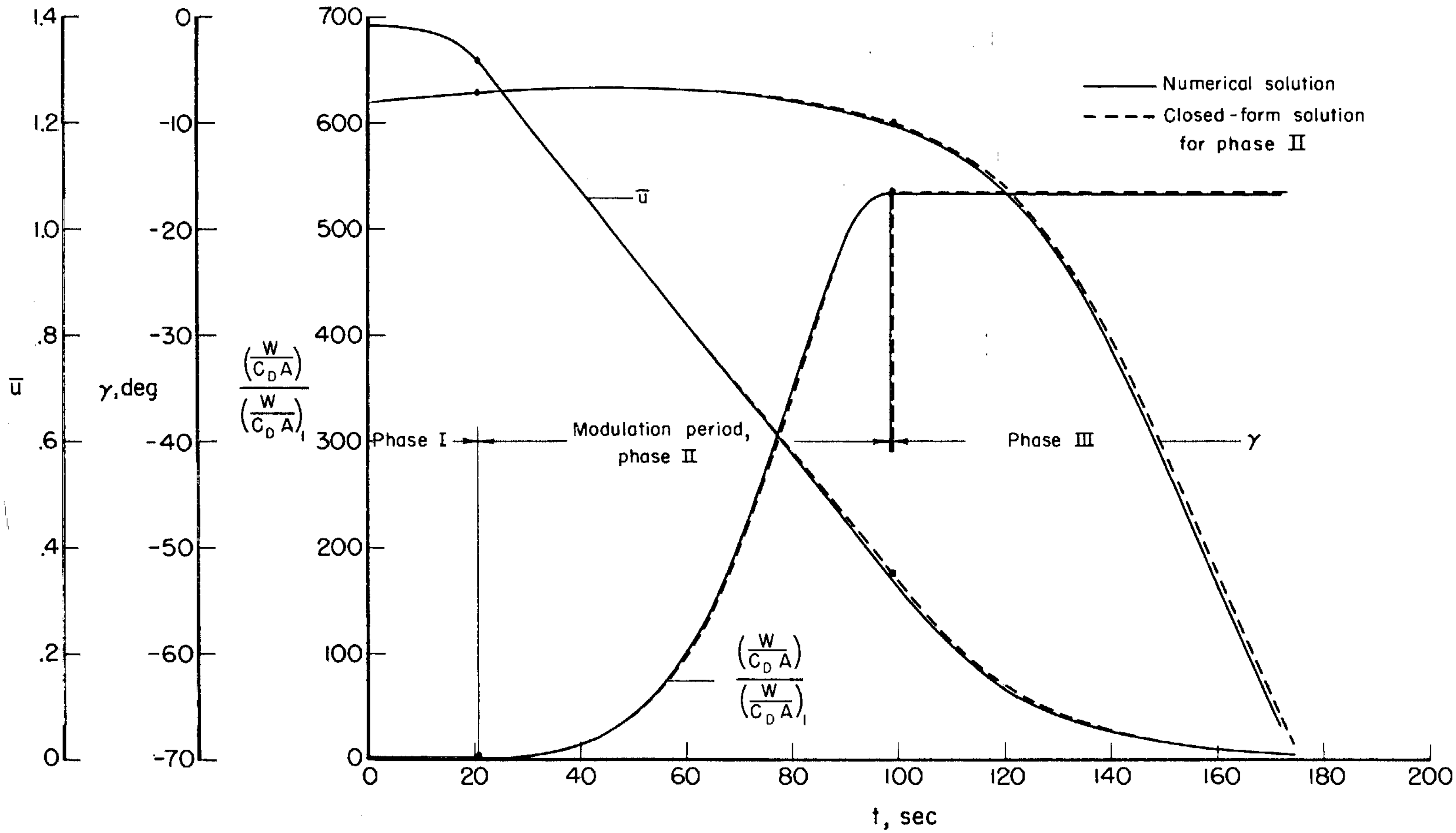
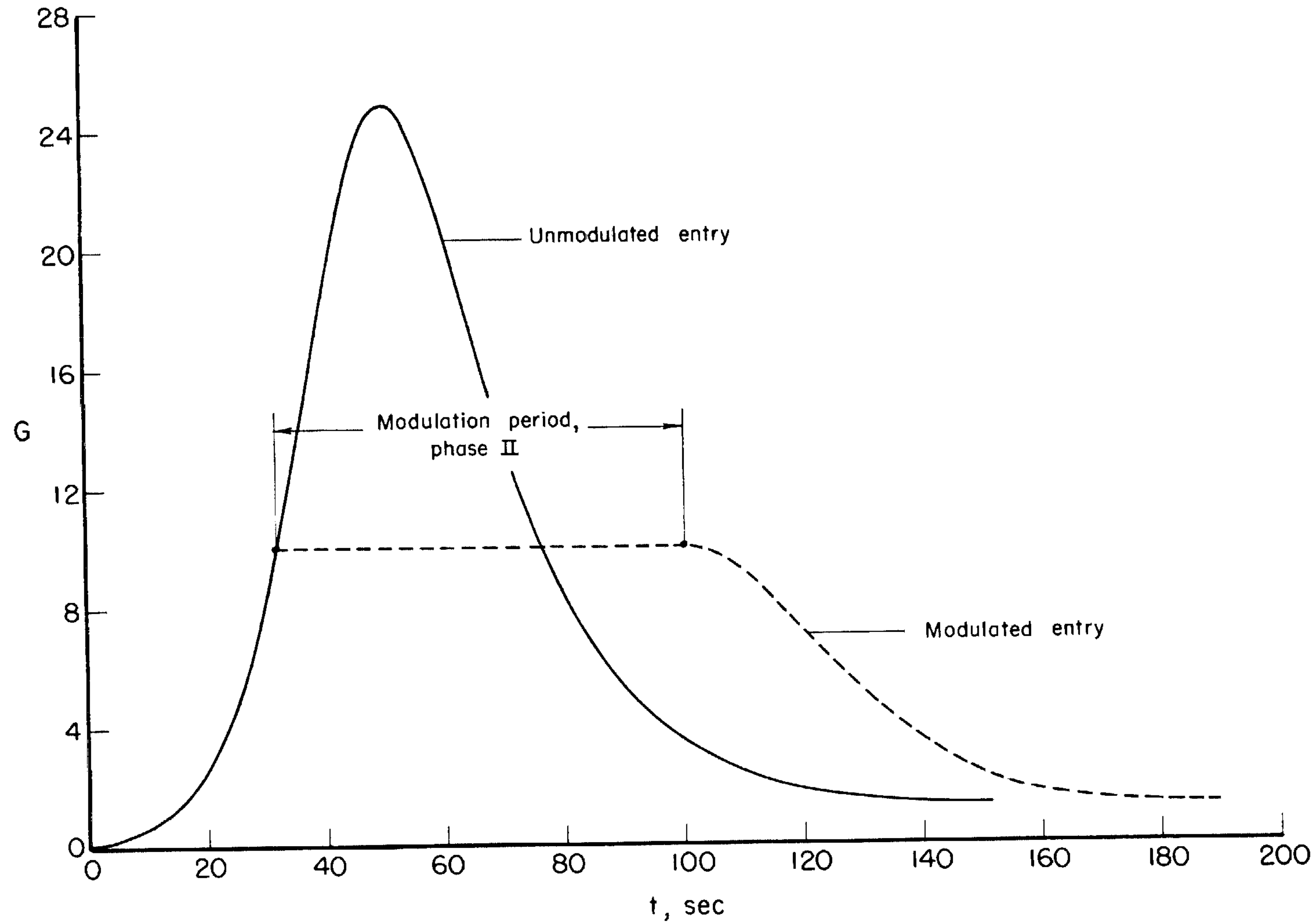


