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POINT RETURN FROM A LUNAR MISSION FOR A VEHICLE THAT
MANEUVERS WITHIN THE EARTH'S ATMOSPHERE

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SUMMARY

An investigation has been made of point return of a vehicle with a lift-to-drag ratio of $1/2$, returning from a lunar mission. It was found that the available longitudinal and lateral range allowed considerable tolerances in entry conditions for a point return. Longitudinal range capability for a vehicle that was allowed to skip to an altitude not exceeding 400 miles was about $3-1/2$ times greater than the range capability of a vehicle that was restricted to remain in the atmosphere after entry. Longitudinal range is very sensitive to changes in both velocity and flight-path angle at the bottom of the first pull-out and at exit. An investigation showed that after a skip a vehicle could be placed in a circular orbit for a relatively modest weight penalty. A skip maneuver was found to have no effect on lateral range when the roll was initiated at a velocity near satellite speed after the vehicle had re-entered the atmosphere. However, when the roll was initiated at the earliest possible time along the undershoot boundary, lateral range was increased by a factor of about $2-1/2$. The tolerable errors in time of arrival and in inclination of the orbital plane at point of entry were greater for the skip trajectory than for the no-skip trajectory.

INTRODUCTION

It is desirable that a vehicle entering the earth's atmosphere on return from a lunar mission be able to land at a predetermined site. The ability to land at a desired destination, for the purpose of the present paper, is termed "point return." The point return of a vehicle critically depends on the inclination of the orbital plane, the flight-path angle, velocity, and time of entry into the atmosphere. Point return is possible for any vehicle if the desired entry trajectory parameters can be attained by midcourse guidance. In practice, these parameters can be expected to be in error. Errors in time of arrival and/or orbital-plane orientation must be adjusted by maneuvering the vehicle within the earth's atmosphere. Conversely, maneuvering of a vehicle allows guidance errors in the entry trajectory parameters (ref. 1).

Limited studies of the lateral and longitudinal ranges of vehicles have been reported. An approximate analytical method for determining the lateral range of a vehicle is reported in reference 2. The range of a vehicle with a lift-to-drag ratio of $1/2$ that is restricted to remain in the atmosphere after entry is reported in reference 3. A logical way to achieve greater range would be to consider the range made possible by a single skip maneuver. A further extension of the skip maneuver would be to place the vehicle in a near-earth orbit in which it could wait for the desired location to move into range before beginning the final descent.

It is the purpose of the present paper to determine the point return of a vehicle with a lift-to-drag ratio of $1/2$, and also to examine briefly the effect on range of increasing the L/D to 2. The point return of a vehicle restricted to remain in the atmosphere after entry will be compared to that of a vehicle allowed to skip to an apogee altitude of 400 miles. Fuel requirements for placing a vehicle in a circular near-earth orbit will be studied. The consequence of the maneuvering capability of a vehicle on the entry trajectory parameters will be investigated.

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NOTATION

a	deceleration, g
A	reference area of vehicle, ft ²
C _D	drag coefficient
g	earth's gravitational acceleration, ft/sec ²
i	inclination of orbital plane with respect to earth's equatorial plane, deg
I _{sp}	specific impulse, sec
$\frac{L}{D}$	lift-drag ratio
m	mass of vehicle, slugs
m _F	mass of fuel, slugs
q _∞	free-stream dynamic pressure, lb/ft ²
r	radius from center of earth, ft
R	longitudinal range, statute miles
t	time, sec

V	velocity of vehicle, ft/sec
V_s	local satellite velocity, ft/sec
W	weight of vehicle, lb
W_f	weight of fuel, lb
y	altitude, ft or miles
γ	flight-path angle, deg
ϕ	roll angle, deg

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Subscripts

a	conditions at apogee of skip
E	conditions at entry, 400,000 ft
o	conditions at surface of earth, along the equator
x	conditions at exit, 400,000 ft

RANGE CAPABILITY

Trajectory Analysis

The equations of motion used to calculate the trajectories discussed in this report are given in the appendix. Results were obtained for the simplified case of a nonrotating earth. Entry was assumed to occur at an altitude of 400,000 feet and a velocity of 36,068 feet per second with the vehicle heading along the equator. The $W/C_D A$ for the vehicle was assumed constant at a value of 50 pounds per square foot. The atmosphere model used was that proposed by ARDC in 1959 (ref. 4).

Additional conditions and definitions are as follows:

(a) The undershoot boundary was limited to the largest flight-path angle at entry such that the deceleration would not exceed 10 g.

(b) The overshoot boundary was limited to the smallest flight-path angle at entry such that the vehicle was "captured" by the earth's atmosphere while maximum L/D was applied in a downward direction.

(c) For some trajectories, a single skip maneuver was permitted in which the skip was limited to an altitude of 400 miles.

(d) Instantaneous modulations of L/D were applied at discrete points along the trajectories.

(e) The vehicle was considered to be at its destination when the velocity had decreased to 1000 ft/sec.

Longitudinal Range, $L/D = 1/2$

For the above conditions with L/D of $1/2$, the flight-path angles at entry for overshoot and undershoot boundaries were found to be -4.62° and -7.48° , respectively. The minimum and maximum ranges along both boundaries are shown in figure 1. The difference between the maximum range along the undershoot boundary and the minimum range along the overshoot boundary can be considered the available range from any entry in the corridor. No attempt was made to find the optimum modulation technique, so that the values of range at the extremes of both boundaries cannot be considered as truly maximum or minimum.

Values of range as large as 15,500 miles on the undershoot boundary and 26,800 miles on the overshoot boundary can be obtained if the vehicle is allowed to skip to an altitude not exceeding 400 miles. This altitude was considered to be a conservative estimate of the lower limit of the Van Allen radiation belt. For a vehicle restricted to the sensible atmosphere (below 400,000 ft) after entry, the maximum range on the boundaries is 6300 miles.

The trajectories used to obtain the extreme values of range shown in figure 1 are presented in figure 2. The trajectory parameters are plotted as a function of time. Figures 2(a), (b), and (c) show the trajectories along the undershoot boundary, and figures 2(d), (e), and (f) show the trajectories along the overshoot boundary. The minimum- and maximum-range trajectories for a vehicle restricted to the sensible atmosphere after entry are shown in figures 2(a), (b), (d), and (e). The maximum-range trajectories for a vehicle that is allowed to skip are plotted in figures 2(c) and (f).

For a vehicle that is allowed to skip, the apogee altitude and the range outside the atmosphere can be computed in closed form from Newton's equations for a two-body drag-free trajectory, and are as follows:

$$\frac{r_a}{r_x} = \frac{1 + \sqrt{1 - \frac{V_x^2}{g_x r_x} \left(2 - \frac{V_x^2}{g_x r_x}\right) \cos^2 \gamma_x}}{2 - \frac{V_x^2}{g_x r_x}} \quad (1)$$

$$R = 2r_0 \sin^{-1} \frac{\frac{V_x^2}{g_x r_x} \sin \gamma_x \cos \gamma_x}{\sqrt{1 - \frac{V_x^2}{g_x r_x} \left(2 - \frac{V_x^2}{g_x r_x}\right) \cos^2 \gamma_x}} \quad (2)$$

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Figure 3 shows the longitudinal range available outside the earth's atmosphere as a function of exit conditions, computed from equation (2). The limiting line to the right of the figure was obtained by setting $r_a = r_0 + 400$ miles in equation (1). It is apparent from figure 3 that range is extremely dependent on changes in exit velocity, particularly in the vicinity of local satellite velocity. The effects on range of changes in velocity and flight-path angle have been evaluated and are discussed below.

Effects of V and γ on Longitudinal Range

The dependence of range outside the atmosphere on changes in exit flight-path angle or exit velocity is shown more explicitly in figure 4 and has been evaluated by partial differentiation of equation (2), to obtain equations (3) and (4).

$$\frac{\partial R}{\partial \gamma_x} = -2r_0 \frac{V_x^2}{g_x r_x} \left[\frac{1 - \left(2 - \frac{V_x^2}{g_x r_x}\right) \cos^2 \gamma_x}{1 - \frac{V_x^2}{g_x r_x} \left(2 - \frac{V_x^2}{g_x r_x}\right) \cos^2 \gamma_x} \right] \quad (3)$$

and

$$\frac{\partial R}{\partial V_x} = 4r_0 \frac{V_x}{g_x r_x} \left[\frac{\sin \gamma_x \cos \gamma_x}{1 - \frac{V_x^2}{g_x r_x} \left(2 - \frac{V_x^2}{g_x r_x}\right) \cos^2 \gamma_x} \right] \quad (4)$$

Figure 4 shows that for a trajectory that is intended to exit the atmosphere, small errors in exit flight-path angle or exit velocity can have a considerable effect on the longitudinal range of the vehicle.

The dependence of range on errors in flight-path angle and velocity at exit leads to further consideration of the effect of comparable errors in flight-path angle or velocity at earlier times in the trajectory. To investigate this point, two undershoot trajectories were taken for comparison, maximum range with no skip (fig. 2(b)), and maximum range with skip (fig. 2(c)). Errors in flight-path angle or velocity were introduced at a time when the dynamic pressure was a maximum (first pull-out, flight-path angle near zero), and the L/D modulations were maintained independent of the errors introduced in either γ or V. The results are shown in figure 5, where the change or error in longitudinal range, ΔR , is plotted as a function of the error in flight-path angle, $\Delta\gamma$, and the error in velocity, ΔV . Figure 5(a) (maximum range, no-skip trajectory) shows that any positive increment in either γ or V charges the trajectory into one that exits the atmosphere. The maximum altitude exceeds 400 miles at $\Delta\gamma = 0.5^\circ$. In figure 5(b) (maximum range, skip trajectory), small positive increments in γ or V result in an apogee altitude above 400 miles, since the maximum altitude originally was designed to approach 400 miles. Comparison of figures 5 and 4 shows that errors in γ or V at the time of maximum dynamic pressure can result in charges of longitudinal range of the same order of magnitude as comparable errors in γ or V at exit. It is also evident by comparison of figures 5(a) and 5(b) that errors of a comparable magnitude in γ or V early in the entry are as significant for a nonskipping trajectory as for a skipping trajectory.

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Near-Earth Holding Orbit

It has been shown that the skip maneuver can be used to increase longitudinal range by a substantial amount compared to the no-skip maneuver. A further extension of the skip maneuver is the near-earth holding orbit. In this case, the longitudinal range becomes unlimited. As a result of rotation of the earth, the choice of landing point is particularly flexible for orbits of large inclination to the equator. However, the holding orbit requires an expenditure of fuel to give an increment of velocity to establish the orbit.

For simplicity, the circular near-earth holding orbit will be discussed. The holding-orbit maneuver will be executed at the apogee of the skip, and the increment of velocity needed to establish the orbit will be calculated at this point.

The following equation relates the increment of velocity to the weight of fuel required to achieve a holding orbit.

$$\left. \begin{aligned}
 \text{Thrust} &= m \frac{dV}{dt} = -I_{sp}g_0 \frac{dm}{dt} \\
 \int_{V_a}^{V_s} dV &= -I_{sp}g_0 \int_m^{(m-m_f)} \frac{dm}{m} \\
 \frac{m_f}{m} = \frac{W_f}{W} &= 1 - e^{-\left(\frac{V_s - V_a}{I_{sp}g_0}\right)}
 \end{aligned} \right\} \quad (5)$$

Apogee velocity was calculated from equation (6),

$$V_a^2 = 2g_0 r_0^2 \frac{r_x - r_a}{r_x r_a} + V_x^2 \quad (6)$$

Figure 6 shows the ratio of fuel weight to total weight of the entry vehicle that is required for circular orbit at exit velocities from 24,000 to 26,000 ft/sec. The ratios W_f/W were evaluated for three apogee altitudes, 400, 300, and 200 miles, and for two values of specific impulse, 400 and 300 seconds. The results of the calculations shown in the figure indicate that at the higher exit velocity, circular orbit can be achieved for a relatively modest weight penalty, the order of 200 pounds of fuel for an entry vehicle weighing 5000 pounds.

$$\text{Lateral Range, } L/D = 1/2$$

For the purpose of the present study, only roll maneuvers, with roll angle, ϕ , held constant from the time of initiation of roll to touchdown, were considered for the determination of lateral range.

