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PROJECT ECHO — SYSTEM CALCULATIONS

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SUMMARY

The primary experimental objective of Project Echo was the transmission of radio communications between points on the earth by reflection from the balloon satellite. This paper describes system calculations made in preparation for the experiment and their adaptation to the problem of interpreting the results. The calculations include path loss computations, expected audio signal-to-noise ratios, and received signal strength based on orbital parameters.

PREFACE

The Project Echo communications experiment was a joint operation by the Goddard Space Flight Center of the National Aeronautics and Space Administration (NASA), the Jet Propulsion Laboratory (JPL), the Naval Research Laboratory (NRL), and the Bell Telephone Laboratories (BTL). The equipment described herein, although designed by BTL as part of its own research and development program, was operated in connection with Project Echo under contract NASW-110 for NASA. Overall technical management of Project Echo was the responsibility of NASA's Goddard Space Flight Center.

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INTRODUCTION

A satellite communication system such as that of the Project Echo experiment is influenced by many factors such as power output, frequency, antenna gain, free-space path loss, receiver noise temperature, and method of modulation — factors which are also common to point-to-point microwave systems. Three other factors must be considered in the design of satellite communication systems; all three are functions of satellite position in the region of mutual visibility. They are:

1. Variations in free-space path loss
2. Variations in sky noise temperature
3. Loss in the earth's atmosphere.

This report shows how these system parameters are used to predict the performance of the voice circuits that constitute the Echo communications experiment. This discussion assumes normal propagation conditions and does not take into account statistical occurrences such as attenuation due to rainfall or multipath fading, which are beyond the scope of this paper.

FREE-SPACE PATH LOSS FORMULA AND COMPUTATIONS

Assume a transmitting antenna with actual gain G_1, radiating a power of P_T watts. The power density φ_1 at a distance d_1 will then be

$$\varphi_1 = G_1 \frac{P_T}{4\pi d_1^2} .$$

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The amount of power intercepted by an object of projected area σ will then be

$$P_1 = \phi_1 \sigma .$$

A sphere, in effect, radiates this energy isotropically; hence the power density ϕ_2 at a distance d_2 from the sphere will be

$$\phi_2 = \frac{P_1}{4\pi d_2^2} .$$

The amount of power P_R received by an antenna with effective aperture area A_2 in this field is

$$P_R = \phi_2 A_2 = \phi_2 \frac{\lambda^2 G_2}{4\pi} ,$$

where λ is the wavelength and

$$G_2 = \frac{4\pi A_2}{\lambda^2} .$$

After suitable substitutions the received power is

$$P_R = G_1 G_2 \frac{\lambda^2 \sigma P_T}{(4\pi)^3 \, d_1^2 \, d_2^2} . \tag{1}$$

Rearranging Equation 1 gives the free-space path loss L :

$$L = \frac{P_T}{P_R} = \frac{(4\pi)^3 \, d_1^2 \, d_2^2}{G_1 G_2 \lambda^2 \sigma} . \tag{2}$$

This expression serves to calculate the expected free-space path loss, provided that the various parameters can be determined to sufficient accuracy. The presence of $d_1^2 d_2^2$ in Equation 2 shows that the expected free-space path loss L is a function of the satellite position.

In order to compute the free-space path loss, antenna gains and frequencies of operation are required. These constants are given in Table 1.

Table 1

Antenna Gains and Frequencies of Operation
for Computing Free-Space Path Loss

Antenna	Gain (db)	Line Loss (db)	Net Gain (db)
BTL 2390 Mc horn	43.3 ± 0.16	0	43.3 ± 0.16
BTL 960 Mc dish	43.1 ± 0.1	0.5	42.6 ± 0.1
BTL 961 Mc dish	32.6 ± 0.2	0	32.6 ± 0.2
JPL 2390 Mc dish	53.7*	0.4	53.3
JPL 960 Mc dish	45.8 ± 0.6	0.2	45.6 ± 0.6
NRL 2390 Mc dish	50.2*	1.6*	48.6

*Estimated values, not measured.