Figure 1.- Comparison of trajectory parameters obtained from numerical and closed-form solutions for a modulated entry; $\bar{V}_i = 1.4$, $L/D = 0$, $\gamma_i = -8^\circ$, $G_{max} = 10$, earth.



(a) Resultant deceleration

Figure 2.- Effect of modulation on entry trajectory parameters; $\overline{V}_i = 1.4$, $L/D = 0$, $\gamma_i = -6^{\circ}$, earth.

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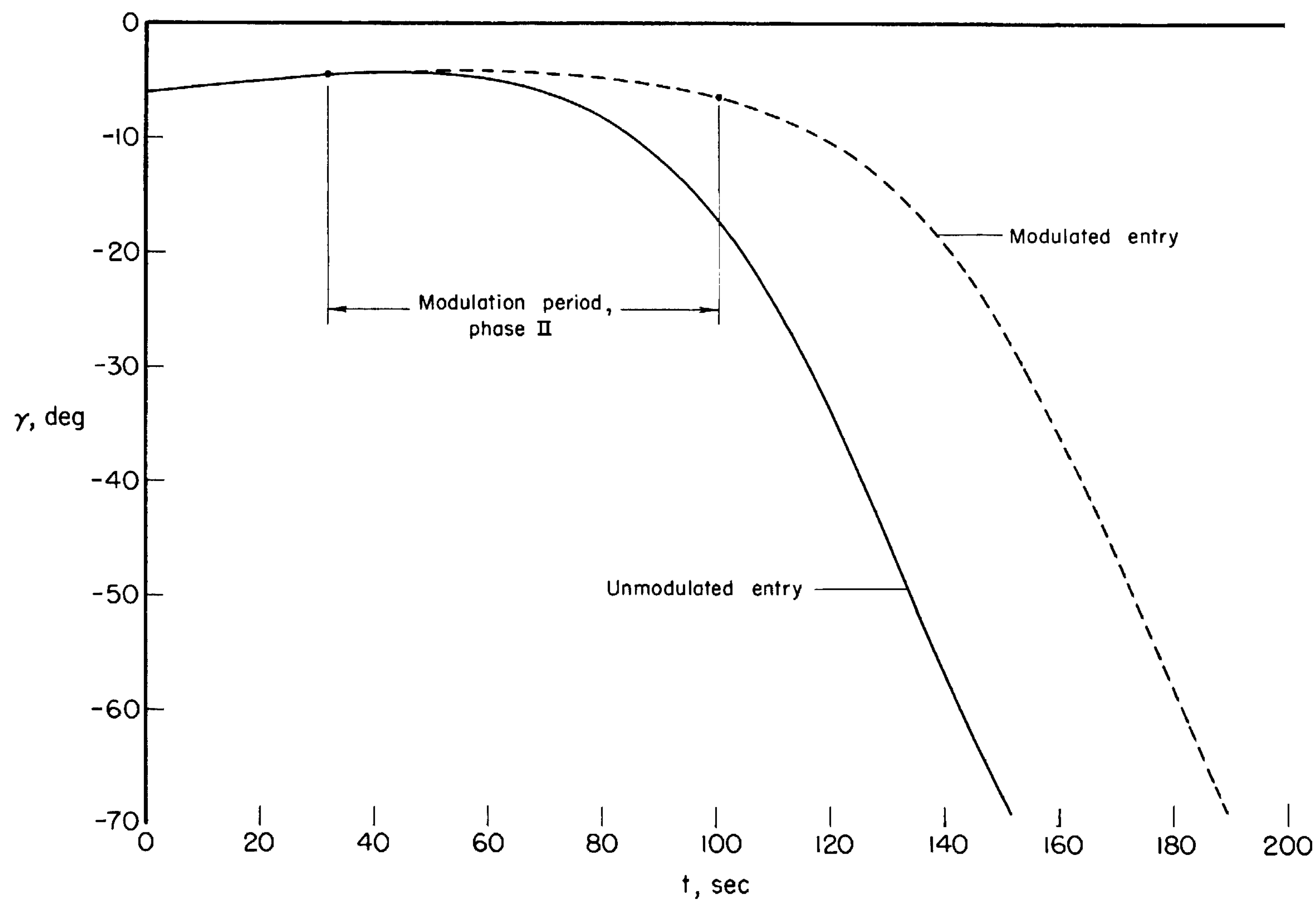
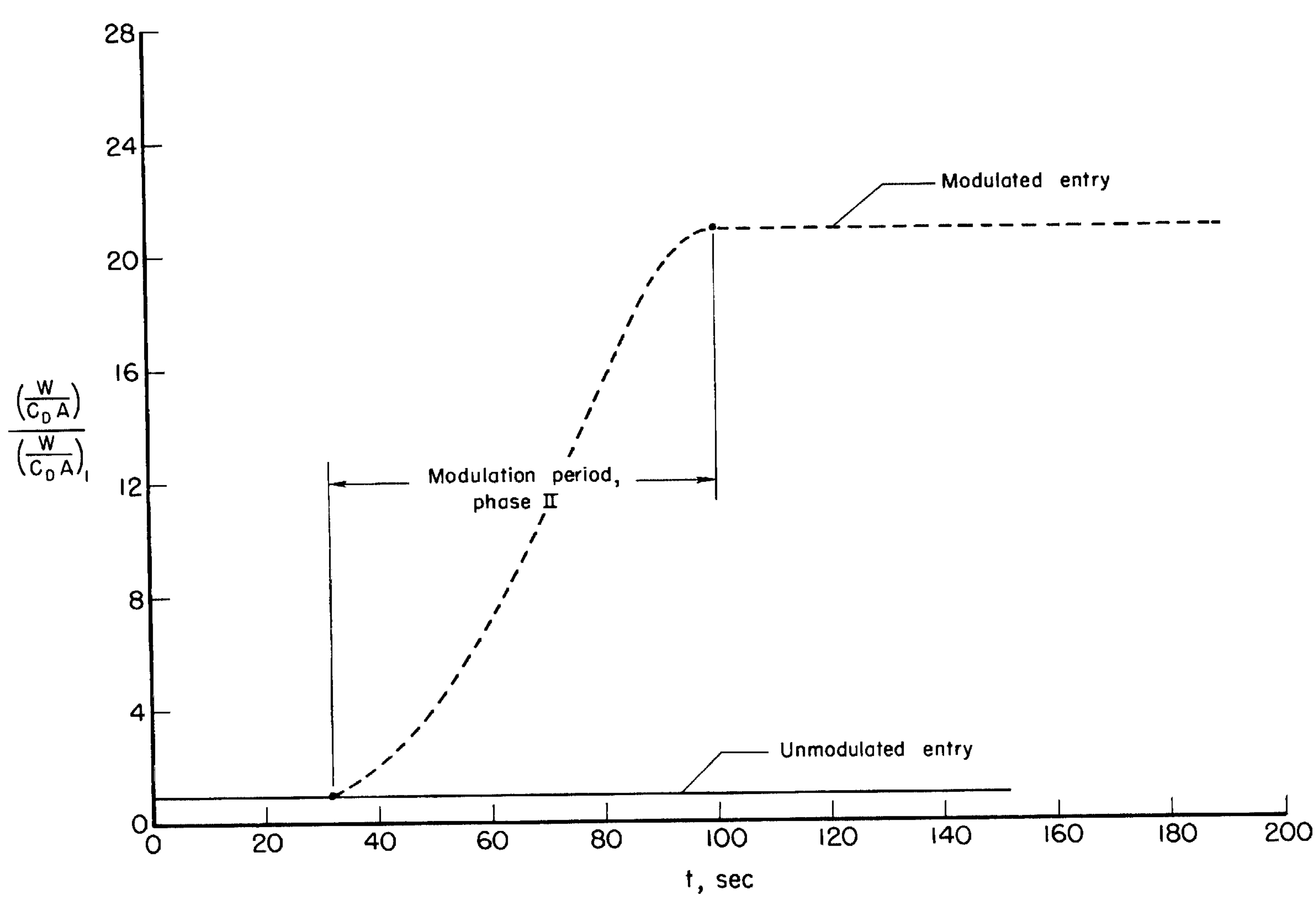


