NASA TN D-1037

TECHNICAL NOTE

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THE USE OF DRAG MODULATION TO LIMIT THE RATE AT WHICH DECELERATION INCREASES DURING NONLIFTING ENTRY

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

WASHINGTON September 1961

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SUMMARY

The method developed in NASA TN D-319 for studying the atmosphere entry of vehicles with varying aerodynamic forces has been applied to obtain a closed-form solution for the motion, heating, range, and variation of the vehicle parameter m/C_DA for nonlifting entries during which the rate of increase of deceleration is limited. The solution is applicable to vehicles of arbitrary weight, size, and shape, and to arbitrary atmospheres. Results have been obtained for entries into the earth's atmosphere at escape velocity during which the maximum deceleration and the rate at which deceleration increases were limited. A comparison of these results with those of NASA TN D-319, in which only the maximum deceleration was limited, indicates that for a given corridor depth, limiting the rate of increase of deceleration and the maximum deceleration requires an increase in the magnitude of the change in m/C_DA and results in increases in maximum heating rate, total heat absorbed at the stagnation point, and range.

INTRODUCTION

Man's performance of useful duties during atmosphere entry requires that the maximum deceleration and the rate at which the deceleration increases be held within prescribed limits (see refs. 1, 2, and 3). The problem of limiting the maximum deceleration has been studied by several investigators. In the study reported in NASA TN D-319 (ref. 4), a differential equation applicable to vehicles of arbitrary weight, size, and shape, and to arbitrary atmospheres was developed for entries during which the aerodynamic forces vary. A closed-form solution for nonlifting entries was obtained for the motion, heating, range, and variation in m/C_DA required to maintain specified maximum deceleration limits. The purpose of the present investigation is to obtain a solution of the equation developed in reference 4 for the case of specified deceleration-rate limits as well as specified maximum decelerations. The solution is used to determine the effect of limiting the deceleration and the rate of increase of deceleration (by varying m/C_DA) on the corridor depth, heating characteristics, and range for vehicles making shallow nonlifting entries into the earth's atmosphere at escape velocity.

NOTATION

A	reference area for drag, sq ft
C_D	drag coefficient, $\frac{2D}{\rho V^2 A}$
D	drag force, lb
g	gravitational acceleration, ft sec^{-2}
g_c	gravitational conversion constant, 32.2 ft sec^{-2}
G	deceleration in g units
K	specified rate of increase of deceleration, $\frac{dG}{dt}$, g/sec
L	lift force, lb
m	mass of vehicle, slugs
q_s	convective heating rate per unit area at the stagnation point, Btu ft^{-2} sec^{-1}
$\overline{q}$	dimensionless function proportional to convective heating rate (eq. (26))
Q_s	total convective heat absorbed per unit area at the stagnation point, Btu ft^{-2}
$\overline{Q}$	dimensionless function proportional to total heat absorbed, (eq. (28))
r	distance from planet center, ft
R	radius of curvature of vehicle surface, ft
Δs	range measured from entry point, ft (see eq. (29))
t	time, sec
u	tangential velocity component normal to a radius vector, ft sec^{-1}
$\overline{u}$	dimensionless velocity ratio, $\frac{u}{\sqrt{gr}}$
V	resultant velocity, $\frac{u}{\cos \gamma}$
$\overline{V}$	dimensionless velocity ratio, $\frac{V}{\sqrt{gr}}$

W	weight of vehicle at earth's surface, mg_c, lb
y	altitude, ft
Z	dimensionless function of $\bar{u}$ (eq. (4))
β	atmosphere density decay parameter, ft^{-1}, 1/23,500 ft^{-1} for earth
γ	flight-path angle relative to the local horizontal, negative for descent
Δ	vehicle parameter, $\frac{m}{C_D A}$
Δy_p	corridor depth, statute miles
ρ	atmosphere density, slugs ft^{-3}
$\bar{\rho}_o$	mean value for exponential approximation to atmosphere density-altitude relation, slugs ft^{-3}, 0.0027 slugs ft^{-3} for earth

Subscripts

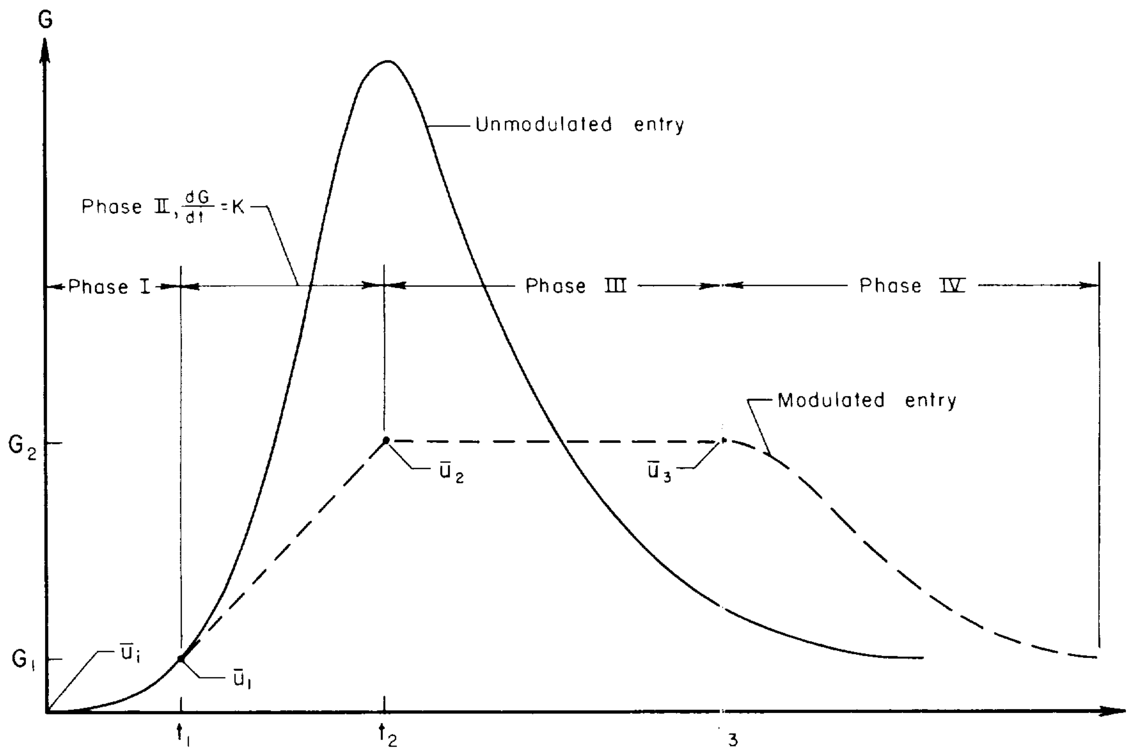
i	initial
max	maximum
mod	modulated entry
o	surface of a planet
unmod	unmodulated entry
1, begin	point in the trajectory where modulation begins
2	point in the trajectory where modulation for constant deceleration rate ends
3, end	point in the trajectory where modulation ends
I,II,III	phase of a trajectory

Superscript

differentiation with respect to $\bar{u}$

ANALYSIS

In the present analysis of shallow nonlifting entries, m/C_DA will be varied so that both the maximum deceleration and the rate at which the deceleration increases do not exceed specified limiting values. The trajectory for this type of modulated entry generally consists of the four distinct phases illustrated below. The deceleration-time histories



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for an unmodulated entry and a modulated entry are shown for the same initial conditions. During phase I the vehicle enters the atmosphere with a given velocity ($\bar{u}_i$) and entry angle (γ_i) and maintains a constant m/C_DA. The deceleration increases monotonically until the specified value of the deceleration rate is attained at velocity $\bar{u}_1$, at which velocity the magnitude of the deceleration is G_1. At this velocity phase II begins with the modulation of m/C_DA in such a manner that the deceleration rate is held constant at a specified value K; that is,

$$G = G_1\left[1 + \frac{K}{G_1}(t - t_1)\right] \quad \text{for} \quad t_1 \leq t \leq t_2 \tag{1}$$

The velocity $\bar{u}_2$, at which phase III begins, is determined as that velocity at which the specified value of the maximum deceleration G_2 is first attained. During phase III, m/C_DA is modulated in such a manner that the deceleration is held constant until the velocity is reduced to the value $\bar{u}_3$. This velocity ($\bar{u}_3$) is determined by specifying that termination of the modulation will allow the deceleration to decrease monotonically as the vehicle completes its entry. Phase IV is this final phase of the entry. It should be noted that phases I, III, and IV of this report correspond to phases I, II, and III, respectively, of reference 4.

The general differential equation which defines the trajectory for the four phases of the modulated entry (and the unmodulated entry) is obtained from reference 4 as

$$\bar{u}Z'' - \left(Z' - \frac{Z}{\bar{u}}\right) + \bar{u}Z\left[\frac{\Delta''}{\Delta} - \left(\frac{\Delta'}{\Delta}\right)^2\right] + \bar{u}Z'\left(\frac{\Delta'}{\Delta}\right) - \frac{1 - \bar{u}^2}{\bar{u}Z}\cos^4\gamma = 0 \qquad (2)$$

where

$$\bar{u} = \frac{u}{\sqrt{gr}} \qquad (3)$$

is the dimensionless independent variable,

$$Z = \frac{\bar{\rho}_o\sqrt{\frac{r}{\beta}}}{2\Delta}\,\bar{u}\,e^{-\beta y} \qquad (4)$$

is the dimensionless dependent variable first presented in reference 5, and

$$\Delta \equiv \frac{m}{C_DA} = \Delta(\bar{u}) \qquad (5)$$

The accompanying expression for the flight-path angle is

$$\sqrt{\beta r}\,\sin\gamma = Z' - \frac{Z}{\bar{u}} + Z\left(\frac{\Delta'}{\Delta}\right) \qquad (6)$$

Equation (2) is applicable to vehicles of arbitrary weight, size, and shape, and to arbitrary atmospheres. The solution of equation (2) for the present analysis consists primarily of determining the solution for phase II of the modulated portion of the trajectory, since the solution for phase III is indicated in reference 4 and the solutions for the unmodulated portions of the trajectory (phases I and IV) are indicated in reference 5.