If roll is initiated at a time when the velocity is near the local satellite speed, after the vehicle re-enters the atmosphere, the magnitude of the lateral range is about ± 200 miles. This is shown in figure 7, where lateral range is plotted as a function of longitudinal range for three undershoot trajectories. Since trajectories along the corridor boundaries are the most critical, the undershoot boundary has been chosen for study of lateral-range capability. A skip maneuver has no effect on lateral range if the roll is initiated at a velocity near satellite speed after the vehicle has re-entered the atmosphere. In effect, a rectangular print is obtained which extends from about 1500 to 15,000 miles with a total width of 400 miles.

Since all available lift is used to keep from exceeding a deceleration of 10 g during the initial part of the entry on the undershoot trajectory, lift for lateral range is not available until after the time of maximum deceleration. If the roll is initiated at the first possible moment

(immediately after maximum deceleration), the lateral range is increased to about ± 500 miles. This is shown in figure 8, where lateral range is plotted as a function of longitudinal range. This print was obtained by initiation of various roll angles immediately subsequent to maximum deceleration along the undershoot boundary with the resultant L/D held constant at $+1/2$. Because of the positive L/D , the apogee altitude exceeded 400 miles for roll angles less than 15° . Included in figure 8 is the envelope of lateral range with roll initiated near satellite velocity from figure 7. The initiation of roll at the earlier time, before the vehicle exited the atmosphere, resulted in substantial increase in lateral range.

Longitudinal and Lateral Range, $L/D = 2$

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Up to this point, the paper has been concerned with the range of a capsule-type vehicle with an $L/D = 1/2$. It is of interest to compare this with the range of a glider with $L/D = 2$. It was felt that maintaining constant values of $W/C_D A$, as was done for the $L/D = 1/2$ vehicle, would be unrealistic for a glide vehicle; consequently, C_D and L/D were allowed to vary in the manner suggested in reference 5, while W/A was held constant at 30 lb/ft^2 . The trajectory chosen for comparison is shown in figure 9. No effort was made to achieve maximum range for this trajectory. The vehicle entered along an undershoot boundary with $L/D = 1/2$ in order to avoid the excessive heating that would occur if it entered at maximum L/D . The L/D was changed to $-1/2$ after maximum deceleration; and finally, when it was assured that the vehicle would not skip out of the atmosphere, the L/D was changed to 2. These modulations resulted in a trajectory for which the longitudinal range was about one-half an earth circumference.

The lateral range achieved when roll was initiated near satellite velocity, is shown in figure 10. Included in the figure is the envelope of lateral range for a vehicle with $L/D = 1/2$. The advantage of the $L/D = 2$ vehicle is apparent.

TOLERANCES ON ENTRY CONDITIONS

The preceding sections were concerned primarily with the range of a vehicle entering the earth's atmosphere at parabolic speed. It was shown that longitudinal range can be extended by a skip maneuver and that lateral range can be extended by initiation of roll at the earliest possible time. The consequence of these extensions of longitudinal and lateral range is reflected in a relaxation of midcourse guidance requirements. The extent to which these requirements could be relaxed was evaluated by applying the present results to the analysis presented in

reference 1. For a vehicle with a given range capability, we wish to evaluate the maximum allowable error in time of arrival at entry, Δt , and the maximum allowable error in inclination of the orbital plane, Δi , that will permit the vehicle to land at a predetermined destination.

As an example problem let us assume that the landing site is located at 35° N latitude. Since it was shown in reference 1 that the relation between the latitude of the target and that of the entry point has a strong influence on the allowable errors at entry, results will be presented for entry at two latitudes, the equator and 30° N latitude. For all cases under consideration, it was assumed that touchdown could not be achieved in less than 2500 miles from the point of entry (minimum range, overshoot trajectory). For the vehicle with $L/D = 1/2$ under study, the following cases were investigated.

(a) The vehicle was restricted to the sensible atmosphere after entry, and roll was initiated at a velocity near local satellite speed.

(b) The vehicle was allowed to skip to an altitude not exceeding 400 miles, and roll was initiated at a velocity near satellite speed after the vehicle had re-entered the atmosphere.

(c) The vehicle was allowed to skip to an altitude not exceeding 400 miles, and roll was initiated immediately after maximum deceleration before the vehicle had exited the atmosphere.

The results are tabulated in the table below.

Case	Entry latitude, deg	Longitudinal range, miles	Lateral range, miles	Δt , hr	Δi , deg
(a)	0	6,300	± 200	± 0.8	± 5.0
(b)	0	15,000	± 200	± 2.4	± 5.0
(c)	0	8,000	± 500	± 3.0	± 12.0
(a)	30 N	6,300	± 200	± 1.1	± 2.2
(b)	30 N	15,000	± 200	± 1.1	± 2.2
(c)	30 N	8,000	± 500	± 2.1	± 5.5

These results show that a vehicle of the assumed maneuver capability has substantial tolerance in time of arrival and orbital plane inclination. For a lateral range of ± 200 miles and entry at the equator, the permissible time-of-arrival error is increased by a factor of three for the skipping vehicle compared to that for the nonskipping vehicle, whereas there is no change in the allowable error of the inclination of the orbital plane. When lateral range is increased to ± 500 miles, Δt and Δi are both increased over the previous cases.

For entry at 30° N latitude, no advantage was realized for the skip-ping vehicle compared to the nonskipping vehicle for a lateral range of ± 200 miles. Again, increasing the lateral range to ± 500 miles increased both Δt and Δi .

CONCLUDING REMARKS

The point return of a vehicle, with a lift-to-drag ratio of $1/2$, returning from a lunar mission has been investigated. It was found that the available longitudinal and lateral ranges allowed considerable tolerances in time of entry and inclination of the orbital plane.

It was found that if the vehicle is allowed to skip to an altitude not exceeding 400 miles, a longitudinal-range variation in excess of 13,000 miles can be achieved on any entry in the corridor; whereas, if the vehicle is restricted to the sensible atmosphere after entry, the longitudinal-range variation is about 4,000 miles.

Longitudinal range for a skip trajectory is very sensitive to errors in either exit velocity or exit flight-path angle, particularly for exit velocities in the vicinity of local satellite velocity. However, dependence of longitudinal range on errors in either velocity or flight-path angle appears to be as critical at the bottom of the first pull-out as at exit. These errors can transform trajectories that are not intended to exit the atmosphere into skip trajectories.

Calculations of the weight of fuel required to achieve a near-earth holding orbit showed that circular orbit can be achieved under some conditions for a relatively modest weight penalty.

The lateral range for the case where roll is initiated near local satellite velocity, after the vehicle has re-entered the atmosphere, is about ± 200 miles regardless of the type of trajectory (skip or no skip). For roll initiated immediately subsequent to maximum deceleration along the undershoot boundary, before the vehicle exits the atmosphere, lateral range is increased to about ± 500 miles.

The consequence of increased lateral and longitudinal range is reflected in the relaxation of guidance requirements at entry. For the examples discussed for the vehicle with $L/D = 1/2$ required to land at a predetermined destination, tolerable errors in time of entry and inclination of the orbital plane are substantially greater for the skip trajectory than for the no-skip trajectory.

Ames Research Center
National Aeronautics and Space Administration
Moffett Field, Calif., Sept. 8, 1961

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APPENDIX

EQUATIONS OF MOTION

The complete equations of motion (ref. 6) are shown below in the nomenclature of this paper.

$$\begin{aligned} \frac{dV}{dt} = & -\frac{C_{DA}}{m} q_{\infty} + G_r \sin \gamma + G_{\delta} \cos \gamma \cos \beta \\ & + \omega^2 r \cos \delta (\sin \gamma \cos \delta - \cos \gamma \cos \beta \sin \delta) \end{aligned} \quad (A1)$$

$$\begin{aligned} V \frac{d\gamma}{dt} = & \frac{L}{D} \frac{C_{DA}}{m} q_{\infty} \cos \varphi + \frac{V^2 \cos \gamma}{r} + G_r \cos \gamma - G_{\delta} \sin \gamma \cos \beta \\ & + 2\omega V \sin \beta \cos \delta + \omega^2 r \cos \delta (\cos \gamma \cos \delta + \sin \gamma \cos \beta \sin \delta) \end{aligned} \quad (A2)$$

$$\begin{aligned} V \cos \gamma \frac{d\beta}{dt} = & \frac{L}{D} \frac{C_{DA}}{m} q_{\infty} \sin \varphi + \frac{V^2 \cos^2 \gamma \sin \beta \sin \delta}{r \cos \delta} - G_{\delta} \sin \beta \\ & + 2\omega V (\cos \gamma \sin \delta - \sin \gamma \cos \beta \cos \delta) + \omega^2 r \sin \beta \sin \delta \cos \delta \end{aligned} \quad (A3)$$

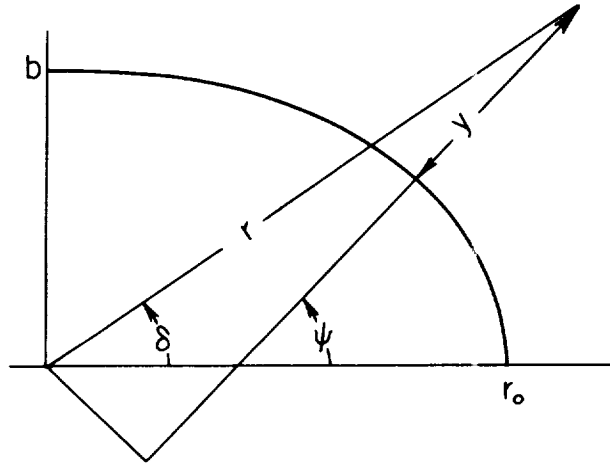
where the gravitational acceleration components are

$$\left. \begin{aligned} G_r &= -g \left[1 + J \left(\frac{r_0}{r} \right)^2 (1 - 3 \sin^2 \delta) \right] \\ G_{\delta} &= -2Jg \left(\frac{r_0}{r} \right)^2 \sin \delta \cos \delta \\ J &= 0.0016232, \text{ oblateness constant} \end{aligned} \right\} \quad (A4)$$

For the calculations of the trajectories discussed in this paper, it was assumed that the rotation of the earth, ω , equaled zero; therefore, all terms in equations (A1) through (A3) which include ω as a factor become zero.

The bearing of the velocity vector, β , is measured from a northerly direction; hence, for a vehicle that is heading east, $\beta = 90^\circ$. Because of the earth's oblateness, the equatorial radius, r_0 , is greater than the

polar radius, b . The sketch below, which represents part of a cross section of the earth with the oblateness greatly exaggerated, shows the relationship between the declination, δ , and the latitude, ψ .



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If the position is known in terms of altitude, y , and latitude, ψ , the geocentric distance, r , and the declination, δ , are computed from

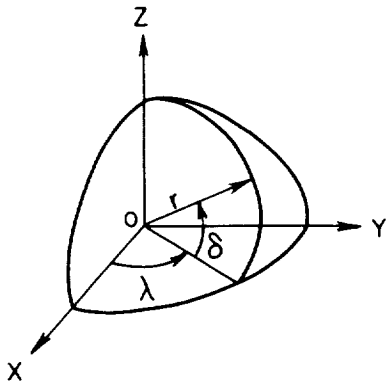
$$\left. \begin{aligned} r \cos(\psi - \delta) &= y + \sqrt{r_0^2 \cos^2 \psi + b^2 \sin^2 \psi} \\ (r_0^2 - b^2) \cos \psi &= r \sqrt{r_0^2 \cos^2 \psi + b^2 \sin^2 \psi} \sin(\psi - \delta) \end{aligned} \right\} \quad (A5)$$

The following relations can be used to write the equations of motion in spherical coordinates (see sketch below):

$$\frac{dr}{dt} = V \sin \gamma$$

$$\frac{d\lambda}{dt} = \frac{V \cos \gamma \sin \beta}{r \cos \delta}$$

$$\frac{d\delta}{dt} = \frac{V \cos \gamma \cos \beta}{r}$$



The equatorial plane is XOY.
 The meridian of Greenwich is XOZ.
 The north pole is in the direction OZ.
 The longitude, λ , is positive east.