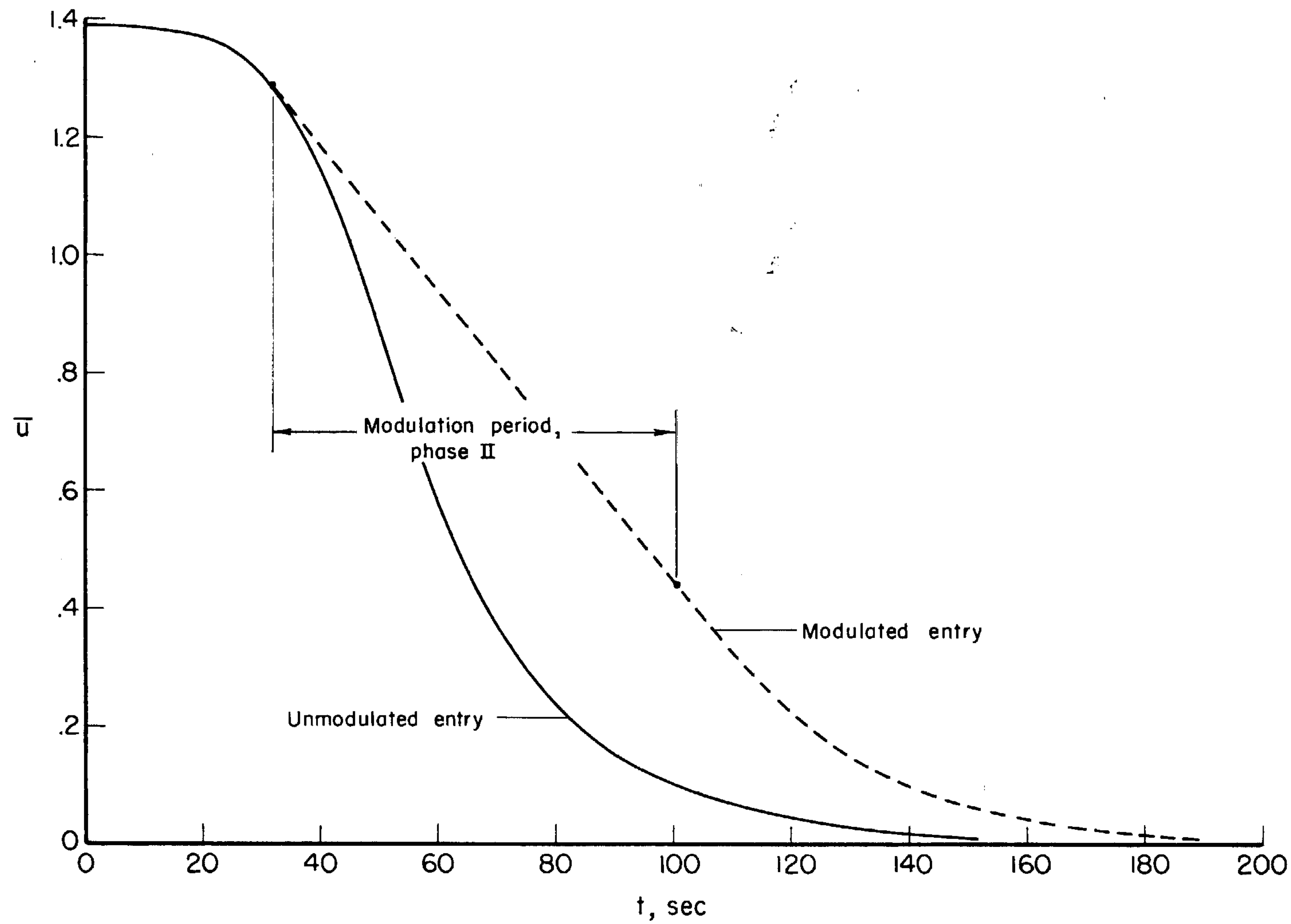
(b) Flight-path angle

Figure 2.- Continued.



(c) Modulation parameter

Figure 2.- Continued.