There are four basic steps to the solution of equation (2) for phase II. First, by eliminating Z and its derivatives from equation (2) there results a differential equation which defines the required variation in m/C_DA. Second, the solution of this differential equation is obtained. Third, the duration of phase II is determined (i.e., the velocity $\bar{u}_2$ is determined). Finally, the solution for phase II is matched with the solutions for phases I and III.

The first step is accomplished by means of the equation from reference 4 for the resultant deceleration during shallow nonlifting entries,[1] which is

$$G = \sqrt{\beta r}\,\bar{u}Z \tag{7}$$

An expression for the Z function during phase II can be obtained by eliminating G from equation (7) in the following manner. The first derivative of equation (1) can be written

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$$\frac{dG}{dt} = K = \frac{dG}{d\bar{u}}\frac{d\bar{u}}{dt} \tag{8}$$

Substitution of $dG/d\bar{u}$, as obtained from differentiation of equation (7), and $d\bar{u}/dt$, as obtained from reference 4 for $L/D = 0$,

$$\frac{d\bar{u}}{dt} = -\sqrt{\beta g}\,\bar{u}Z \tag{9}$$

in equation (8) yields the differential equation for the Z function during phase II

$$\bar{u}Z(\bar{u}Z' + Z) = -\frac{K}{\sqrt{\beta g}\,\sqrt{\beta r}} \tag{10}$$

If it is noted from equation (7) that

$$\frac{d\left(\frac{G^2}{\beta r}\right)}{d\bar{u}} = \frac{d(\bar{u}Z)^2}{d\bar{u}} = 2\bar{u}Z(\bar{u}Z' + Z) \tag{11}$$

and that at $\bar{u} = \bar{u}_1$, $(\bar{u}Z)^2 = {G_1}^2/\beta r$, equation (10) can be integrated to yield the desired expression for Z during phase II

$$Z = \frac{B_1}{\bar{u}}\sqrt{a + b\bar{u}} \tag{12}$$

where

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[1]In this report a shallow entry implies that during modulation the flight-path angle is sufficiently small that $\cos\gamma \approx 1$ is a valid approximation.

$$\left.\begin{aligned} a &= 1 - b\bar{u}_1 \\ b &= -\frac{2K}{\sqrt{\beta g}\,\sqrt{\beta r}\,B_1{}^2} \\ B_1 &= \frac{G_1}{\sqrt{\beta r}} \end{aligned}\right\} \quad (13)$$

Substituting equation (12) and its derivatives in equation (2) gives the differential equation which describes the variation in m/C_DA required to maintain a constant deceleration rate

$$\frac{\Delta''}{\Delta} - \left(\frac{\Delta'}{\Delta}\right)^2 + \left[\frac{b}{2(a + b\bar{u})} - \frac{1}{\bar{u}}\right]\left(\frac{\Delta'}{\Delta}\right) + \frac{4}{\bar{u}^2} - \frac{3b}{2\bar{u}(a + b\bar{u})} - \frac{b^2}{4(a + b\bar{u})^2} - \frac{1 - \bar{u}^2}{B_1{}^2(a + b\bar{u})} = 0 \quad (14)$$

and the related expression for the flight-path angle is obtained from equation (6) as

$$\sin \gamma = \frac{B_1}{\sqrt{\beta r}} \frac{\sqrt{a + b\bar{u}}}{\bar{u}} \left[\frac{b}{2(a + b\bar{u})} - \frac{2}{\bar{u}} + \frac{\Delta'}{\Delta}\right] \quad (15)$$

The solution of equation (14) for the variation of m/C_DA, or Δ, with velocity during phase II can be obtained in closed form as

$$\begin{aligned} \left(\frac{\Delta}{\Delta_1}\right)_{II} = \exp \Bigg\{ & \frac{2}{3\sqrt{a}\, b^2 B_1{}^2} X(\xi)(2a - b\bar{u}_1\xi) \ln \frac{\sqrt{a} + X(\xi)}{\sqrt{a} - X(\xi)} - \ln X(\xi) \\ & - \frac{4aC}{3b^2}\left[X(\xi) - 1\right] + \frac{2C\bar{u}_1}{3b}\left[\xi X(\xi) - 1\right] \\ & + \left(\frac{4a}{3b^2 B_1{}^2} + 2\right) \ln \xi + \frac{2\bar{u}_1}{3bB_1{}^2}(1 - \xi) \\ & - \frac{2a\bar{u}_1{}^2}{3b^2 B_1{}^2}(1 - \xi^2) + \frac{2\bar{u}_1{}^3}{9bB_1{}^2}(1 - \xi^3) \\ & - \frac{2(3a - 1)}{3\sqrt{a}\, b^2 B_1{}^2} \ln \frac{\sqrt{a} + 1}{\sqrt{a} - 1} \Bigg\} \end{aligned} \quad (16)$$

where

$$
\left.\begin{aligned}
\xi &= \frac{u}{\bar{u}_1} \\
X(\xi) &= \sqrt{1 - b\bar{u}_1(1 - \xi)} \\
C &= \frac{\sqrt{\beta r}}{B_1} \sin \gamma_1 + \frac{1}{\sqrt{a}\, B_1{}^2} \ln \frac{\sqrt{a} + 1}{\sqrt{a} - 1} - \frac{2(3a - 1)}{3b^2 B_1{}^2}
\end{aligned}\right\} \quad (17)
$$

The details of the derivation of equation (16) are presented in the appendix to this report. The expression for the flight-path angle obtained from equation (15) and the derivative of equation (16) is

$$
\sin \gamma_{II} = - \frac{B_1}{\sqrt{\beta r}} \left[\frac{1}{\sqrt{a}\, B_1{}^2} \ln \frac{\sqrt{a} + X(\xi)}{\sqrt{a} - X(\xi)} - \frac{4a}{3b^2 B_1{}^2} X(\xi) + \frac{2\bar{u}_1}{3bB_1{}^2} \xi X(\xi) - C \right] \tag{18}
$$

The velocity $\bar{u}_2$ at which phase II is terminated is determined from the deceleration-velocity relation as that value of velocity at which the specified maximum deceleration G_2 is first attained. From equations (7), (12), (13), and (17), the deceleration-velocity relation is found to be

$$
G = G_1 \sqrt{1 - b\bar{u}_1(1 - \xi)} \tag{19}
$$

and from equation (19), the value of ξ_2 (and hence $\bar{u}_2$) at which phase II is terminated is

$$
\xi_2 = 1 - \frac{1}{b\bar{u}_1} \left[1 - \left(\frac{G_2}{G_1} \right)^2 \right] \tag{20}
$$

For modulated entries during which only the deceleration rate is limited, the velocity $\bar{u}_2$ is determined as indicated in the appendix.

It now remains to match the solution for phase II with the solutions for phases I and II. This is done by matching the attitude and flight-path angle at the velocities for the beginning and end of phase II ($\bar{u}_1$ and $\bar{u}_2$, respectively). The quantities at the beginning of modulation Z_1, or Δ_1, and γ_1 are known from phase I (obtained as indicated in ref. 5); hence, with the aid of equation (15) and the fact that at $\bar{u} = \bar{u}_1$, $a + b\bar{u} = 1$, the solutions for phases I and II are matched by satisfying the conditions

$$\left.\begin{aligned}\Delta_{II_i} &= \Delta_1 \\ \left(\frac{\Delta'}{\Delta}\right)_{II_i} &= \frac{\sqrt{\beta r}}{B_1}\,\bar{u}_1 \sin\gamma_1 + \frac{2}{\bar{u}_1} - \frac{b}{2}\end{aligned}\right\} \quad (21)$$

where the subscript II_i denotes initial values for phase II. At the end of phase II, Δ_2, and γ_2 are known (eqs. (16) and (18)), and the Z function and the equation for the flight-path angle are known for phase III (see ref. 4); hence, the solutions for phases II and III are matched by satisfying the conditions

$$\left.\begin{aligned}\Delta_{III_i} &= \Delta_2 \\ \left(\frac{\Delta'}{\Delta}\right)_{III_i} &= \frac{\sqrt{\beta r}}{B_2}\,\bar{u}_2 \sin\gamma_2 + \frac{2}{\bar{u}_2}\end{aligned}\right\} \quad (22)$$

where

$$B_2 = \frac{G_2}{\sqrt{\beta r}} \quad (23)$$

For modulated entries during which only the deceleration rate is limited, the solutions for phases II and III are matched as indicated in the appendix.

The solution of equation (2) is known for the remainder of the trajectory; that is, the closed-form solution for the variation of m/C_DA required to maintain constant maximum deceleration during phase III is given in reference 4 and the solution for phase IV is given in reference 5. However, it should be noted that in the present analysis, for phase III

$$\left(\frac{\Delta}{\Delta_1}\right)_{III} = \left(\frac{\Delta_2}{\Delta_1}\right)_{II}\left(\frac{\Delta}{\Delta_2}\right)_{III} \quad (24)$$

where $(\Delta_2/\Delta_1)_{II}$ is determined from equation (16) with $\xi = \xi_2$, and $(\Delta/\Delta_2)_{III}$ is determined from the closed-form solution of reference 4. As noted earlier, the closed-form solution of reference 4 for the variation of Δ required to maintain a constant deceleration is for phase II of that reference.