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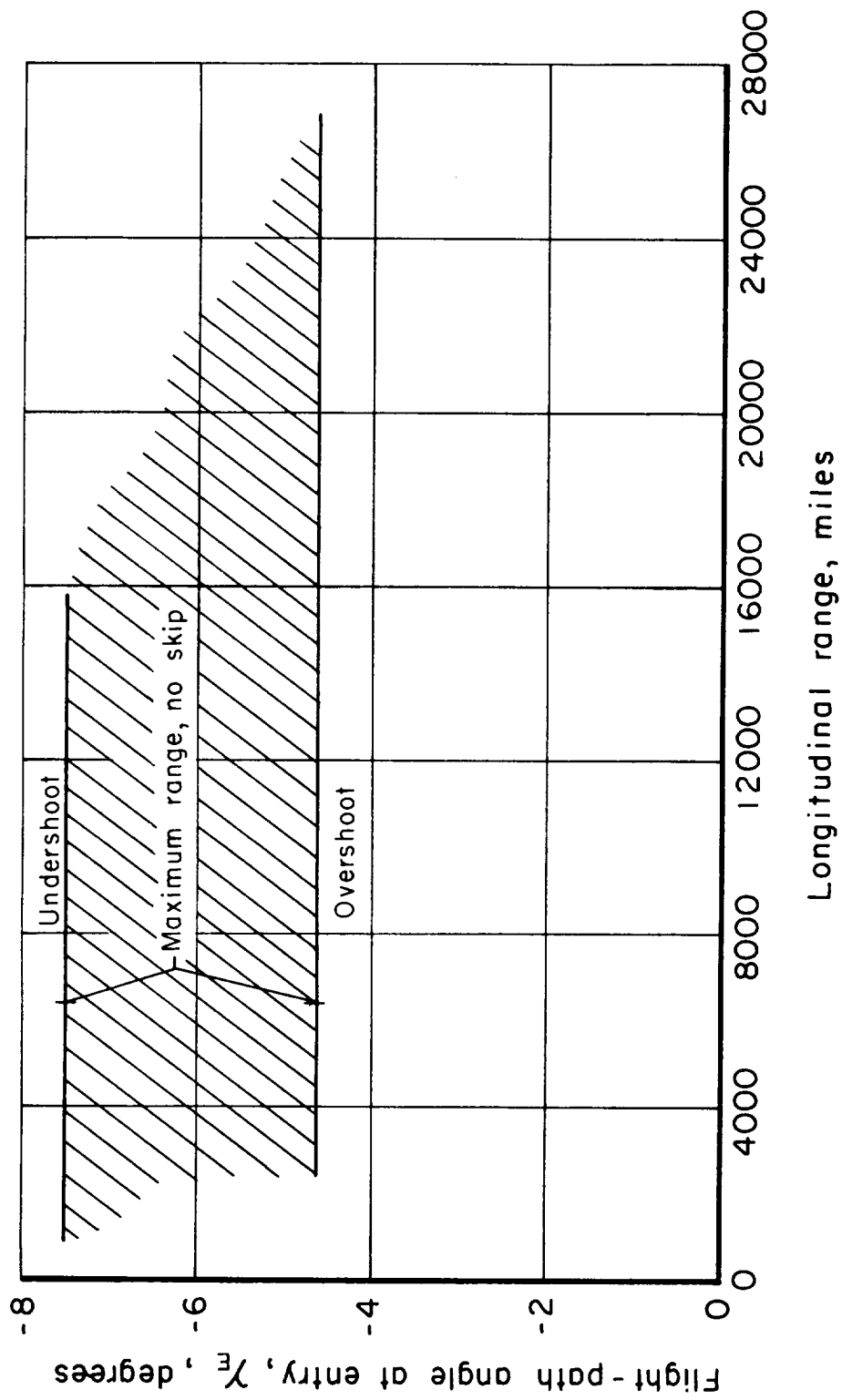
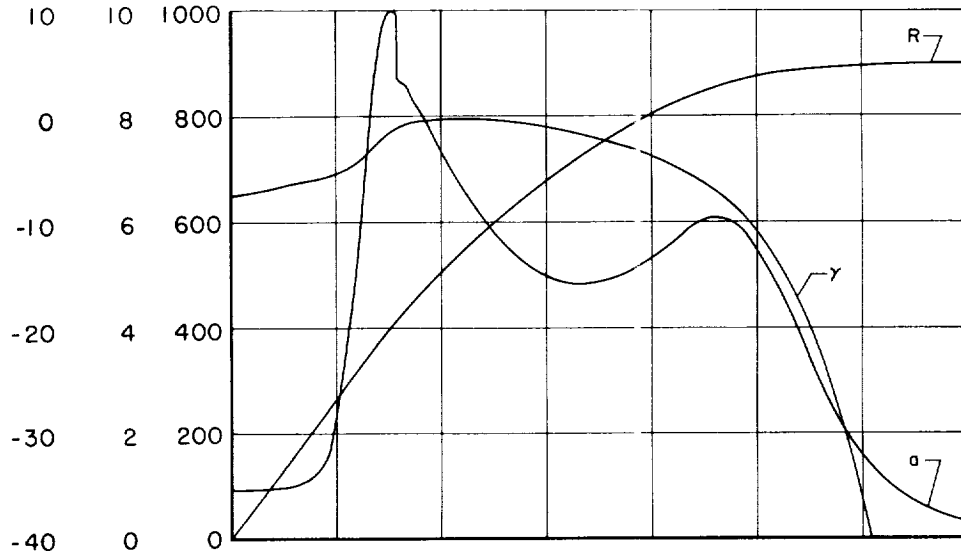
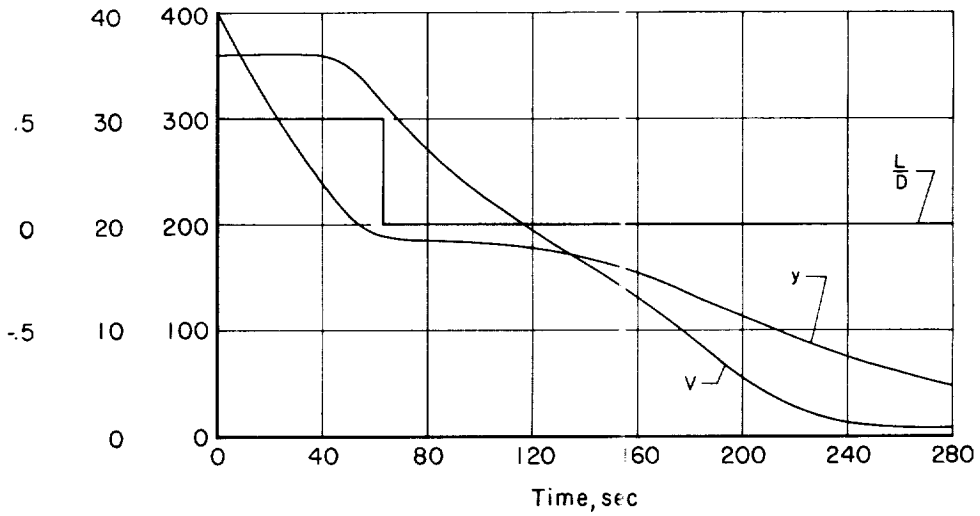


Figure 1.- Longitudinal range along corridor boundaries.

γ α R
(degrees) (g) (miles)



$\frac{L}{D}$ $V \times 10^{-3}$ $y \times 10^{-3}$
(ft/sec) (ft)

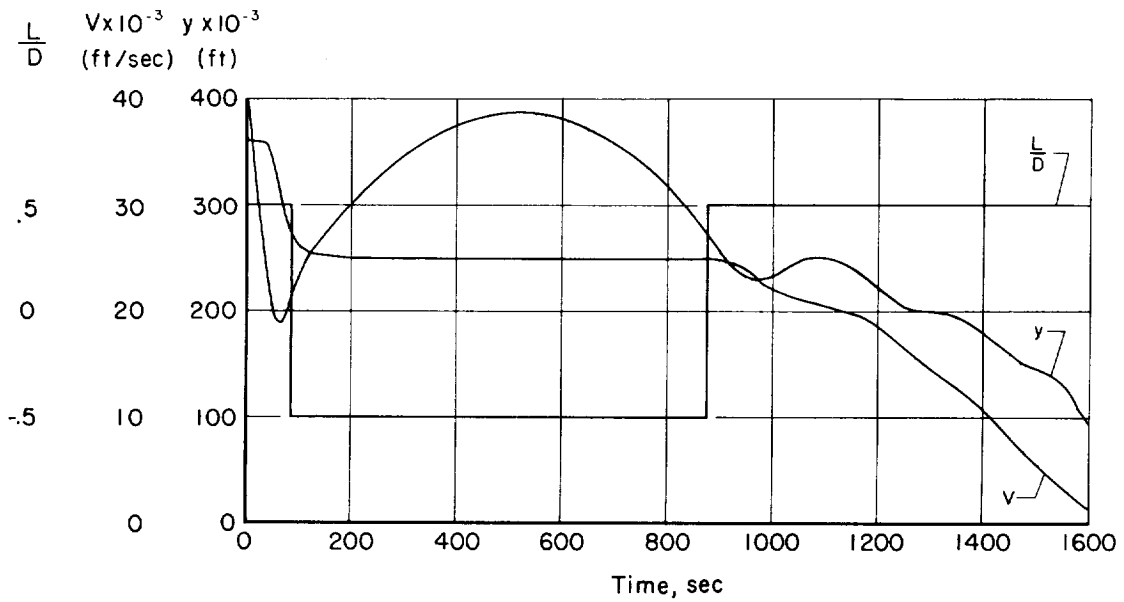
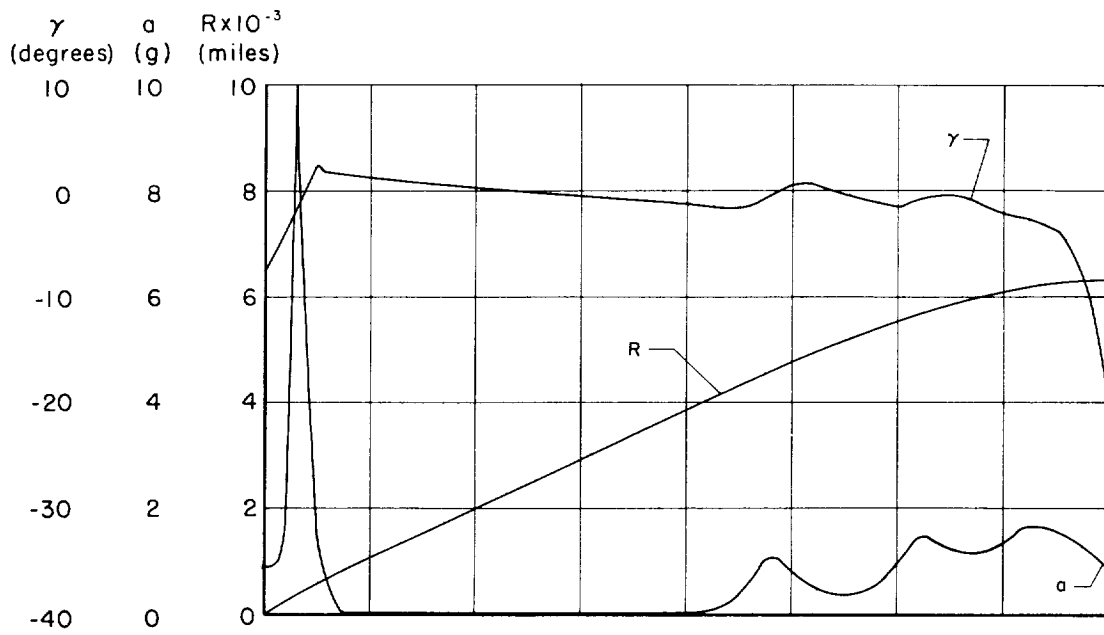


(a) Undershoot, minimum range.