(d) Tangential velocity

Figure 2.- Concluded

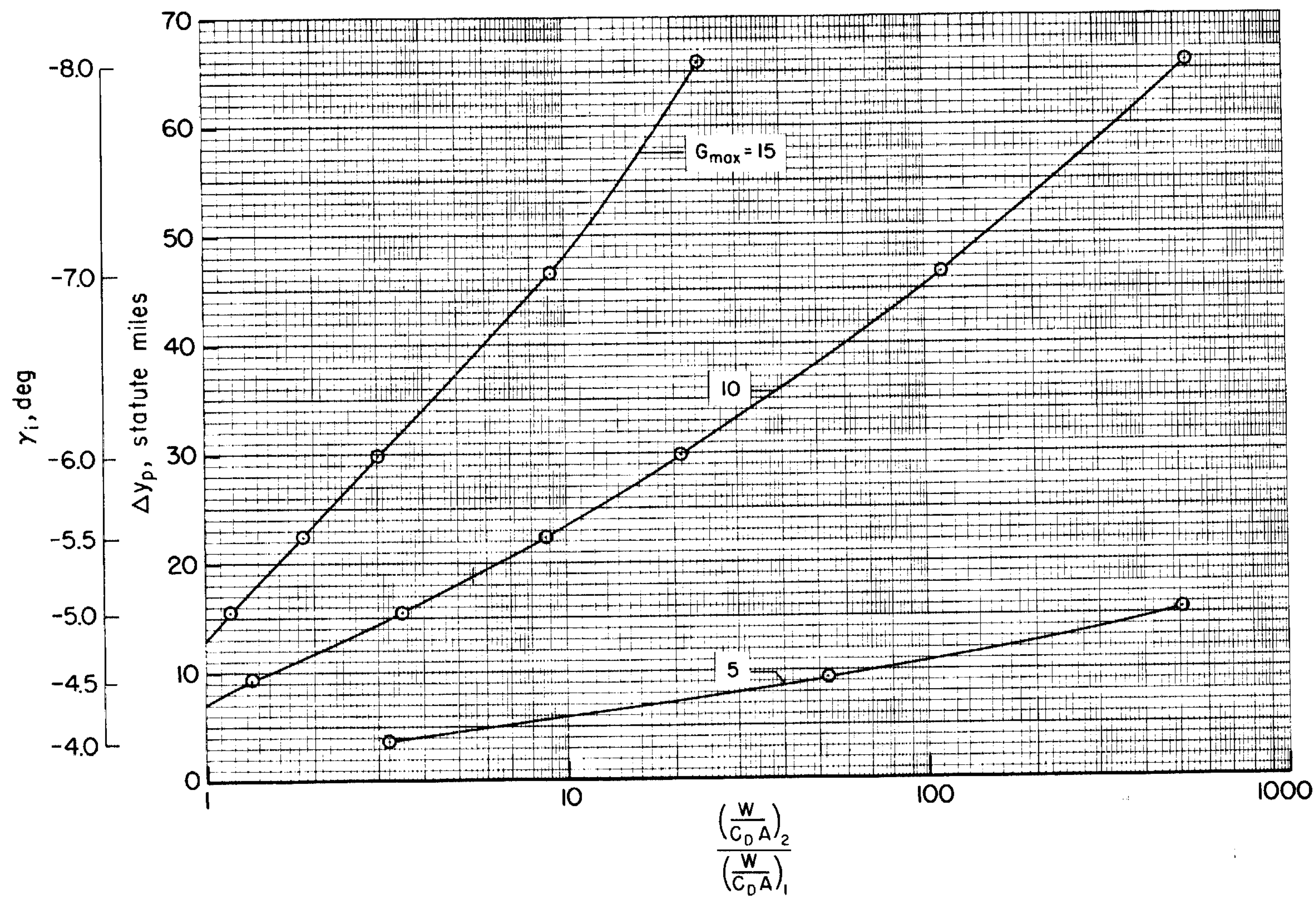


Figure 3.- Effect of varying W/C_DA on the deceleration-limited