RESULTS AND DISCUSSION

The closed-form solution of this report and that of reference 4 have been employed to study the effect on nonlifting entry trajectories of maintaining a specified deceleration rate and a specified maximum deceleration by varying W/C_DA during entry into the earth's atmosphere at escape velocity.[2] In what follows, the effect of modulation on the time history of W/C_DA, flight-path angle, deceleration, and velocity is discussed first. The effect of modulation on the corridor depth, heating, and range is then discussed.

Trajectory Parameters

Typical time histories of W/C_DA, flight-path angle, deceleration,
and velocity for nonlifting vehicles are presented in figure 1 for unmod- A
ulated and modulated shallow entries into the earth's atmosphere at 5
escape velocity ($\bar{V}_i = 1.4$). The curves shown are for an entry angle 0
$\gamma_i = -6^\circ$. The modulated entries are for a 10g maximum deceleration limit 2
only (for which $(dG/dt)_{max} = 0.95$ when $G = 10$), and for the combined limits of a 0.25g/sec deceleration rate and a 10g maximum deceleration. It is interesting to note that the maximum deceleration rate for an unmodulated 10g entry ($\gamma_i = -4.32^\circ$) is 0.22g/sec. The data in figure 1 for the unmodulated entry and the entry modulated to limit the maximum deceleration only are reproduced from reference 4. For the entry considered above, it can be seen in figure 1(a) that adding the restriction of a 0.25g/sec deceleration-rate limit to a 10g maximum deceleration limit requires that the W/C_DA change capability of the vehicle must be 32 rather than 21. By beginning the modulation at an earlier time to satisfy the added restriction of the deceleration-rate limit, the flight-path curvature is slightly reduced relative to that for the entry which satisfies only the maximum deceleration limit (see fig. 1(b)) and this results in the reduced deceleration rate shown in figure 1(c) during phase II of the modulation period.

Corridor Depth

In this report the overshoot boundary is defined by the trajectory for which a vehicle would pass through just enough atmosphere to reduce the velocity to local circular as the vehicle is about to exit from the atmosphere (see ref. 5). The undershoot boundary is defined by the trajectory for which a specified deceleration rate and/or maximum deceleration is not exceeded. For vehicles for which the initial W/C_DA is identical for both the overshoot and undershoot boundaries, the corridor depth, for successful entry into an exponential atmosphere, depends only upon the allowable entry angle (see ref. 4). Changes in corridor depth considered herein result from the use of drag modulation to maintain specified deceleration time histories during entry along the

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[2]Since the remainder of the report is concerned only with entries into the earth's atmosphere W/C_DA rather than m/C_DA is used. Also, for earth, the value of 30 is used for $\sqrt{\beta r}$, and 27 is used for $1/\sqrt{\beta g}$.

undershoot boundary. For example, increased corridor depths result from lowering the undershoot boundary by the use of modulation which permits entries at steeper angles than are possible without modulation.

Corridor depths and the corresponding entry angles attainable by modulating W/C_DA during shallow nonlifting entries into the earth's atmosphere at escape velocity are shown in figure 2 as a function of the ratio of W/C_DA at the end of modulation to W/C_DA at the beginning of modulation. Results are shown for modulated entries for which the maximum deceleration is limited to 5.0, 7.5, and 10.0g. For these entries the maximum deceleration rate is not limited and increases with increases in entry angle. For the above specified deceleration limits, results are shown which indicate the effect of imposing the additional restrictions of deceleration-rate limits of 0.10, 0.25, and 0.50g/sec. With the restriction of a deceleration-rate limit of 0.10g/sec the maximum deceleration for entry angles less than -5.03° is less than 10g, and the maximum deceleration for entry angles less than -4.32° is less than 7.5g. Therefore, the data in figure 2 (and in all subsequent figures) representing these conditions are faired with a short dashed curve. It can be seen in the figure that the additional restriction of a deceleration-rate limit generally reduces the corridor depth. For example, a vehicle with a change capability in W/C_DA of 21 can enter the earth's atmosphere at escape velocity through a corridor up to 30 miles in depth ($\gamma_i = -6^{\circ}$) and not exceed a maximum deceleration of 10.0g. However, the additional restriction of a deceleration-rate limit of 0.25g/sec reduces the corridor depth to approximately 27 miles, which corresponds to an entry angle of approximately -5.8°. On the other hand, in order to attain a 30-mile corridor and comply with this additional restriction, the change capability in W/C_DA must be increased from 21 to 32. It is interesting to note from reference 6 that to use lift to attain a 10.0g-limited, 30-mile corridor, without modulation, the entering vehicle must have an L/D capability of at least ±0.3. For this unmodulated entry along the undershoot boundary, however, the deceleration rate reaches a maximum value of 0.42g/sec. To limit an unmodulated lifting entry along the undershoot boundary to 10.0g and 0.25g/sec, the L/D must be restricted to 0.05. These restrictions ($-0.3 \leq L/D \leq 0.05$, $K_{max} = 0.25$, and $G_{max} = 10$), with no modulation, provide only a 17-mile entry corridor. This entry corridor, however, could be increased by modulating L/D.

Heating

For the entries considered herein, peak heating occurs during the high-altitude portion of the trajectory where the density and the attendant Reynolds numbers are low; consequently, the following discussion of heating during entry will cover only those results obtained when a laminar boundary layer is considered.

Convective heating rate at the stagnation point.- The convective heating rate per unit area at the stagnation point during entry into the earth's atmosphere, as given in reference 5, is

$$q_s = \frac{590}{\sqrt{g_c}}\sqrt{\frac{W}{C_D AR}}\,\frac{\bar{q}}{\cos^3\gamma} \tag{25}$$

where for entries considered here with limited values of deceleration rate and maximum deceleration

$$\bar{q} = \left\{ \begin{array}{ll} \bar{u}^{5/2}Z^{1/2} & \text{for phases I and IV} \\ B_1^{1/2}\bar{u}^2(a + b\bar{u})^{1/4} & \text{for phase II} \\ B_2^{1/2}\bar{u}^2 & \text{for phase III} \end{array} \right\} \tag{26}$$

A
5
0
2

Equation (25) has been used to calculate the stagnation-point heat-transfer rate for modulated and unmodulated entries which have the same maximum deceleration. Results for the maximum heating rate as a function of corridor depth are shown in figure 3 for deceleration limits of 7.5 and 10.0g, and for deceleration-rate limits of 0.10, 0.25, and 0.50 g/sec. Also shown are the values of maximum deceleration rate $(dG/dt)_{max}$ for the entries which are modulated to satisfy only a maximum deceleration limit. In making these heating-rate calculations it was assumed that the radius of curvature R of the vehicle surface at the stagnation point was identical and constant for all entries. It was also assumed that the initial W/C_DA was the same for all entries. It should be noted that the data presented in figure 3 can be used to calculate the ratio of stagnation-point heating rates for the same modulated and unmodulated entries (same γ_i) for which $R_{mod} \neq R_{unmod}$, provided R remains constant during entry. This is done by multiplying the ratio of heating rates shown in figure 3 by $(R_{unmod}/R_{mod})^{1/2}$. If R varies during entry, the stagnation-point heating rate will depend upon the variation of R with time or velocity (see eqs. (25) and (26)).

The heating-rate penalty incurred by the use of modulation to increase corridor depth can be seen in figure 3. For example, the maximum heating rate for the 10g-limited modulated entry (10g only) which provides a 30-mile corridor is about twice that for the unmodulated entry which provides a 7-mile corridor. Adding the further restriction of limiting the deceleration rate to 0.25g/sec results in a maximum heating rate of almost two-and-a-half times that for the unmodulated entry.

Total convective heat absorbed at the stagnation point.- The total convective heat absorbed per unit area at the stagnation point during a modulated entry into the earth's atmosphere can be written (see ref. 5)

$$Q_s = \frac{15,900}{\sqrt{g_c}} \int \sqrt{\frac{W}{C_D A R}} \left(\frac{d\bar{Q}}{d\bar{u}}\right) d\bar{u} \tag{27}$$

where

$$\frac{d\bar{Q}}{d\bar{u}} = \left\{ \begin{array}{ll} \dfrac{\bar{u}^{3/2}}{Z^{1/2}\cos^2\gamma} & \text{for phases I and IV} \\[2ex] \dfrac{\bar{u}^2}{B_1{}^{1/2}(a + b\bar{u})^{1/4}} & \text{for phase II} \\[2ex] \dfrac{\bar{u}^2}{B_2{}^{1/2}} & \text{for phase III} \end{array} \right\} \tag{28}$$

Equation (27) has been used to calculate the total heat absorbed per unit area at the vehicle stagnation point for modulated and unmodulated entries which have the same maximum deceleration limit. The results as a function of corridor depth are shown in figure 4 for deceleration limits of 7.5 and 10.0g, and for deceleration-rate limits of 0.10, 0.25, and 0.50g/sec. In all calculations the same assumptions were made with regard to vehicle shape and W/C_DA that were made in calculating the heating-rate data of figure 3 (i.e., R is identical and constant during entry, and the initial W/C_DA is identical for all entries). For vehicles which satisfy these assumptions, the penalties in total heat absorbed at the stagnation point can be seen in figure 4. For example, in the case cited previously, in which modulation is used to increase the depth of a 10g-limited corridor from 7 to 30 miles, the total heat absorbed at the stagnation point during the modulated entry which limits only the maximum deceleration is 1.8 times that for the unmodulated entry. Adding the further restriction of limiting the deceleration rate to 0.25g/sec results in the absorption of 2.3 times as much total heat as that for the unmodulated entry. As in the case of the heat-transfer rate, the data of figure 4 may be used, similarly, to calculate the total heat absorbed at the stagnation point for different vehicles where $R_{mod} = \text{constant} \neq R_{unmod}$; otherwise, $R(\bar{u})$ must be available for use in equation (27).

