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EFFECTS OF MACH NUMBER, LEADING-EDGE BLUNTNESS,
AND SWEEP ON BOUNDARY-LAYER TRANSITION
ON A FLAT PLATE

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Moffett Field, Calif.

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EFFECTS OF MACH NUMBER, LEADING-EDGE BLUNTNESS, AND SWEEP ON BOUNDARY-LAYER TRANSITION ON A FLAT PLATE

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SUMMARY

The effects of leading-edge bluntness and sweep on boundary-layer transition on flat plate models were investigated at Mach numbers of 2.00, 2.50, 3.00, and 4.00. The effect of sweep on transition was also determined on a flat plate model equipped with an elliptical nose at a Mach number of 0.27. Models used for the supersonic investigation had leading-edge radii varying from 0.0005 to 0.040 inch. The free-stream unit Reynolds number was held constant at 15 million per foot for the supersonic tests and the angle of attack was 0°. Surface flow conditions were determined by visual observation and recorded photographically. The sublimation technique was used to indicate transition, and the fluorescent-oil technique was used to indicate flow separation. Measured Mach number and sweep effects on transition are compared with those predicted from shock-loss considerations as described in NACA Rep. 1312.

For the models with the blunter leading edges, the transition Reynolds number (based on free-stream flow conditions) was approximately doubled by an increase in Mach number from 2.50 to 4.00; and nearly the same result was predicted from shock-loss considerations. At all supersonic Mach numbers, increases in sweep reduced the transition Reynolds number and the amount of reduction increased with increases in bluntness. The shock-loss method considerably underestimated the sweep effects, possibly because of the existence of crossflow instability associated with swept wings. At a Mach number of 0.27, no reduction in the transition Reynolds number with sweep was measured (as would be expected with no shock loss) until the sweep angle was attained where crossflow instability appeared.

INTRODUCTION

At supersonic speeds the accurate estimation of the performance of airplanes and missiles depends to a large extent on how accurately the skin friction and heat transfer can be estimated. It follows that knowledge of the transition location and the influence on transition of such factors as Mach number, leading-edge sweep, and bluntness (which are known to affect transition) are of considerable importance. Although the effects of these factors on transition have been the subject of several investigations, the influence of each factor on transition is still not completely defined and understood. It was shown in references 1 and 2 that slight blunting of the leading edge can produce large increases in the length of laminar flow at supersonic speeds. A marked increase in the free-stream transition Reynolds number was reported in reference 2 for a hollow cylinder with a leading-edge radius of 0.0010 to 0.0015 inch when the Mach number was increased from 3 to 8. A smaller increase in the transition Reynolds number was reported in reference 3 for about the same increase in Mach number, but for a hollow cylinder with about 1/10 the bluntness of the model of reference 2. In reference 1, the adverse effect of sweep on the length of laminar flow at a Mach number of 4.04 was shown for wings with flat surfaces and flat blunted leading edges.

In an attempt to explain the effects of Mach number, leading-edge sweep, and bluntness on the length of laminar flow, Moeckel suggested in reference 4 that these effects could be attributed to changes in the local unit Reynolds number as produced by a detached shock wave at the leading edge. From this hypothesis it can be reasoned that for a given free-stream unit Reynolds number, increases in Mach number, increases in bluntness, or decreases in sweep should have the effect of reducing the local unit Reynolds number and thereby increasing the length of laminar flow. In reference 5 this shock concept is discussed in detail and equations are presented by which sweep and Mach number effects on transition can be calculated.

Another cause of the decrease in length of laminar flow with increase in sweep is the spanwise flow known to exist in the boundary layer on swept curved surfaces. For the case of swept surfaces with blunt leading edges at supersonic speeds, Chapman in reference 5 indicates that only a very small amount of secondary (spanwise) flow is necessary to produce boundary-layer instability and transition. This crossflow phenomenon and its relationship to transition is also discussed by Boltz, Kenyon, and Allen in reference 6 for swept wings at subsonic speeds.

In the present investigation measurements were made of the changes in the length of laminar flow that were caused by changes in certain test variables known to alter the shock strength. The variables that were changed independently, while all other variables were held constant, were the Mach number, leading-edge sweep, and bluntness. A flat plate with nearly a zero pressure gradient was also tested at a subsonic Mach number, where sweep variation does not change local unit Reynolds number, to determine whether the length of laminar flow changed with sweep.

NOTATION

l_T	length of laminar flow
M	Mach number
p	static pressure
p_{t_2}	total pressure behind a shock wave
R_T	transition Reynolds number based on free-stream conditions, $\frac{U_\infty l_T}{\nu_\infty}$
T	static temperature
T_t	total temperature
U	velocity in the stream direction
α	angle of attack
μ	viscosity
ν	kinematic viscosity
ρ	mass density

Subscripts

M	Mach number
ref	reference Mach number
s	surface values in inviscid flow
Λ	sweep angle
$\Lambda = 0$	sweep angle of 0
∞	free-stream conditions

APPARATUS AND MODEL DESCRIPTION

Facilities

The tests were conducted at Ames Research Center in two different wind tunnels. For the supersonic tests at Mach numbers from 2.00 to 4.00, an 8-inch supersonic nozzle was used. This facility has a translating block for changing Mach number and is a nonreturn blowdown type wind tunnel. For the subsonic tests at a Mach number of 0.27, one of the Ames 7- by 10-foot return type wind tunnels was used.

Models

The models tested at supersonic speeds were supported by a sting attached to the lower surface of the models. Each model had a flat upper surface and a semicircular leading edge. Several different leading-edge radii were obtained by progressively blunting the leading edge. Dimensional data for the model used in the variable sweep tests are given in figure 1(a). The leading-edge sweep of this model could be varied in multiples of 15° by adjustment of an index head which had its rotational axis near the center of the model; thus, the model remained in the center of the wind-tunnel nozzle. This model was tested with leading-edge radii of 0.0005, 0.0025, and 0.020 inch. A second model, with an unswept leading edge, was also used in the supersonic tests in which the Mach number was varied. The leading-edge radii tested on this model were 0.0025, 0.020, and 0.040 inch. A sketch of this model is shown in figure 1(b). Both models were made of steel and had ground finishes for which profilometer measurements indicated a roughness range of about 5 to 25 microinches (rms).

The model used for the subsonic tests had an elliptical nose and was mounted on a single strut. The sweep of this model was changed both by rotating the turntable in the wind-tunnel floor upon which the strut was mounted and by adjusting an index head similar to the one used for the supersonic tests. This model was also made of steel. A cross-sectional sketch of the nose and a dimensional sketch of the model are presented in figure 1(c). The finish of this model was approximately the same as the finish of the models used for the supersonic tests.

TEST CONDITIONS

At Mach numbers of 2.00, 2.50, 3.00, and 4.00, with the angle of attack maintained at 0°, the sweep angle of the 4-inch model was varied from 0° to 75° by increments of 15°. To determine bluntness effects on transition, this model was tested with leading-edge radii of 0.0005, 0.0025, and 0.020 inch. Supersonic Mach number effects on transition were also determined for the 7-inch unswept model throughout the Mach number range of 2.00 to 4.00 by increments of 0.50. This model was tested with nose radii of 0.0025, 0.020, and 0.040 inch. The unit Reynolds number based on free-stream flow conditions was maintained at 15 million per foot for all the supersonic Mach numbers.

At a Mach number of 0.27, the sweep angle of the 37.5-inch model was varied from 0° to 80°. The angle of attack was held constant throughout two separate phases of these tests. The first phase was conducted at an angle of attack of -2.3° measured in a vertical plane in the stream direction. The second phase of the test was conducted at an angle of attack of -2.3° measured in a vertical plane perpendicular to the model leading edge. For these subsonic tests, the unit Reynolds number was maintained at 1.8 million per foot.

VISUAL-FLOW TECHNIQUES

Transition was indicated by the sublimation technique described in reference 7. After each spraying operation the sublimable material was carefully smoothed with a sheet of paper to reduce the possibility of the material from causing premature transition. For the supersonic tests, naphthalene was used as the sublimable material with petroleum ether as the carrying agent. For the subsonic tests, a slower evaporating material, tetrachlorobenzene, was substituted for the naphthalene. In all cases the average length of laminar flow as indicated by the material remaining on the central portions of the models is presented.

During part of the tests the fluorescent-oil technique, described in reference 8, was used to indicate possible flow separation. A mixture of about 80 parts of SAE 40 oil and 1 part of green, fluorescent, oil-soluble powder was brushed on the models and photographed under ultraviolet lights.

RESULTS AND DISCUSSION

Effects of Mach Number at a Sweep Angle of 0°

Experimental results.- Typical photographs showing the increase in the length of laminar flow with an increase in Mach number from 2.00 to 4.00 are presented in figure 2. The variation of transition Reynolds number (based on free-stream flow conditions) with Mach number, as computed from photographs similar to those in figure 2, is presented in figure 3 for both the 4- and 7-inch models with different amounts of leading-edge bluntness. At a Mach number of 2.00, the transition Reynolds number increased with an increase in leading-edge radius from 0.0005 to

0.0025 inch but decreased with additional leading-edge blunting. This result of slight blunting having a favorable effect and additional blunting having an unfavorable effect on the length of laminar flow was also reported in reference 9 at a Mach number of 2.01. In the present investigation at a Mach number of 2.00, however, it was observed that the models vibrated considerably; therefore, it is not clear whether the results at Mach number of 2.00 were influenced by the unsteady flow in the wind tunnel. At Mach numbers of 2.50 and above, the observed model vibration was considerably less, probably making the Mach number effects on transition measured for the higher Mach numbers more reliable. It should be noted that, in general, the models with the blunter leading edges showed a greater increase in the transition Reynolds number with Mach number than the model with the sharpest leading edge. Data taken from references 2 and 3 for a larger range of Mach numbers are also presented in figure 3. Between Mach numbers of 2.8 and 4.1, the data from reference 3 were obtained by firing hollow cylinders, having leading-edge thicknesses of about 0.0003 to 0.0004 inch, through still air. The Mach number effects on transition as measured in the present investigation for the leading-edge radius of 0.0005 inch are about the same as reported in reference 3 even though factors known to affect transition, such as wind-tunnel turbulence and wall interference, were very different for the two investigations.