The free-space path loss has been computed from Equation 2 for the Echo I satellite balloon as a function of position for the two-way path between the Jet Propulsion Laboratory (JPL) facility at Goldstone Lake, California and the Bell Telephone Laboratories (BTL) station at Holmdel, New Jersey. The results are given in Figures 1 and 2.

The balloon scattering cross section σ was assumed to be that of a 100-foot-diameter sphere, perfectly conducting and many wavelengths in diameter, so that

$$\sigma = \frac{\pi(100)^2}{4} = 7854 \text{ ft}^2 .$$

The frequency in the east-west direction was 960 Mc; in the west-east direction it was 2390 Mc. Figure 1 is a plot of the free-space path loss versus satellite altitude when the balloon was midway between these terminals.

Figure 2 shows contours of constant free-space path loss relative to the loss at mid-path for a satellite height of 1000 statute miles, with the radius of the earth taken to be 3950 miles. The contours, in steps of 1 db, are plotted on a stereographic projection. The orbital inclination of Echo I is 47.27 degrees, which limits the northern extent of mutual visibility. The equations necessary for these computations are derived in Appendix A.

Because the Naval Research Laboratory (NRL) facility at Stump Neck, Maryland is only 200 miles from Holmdel, New Jersey, the free-space path loss from Stump Neck to Holmdel can be computed as the round-trip loss from either of these two locations to the satellite. The error in this assumption is less than 0.6 db (Appendix B) for any position in the area of mutual visibility. Figure 3 shows the contours of constant free-space path loss for this case. The free-space path loss is 178.7 db at 2390 Mc for a satellite altitude of 1000 miles.

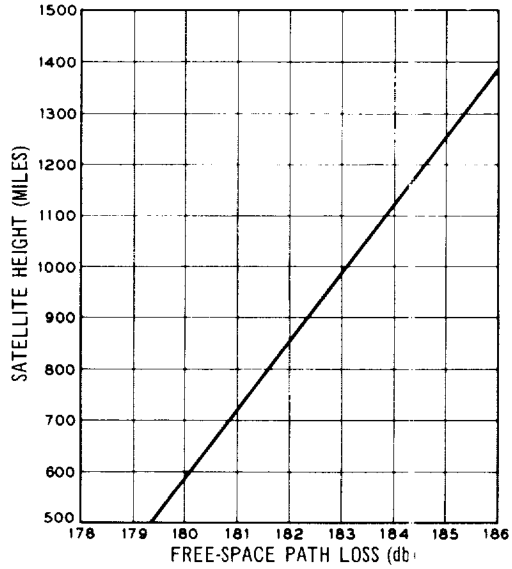

Figure 1 - Free-space path loss versus satellite altitude for Echo I midway between Holmdel, New Jersey and Goldstone Lake, California (east-west direction; for west-east, subtract 0.5 db, owing to the difference in frequency and antenna gain)