Figure 2.- Trajectories.

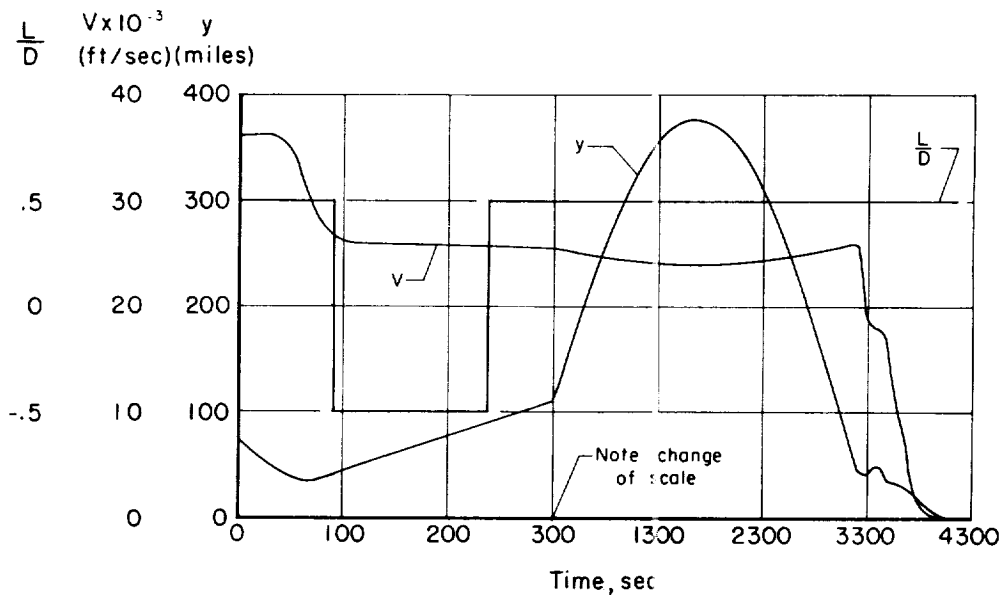
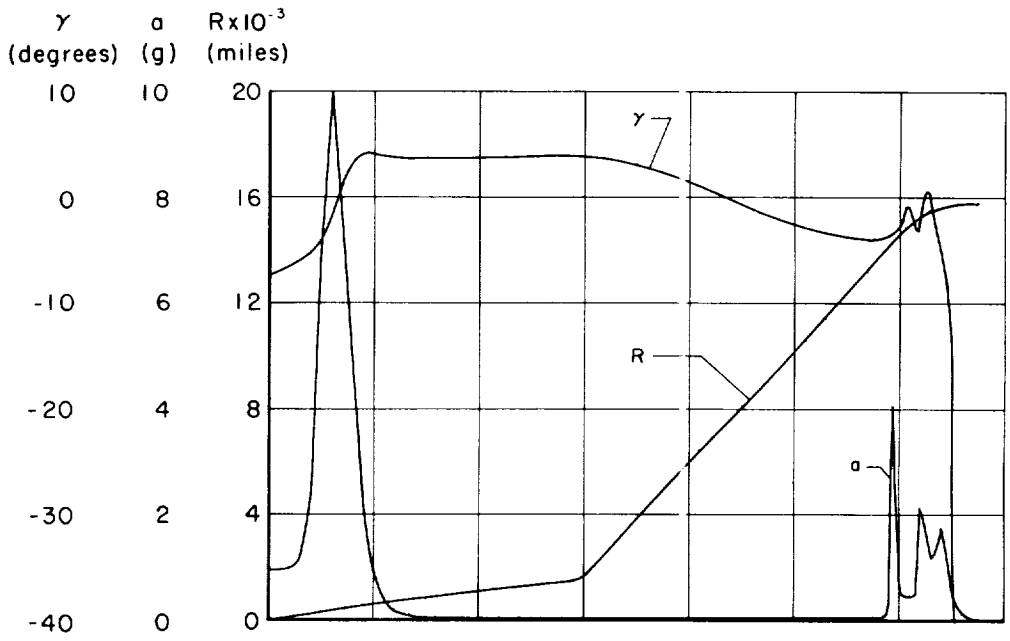
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(b) Undershoot, maximum range, no skip.

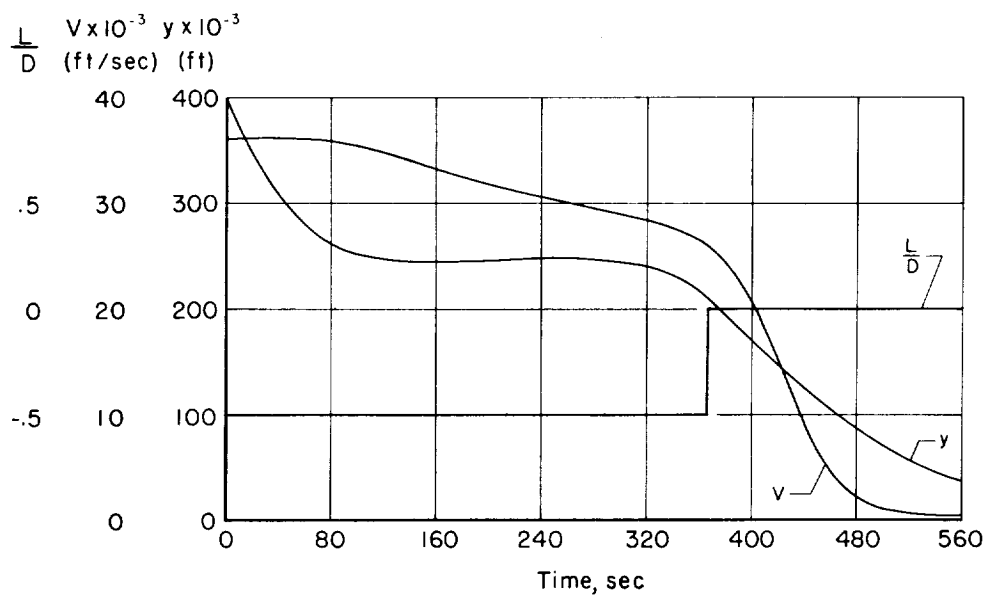
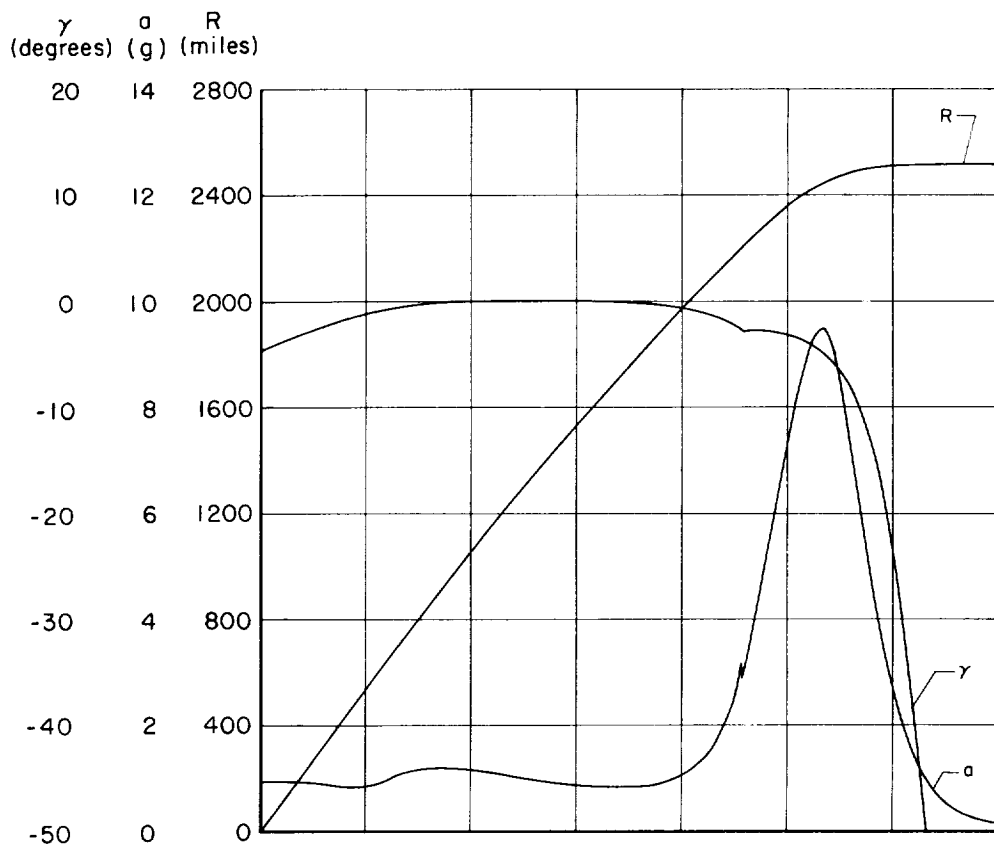
Figure 2.- Continued.



(c) Undershoot, maximum range, skip.

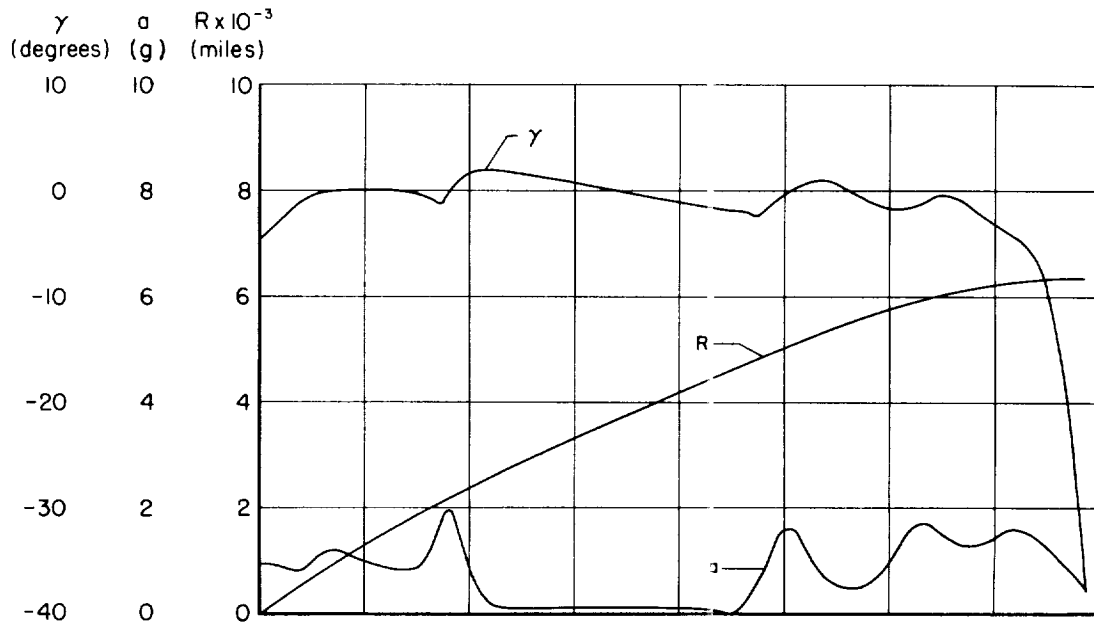
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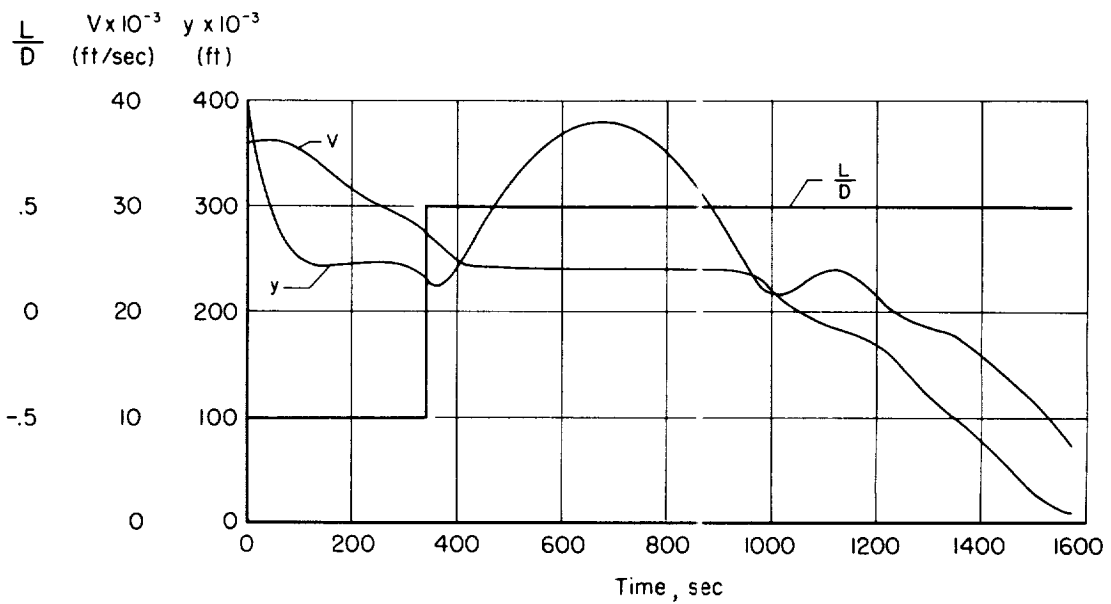