entry corridor; $\overline{V}_i = 1.4$, $L/D = 0$, earth.

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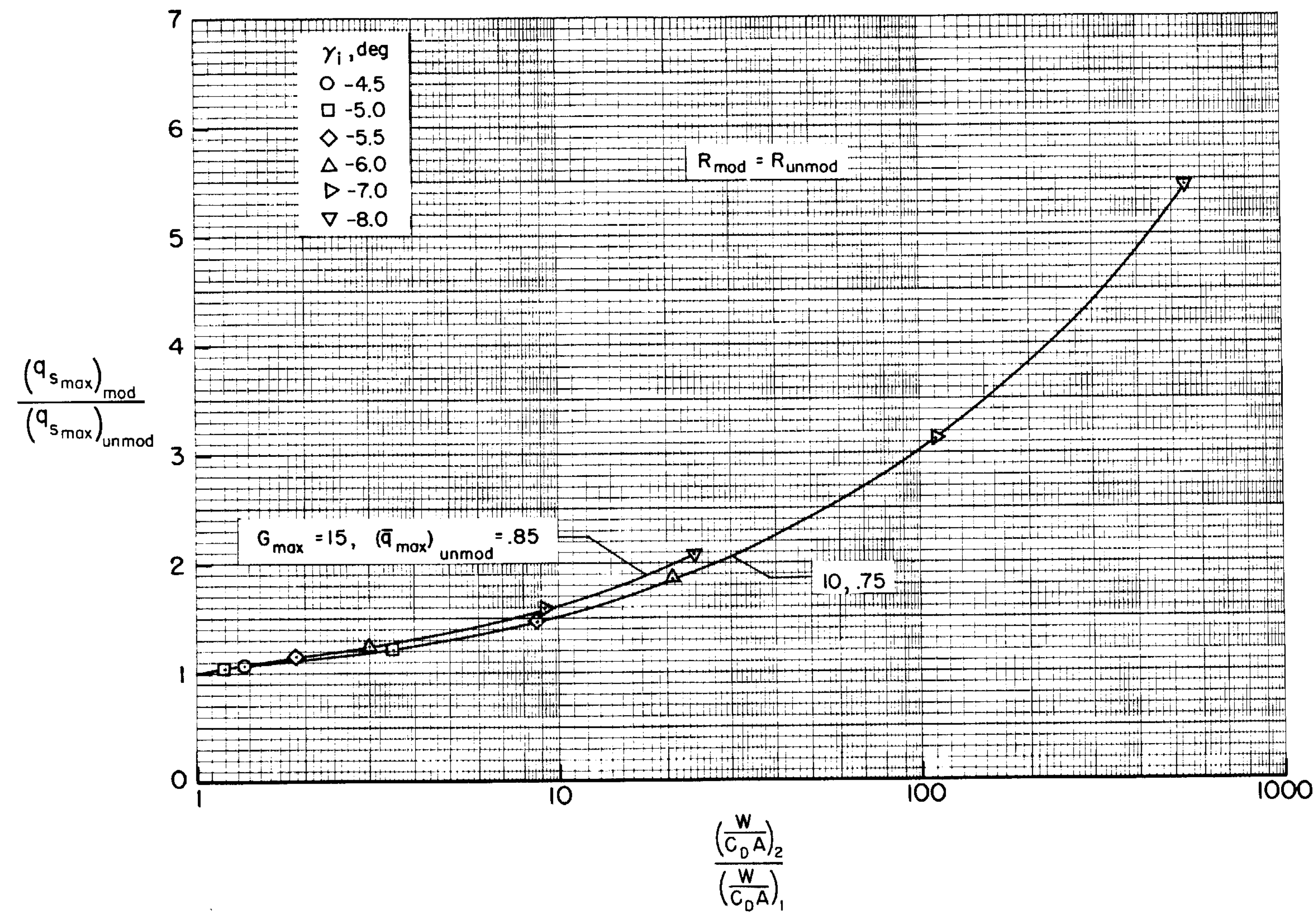