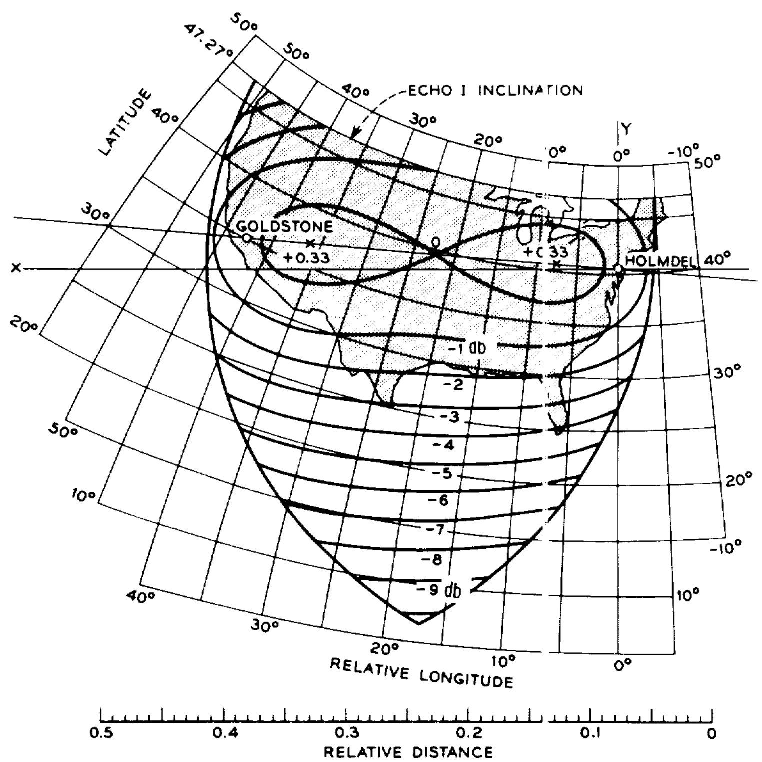

Figure 2 - Contours of constant free-space path loss relative to the loss at midpath between Holmdel, New Jersey and Goldstone Lake, California for a satellite height of 1000 miles, with the earth's radius assumed to be 3950 miles