<u>Predicted results</u>.- The hypothesis is made in reference 4 that the transition Reynolds number based on local properties is substantially unchanged when a sharp leading edge is blunted and, therefore, that the downstream movement of transition is inversely proportional to the ratio of Reynolds number on a blunt leading-edged surface to Reynolds number on a sharp leading-edged surface. This effect then would be most pronounced at the higher Mach numbers for which greater changes in the surface Reynolds number would result from the associated higher shock losses. It is also suggested in reference 4 that at each supersonic Mach number there is a minimum bluntness required to produce the full bluntness effect on transition, but that increases in bluntness beyond this value should have a negligible effect on transition. Although methods for calculating the effect of leading-edge blunting on transition are presented in references 4 and 5, equations for calculating this effect are also presented in appendix A for the sake of completeness. The predicted variation of transition Reynolds number with Mach number, for full leading-edge bluntness effect, as described in appendix A, is presented in figure 3. The transition Reynolds number is assumed equal to the experimental values at a Mach number of 2.50. The latter assumption is required at this time, since it is possible only to predict the change in transition caused by changes in Mach number. As shown in figure 3, the model with the two blunter leading edges nearly attained the predicted effect of Mach number on transition (Mach number increase from 2.50 to 4.00 approximately doubled the length of laminar flow) but the models with the sharper edges showed that the effect of Mach number was somewhat less than the predicted effect. Apparently, neither the model with the two smaller leading-edge radii nor the model of reference 3 was blunt enough to produce the full bluntness effect. However, the data of

reference 2 show that nearly the full bluntness effect was realized in that investigation between Mach numbers of about 6 to 8. The present data were used in reference 5 to show that at a Mach number of 4.00, only a ratio of inviscid shear layer (defined in ref. 4) to boundary-layer thickness of about 0.30 is required to attain nearly the full bluntness effect.

Effects of Sweep at a Constant Mach Number

Experimental results.- Typical photographs showing a reduction in length of laminar flow with an increase in sweep angle at a Mach number of 3.00 are presented in figure 4. The variation of transition Reynolds number (based on free-stream conditions) with sweep at various Mach numbers, as determined from photographs similar to those in figure 4, is presented in figure 5. These data have been normalized by the transition Reynolds number at a sweep angle of 0°. It should be noted in figure 5 that increases in sweep reduced the transition Reynolds number at all supersonic Mach numbers and that increases in bluntness accentuated this effect of sweep. For supersonic Mach numbers of 3.00 and below, and sweep angles above those corresponding to a sonic leading edge (marked on each figure), a discontinuity can be observed in some of the data (e.g., see figs. 5(b) and 5(d)). In the flow visualization studies it was observed that with the leading edge subsonic, fluorescent oil accumulated along the leading edge, indicating local flow separation or a "bubble." Only a few points are presented above these critical sweep angles, since these points do not represent the extent of laminar flow but instead represent the extent of the leading-edge bubble.

At a Mach number 0.27 and a sweep angle of 0°, this same type of leading-edge bubble existed at an angle of attack of 0°. A separation bubble also occurred on this model at an angle of attack of -2.3° when the sweep angle was 75°. Transition data for these conditions are not presented since, again, they would not represent the length of laminar flow. Representative fluorescent-oil photographs are presented in figure 6, showing evidence of the leading-edge bubble at Mach numbers of 0.27 and 3.00. It was found necessary, therefore, to test the model at a Mach number of 0.27 at a small negative angle of attack to eliminate the bubble and to obtain a nearly zero pressure gradient. At various angles of sweep at a Mach number of 0.27, the angle of attack was held at -2.3°, first as measured in the stream direction and then as measured in a vertical plane perpendicular to the wing leading edge. As shown in figure 5(a) this change in angle of attack caused only a small difference in the results. For the flow conditions of this test, it can be seen that sweep had only a small effect on the transition Reynolds number until the sweep angle was greater than about 45°. The reduction in length of laminar flow at a sweep angle of 60° is believed to be associated with the crossflow instability discussed in references 5 and 6. It should be realized that the sweep angle at which crossflow instability first occurred is uniquely defined by a critical crossfl Reynolds number and, therefore, depends on the local Mach number, Reynolds number, and wall temperature.

Although all the data presented herein were obtained on steel models, a limited investigation was conducted at a Mach number of 3.00 on a model geometrically similar to the 4-inch model but made of nylon, to determine possible effects on transition which might be attributable to heat conduction within the model. Even though the conductivity of steel is approximately eight times that of nylon, no measurable differences were found in the effect of sweep on transition.

Predicted results.- It is suggested in reference 4 that the difference in the transition movement caused by blunting of swept and unswept flat surfaces at supersonic speeds can be attributable to the difference in the shock strengths for these two cases. From this shock concept, it can be reasoned that blunting should have a larger favorable effect on the length of laminar flow for an unswept model than for a swept model. This can be reasoned from the fact that the local unit Reynolds number of the unswept model would be reduced to a greater extent when consideration is given to the flow changes associated with the stronger normal shock for the unswept model. Equations are given in appendix B by which the maximum change in transition caused by sweep can be estimated according to the shock method of reference 4. Similar equations are also presented in reference 5.

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The maximum change in transition Reynolds number with sweep as predicted from the method of reference 4 is shown for each Mach number in figure 5. It can be noted that, in general, for the model with leading-edge radii of 0.0025 and 0.020 inch, a larger decrease in transition Reynolds number with sweep was measured as the supersonic Mach number was increased as would be expected from shock-loss considerations alone; however, the experimental values are considerably below the predicted values. For the model with the leading-edge radius of 0.0005 inch, the predicted change in the variation of transition Reynolds number with sweep due to Mach number changes was not realized. This difference in the experimental and predicted results might again be explained on the basis that the model with 0.0005-inch leading-edge radius had insufficient bluntness to obtain the full bluntness effect, particularly at higher Mach numbers for which reference 4 indicates that greater bluntness is required. The reason that sweep had a larger detrimental effect on transition than expected from shock-loss considerations for the model with the two blunter leading edges is believed to be connected with the crossflow phenomena associated with swept wings, discussed in references 5 and 6. Photographic evidence of the crossflow and resultant longitudinal vortices existing over the model is given in figure 7 in which striations similar to those shown in reference 6 are discernible. Similar striations were observed at all angles of sweep except 0°. It is believed evident, then, that two important adverse effects on transition can be induced by sweep, even on a flat plate with a leading-edge radius of only 0.0005 inch, one effect related to the leading-edge shock and the other related to the crossflow and associated vortex flow.

As expected with no shock losses, no change in transition Reynolds number with sweep was measured as shown in figure 5(a) at a Mach number of 0.27 until a sweep angle of about 60° was obtained. At this sweep angle the crossflow effect probably became dominant. When this model was swept, striations were also observed in the sublimable material, showing evidence of the three-dimensionality of the flow.

A summary plot showing the change in transition Reynolds number with sweep at various Mach numbers, as computed with the equations for the shock method in appendix B, is presented in figure 8. The line defining the sweep angles at which the leading edge becomes sonic is also shown in this figure. For a given Mach number, no laminar flow was noted in this investigation at sweep angles at and above those for which the leading edge was sonic. Because of the probable existence of some crossflow on the surface and the adverse effect of this flow on transition, figure 8 simply provides an estimate of the maximum effect of sweep on a flat plate to be expected from shock-loss considerations alone.

SUMMARY OF RESULTS

The following results were obtained from an investigation of the effects of Mach number, leading-edge bluntness, and sweep on the transition Reynolds number (based on free-stream properties) of a flat plate.

1. For the model with nose radii of 0.020 and 0.040 inch the transition Reynolds number was approximately doubled by an increase in Mach number from 2.50 to 4.00, a result approximately in agreement with the increase predicted from shock-loss considerations in NACA Rep. 1312 for flat plates with sufficient bluntness to realize the full bluntness effect.

2. At all supersonic Mach numbers, increases in sweep reduced the transition Reynolds number, and increases in bluntness accentuated this effect of sweep. At a Mach number of 0.27 and a unit Reynolds number of 1.8 million, no significant reduction in the transition Reynolds number with sweep was measured for sweep angles up to 45°.

3. At supersonic Mach numbers whenever the leading edge became subsonic, flow separation (a bubble) occurred at the leading edge and the reattached boundary layer was always turbulent.

4. For the model with leading-edge radii of 0.0025 and 0.020 inch, sweeping the leading edge decreased the length of laminar flow more than the amount that would be predicted from shock-loss considerations alone.

This result may be caused by the crossflow instability associated with a swept leading edge, since the sublimation studies showed evidence of streamwise vortices existing over the model surface when the model was swept.

Ames Research Center
National Aeronautics and Space Administration
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APPENDIX A

DEVELOPMENT OF EQUATION FOR THE EFFECT OF MACH NUMBER ON FREE-STREAM TRANSITION REYNOLDS NUMBER

An equation is derived below from which the effects of Mach number on transition Reynolds number (based on free-stream flow conditions) can be computed. In accordance with reference 4, the following assumptions will be made:

(1) The leading edge is sufficiently blunt that the Reynolds number at the outer edge of the boundary layer is the same as would be computed for the inviscid shear layer at the model surface for the entire length of the laminar flow.

(2) Transition occurs far enough downstream from the leading edge that the surface static pressure is equal to the free-stream static pressure.

(3) Any changes in free-stream transition Reynolds number can be wholly explained on the basis of changes in the local unit Reynolds number produced by the normal shock at the leading edge.