Figure 4.- Effect of varying W/C_DA on the maximum, laminar, convective, stagnation-point heating rate for entry along a deceleration-limited undershoot boundary; $\bar{V}_i = 1.4$, $L/D = 0$, earth.

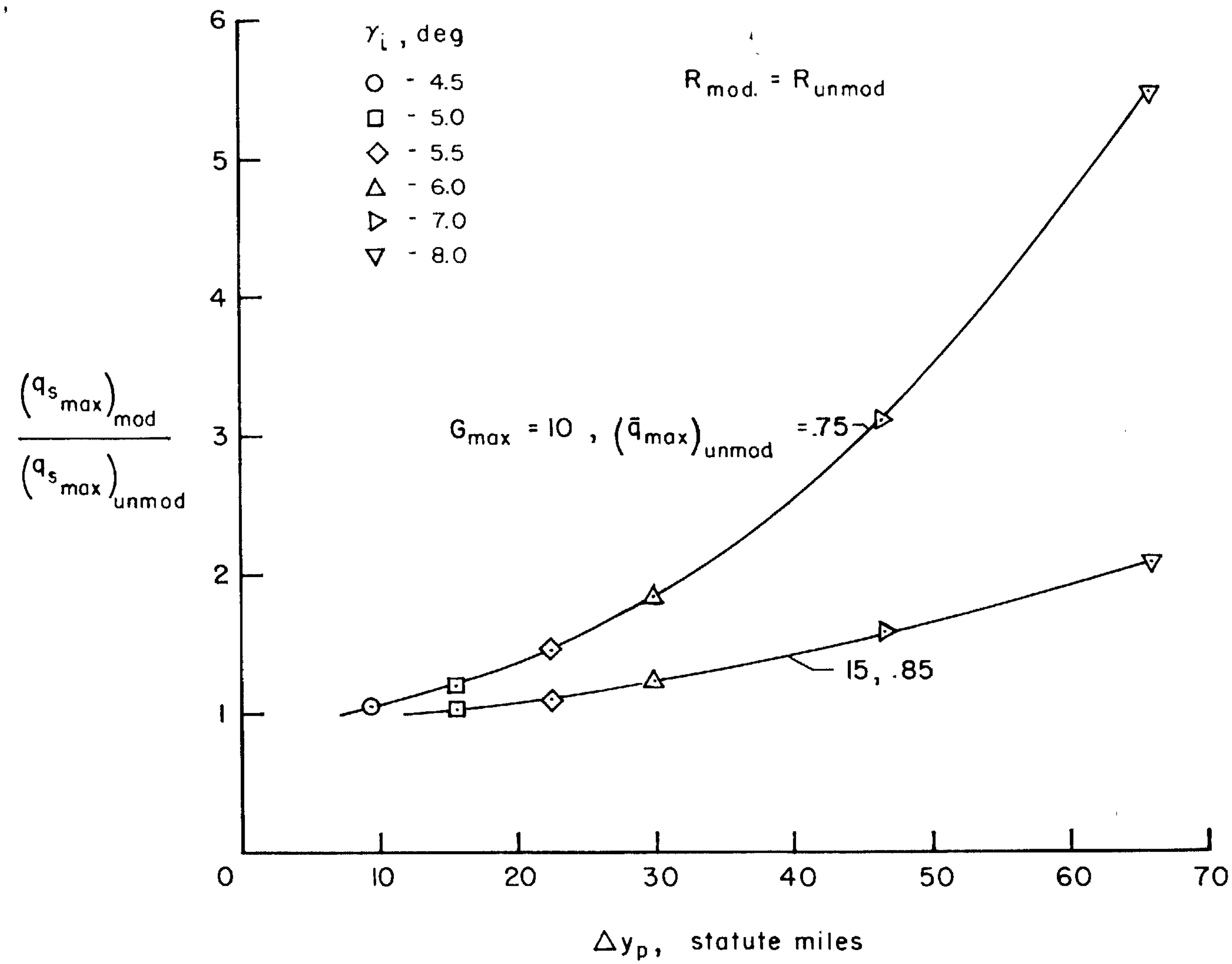

Figure 5.- Heating rate associated with the use of modulation to increase entry corridor depth; $\overline{V}_i = 1.4$, $L/D = 0$, earth.