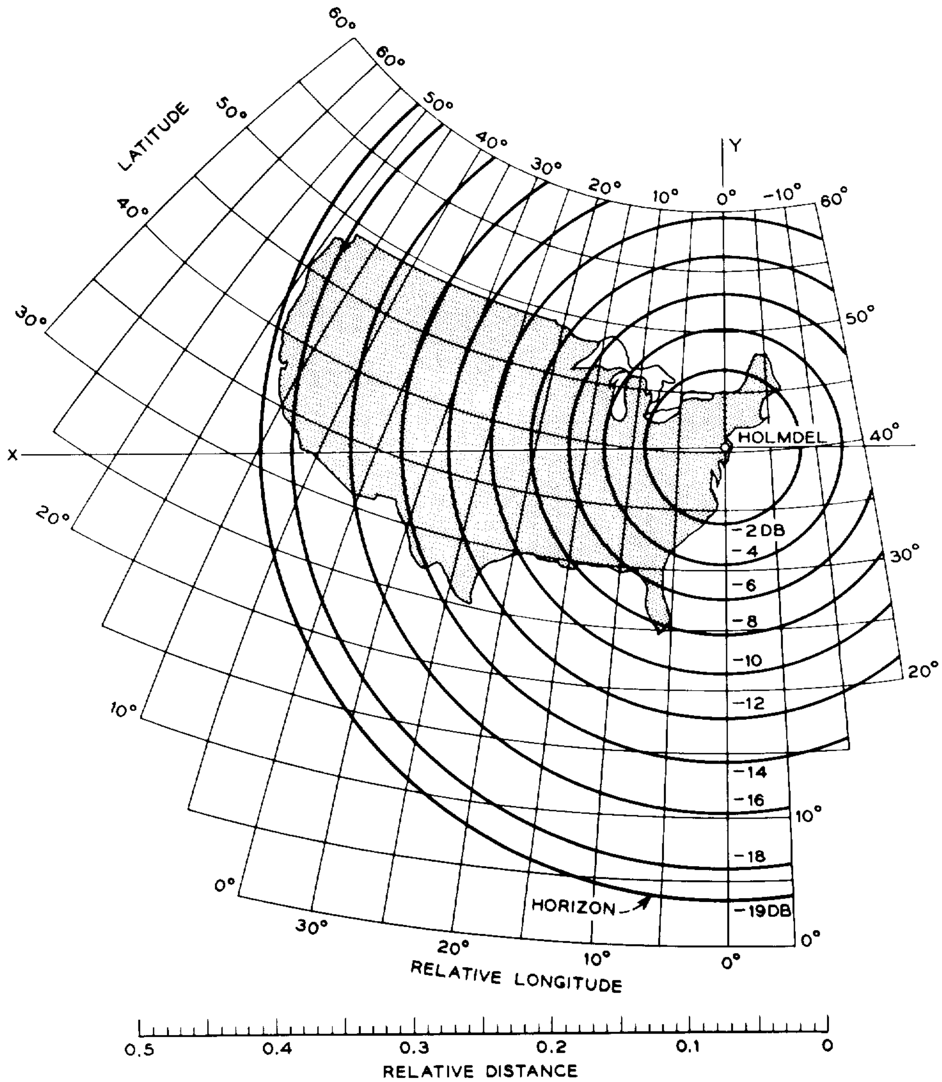

Figure 3 - Contours of constant free-space path loss relative to the loss at midpath between Stump Neck, Maryland and Holmdel, New Jersey

It can be noted from Figures 2 and 3 that the difference between maximum and minimum free-space path loss on the JPL-BTL path is about 10 db, while for the NRL-BTL path this difference is 19 db. It should be also noted that between NRL and BTL this maximum difference is encountered twice on every pass, while the maximum difference almost never occurs on the JPL-BTL path.

EXPECTED SIGNAL-TO-NOISE RATIOS IN VOICE CIRCUITS

The signal-to-noise power ratios (S/N) to be expected depend on the type of modulation technique employed, as well as upon the received carrier-to-noise power ratio (C/N). The first step is to compute the C/N at the receiver for suitable conditions, and then to discuss the modulation methods and voice band signal-to-noise performance. This has been done for the two-way voice path between JPL and BTL, and the results are given in Table 2. The satellite is assumed to be midway between the terminals.

Table 2
Communication Parameters for Echo I Midway Between Goldstone Lake, California and Holmdel, New Jersey

Parameter	East-to-West	West-to-East
Transmitted power:	+70 dbm (10 kw)	+70 dbm (10 kw)
Frequency:	960.05 Mc	2390 Mc
Transmitting antenna net gain:	42.6 db	53.3 db
Receiving antenna: net gain:	45.6 db	43.3 db
Free-space path loss:	183.1 db	182.6 db
Loss through atmosphere:	0 db	0 db
Received carrier power:	-113.1 dbm	-112.6 dbm
Receiver system noise temperature:	350 °K	25 °K
Receiver noise power in 6-kc band:	-135.4 dbm	-146.8 dbm
Carrier-to-noise ratio at receiver:	22.3 db	34.2 db

For other positions of the Echo I balloon in the region of mutual visibility, the C/N ratio is modified by three effects:

1. Variations in free-space path loss
2. Variations in sky noise temperature
3. Loss in the earth's atmosphere.

The first effect has been discussed earlier and the correction for position can be made from Figure 2 for any satellite position. The remaining two effects have been discussed by Hogg[1] and by DeGrasse, Hogg, Ohm, and Scovil.[2] For example, the sky noise temperature and atmospheric loss can be calculated when the antennas are pointed at the horizon. The loss through the atmosphere and system noise temperature are then 3.2 db and 435 °K for the east-west path, and 4.2 db and 110 °K for the west-east path; this would be the worst case. For elevation angles above 10 degrees, however, these effects are essentially negligible.

The audio S/N depends to a considerable extent on the modulation technique. Three techniques are considered here: single-sideband (SSB), FM, and FM with feedback (FMFB).

The transmitters are assumed to be peak-power-limited and the audio signal to be the maximum rms sine wave obtainable. The audio bandwidth is 3 kc, and the noise bandwidth is assumed to be 6 kc.