(d) Overshoot, minimum range.

Figure 2.- Continued.

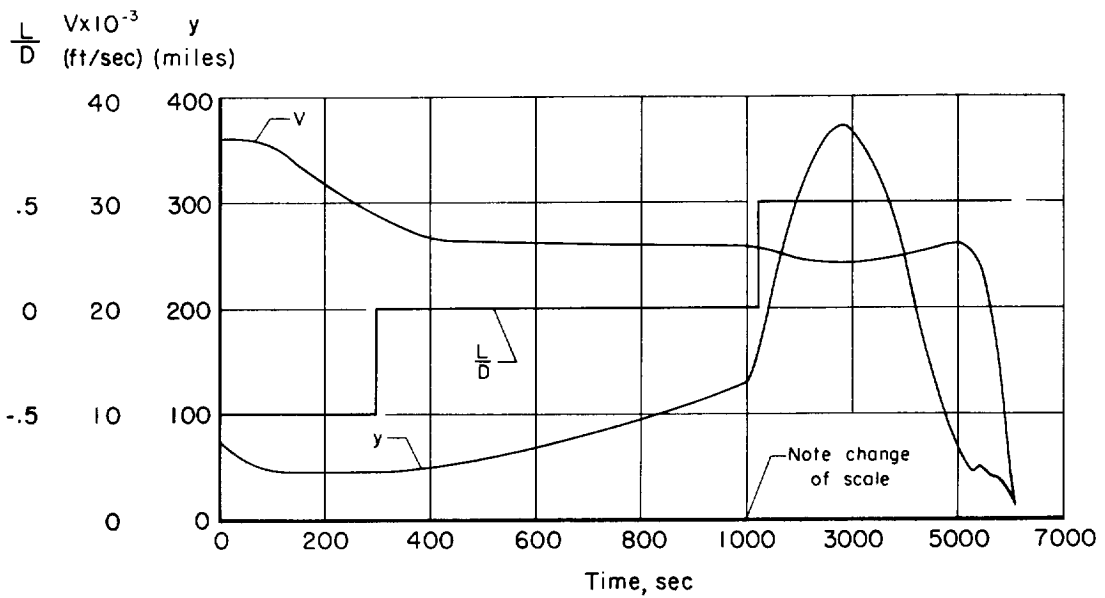
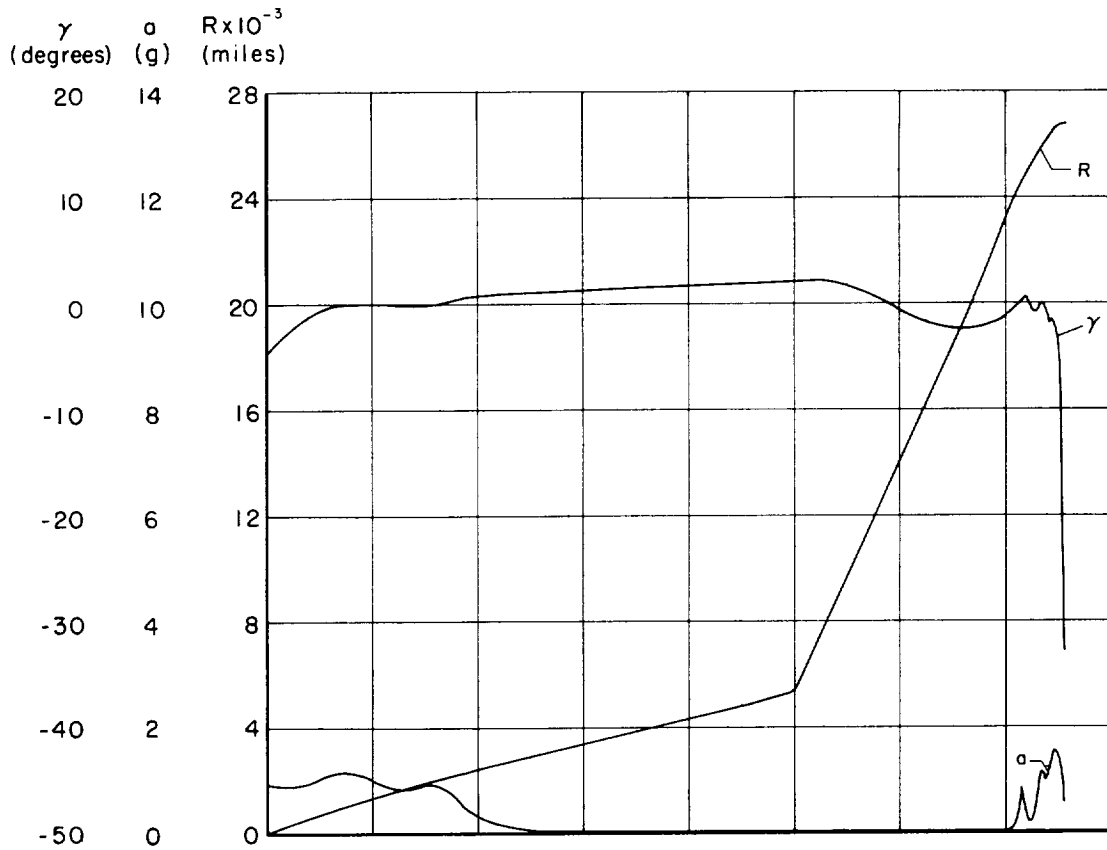


A
5
5
3



(e) Overshoot, maximum range, no skip.

Figure 2.- Continued.



(f) Overshoot, maximum range, skip.

Figure 2.- Concluded.