At a given Mach number, the ratio of the unit Reynolds number with a blunted leading edge to that for an unblunted leading edge can be written as

$$\frac{U_s/\nu_s}{U_\infty/\nu_\infty} \equiv \frac{\rho_s U_s \mu_\infty}{\rho_\infty U_\infty \mu_s} \tag{A1}$$

where, from reference 10,

$$\frac{U_s}{U_\infty} = \frac{M_s}{M_\infty}\sqrt{\frac{T_s}{T_\infty}} \tag{A2}$$

and, from Sutherland's formula, in reference 10,

$$\frac{\mu_\infty}{\mu_s} = \frac{T_s + 198.6}{T_\infty + 198.6}\left(\frac{T_\infty}{T_s}\right)^{3/2} \tag{A3}$$

and, from the equation of state, since it is assumed that $p_s = p_\infty$,

$$\frac{\rho_s}{\rho_\infty} = \frac{T_\infty}{T_s} \tag{A4}$$

Substituting equations (A2), (A3), and (A4) into equation (A1), we obtain

$$\frac{U_s/\nu_s}{U_\infty/\nu_\infty} = \left(\frac{T_\infty}{T_s}\right)^2 \left(\frac{M_s}{M_\infty}\right) \left(\frac{T_s + 198.6}{T_\infty + 198.6}\right) \tag{A5}$$

Substituting the relationship for temperature and Mach number as given in reference 10 (noting that the enthalpy is constant through a shock wave) in equation (A5), we obtain

$$\frac{U_s/\nu_s}{U_\infty/\nu_\infty} = \left(\frac{1 + 0.2\ M_s^2}{1 + 0.2\ M_\infty^2}\right)^2 \left(\frac{M_s}{M_\infty}\right) \left(\frac{\dfrac{T_t}{1 + 0.2\ M_s^2} + 198.6}{\dfrac{T_t}{1 + 0.2\ M_\infty^2} + 198.6}\right) \tag{A6}$$

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where the relationship between M_s and M_∞ can be derived by equating the Rayleigh pitot formula (100) in reference 10 to the pressure versus Mach number formula (44) to obtain

$$M_s = \left[(6M_\infty^2)\left(\frac{6}{7M_\infty^2 - 1}\right)^{5/7} - 5\right]^{1/2} \tag{A7}$$

It is also possible to obtain M_s for a given M_∞ from table II of reference 10. This Mach number, M_s, is then obtained by entering column 2 in table II at the value of p_1/p_{t_2} given in column 16 corresponding to Mach number M_∞. Since the length of laminar flow or the transition Reynolds number based on stream conditions will vary inversely as the ratio given by equation (A6), the following expression can be written.

$$\frac{(R_T)_M}{(R_T)_{M_{ref}}} = \frac{\left[(U_\infty/\nu_\infty)/(U_s/\nu_s)\right]_M}{\left[(U_\infty/\nu_\infty)/(U_s/\nu_s)\right]_{M_{ref}}} \tag{A8}$$

Substituting equation (A6) into equation (A8), we obtain

$$\frac{(R_T)_M}{(R_T)_{M_{ref}}} = \frac{\left[\left(\frac{1 + 0.2\ M_s^2}{1 + 0.2\ M_\infty^2}\right)^2 \left(\frac{M_s}{M_\infty}\right)\left(\frac{\frac{T_t}{1 + 0.2\ M_s^2} + 198.6}{\frac{T_t}{1 + 0.2\ M_\infty^2} + 198.6}\right)\right]_{M_{ref}}}{\left[\left(\frac{1 + 0.2\ M_s^2}{1 + 0.2\ M_\infty^2}\right)^2 \left(\frac{M_s}{M_\infty}\right)\left(\frac{\frac{T_t}{1 + 0.2\ M_s^2} + 198.6}{\frac{T_t}{1 + 0.2\ M_\infty^2} + 198.6}\right)\right]_{M}} \tag{A9}$$

where M_s is given by equation (A7).

APPENDIX B

DEVELOPMENT OF THE EQUATION FOR THE EFFECT OF SWEEP

ON FREE-STREAM TRANSITION REYNOLDS NUMBER

The assumptions stated in appendix A will be made for the development of the equation for the effect of sweep on transition Reynolds number (based on free-stream flow conditions). In this case, however, the changes in transition Reynolds number produced by sweep will be associated entirely with changes in the unit Reynolds number on the surface caused by differences in losses through a normal shock for the unswept leading edge and through oblique shocks for the swept leading edges.

At a given Mach number for a constant free-stream unit Reynolds number, the ratio of the unit Reynolds number for the swept case to that for the unswept case can be written as

$$\frac{(U_s/\nu_s)_\Lambda}{(U_s/\nu_s)_{\Lambda=0}} \equiv \frac{(\rho_s)_\Lambda (U_s)_\Lambda (\mu_s)_{\Lambda=0}}{(\rho_s)_{\Lambda=0} (U_s)_{\Lambda=0} (\mu_s)_\Lambda} \tag{B1}$$

where, from reference 10,

$$\frac{(U_s)_\Lambda}{(U_s)_{\Lambda=0}} = \frac{(M_s)_\Lambda}{(M_s)_{\Lambda=0}} \sqrt{\frac{(T_s)_\Lambda}{(T_s)_{\Lambda=0}}} \tag{B2}$$

and, from Sutherland's formula,

$$\frac{(\mu_s)_{\Lambda=0}}{(\mu_s)_\Lambda} = \frac{(T_s)_\Lambda + 198.6}{(T_s)_{\Lambda=0} + 198.6} \left[\frac{(T_s)_{\Lambda=0}}{(T_s)_\Lambda}\right]^{3/2} \tag{B3}$$

and, from the equation of state, noting that $(p_s)_\Lambda = (p_s)_{\Lambda=0}$,

$$\frac{(\rho_s)_\Lambda}{(\rho_s)_{\Lambda=0}} = \frac{(T_s)_{\Lambda=0}}{(T_s)_\Lambda} \tag{B4}$$

Substituting equations (B2), (B3), and (B4) in equation (B1), we obtain

$$\frac{(U_s/\nu_s)_\Lambda}{(U_s/\nu_s)_{\Lambda=0}} = \left[\frac{(T_s)_{\Lambda=0}}{(T_s)_\Lambda}\right]^2 \left[\frac{(M_s)_\Lambda}{(M_s)_{\Lambda=0}}\right] \left[\frac{(T_s)_\Lambda + 198.6}{(T_s)_{\Lambda=0} + 198.6}\right] \tag{B5}$$

Since the enthalpy is constant through a shock, equation (B5) can be rewritten in terms of Mach number from equation (43) of reference 10 as

$$\frac{(U_s/\nu_s)_\Lambda}{(U_s/\nu_s)_{\Lambda=o}} = \left[\frac{1 + 0.2(M_s)_\Lambda^2}{1 + 0.2(M_s)_{\Lambda=o}^2}\right]^2 \left[\frac{(M_s)_\Lambda}{(M_s)_{\Lambda=o}}\right] \left[\frac{\dfrac{T_t}{1 + 0.2(M_s)_\Lambda^2} + 198.6}{\dfrac{T_t}{1 + 0.2(M_s)_{\Lambda=o}^2} + 198.6}\right] \tag{B6}$$

To compute the surface Mach numbers, it is first necessary to compute the ratio of static pressure to total pressure for oblique shocks (eq. (143) in ref. 10) from the following equation

$$\frac{(p_{t_2})_\Lambda}{(p_s)_\Lambda} = \frac{(p_{t_2})_\Lambda}{p_\infty} = \left(\frac{6}{7M_\infty^2\cos^2\Lambda - 1}\right)^{2.5} \left[\frac{6M_\infty^2\cos^2\Lambda(M_\infty^2 + 5)}{5(M_\infty^2\cos^2\Lambda + 5)}\right]^{3.5} \tag{B7}$$

Note that the ratio $(p_{t_2})_\Lambda/(p_s)_\Lambda$ cannot be taken from the tables at $M_\infty \cos \Lambda$ except at $\Lambda = 0$, because one of the Mach number terms of equation (B7) is not multiplied by $\cos \Lambda$. The pressure and Mach number relationship (eq. (44) of ref. 10) and equation (B7) can be used to compute the surface Mach number from the following equation:

$$(M_s)_\Lambda = \sqrt{5\left[\frac{(p_{t_2})_\Lambda}{(p_s)_\Lambda}\right]^{2/7} - 5} \tag{B8}$$

For investigations in which the unit Reynolds number based on free-stream conditions is maintained at a constant value, the length of laminar flow or the free-stream transition Reynolds number will vary with sweep inversely as the ratio of unit Reynolds numbers given in equation (B6). It follows that equation (B6) written in terms of free-stream transition Reynolds number becomes

$$\frac{(R_T)_\Lambda}{(R_T)_{\Lambda=o}} = \frac{(U_s/\nu_s)_{\Lambda=o}}{(U_s/\nu_s)_\Lambda} = \left[\frac{1 + 0.2(M_s)_{\Lambda=o}^2}{1 + 0.2(M_s)_\Lambda^2}\right]^2 \frac{(M_s)_{\Lambda=o}}{(M_s)_\Lambda} \left[\frac{\dfrac{T_t}{1 + 0.2(M_s)_{\Lambda=o}^2} + 198.6}{\dfrac{T_t}{1 + 0.2(M_s)_\Lambda^2} + 198.6}\right] \tag{B9}$$