The maximum transmitted rms sine wave power is 3 db less than the transmitter peak C. However, for the SSB technique, the noise bandwidth may be reduced to 3 kc, resulting in an audio S/N which is equal to C/N:

$$(S/N) = (C/N) .$$

The audio S/N for the case of frequency modulation is given by the standard FM formula, which applies when the receiver input is above the threshold:

$$(S/N) = 3M^2 (C/N) ,$$

where M is the index of modulation. This index for the Echo experiment was 10, so when the receiver was operated above the threshold the S/N is 25 db better than that for SSB. However, the threshold for this receiver occurs at a C/N of approximately 22 db, because the noise bandwidth required to accommodate this signal is about 66 kc.

The audio S/N for the FM receiver with feedback (FMFB) is the same as that for FM when the C/N is above the threshold. However, this receiver[3] has a threshold near C/N = 13 db. At any C/N equal to or greater than 13 db, the audio S/N exceeds that for SSB by 25 db.

Based on the foregoing, the expected audio S/N ratios for the satellite when it is midway between JPL and BTL are as shown in Table 3.

Table 3

Expected Audio Signal-to-Noise Ratios for the Three Modulation Techniques When the Satellite is Midway Between the Terminals (Overhead in the Case of NRL-BTL)

Path of Transmission	Expected Audio S/N (db)	
	SSB	FM or FMFB
BTL-JPL (E-W)	22.3	47.3
JPL-BTL (W-E)	34.2	59.2 (57 db measured*)
NRL-BTL	38.6	63.6 (> 57 db measured*)

*The maximum S/N obtainable in this receiver is limited by the audio amplifier noise. This begins to be significant at a S/N of about 50 db and accounts for the difference between the computed and measured S/N ratios.

The discussion above has been for the JPL-BTL circuit; however, the same general comments would apply to the NRL-BTL circuit with the path loss modified according to the free-space path loss differences shown in Figure 3 and the difference in free-space path loss when the satellite is directly above the terminals. The expected audio S/N when the satellite is directly over the terminals is included in Table 3.

RECEIVED SIGNAL STRENGTH USING ORBITAL PARAMETERS

The foregoing material was based on the assumption that the satellite orbit was circular and the altitude was 1000 miles. After the experiment was underway it was necessary to compute the loss for the known position of the satellite in order to compare the measured and theoretical received signal amplitudes. For this purpose a program was written for the IBM 7090 computer to calculate a path loss parameter L for two given stations using Echo I at the same time:

$$L(t) = 10 \log_{10} \left[(4\pi)^3 \frac{d_1^2(t)\, d_2^2(t)}{\lambda^2 \sigma} \right],$$

where t refers to time. The inputs to this program are the orbital elements, station coordinates, frequency, and balloon cross section. The received power, in decibels, is then

$$(P_R) = 10 \log_{10} (G_1 G_2 P_T) - L(t) .$$

To save computer time, $L(t)$ was calculated for only one frequency, 2390 Mc, since the values only differed by a constant from those of another frequency. Calling this value $L_0(t)$, and using the antenna gains from Table 2, the expressions given in Table 4 for received power in dbm were derived.

Table 4
Received Power for a Transmitted Power of 10 kw

Path	Frequency (Mc)	Received Power P_R (dbm)
BTL-JPL	960	$166.1 - L_0 \pm 0.7$
JPL-BTL	2390	$166.6 - L_0$
NRL-BTL	2390	$162.9 - L_0$
BTL-BTL	961	$-122.0 - 40 \log_{10} d$ (d = slant range in km)

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1. Hogg, D. C., "Effective Antenna Temperatures Due to Oxygen and Water Vapor in the Atmosphere," J. Appl. Phys. 30(9):1417-1419, September 1959

2. DeGrasse, R. W., Hogg, D. C., Ohm, E. A., and Scovil, H. E. D., "Ultra-Low-Noise Antenna and Receiver Combination for Satellite or Space Communications," Proc. Nat. Elect. Conf. 15:370-379, 1959

3. Ruthroff, C. L., "FM Demodulators with Negative Feedback," Bell System Tech. J. 40(4):1149-1156, July 1961; also NASA Technical Note D-1134, 1961