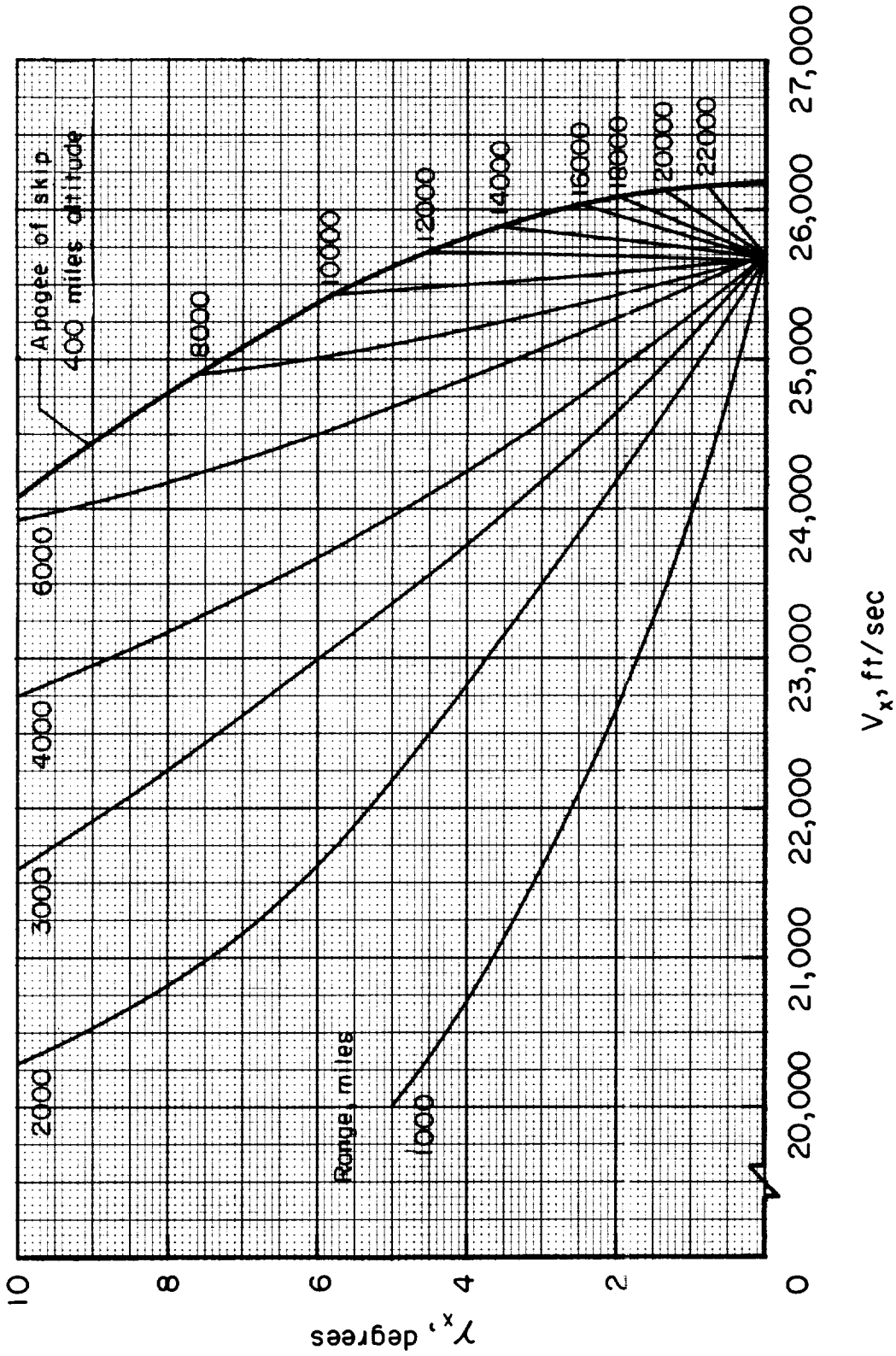
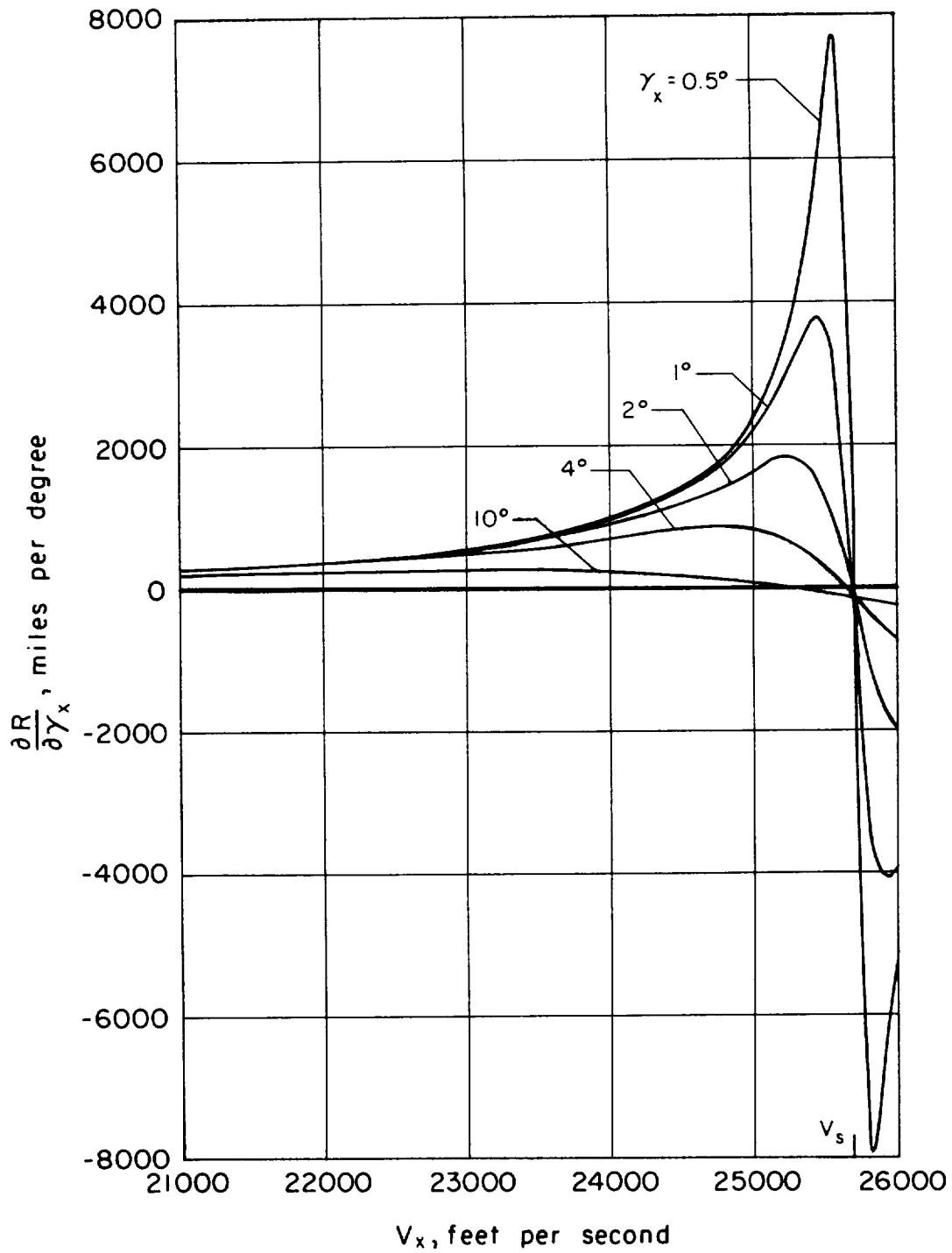
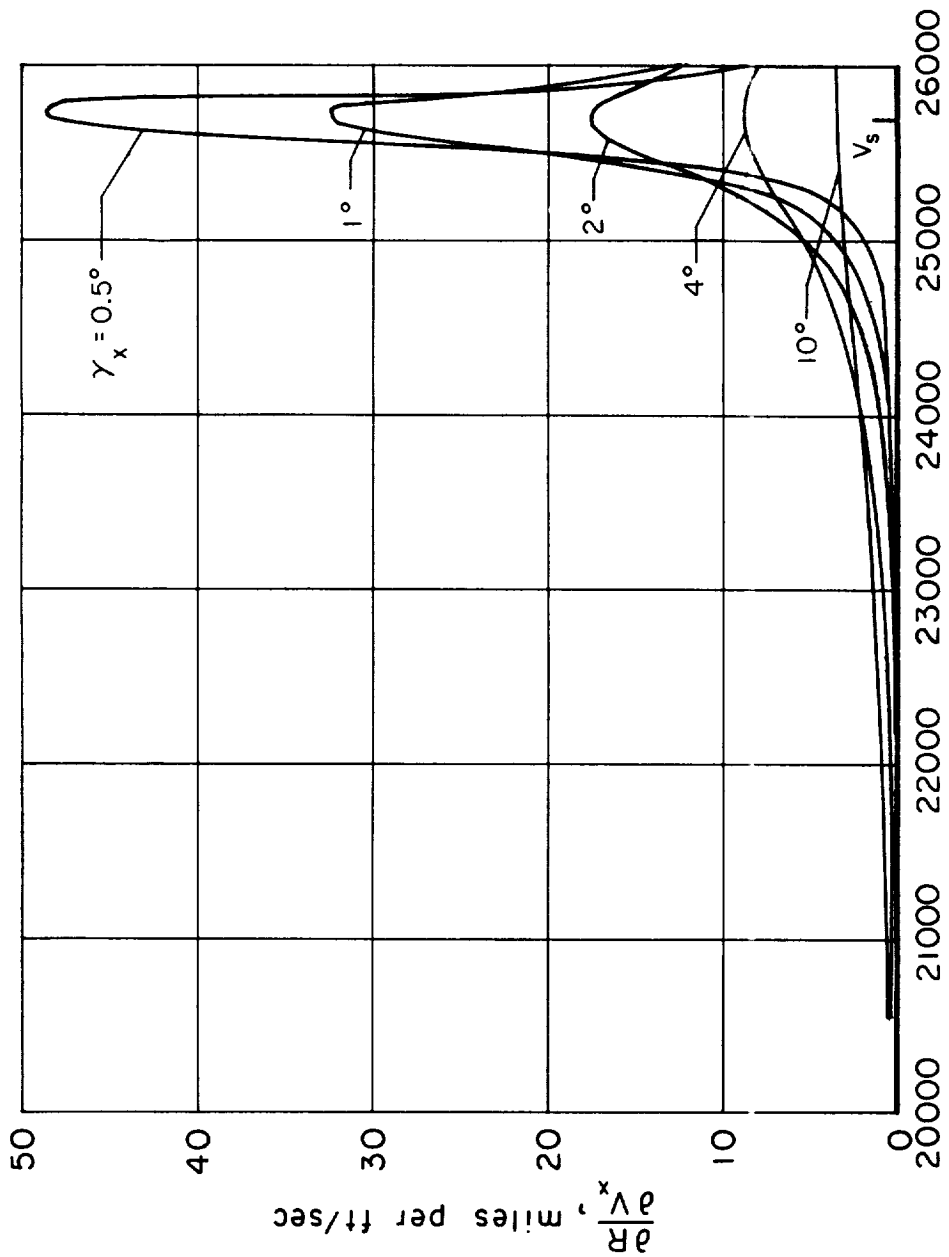


Figure 3.- Longitudinal range outside earth's atmosphere (from 400,000 feet to 400 miles altitude) as a function of exit conditions.



(a) Exit flight path angle.

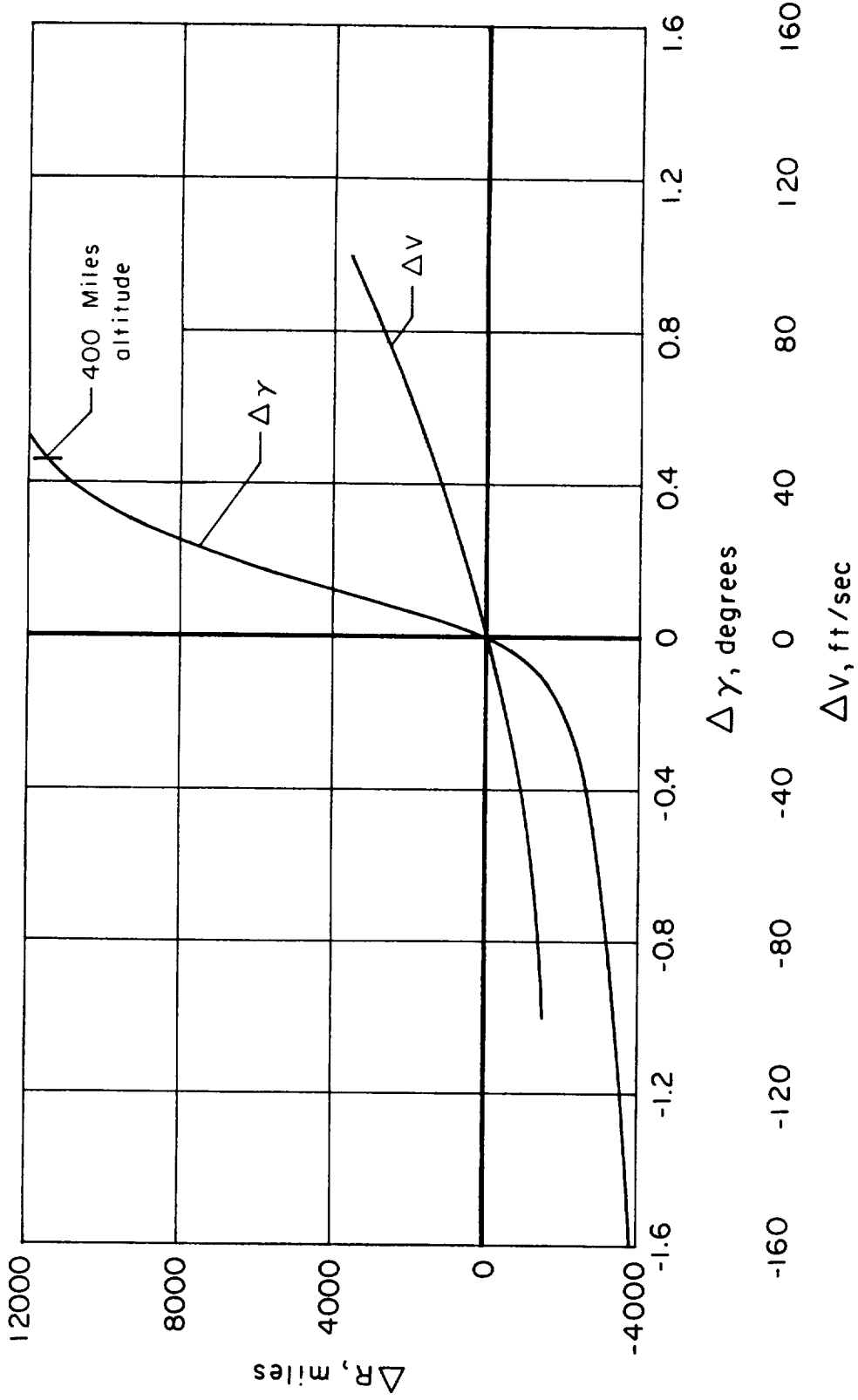
Figure 4.- Dependence of longitudinal range outside the atmosphere to changes in exit angle or exit velocity.