As noted above, the comparison of the heating characteristics of modulated and unmodulated entries was based on the assumption that the initial W/C_DA was the same for all entries. If, instead, it is assumed that the final W/C_DA is the same for all entries, the heating per unit area at the stagnation point will be less for the modulated entries than for the unmodulated entries. Specifically, the data presented in figures 3 and 4 will be reduced approximately by the appropriate ratio of $(\Delta_{begin}/\Delta_{end})^{-1/2}$(i.e., the ordinates of figures 3 and 4 will be less than or equal to unity). The total heat absorbed by the vehicle, however, obtained by integrating the local heat absorbed over the vehicle surface may be greater or less for the modulated entries than for the unmodulated entries, depending upon the vehicle geometry during entry.

Range

In reference 5 the range between any two points of the entry trajectory is defined as the circumferential distance traveled between a point n where the dimensionless velocity is $\overline{u}_n$ and a point $n + 1$ where it is reduced to $\overline{u}_{n+1}$. Thus, from reference 5

$$\frac{\Delta s}{r} = \frac{1}{r}\int_{\overline{u}_n}^{\overline{u}_{n+1}} \overline{u}\left(\frac{dt}{d\overline{u}}\right)d\overline{u} = \frac{1}{\sqrt{\beta r}}\int_{\overline{u}_{n+1}}^{\overline{u}_n} \frac{\cos\gamma}{Z}\,d\overline{u} \tag{29}$$

In the present analysis equation (29) is used to determine the range during phases I and IV. For $\cos\gamma = 1$, $Z(\overline{u})$ as given by equation (12), and $G(\overline{u})$ as given by equation (19), equation (29) can be evaluated to yield the range during phase II as

$$\left(\frac{\Delta s}{r_o}\right)_{II} = \frac{2}{3b^2G_1^{\,2}}\left[(2a - b\overline{u}_2)G_2 - (2a - b\overline{u}_1)G_1\right] \tag{30}$$

Similarly, for $Z(\overline{u})$ as given by equation (7) with constant G, the range during phase III is

$$\left(\frac{\Delta s}{r_o}\right)_{III} = \frac{\overline{u}_2^{\,2}}{2G_2}\left[1 - \left(\frac{\overline{u}_3}{\overline{u}_2}\right)^2\right] \tag{31}$$

Equations (29), (30), and (31) have been used to calculate the range for the same modulated and unmodulated entries for which the corridor depth and heating characteristics have been discussed previously. The results as a function of corridor depth are shown in figure 5 for G_{max} limits of 7.5 and 10.0g, and for the added restrictions of K_{max} limits of 0.10, 0.25, and 0.50g/sec. It can be seen that the range is generally reduced as a result of using modulation to increase corridor depth. However, as shown in figure 5(b) for a 10g-limited corridor, imposing the low deceleration rate $K_{max} = 0.10$ actually increases the range relative to that for the unmodulated 10g-limited entry.

CONCLUDING REMARKS

The differential equation developed in NASA TN D-319 for entries during which the aerodynamic coefficients and vehicle shape vary has been used to obtain a closed-form solution for the motion, heating, range, and variation in W/C_DA for nonlifting entries during which the rate of increase of deceleration is limited. The solution is applicable to vehicles of arbitrary weight, size, and shape, and to arbitrary atmospheres. The solution was used to calculate trajectory parameters for shallow entries into the earth's atmosphere at escape velocity. As an example of the results obtained, the depth of a 10g-limited corridor can be increased from 7 miles for an unmodulated entry to 30 miles for a modulated entry for a vehicle with a W/C_DA change of 21 if no consideration is given to limiting the rate of increase of deceleration. However, if in addition to limiting the maximum deceleration to 10g, the maximum rate of increase of deceleration is limited to 0.25g/sec, a W/C_DA change of 32 is required to obtain the 30-mile corridor. For a given corridor depth, limiting the rate of increase of deceleration and the maximum deceleration results in increases in maximum heating rate, total heat absorbed at the stagnation point, and range, compared with limiting only the maximum deceleration. The heating results were obtained for the case in which the radii of curvature at the stagnation point were constant and equal during all entries, and the initial values of W/C_DA were identical.

Ames Research Center
National Aeronautics and Space Administration
Moffett Field, Calif., May 1, 1961

APPENDIX

CLOSED-FORM SOLUTION FOR m/C_DA, $\bar{u}_2$, AND γ FOR MODULATED SHALLOW NONLIFTING ENTRIES FOR WHICH dG/dt IS CONSTANT

The variation in m/C_DA with velocity required to maintain a constant rate at which the deceleration increases is defined by equation (14). Upon substitution of the dependent variable

$$p = \frac{\Delta'}{\Delta} = \frac{1}{\Delta}\frac{d\Delta}{d\bar{u}} \equiv \frac{1}{m/C_DA}\frac{d(m/C_DA)}{d\bar{u}} \tag{A1}$$

equation (14) reduces to

$$p' + \left[\frac{b}{2(a + b\bar{u})} - \frac{1}{\bar{u}}\right]p = \frac{1 - \bar{u}^2}{{B_1}^2(a + b\bar{u})} + \frac{b^2}{4(a + b\bar{u})^2} + \frac{3b}{2\bar{u}(a + b\bar{u})} - \frac{4}{\bar{u}^2} \tag{A2}$$

The corresponding expression for the flight-path angle is (see eqs. (A1) and (15))

$$\sqrt{\beta r}\,\sin\gamma = B_1\frac{\sqrt{a + b\bar{u}}}{\bar{u}}\left[p + \frac{b}{2(a + b\bar{u})} - \frac{2}{\bar{u}}\right] \tag{A3}$$

Integration of equation (A2) yields ($\sqrt{a + b\bar{u}}/\bar{u}$ is an integrating factor)

$$\left(\frac{\Delta'}{\Delta}\right) = p = \frac{1}{\sqrt{a}\,{B_1}^2}\frac{\bar{u}}{\sqrt{a + b\bar{u}}}\ln\frac{\sqrt{a} - \sqrt{a + b\bar{u}}}{\sqrt{a} + \sqrt{a + b\bar{u}}} + \frac{4a}{3b^2{B_1}^2}\bar{u} - \frac{2}{3b{B_1}^2}\bar{u}^2$$

$$+ \frac{b^2}{2a}\frac{\bar{u}}{(a + b\bar{u})} + \frac{2}{\bar{u}} - \frac{b}{2a} + C\frac{\bar{u}}{\sqrt{a + b\bar{u}}} \tag{A4}$$

where C is the integration constant which can be evaluated in terms of the velocity and flight-path angle at the beginning of modulation; that is, at $\bar{u} = \bar{u}_1$, $a + b\bar{u} = 1$, and from equation (A3),

$$p_1 = \left(\frac{\bar{u}_1}{B_1}\right)\sqrt{\beta r}\,\sin\gamma_1 - \frac{b}{2} + \frac{2}{\bar{u}_1}$$

Thus from equation (A4) evaluated at $\bar{u} = \bar{u}_1$

$$C = \frac{\sqrt{\beta r}}{B_1} \sin \gamma_1 + \frac{1}{\sqrt{a}\, B_1^2} \ln \frac{\sqrt{a} + 1}{\sqrt{a} - 1} - \frac{2(3a - 1)}{3b^2 B_1^2} \tag{A5}$$

Proceeding by integration of equation (A1)

$$\Delta = \frac{m}{C_D A} = \exp\left(\int p \, d\bar{u} + \text{constant}\right) \tag{A6}$$

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Equation (A4) is readily integrable and the integration constant can be evaluated in terms of the velocity and $m/C_D A$ at the beginning of modulation ($\bar{u}_1$ and Δ_1, respectively). Thus, the closed-form solution for the variation of $m/C_D A$ which maintains a constant rate at which the deceleration increases during a shallow nonlifting entry is

$$\begin{aligned}
\frac{\Delta}{\Delta_1} = \frac{m/C_D A}{(m/C_D A)_1} = \exp \Bigg\{ & \frac{2}{3\sqrt{a}\, b^2 B_1^2} X(\xi)(2a - b\bar{u}_1 \xi) \ln \frac{\sqrt{a} + X(\xi)}{\sqrt{a} - X(\xi)} - \ln X(\xi) \\
& - \frac{4aC}{3b^2}\left[X(\xi) - 1\right] + \frac{2C\bar{u}_1}{3b}\left[\xi X(\xi) - 1\right] + \left(\frac{4a}{3b^2 B_1^2} + 2\right) \ln \xi \\
& + \frac{2\bar{u}_1}{3bB_1^2}(1 - \xi) - \frac{2a\bar{u}_1^2}{3b^2 B_1^2}(1 - \xi^2) + \frac{2\bar{u}_1^3}{9bB_1^2}(1 - \xi^3) \\
& - \frac{2(3a - 1)}{3\sqrt{a}\, b^2 B_1^2} \ln \frac{\sqrt{a} + 1}{\sqrt{a} - 1} \Bigg\}
\end{aligned} \tag{A7}$$

where

$$\left.\begin{aligned}
\xi &= \frac{\bar{u}}{\bar{u}_1} \\
X(\xi) &= \sqrt{1 - b\bar{u}_1(1 - \xi)}
\end{aligned}\right\} \tag{A8}$$

and a, b, and B_1 are given by equation (13).