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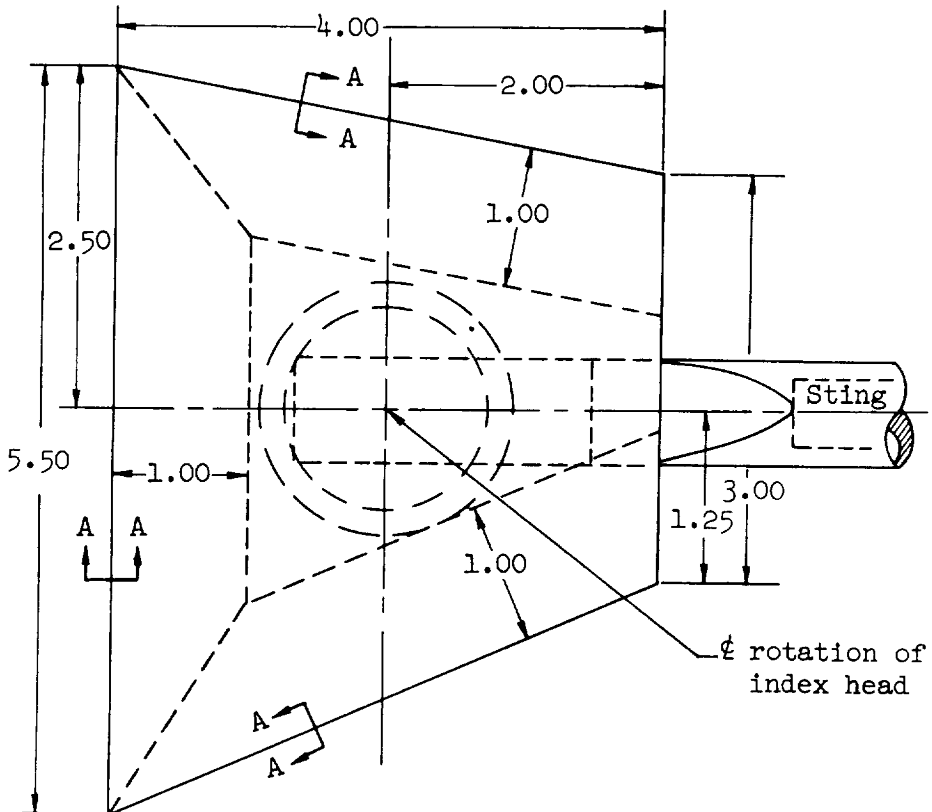
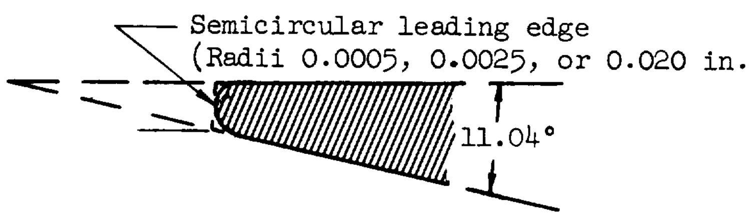
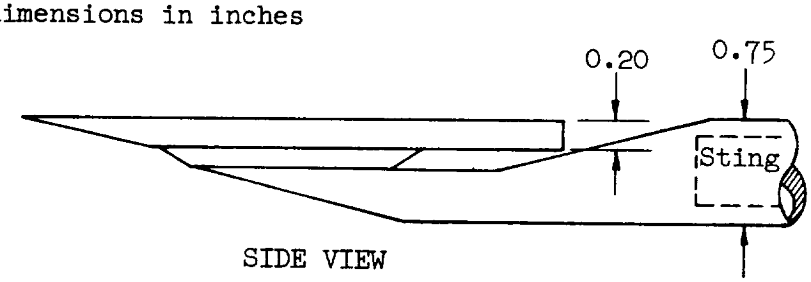


TOP VIEW



Section A-A

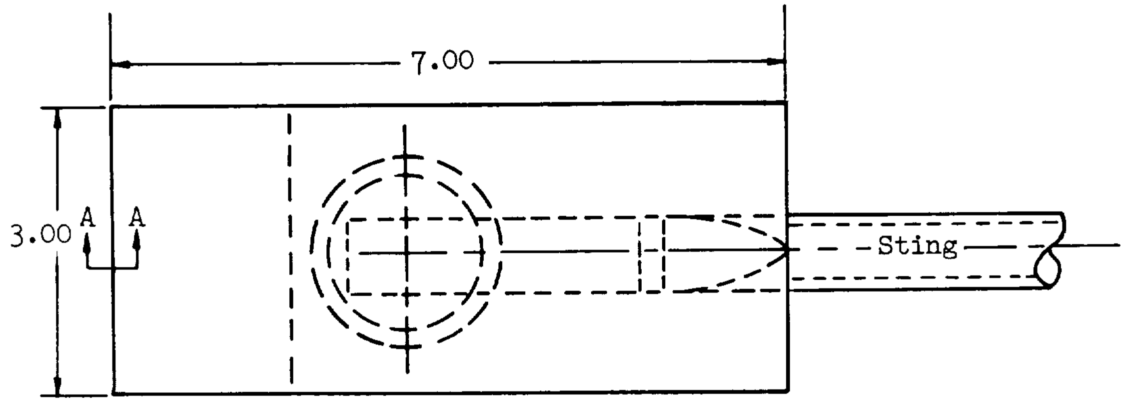
Note: All dimensions in inches



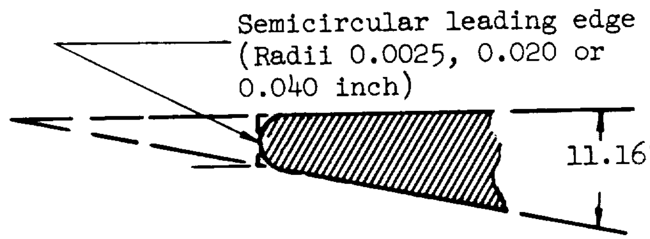
SIDE VIEW

(a) The 4-inch model.

Figure 1.- Model dimensions.

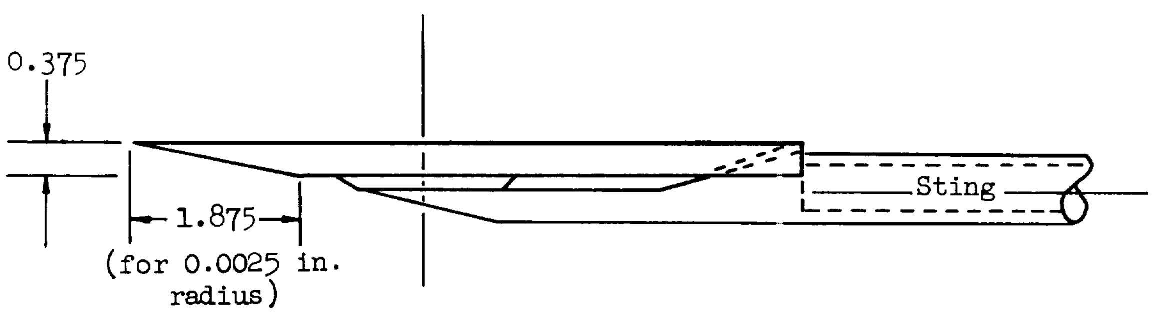


TOP VIEW



Section A-A

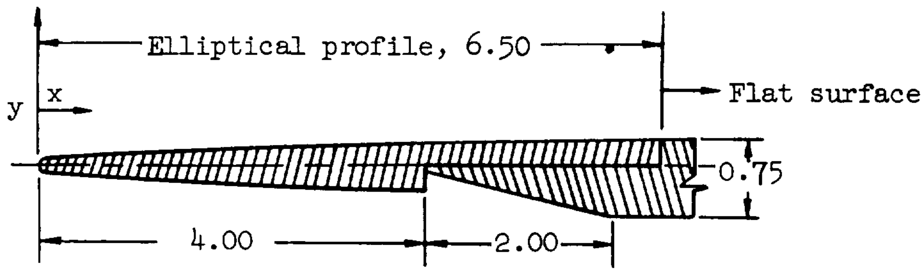
Note: All dimensions in inches



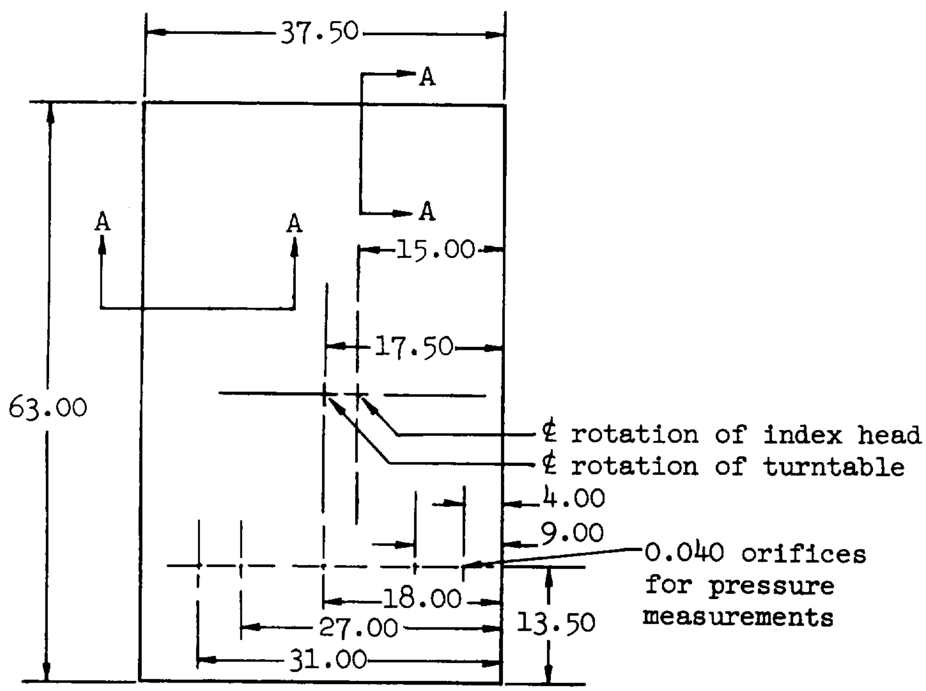
SIDE VIEW

(b) The 7-inch model.

Figure 1.- Continued.