V_x , feet per second

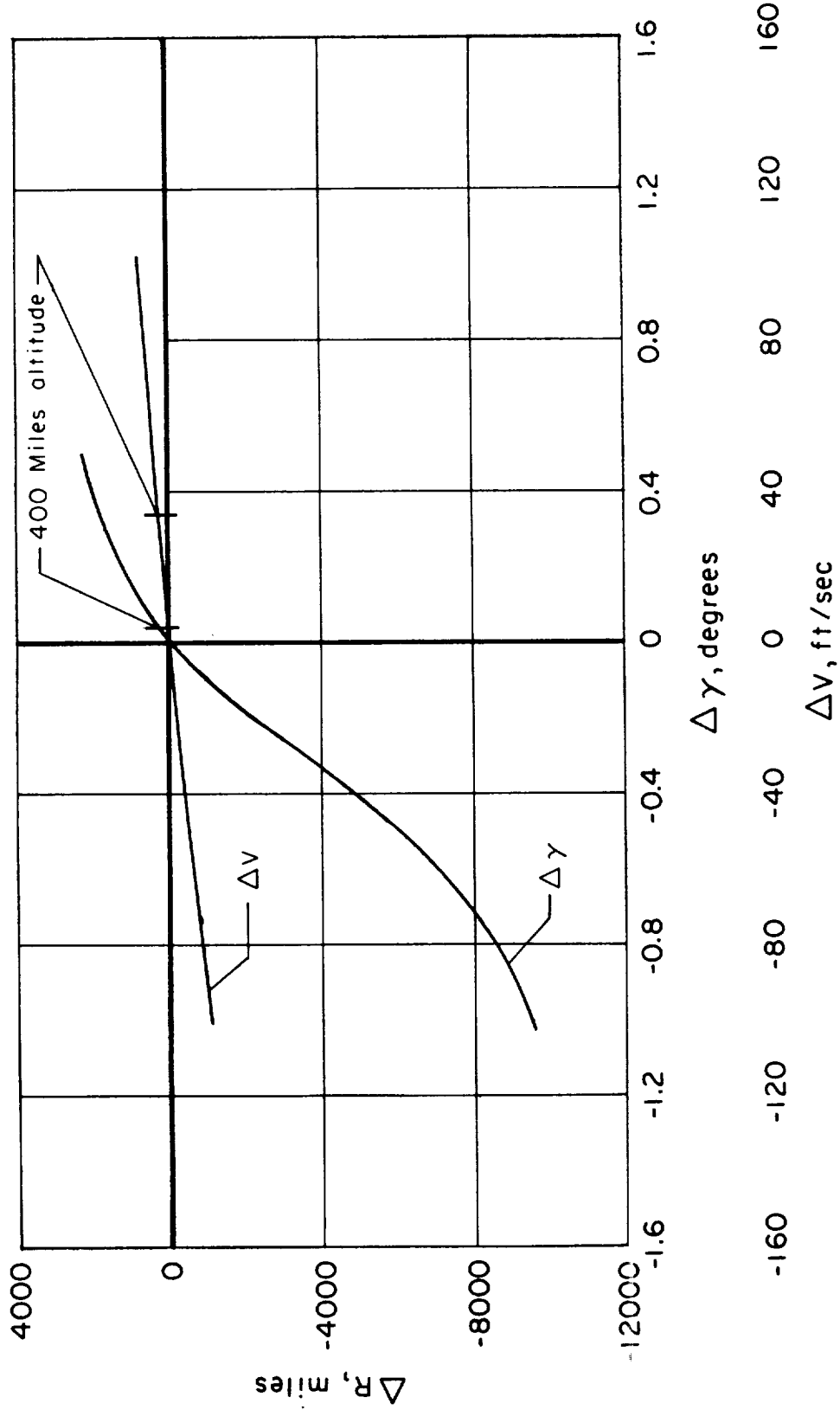
(b) Exit velocity.

Figure 4.- Concluded.



(a) Undershoot, maximum range, no skip.

Figure 5.- Dependence of longitudinal range on flight-path angle or velocity at the time of maximum dynamic pressure (flight-path angle near zero).



(b) Undershoot, maximum range, skip.

Figure 5.- Concluded.

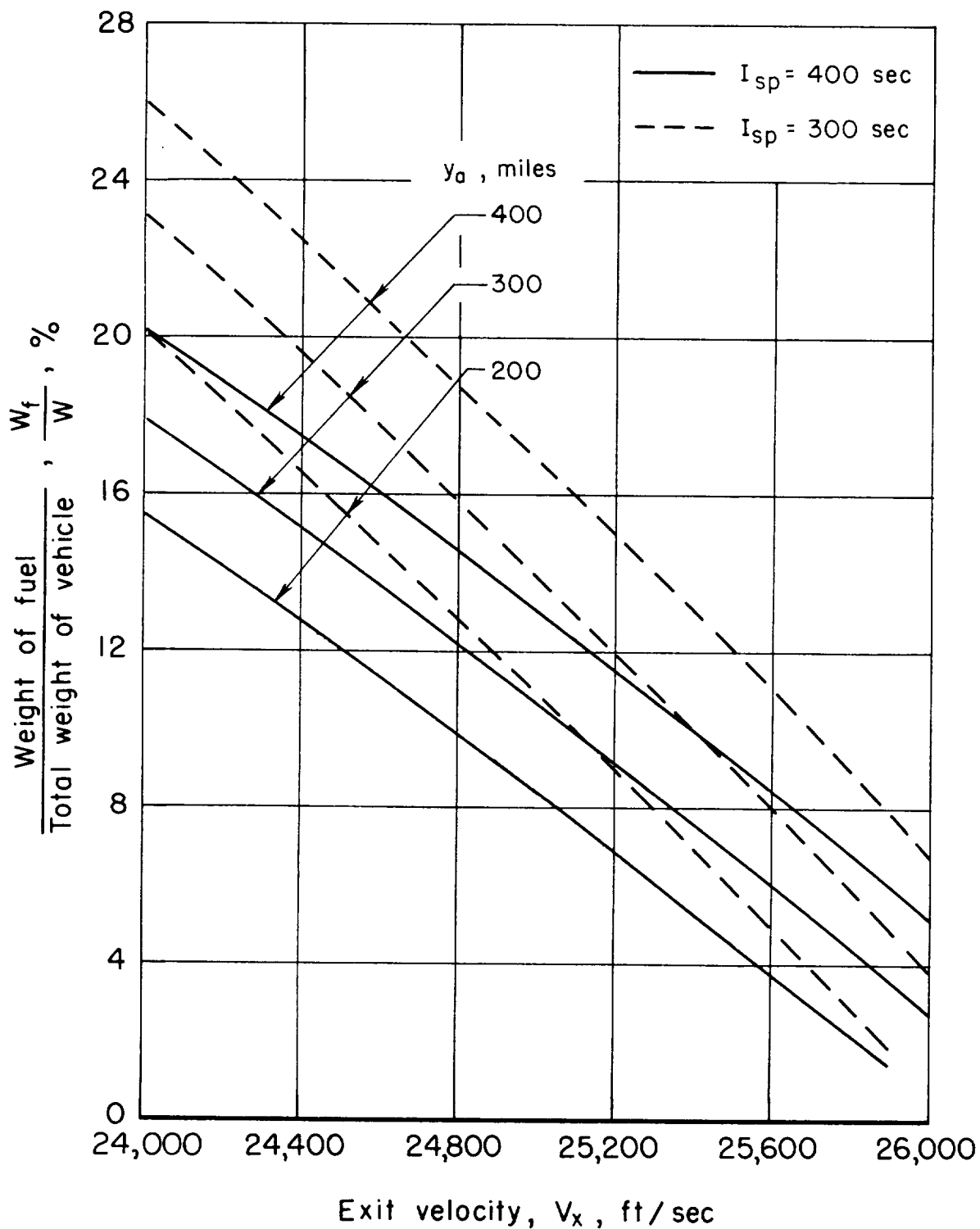


Figure 6.- Fuel required to obtain circular orbit as a function of exit velocity and specific impulse of fuel.

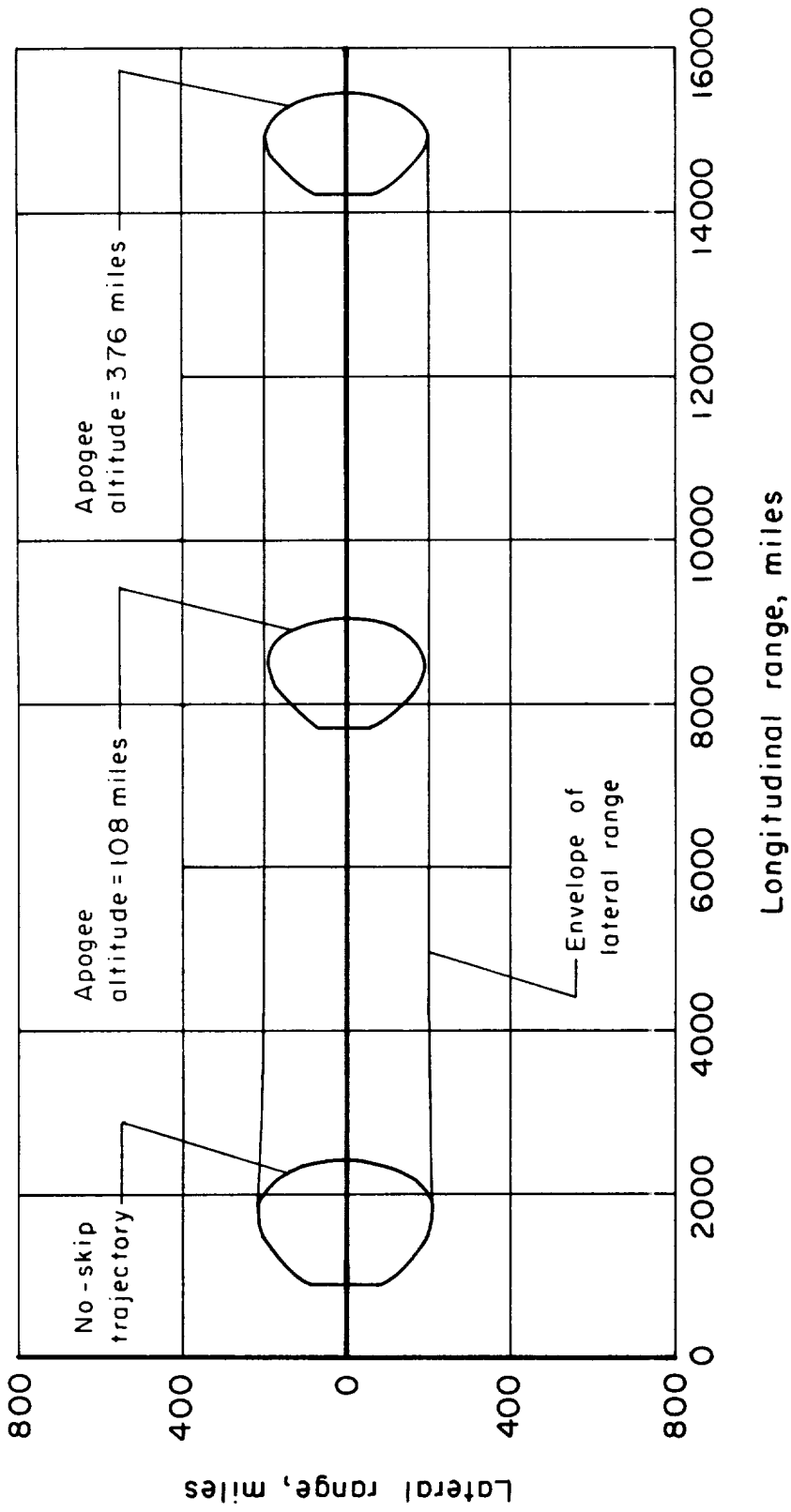


Figure 7.- Lateral range for undershoot trajectories with roll initiated near satellite velocity after the skip maneuver.