The velocity $\bar{u}_2$ at which modulation should be terminated when consideration is given to limiting only the rate at which the deceleration increases is determined in the following manner. For a given set of entry conditions and a specified limiting deceleration rate, the constants in equation (A7) are fixed, and it is a simple matter to determine from this equation that $m/C_D A$ increases to a maximum value and then decreases. Continuation of the modulation after $m/C_D A$ has reached a maximum would sustain a constant deceleration rate above that which would result if the modulation were terminated. Consequently, the value of the velocity $\bar{u}_2$ is determined as that value for which Δ/Δ_1 is a maximum; that is, the value of velocity obtained which satisfies the equation which results from equating the right-hand side of equation (A4) to zero. When it is desired to terminate this modulation at an earlier velocity at which a specified maximum deceleration is attained, as in the present analysis, the velocity $\bar{u}_2$ is determined as indicated in the ANALYSIS section of this report.

The expression for the flight-path angle during the modulation period for constant deceleration rate is obtained from substitution of equation (A4) into equation (A3) as

$$\sin\gamma = -\frac{B_1}{\sqrt{\beta r}}\left[\frac{1}{\sqrt{a}\,B_1^2}\,ln\,\frac{\sqrt{a} + X(\xi)}{\sqrt{a} - X(\xi)} - \frac{4a}{3b^2B_1^2}\,X(\xi) + \frac{2\bar{u}_1}{3bB_1^2}\,\xi X(\xi) - C\right] \tag{A9}$$

For entries during which it is required that only a specified deceleration rate shall not be exceeded, the end of the modulation phase is matched with the final unmodulated phase as follows. The Z function and Δ are known at the end of the modulation period for constant deceleration rate (eqs. (12) and (16)), and $\Delta'/\Delta = p = 0$ at velocity $\bar{u}_2$; hence, with the aid of equation (A3) and the fact that the flight-path angle for an unmodulated entry is given by (see ref. 5)

$$\sqrt{\beta r}\,\sin\gamma = Z' - \frac{Z}{u} \tag{A10}$$

the solution for the modulated deceleration-rate phase (phase II) and the unmodulated final phase (phase III) are matched by satisfying the conditions

$$\left.\begin{aligned} Z_{III_i} &= B_1\,\frac{\sqrt{a + b\bar{u}_2}}{\bar{u}_2} \\ Z'_{III_i} &= -Z_{III_i}\left[\frac{1}{\bar{u}_2} - \frac{b}{2(a + b\bar{u}_2)}\right] \end{aligned}\right\} \tag{A11}$$

Equations (A11) together with the velocity $\bar{u}_2$ constitute the initial conditions from which the trajectory characteristics during the remainder of the entry are calculated as indicated in reference 5. Where it is desired to terminate this modulation at an earlier velocity at which a specified maximum deceleration is attained, as in the present analysis, the various phases of such a trajectory are matched as indicated in the ANALYSIS section of this report.

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REFERENCES

1. Creer, Brent Y., Smedal, Harald A., Capt., USN (MC), and Wingrove, Rodney C.: Centrifuge Study of Pilot Tolerance to Acceleration and the Effects of Acceleration on Pilot Performance. NASA TN D-337, 1961.

2. Edelberg, Robert, Henry, James P., Maciolek, John A., Salzman, Edwin W., and Zuidema, George D.: Comparison of Human Tolerance to Accelerations of Slow and Rapid Onset. Jour. Aviation Medicine, vol. 27, no. 6, Dec. 1956, pp. 482-489.

3. White, William J.: Variations in Absolute Visual Thresholds During Acceleration Stress. Tech. Rep. 60-34, Wright Air Dev. Div., April 1960.

4. Levy, Lionel L., Jr.: An Approximate Analytical Method for Studying Atmosphere Entry of Vehicles With Modulated Aerodynamic Forces. NASA TN D-319, 1960.

5. Chapman, Dean R.: An Approximate Analytical Method for Studying Entry Into Planetary Atmospheres. NASA TR R-11, 1959.

6. Chapman, Dean R.: An Analysis of the Corridor and Guidance Requirements for Supercircular Entry Into Planetary Atmospheres. NASA TR R-55, 1959.

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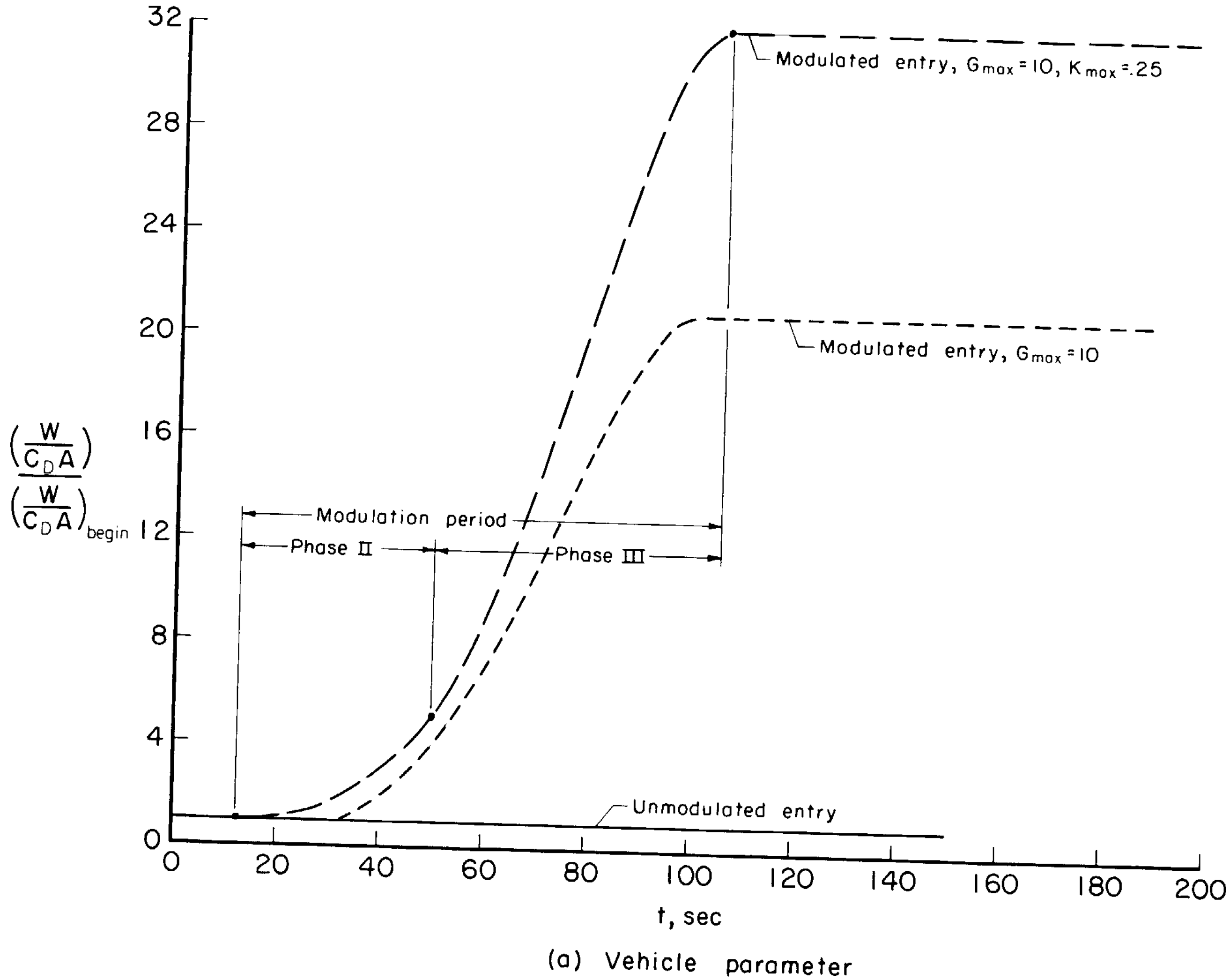
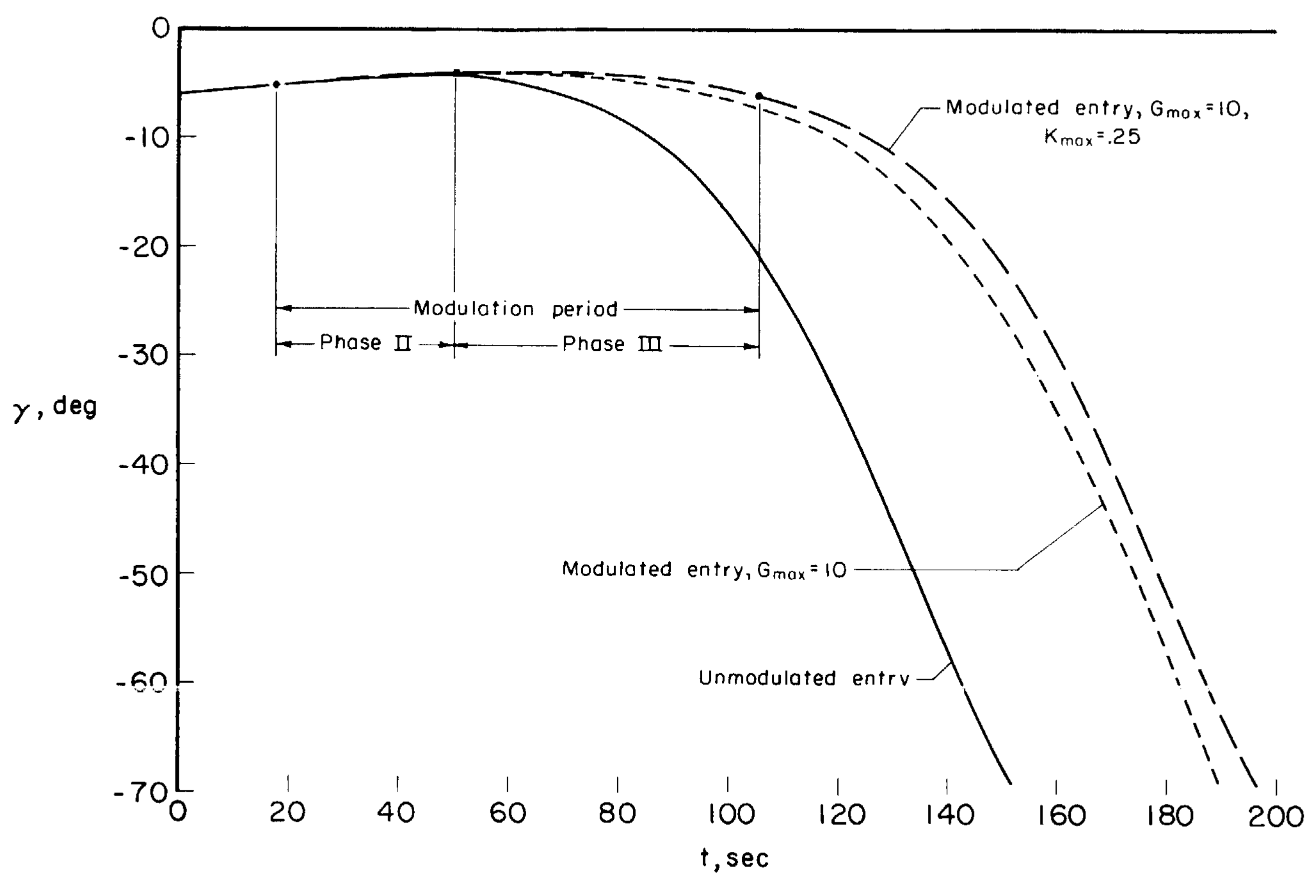