Section A-A



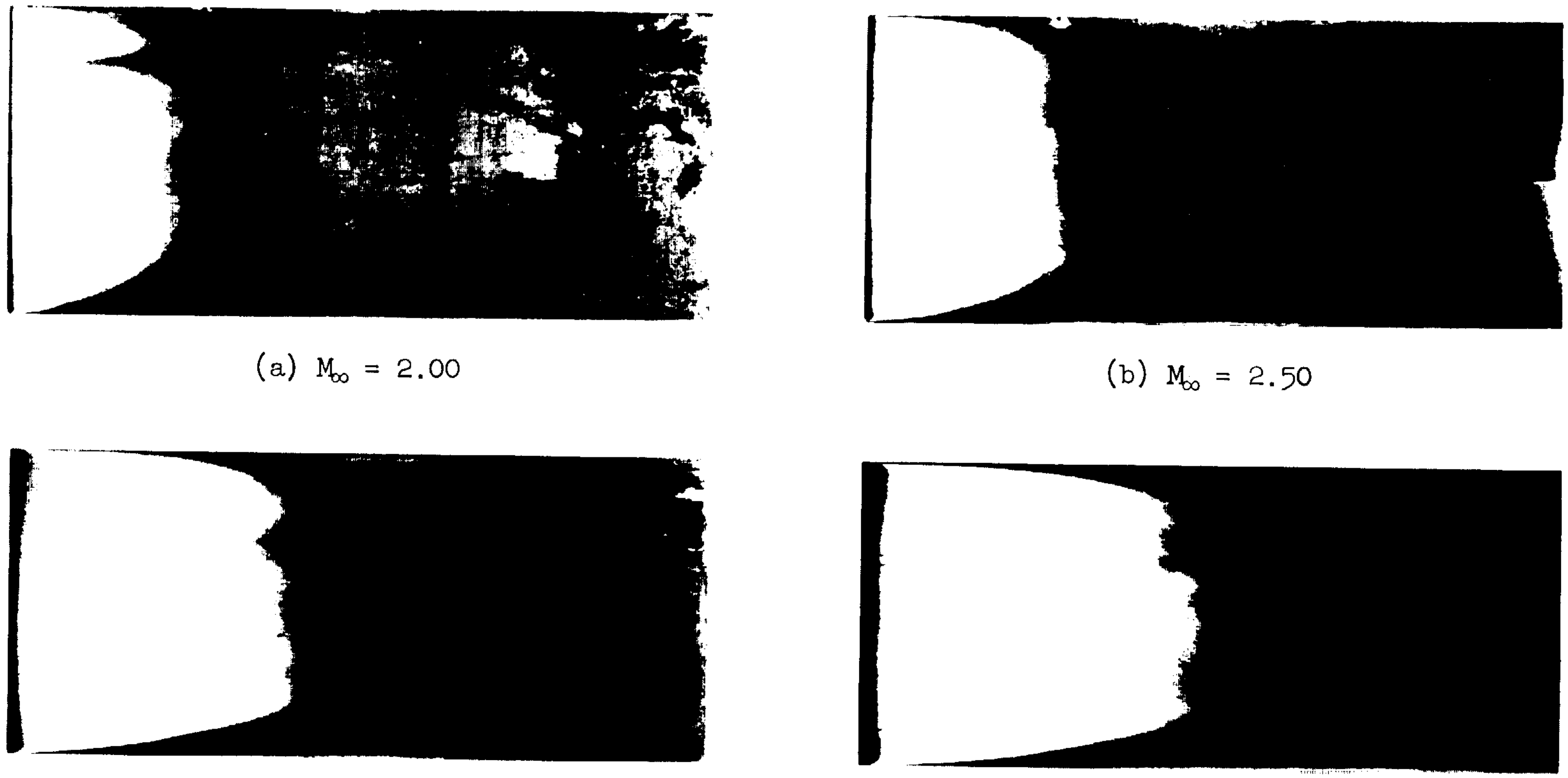
Top view

Nose coordinates	
x, in.	y, in.
0	0
.40	.090
.80	.125
1.20	.150
1.60	.170
2.00	.186
2.40	.200
2.80	.211
3.20	.221
3.60	.229
4.00	.236
4.40	.241
4.80	.245
5.20	.248
5.60	.250
6.00	.250
6.50	.250
L.E. rad.=0.050	

Note: All dimensions in inches

(c) The 37.5-inch model.

Figure 1.- Concluded.



(a) $M_\infty = 2.00$

(b) $M_\infty = 2.50$

(c) $M_\infty = 3.50$

(d) $M_\infty = 4.00$

Figure 2.- Boundary-layer transition on the 7-inch model at various Mach numbers as indicated by the sublimation material; $\Lambda = 0^\circ$, leading-edge radius = 0.0025 inch.

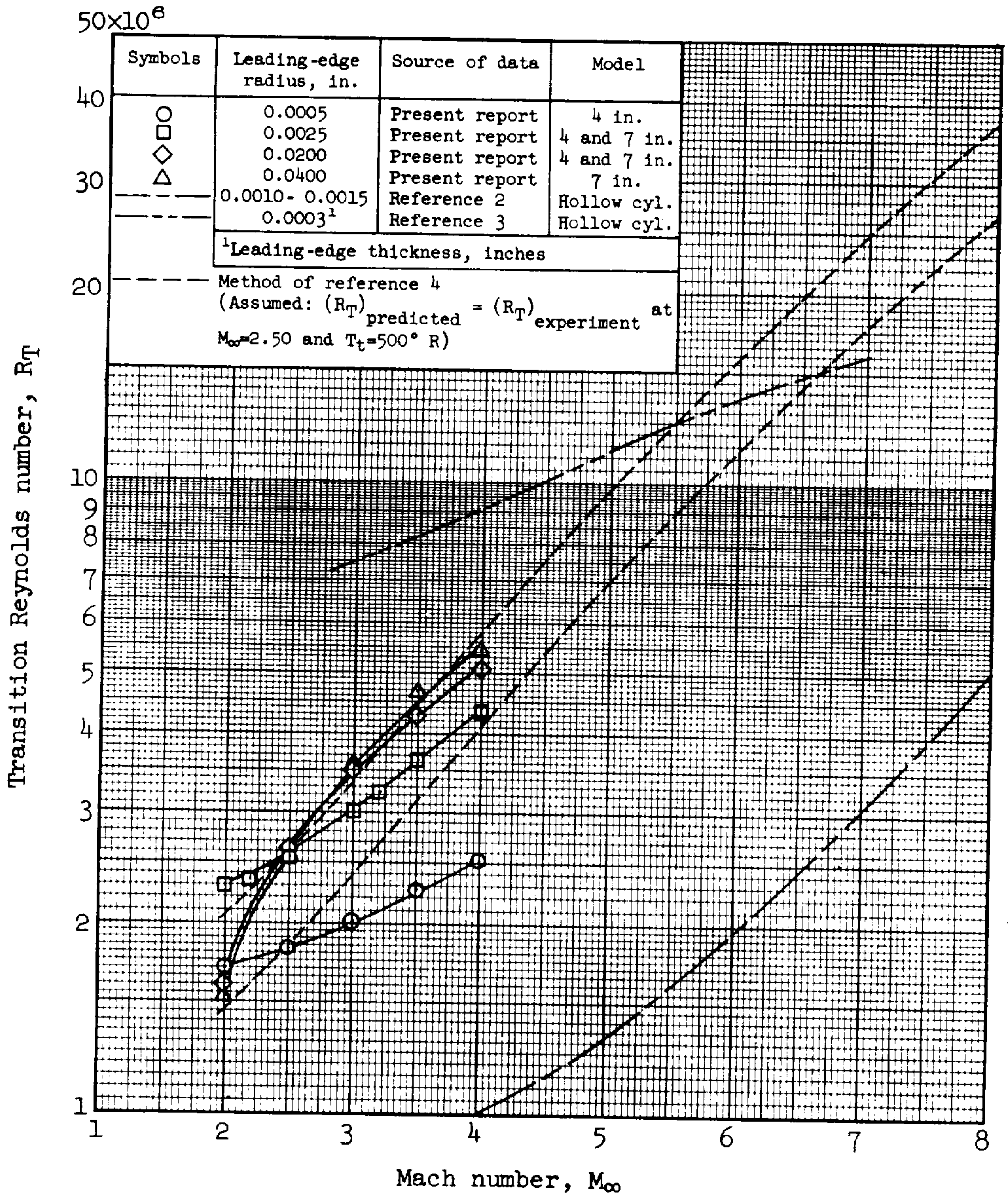
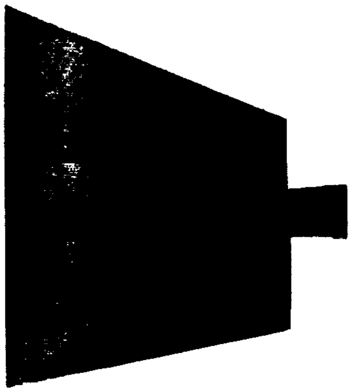
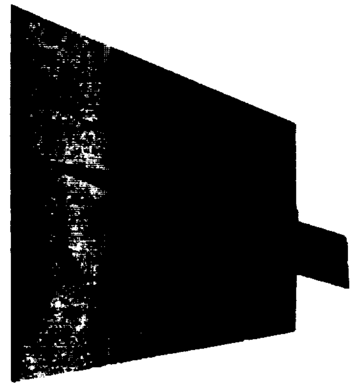


Figure 3.- Variation of the transition Reynolds number with Mach number; $\Lambda = 0^{\circ}$.