APPENDIX A

Free-Space Path Loss as a Function of Satellite Position

The variation in free-space path loss as a function of the position of the satellite in the region of mutual visibility can be understood by examining the behavior of $d_1^2 d_2^2$ in Equation 2 of this report. The geometry is shown in Figure A1. The distances from the satellite to terminals H and G are d_1 and d_2 respectively. The path loss is proportional to $d_1^2 d_2^2$. From the law of cosines,

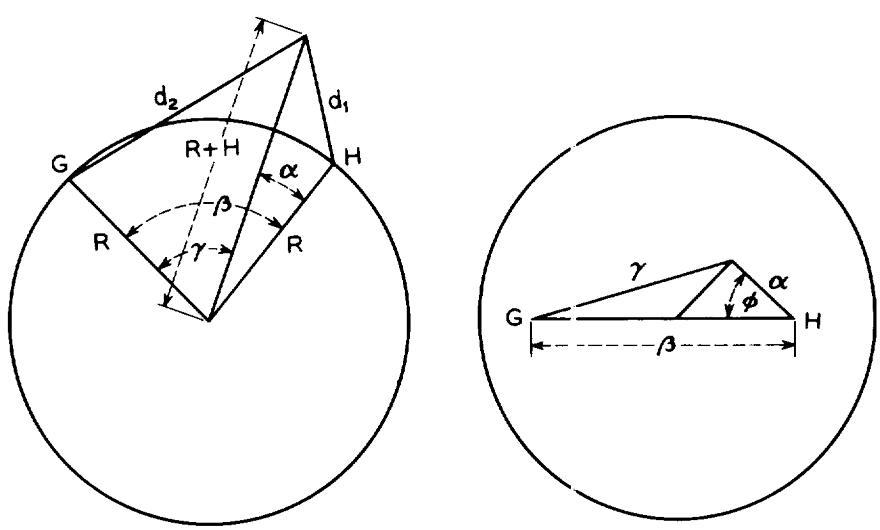
$$\left.\begin{aligned} d_1^2 &= R^2 + (R+h)^2 - 2R(R+h)\cos\alpha = A - B\cos\alpha , \\ d_2^2 &= R^2 + (R+h)^2 - 2R(R+h)\cos\gamma = A - B\cos\gamma , \end{aligned}\right\} \tag{A1}$$

where

$$\cos\gamma = \cos\phi \sin\alpha \sin\beta + \cos\alpha \cos\beta .$$

Now let $M = B/A$ and normalize to A:

$$\frac{d_1^2 d_2^2}{A^2} = \left(1 - M\cos\alpha\right)\left(1 - M\cos\phi\sin\alpha\sin\beta - M\cos\alpha\cos\beta\right) . \tag{A2}$$



(a) (b)

Figure A1 - Satellite geometry for path-loss calculations

Solving for $\cos \phi$ results in

$$\cos \phi = \frac{\left(1 - M \cos \alpha\right)\left(1 - M \cos \alpha \cos \beta\right) - \left(\frac{d_1^2 d_2^2}{A^2}\right)}{M(1 - M \cos \alpha)\ (\sin \alpha \sin \beta)}, \tag{A3}$$

with

$$M = \frac{2R(R + h)}{R^2 + (R + h)^2},$$

$$\beta = \text{central angle between terminals H, G,}$$

$$\frac{d_1^2 d_2^2}{A^2} = \text{normalized path-loss parameters.}$$

We are interested only in points which do not fall below the horizon. Thus, α has a maximum determined by

$$\cos \alpha_{max} = \frac{R}{R + h}. \tag{A4}$$

The normalized path-loss parameter when the satellite is midway between the terminals is found by noting that $\phi = 0$ and $\alpha = \gamma = \beta/2$. Thus

$$\left(\frac{d_1^2 d_2^2}{A^2}\right)_0 = \left(1 - M \cos \frac{\beta}{2}\right)^2. \tag{A5}$$