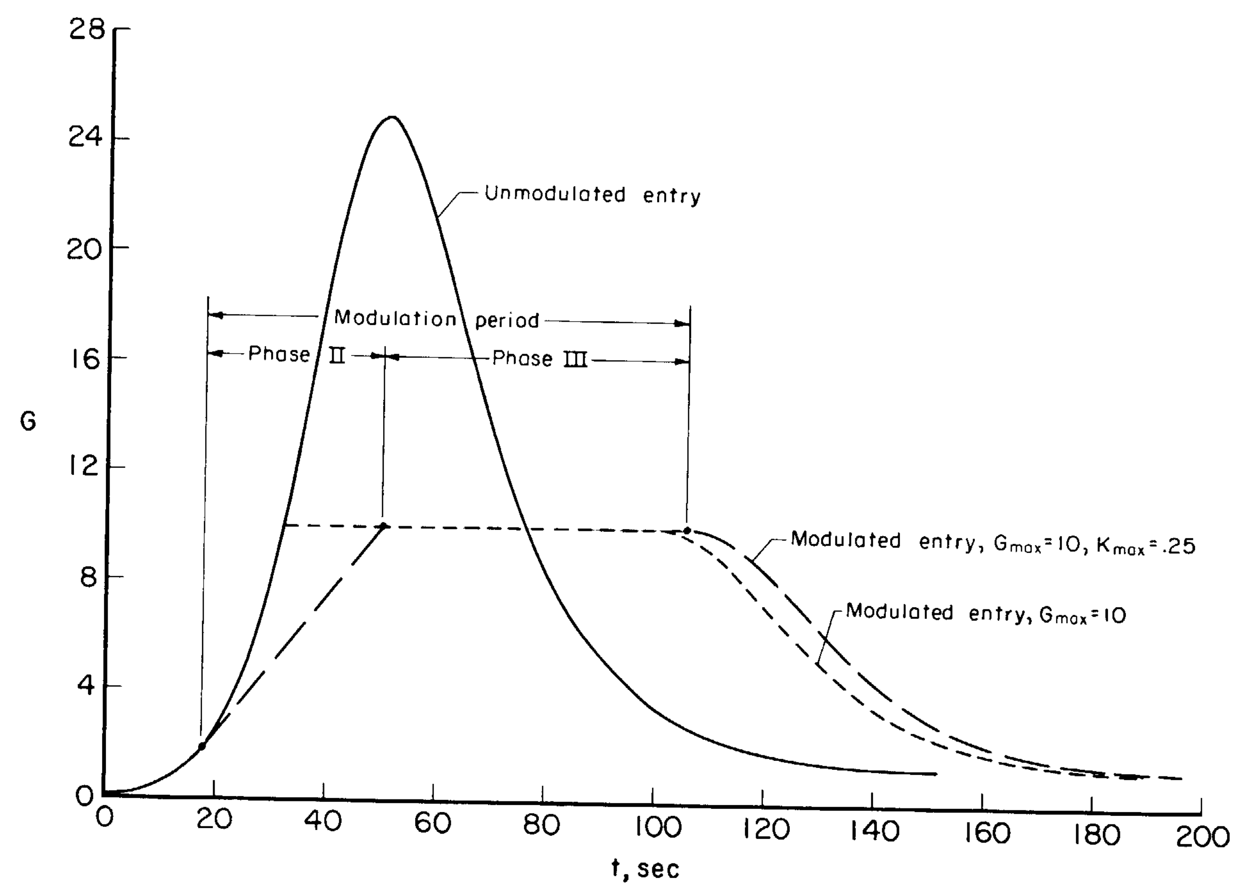
(a) Vehicle parameter.

Figure 1.- Effect of modulation on entry trajectory parameters; $\overline{V}_i = 1.4$, $L/D = 0$, $\gamma_i = -6^\circ$, earth.



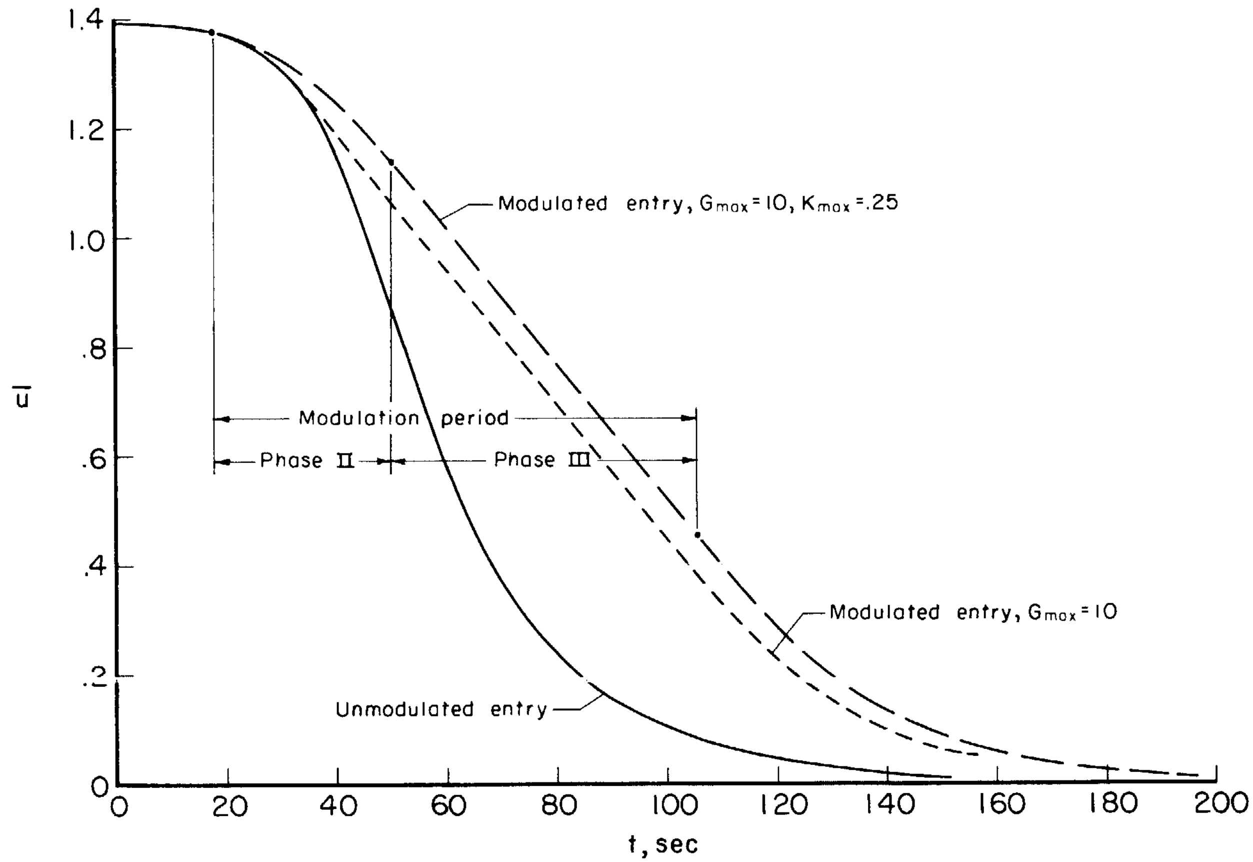
(b) Flight-path angle.

Figure 1.- Continued.



(c) Resultant deceleration.

Figure 1.- Continued.



(d) Tangential velocity.

Figure 1.- Concluded.