(a) $\Lambda = 0^{\circ}$



(b) $\Lambda = 15^{\circ}$

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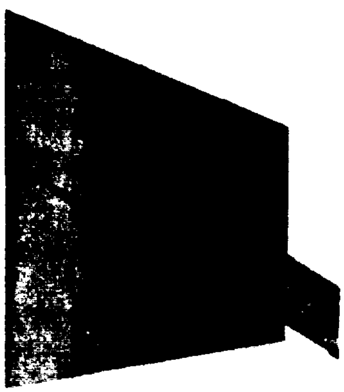
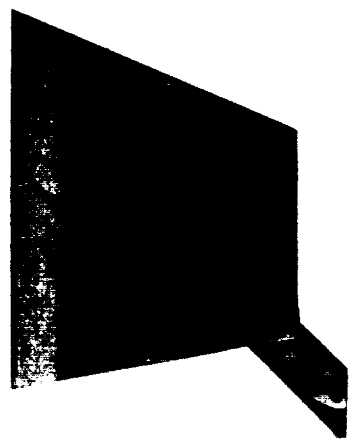
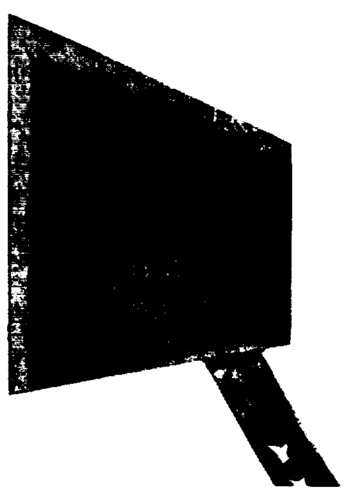


(c) $\Lambda = 30^{\circ}$

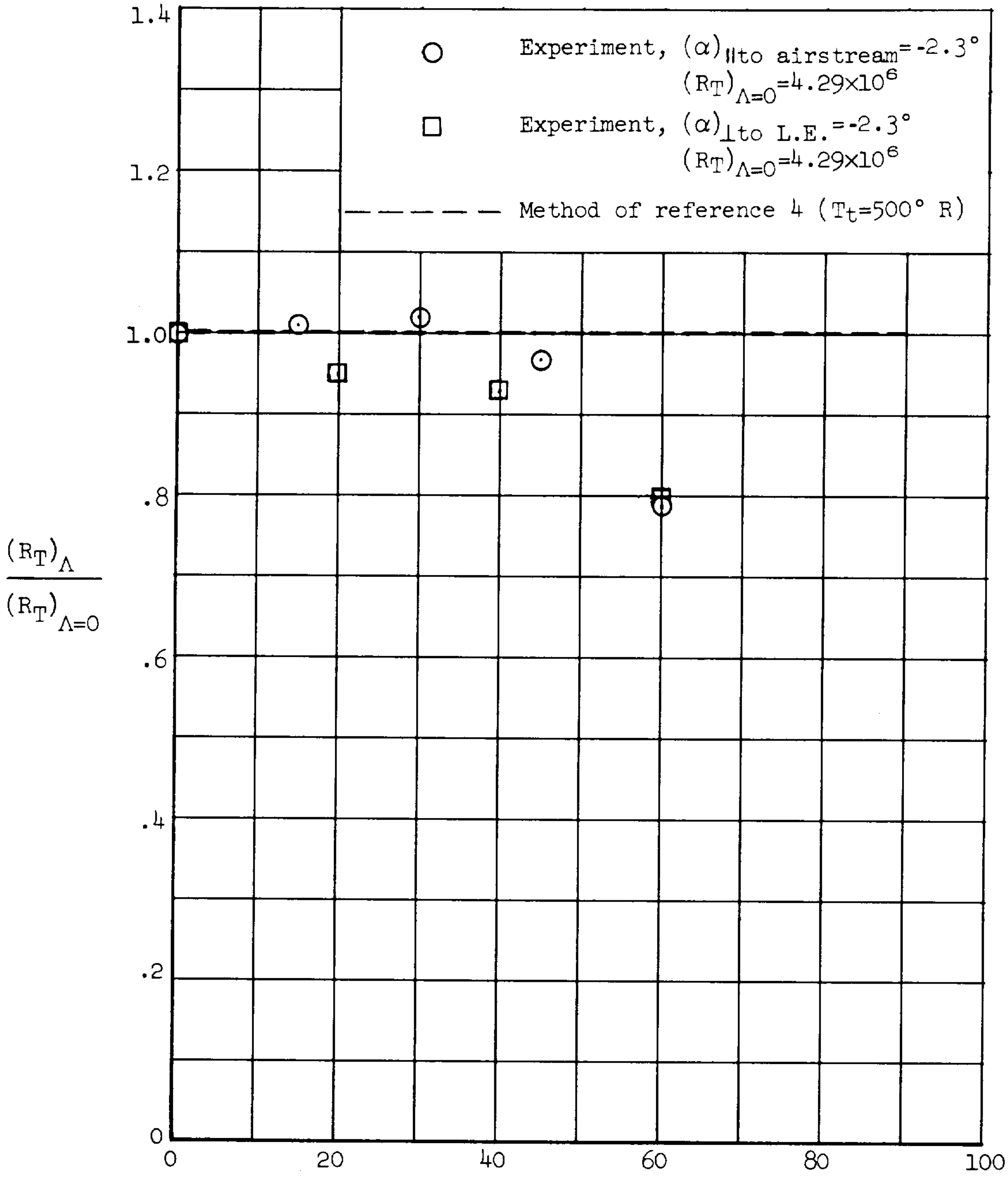


(d) $\Lambda = 45^{\circ}$



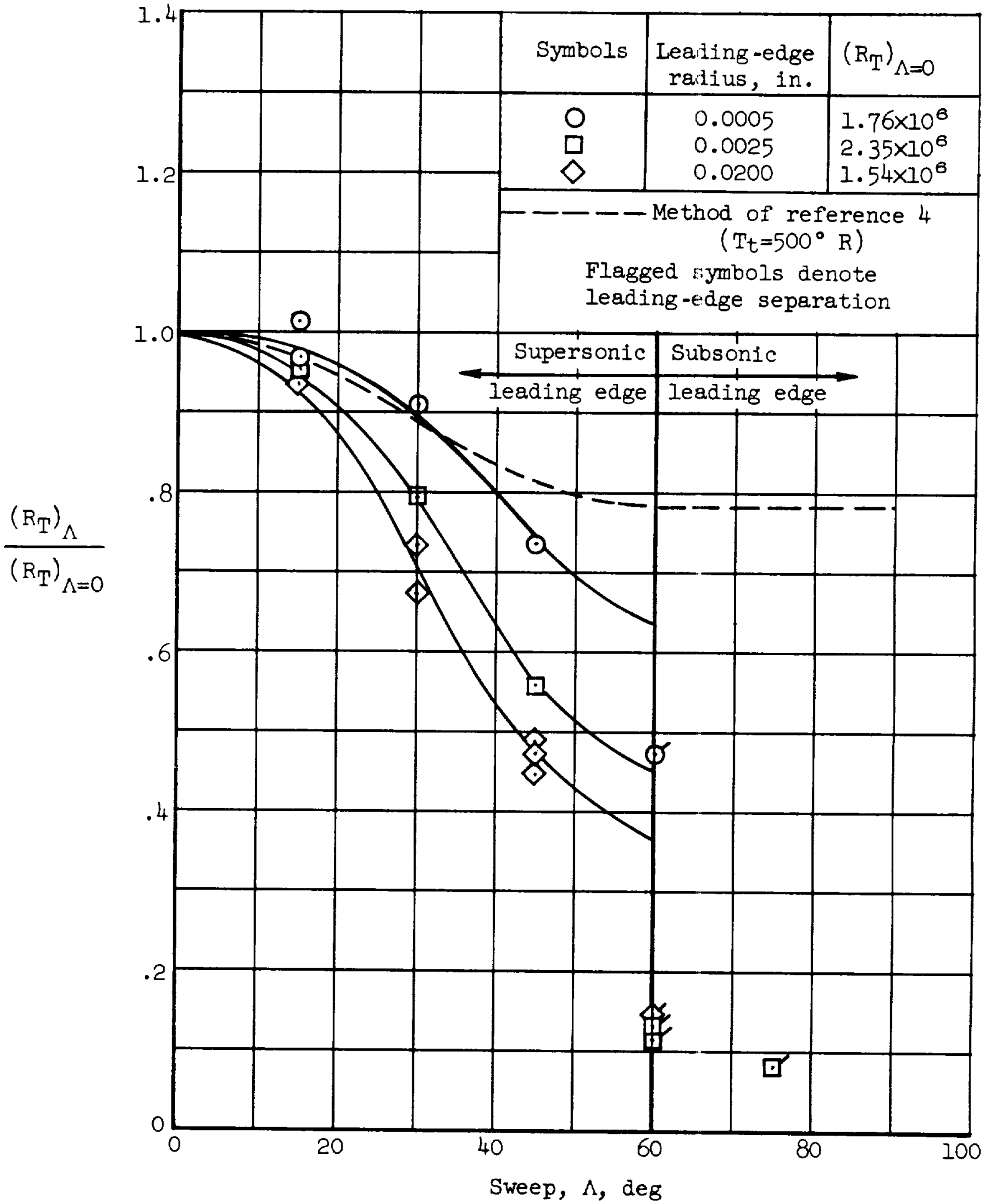
(e) $\Lambda = 60^{\circ}$

Figure 4.- Boundary-layer transition on the 4-inch model at various angles of sweep as indicated by the sublimation material; leading-edge radius = 0.0005 inch.