The maximum value of this parameter occurs when the satellite appears on the horizon to both terminals. For this case, $\gamma = \alpha_{max}$ and

$$\left(\frac{d_1^2 d_2^2}{A^2}\right)_{max} = \left(1 - \frac{MR}{R + h}\right)^2. \tag{A6}$$

When the satellite is at midpath for an altitude of 1000 miles, the values of these parameters are:

$$\begin{aligned} R &= 3950 \text{ miles} \\ \alpha &= \beta/2 = 16.89 \text{ degrees} \\ A &= 4.01 \times 10^7 \text{ miles}^2 \\ B &= 3.91 \times 10^7 \text{ miles}^2 \\ M &= 0.975 \\ d_1^2 d_2^2 &= 7.2 \times 10^{12} \text{ miles}^4. \end{aligned}$$

A program has been prepared for the IBM 704 computer employing Equations A3-A6, and the path-loss contours of Figures 2 and 3 of this report were plotted from these data. Negative signs indicate increasing loss.

The points of minimum path loss are found by setting $\phi = 0$ in Equation A2 and differentiating with respect to α:

$$\frac{d}{d\alpha}\left(\frac{d_1{}^2 d_2{}^2}{A^2}\right) = M \sin \alpha \left[1 - M \cos (\beta - \alpha)\right] - \left(1 - M \cos \alpha\right) M \sin (\beta - \alpha) . \qquad \text{(A7)}$$

The desired points will be solutions of the equation obtained by setting Equation A7 equal to zero; $\alpha = \beta/2$ is a solution, but it is not necessarily a point of minimum loss. By the usual tests we have the following: the midpath point is a maximum loss point if $\cos (\beta/2) - M$ is negative. Conversely, if $\cos (\beta/2) - M$ is positive, the midpath is a minimum loss point.

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APPENDIX B

Error Incurred by Assuming Round-Trip Path Loss from Either Terminal

When the terminals are close together, as are Stump Neck and Holmdel, their central angle β is small, and the path-loss computations are simplified, because $d_1^2 d_2^2 \approx d_1^4$. It is important to derive the maximum error incurred by using this approximation. From Equation A1 of Appendix A,

$$d_2^2 = d_1^2 + 2R(R + h)(\cos \alpha - \cos \gamma) , \qquad \text{(B1)}$$

where

$$\cos \gamma = \cos \phi \sin \alpha \sin \beta + \cos \alpha \cos \beta .$$

Substituting for $\cos \gamma$ results in

$$d_2^2 = d_1^2 + 2R(R + h) \left[\cos \alpha (1 - \cos \beta) - \cos \phi \sin \alpha \sin \beta\right] . \qquad \text{(B2)}$$

If $\beta \ll \pi/2$, this reduces to

$$d_2^2 = d_1^2 - 2R(R + h)\beta \sin \alpha \cos \phi . \qquad \text{(B3)}$$

The ratio of the two sides of the approximation is

$$\frac{d_1^2 d_2^2}{d_1^4} = \frac{d_2^2}{d_1^2} = 1 - \frac{2\beta R(R + h) \cos \phi \sin \alpha}{R^2 + (R + h)^2 - 2R(R + h) \cos \alpha} . \qquad \text{(B4)}$$

The path-loss error in decibels is given by 10 $\log_{10}$ [Equation B4]. The error will be maximum when $\cos \phi = \pm 1$ and $\alpha = \alpha_{max}$. Assuming that $R = 3950$ miles, $h = 1000$ miles, and $\cos \phi = -1$, then $\alpha_{max} = 37.1$ degrees, and

$$\frac{d_2^2}{d_1^2} = 1 + 2.65\beta, \quad \beta \ll \pi/2 . \qquad \text{(B5)}$$

For the NRL-BTL path, $\beta \approx 200/3950 = 0.0506$ radian, and $d_2^2/d_1^2 = 1.134$. The maximum error is 10 $\log_{10} 1.134 = 0.546$ db.

NASA-Langley, 1961