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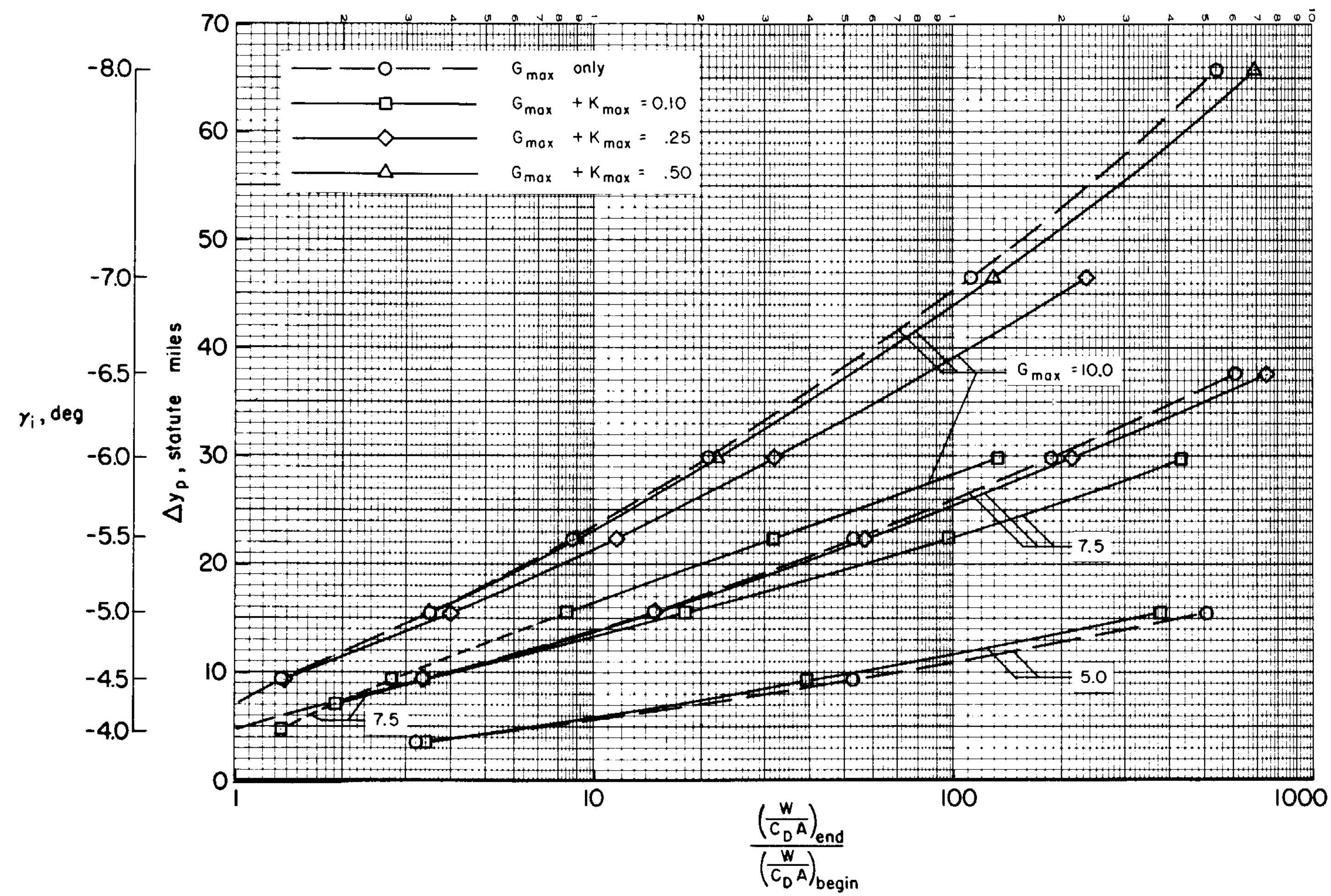
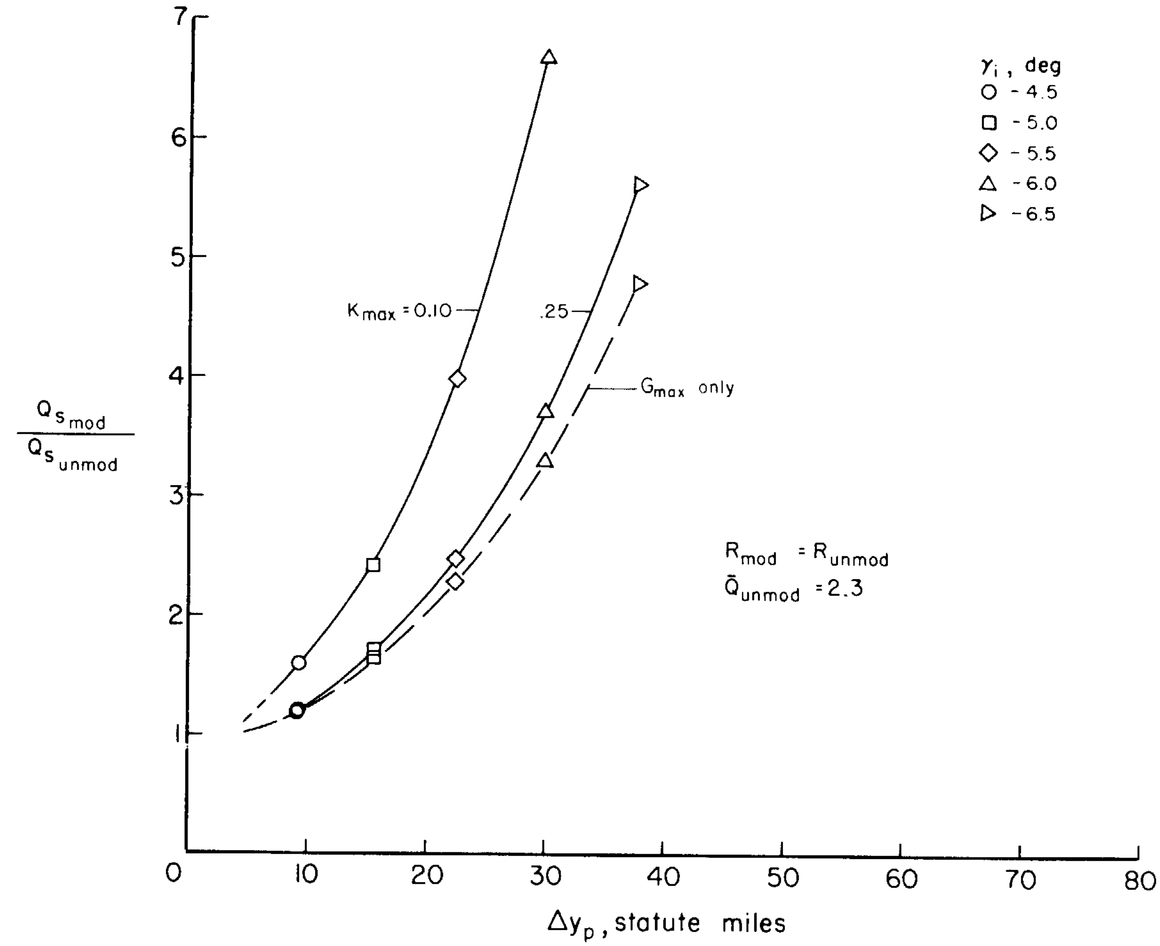
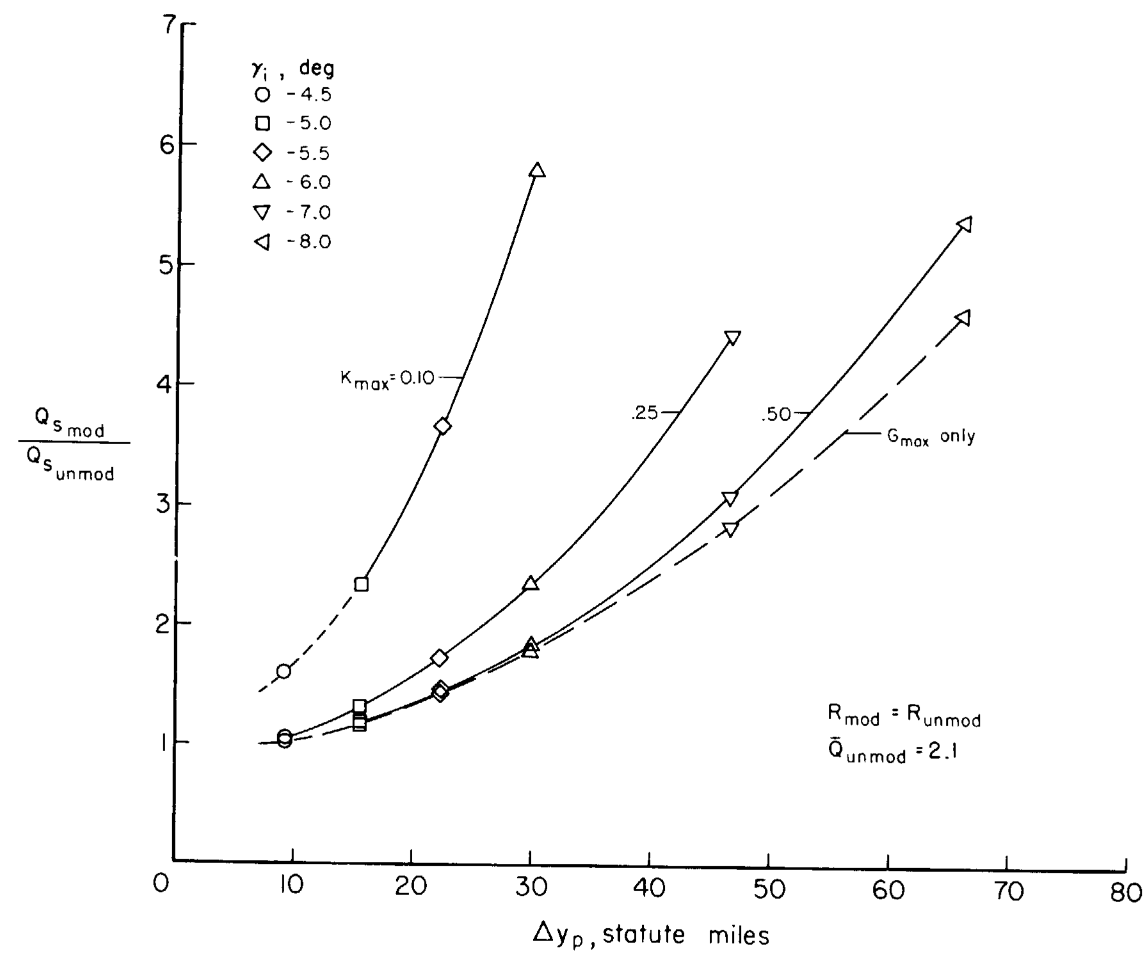