(a) $M_\infty = 0.27$

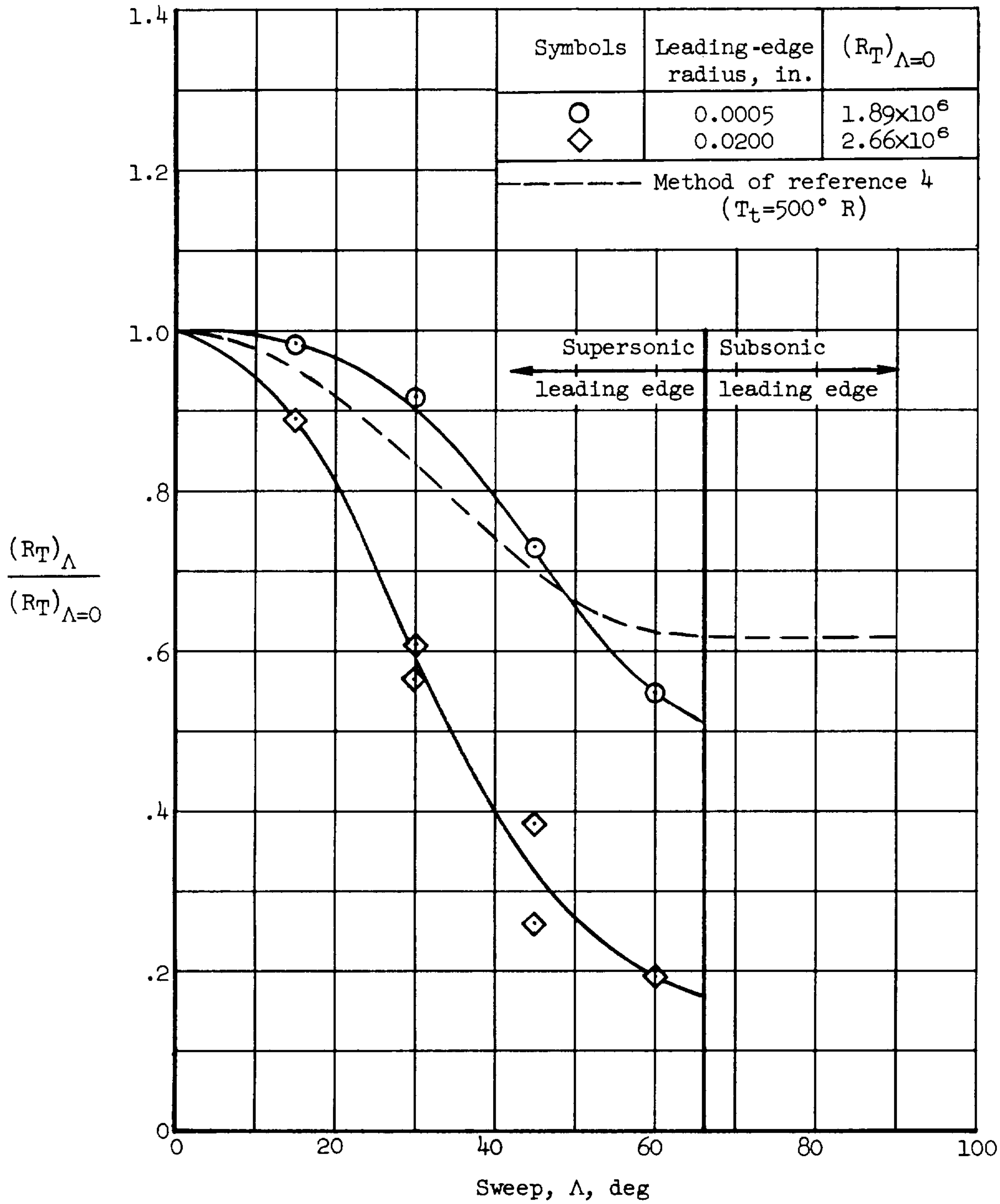
Figure 5.- Variation of the normalized transition Reynolds number with sweep.



(b) $M_\infty = 2.00$

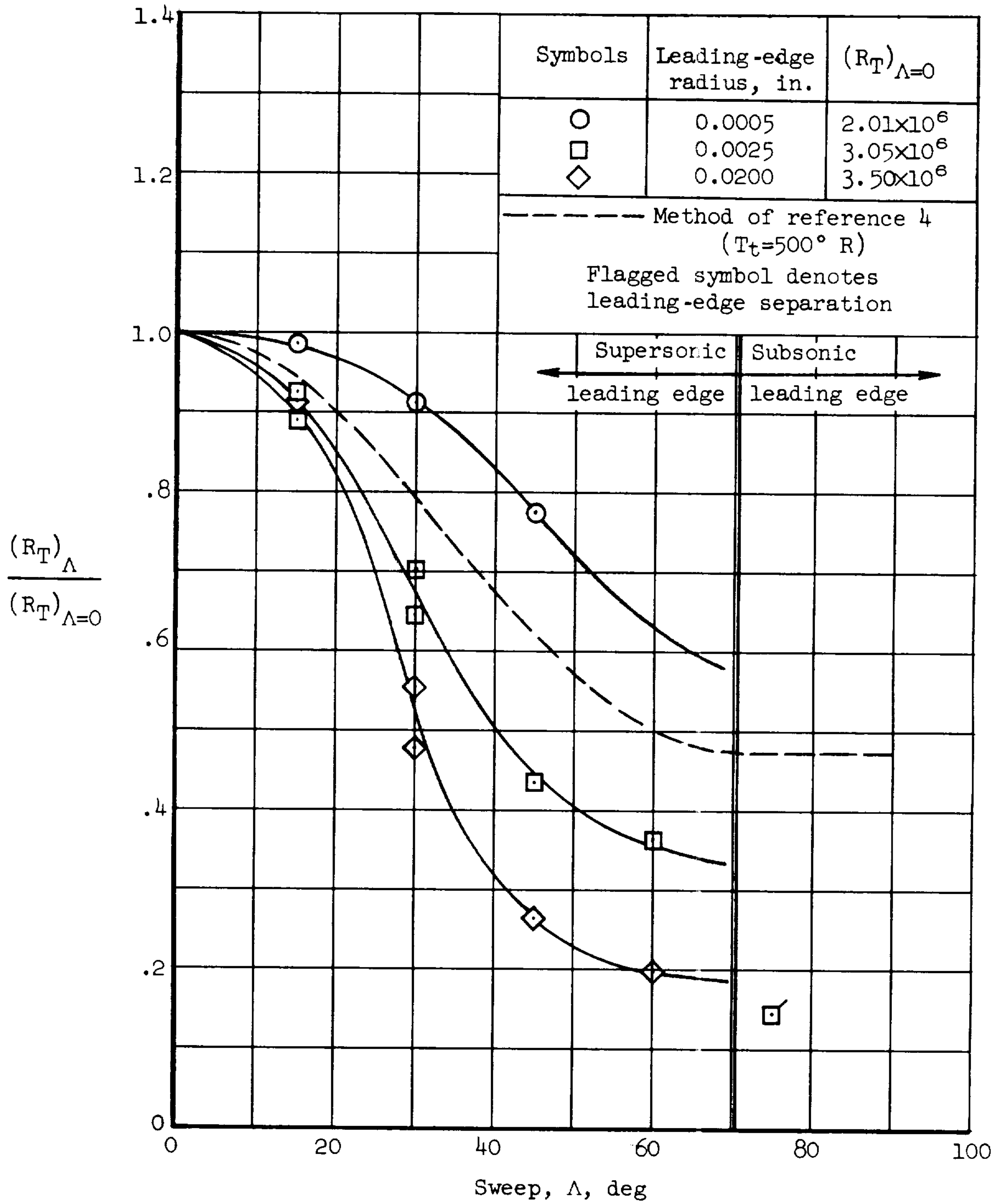
Figure 5.- Continued.

A
4
8
1



(c) M_∞ = 2.50

Figure 5.- Continued.



(d) $M_\infty = 3.00$

Figure 5.- Continued.

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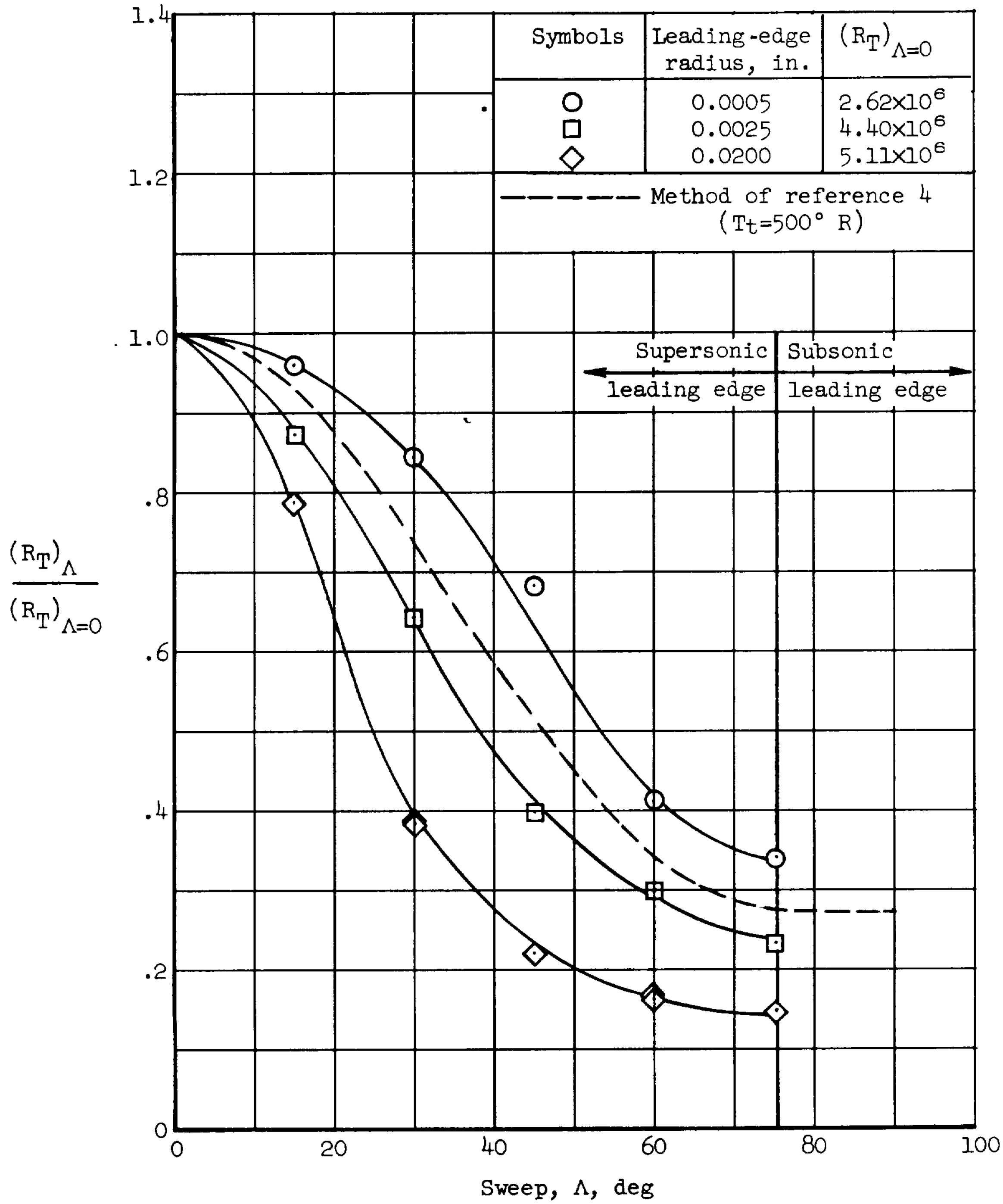
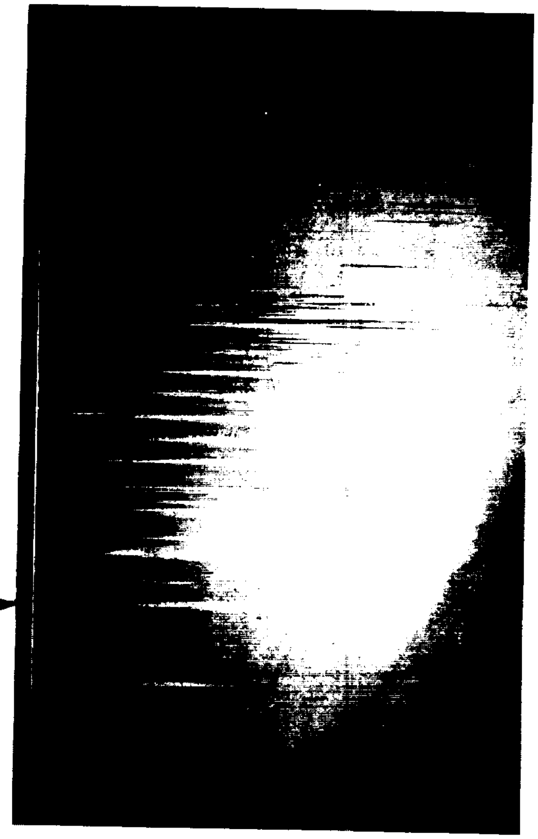