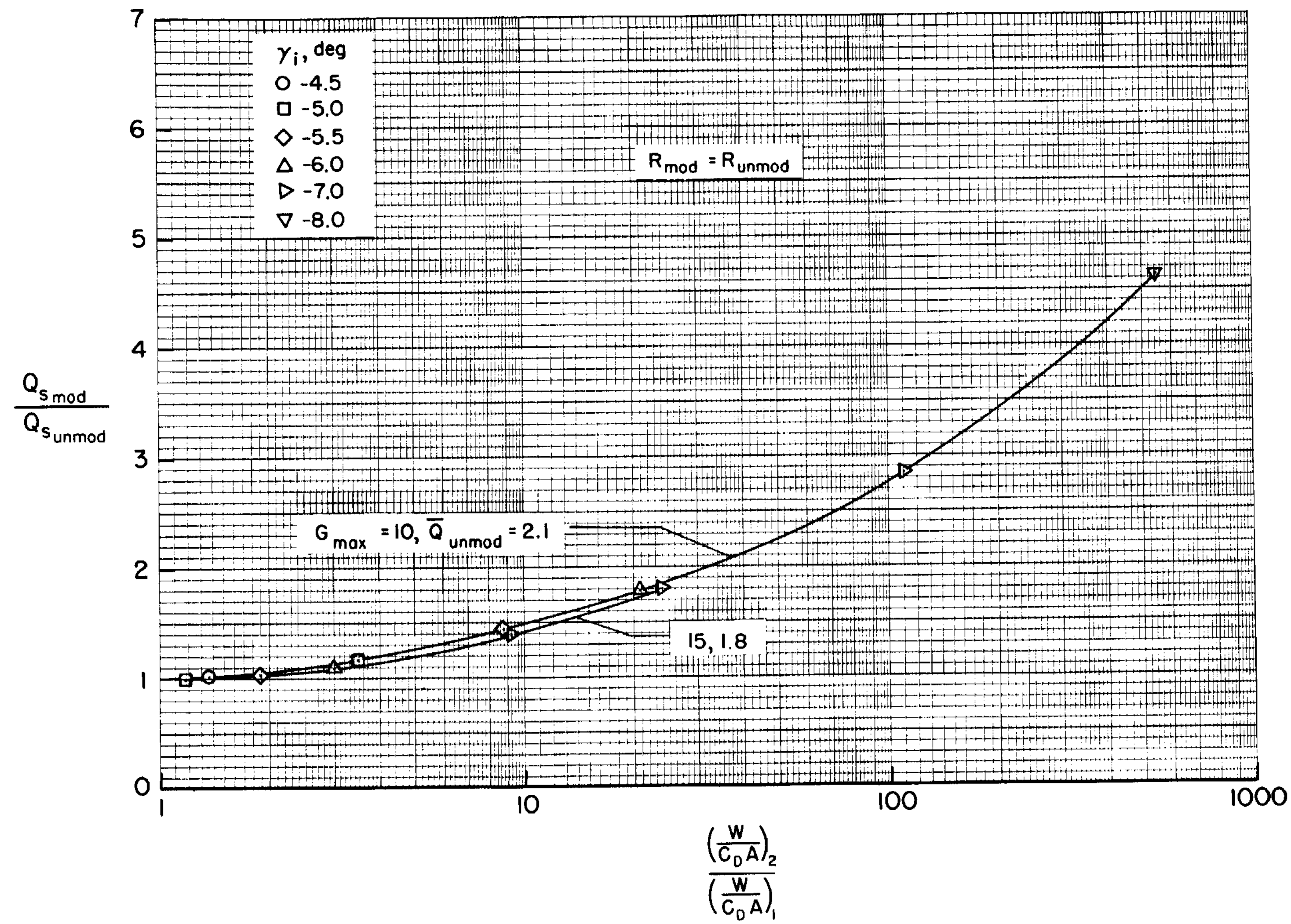

Figure 6.- Effect of varying W/C_DA on the total laminar convective heat absorbed at the stagnation point for entry along a deceleration-limited undershoot boundary; $\bar{V}_i = 1.4$, $L/D = 0$, earth.

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