Figure 2.- Effect of varying W/C_DA on the deceleration- and deceleration-rate-limited entry corridor; $\overline{V}_i = 1.4$, $L/D = 0$, earth.



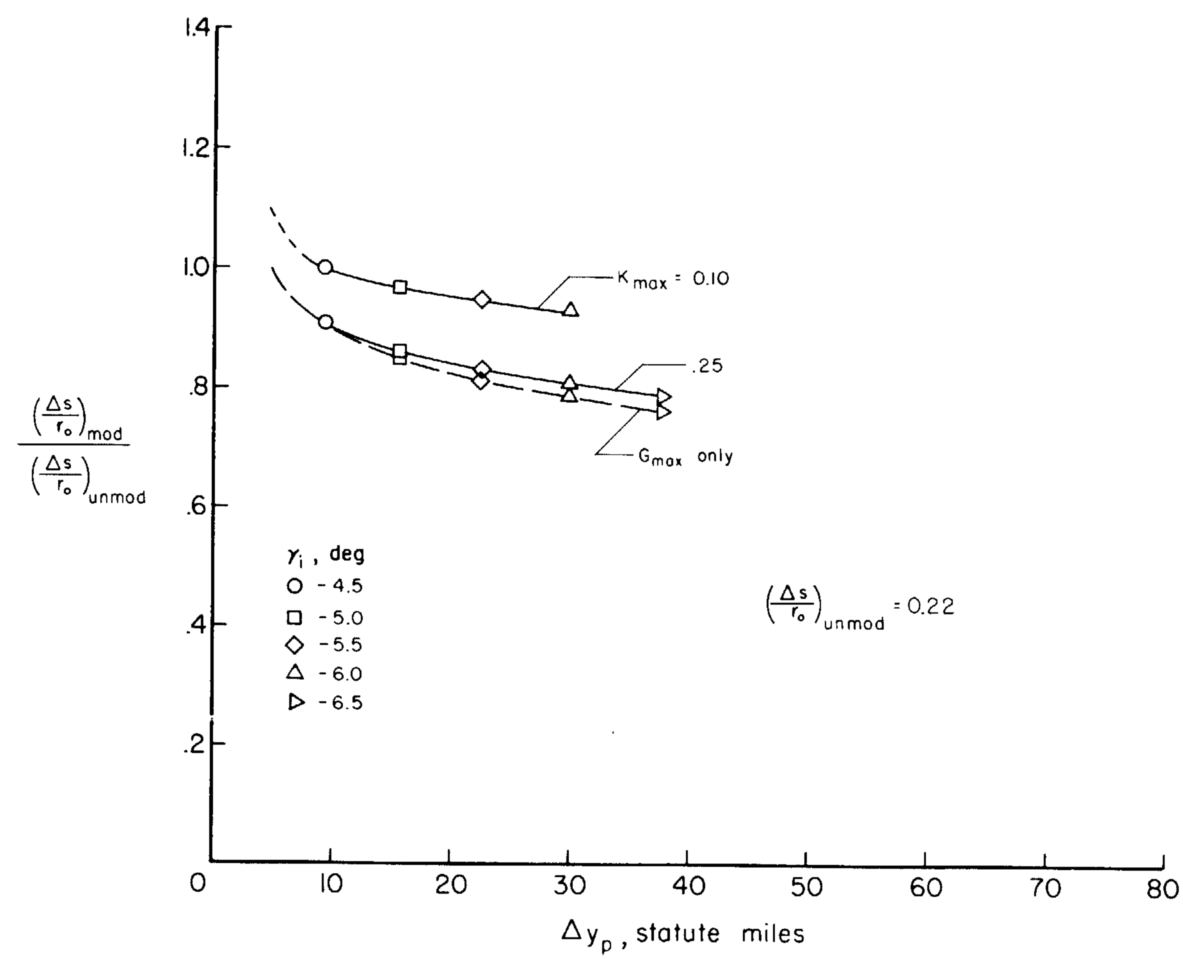
(a) $G_{max} = 7.5$

Figure 4.- Total laminar convective heat absorbed per unit area at the stagnation point associated with the use of modulation along the undershoot boundary of a deceleration- and deceleration-rate-limited entry corridor; $\bar{V}_i = 1.4$, $L/D = 0$, earth.



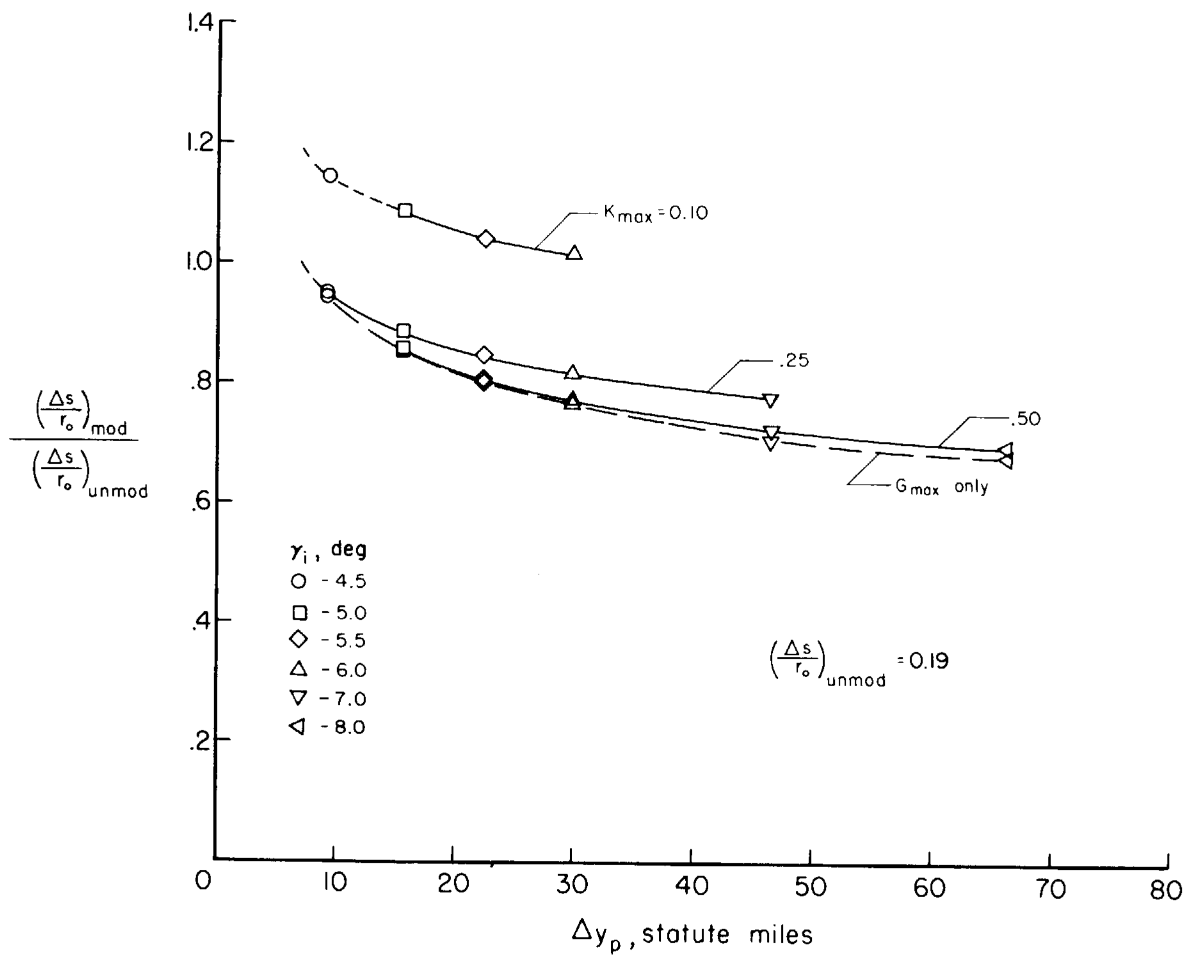
(b) $G_{max} = 10.0$

Figure 4.- Concluded.



(a) $G_{max} = 7.5$

Figure 5.- Range associated with the use of modulation along the undershoot boundary of a deceleration- and deceleration-rate-limited entry corridor; $\bar{V}_i = 1.4$, $L/D = 0$, earth.



(b) G_{max} = 10.0

Figure 5.- Concluded.