(e) $M_\infty = 4.00$

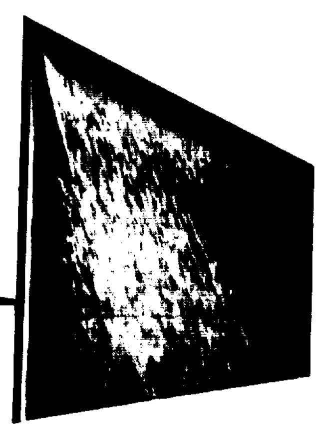
Figure 5.- Concluded.

Leading-edge "bubble"



(a) Subsonic ($M_\infty = 0.27$, $\Lambda = 0^\circ$).

Leading-edge "bubble"



(b) Supersonic ($M_\infty = 3.00$, $\Lambda = 75^\circ$).

Figure 6.- Flow separation at leading edge indicated by fluorescent oil; $\alpha = 0^\circ$.

A
4
8
1

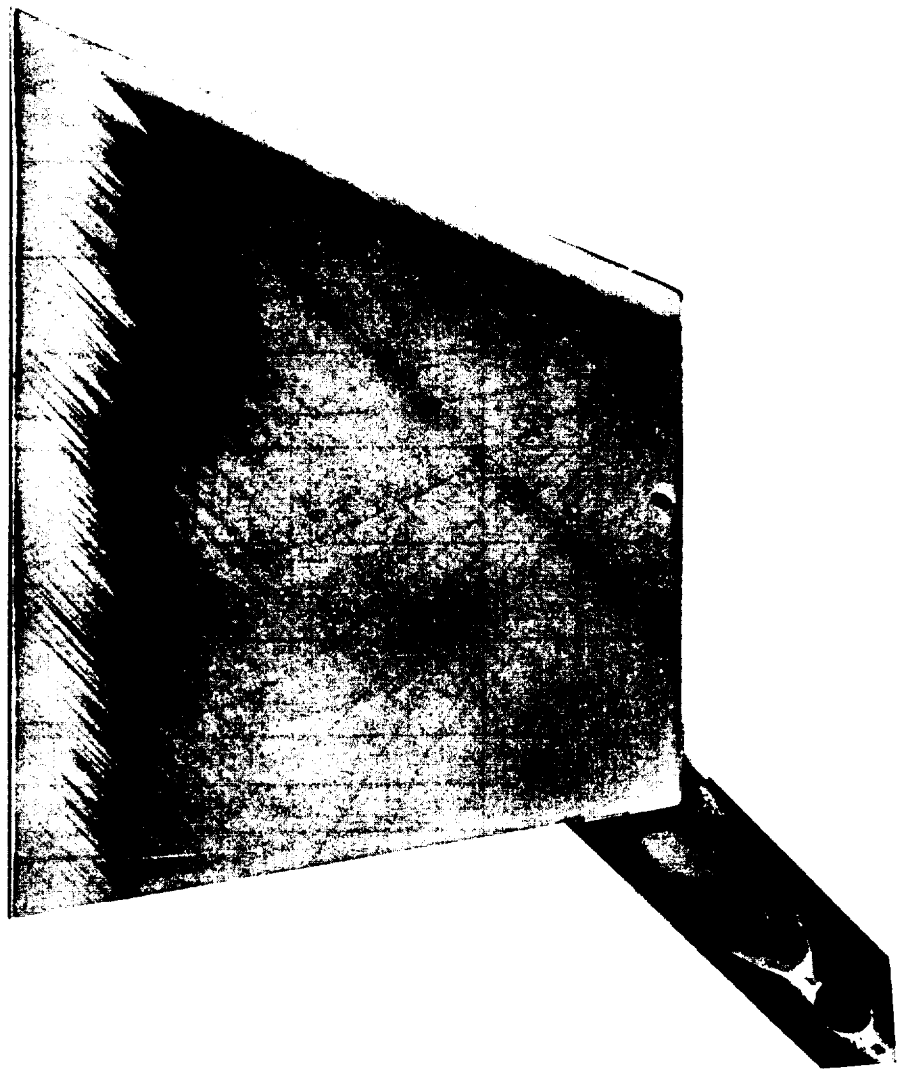


Figure 7.- Evidence of striations on 4-inch model as indicated by the sublimation material; $M_\infty = 4.00$, $\Lambda = 45^\circ$, leading-edge radius = 0.020 inch.

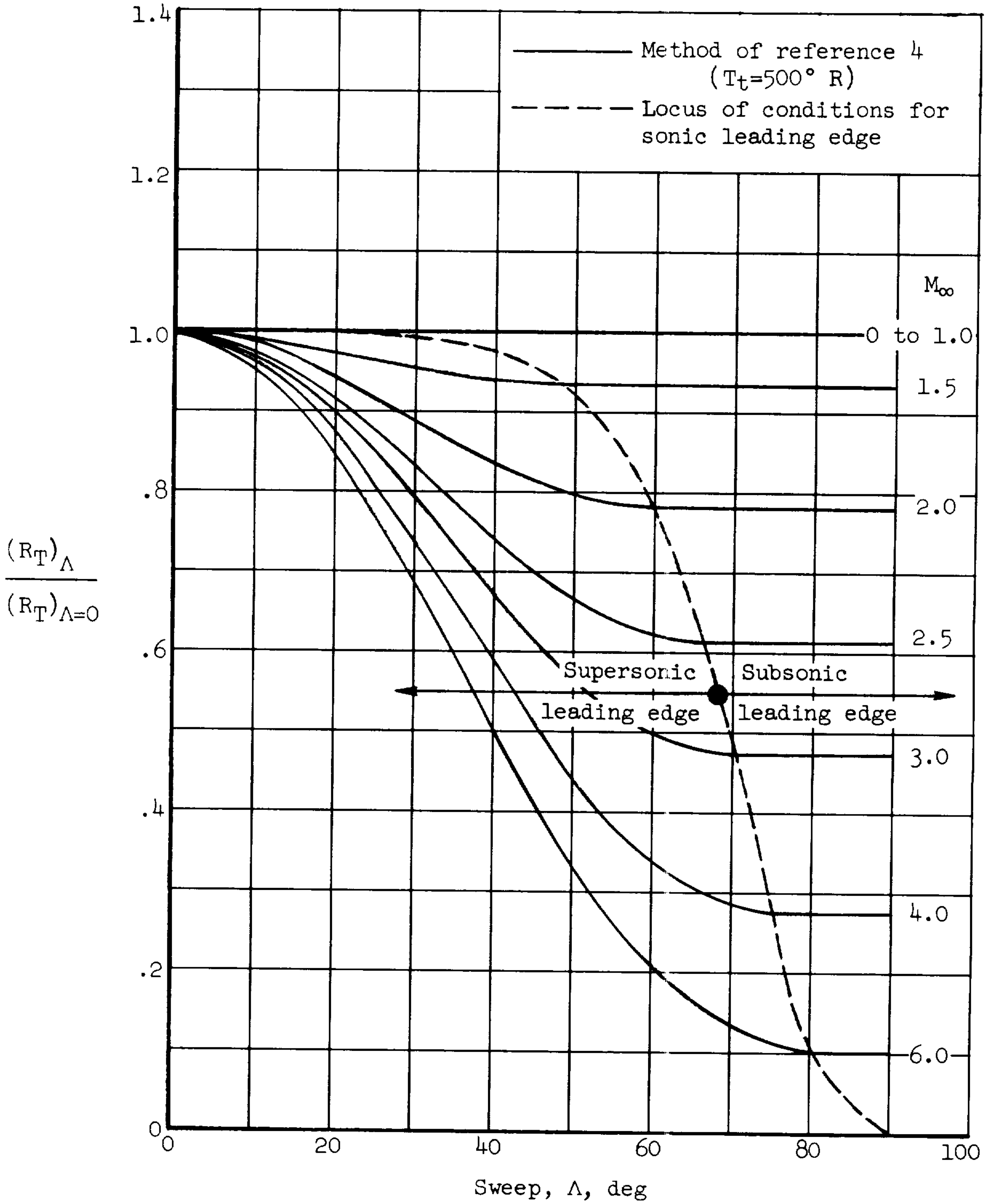


Figure 8.- Estimated variation of the normalized transition Reynolds number with sweep at various Mach numbers.

A 4 8 1

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