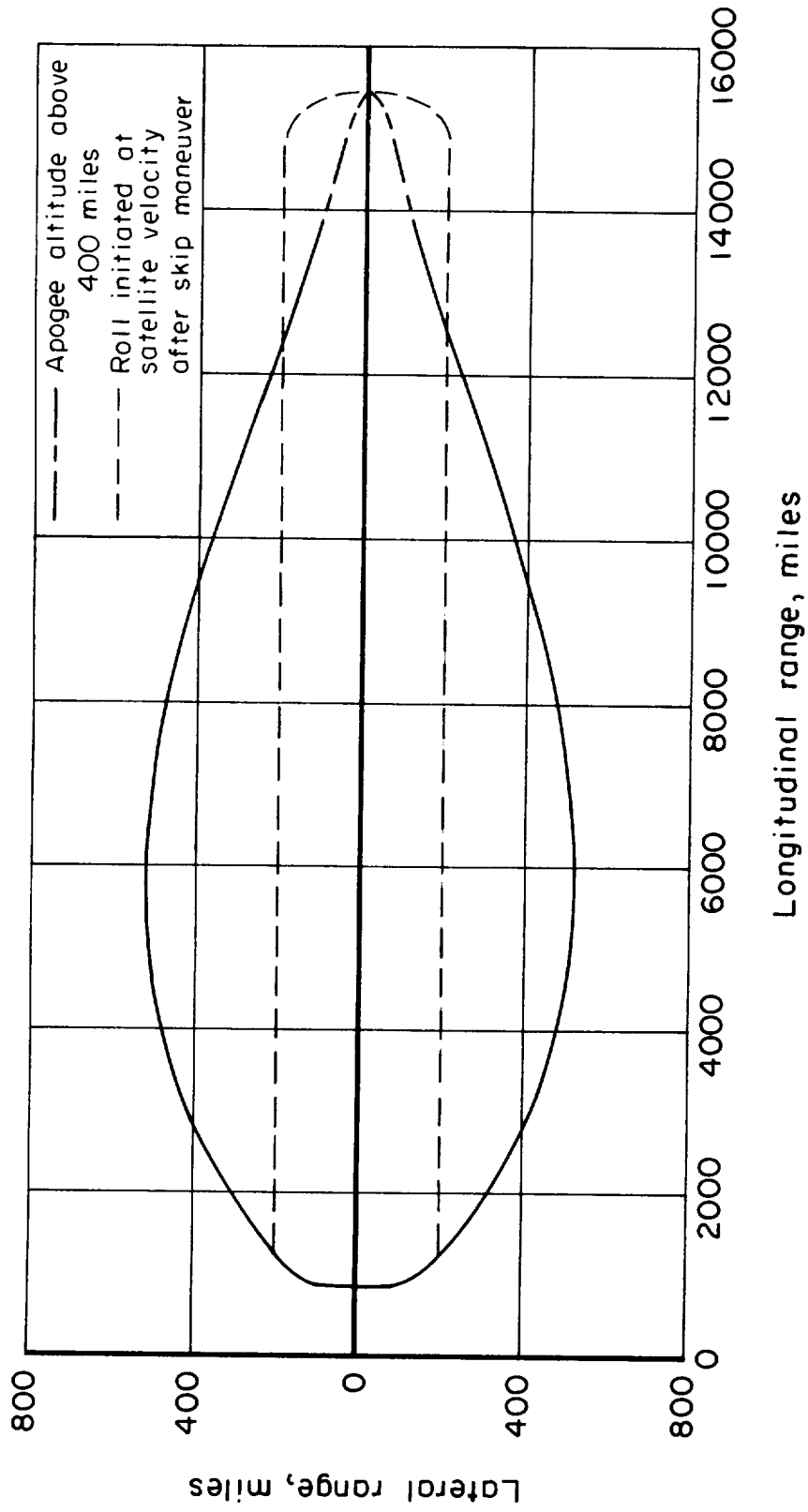


Figure 8.- Lateral range for undershoot trajectory with roll initiated at time of maximum dynamic pressure.

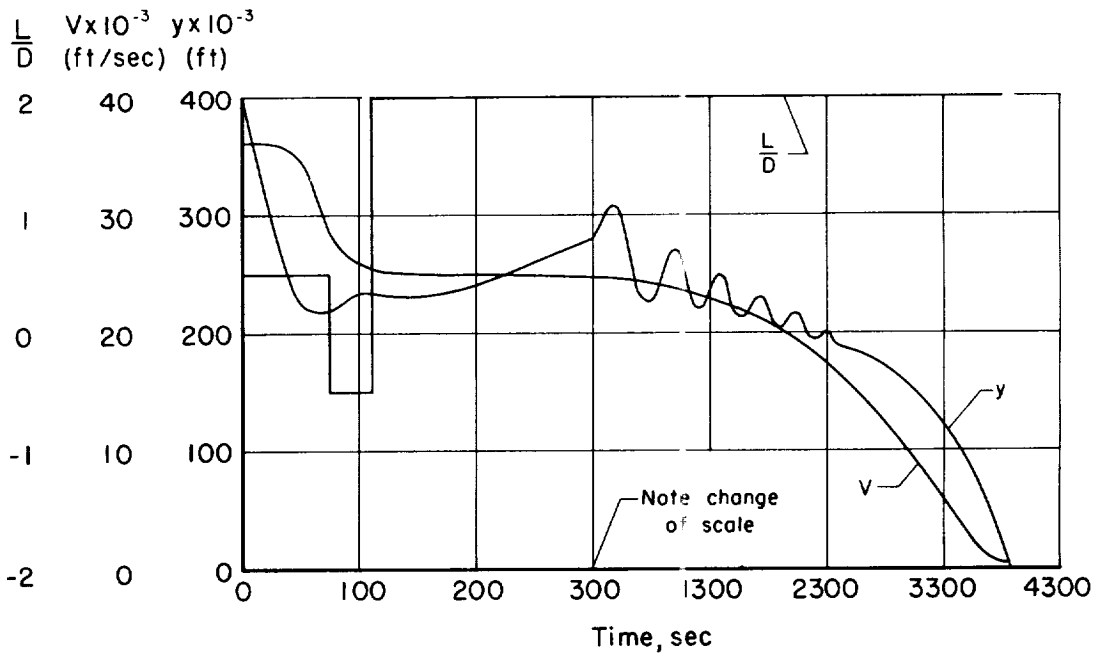
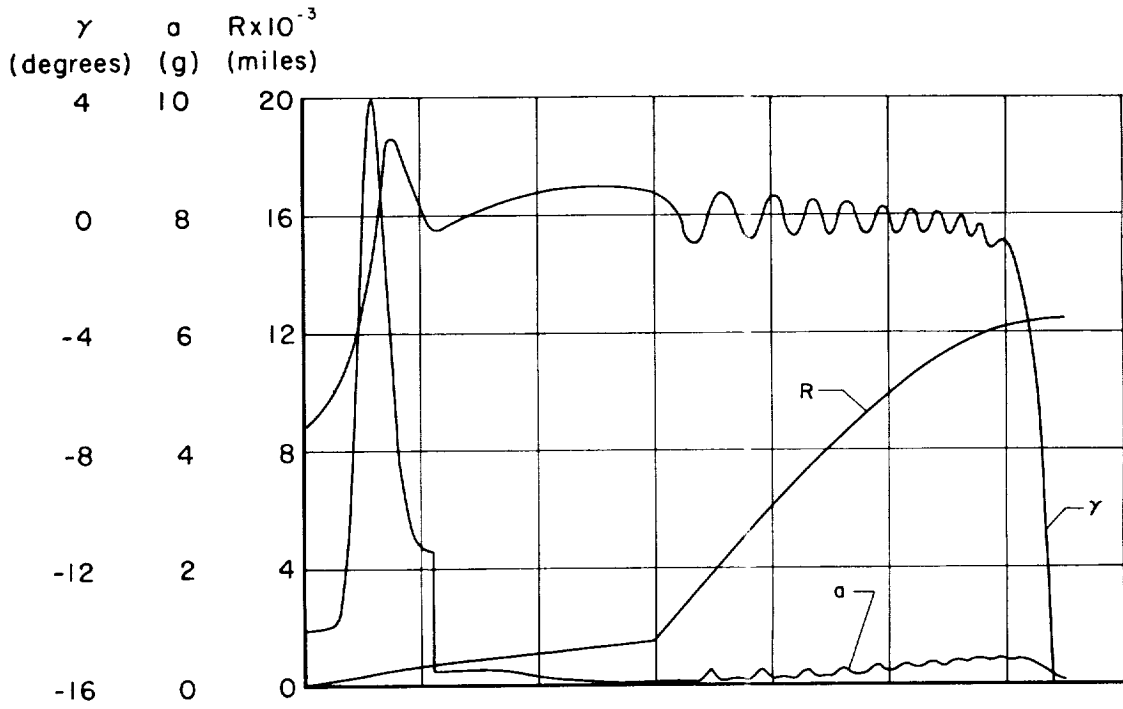


Figure 9.- Undershoot trajectory for vehicle with $L/D = 2$.

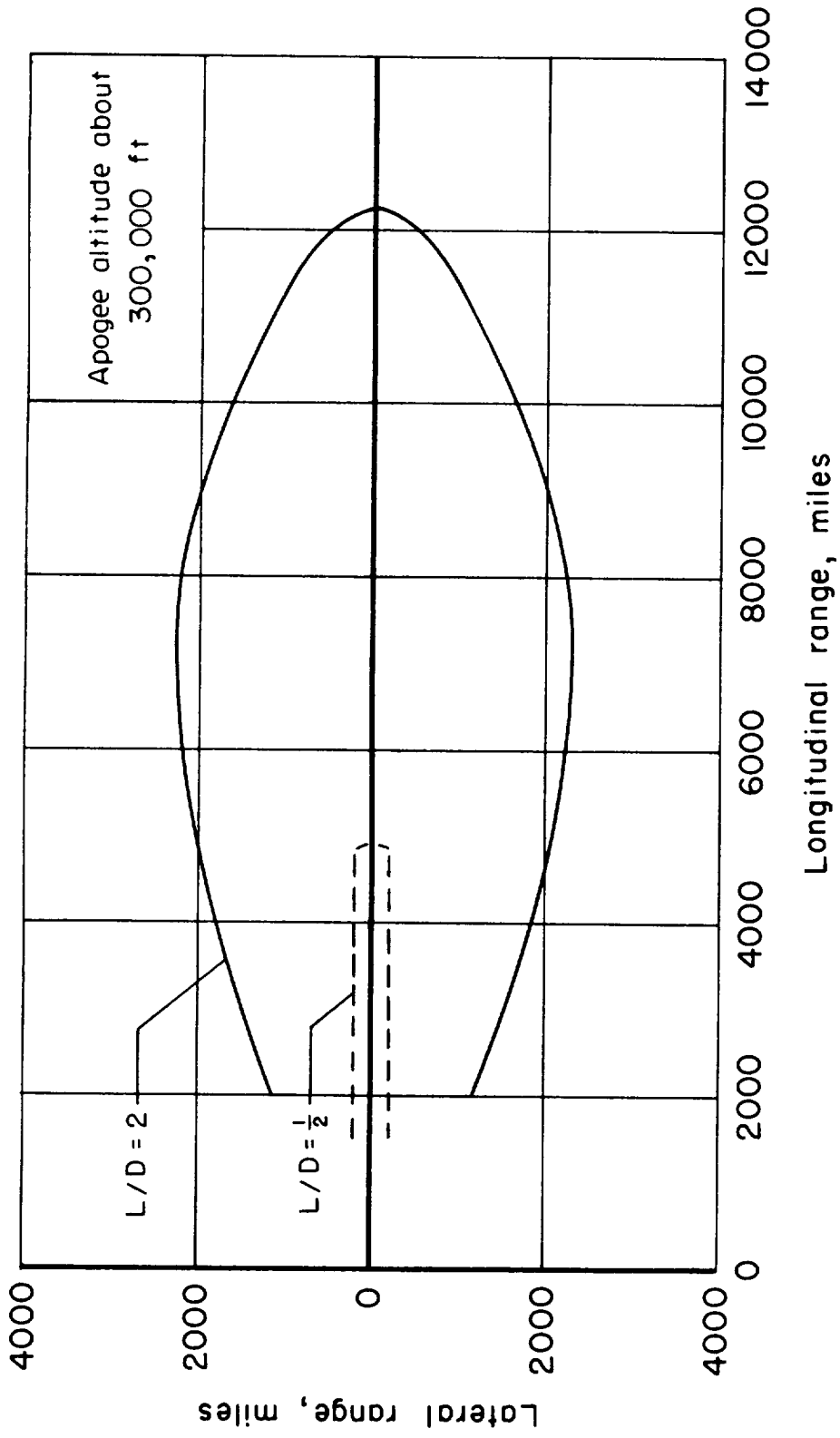


Figure 10.- Range for vehicle with $L/D = 2$.

