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MEMORANDUM

EFFECT OF ARTIFICIAL PITCH DAMPING ON THE LONGITUDINAL
AND ROLLING STABILITY OF AIRCRAFT WITH
NEGATIVE STATIC MARGINS

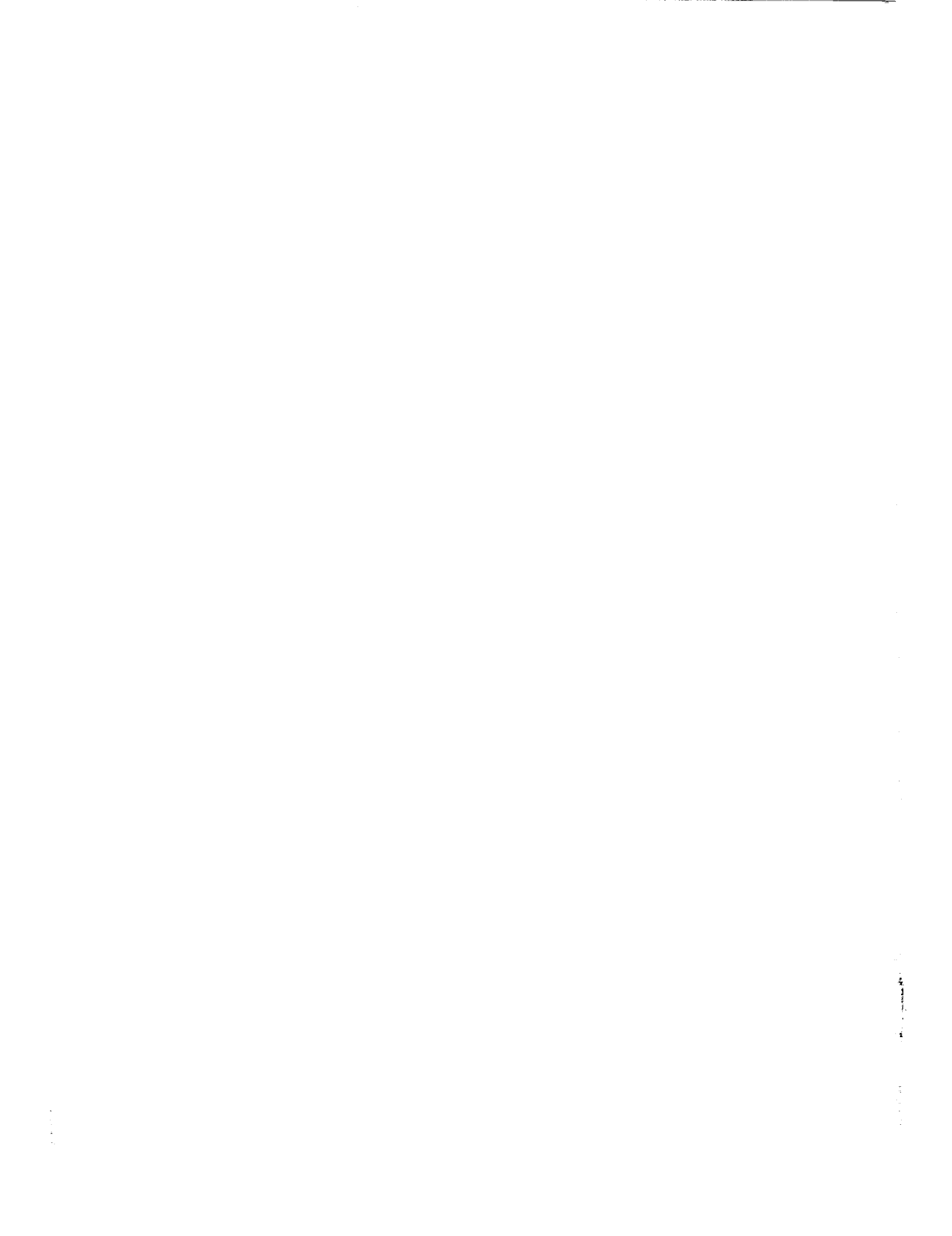
By Martin T. Moul and Lawrence W. Brown

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SUMMARY

A preliminary theoretical investigation has been made of the short-period longitudinal and steady-rolling (inertia coupling) stability of a hypersonic glider configuration for center-of-gravity locations rearward of the airplane neutral point. Such center-of-gravity positions for subsonic flight would improve performance by reducing supersonic and hypersonic static margins and trim drag. Results are presented of stability calculations and a simulator study for a velocity of 700 ft/sec and an altitude of 40,000 feet.

With no augmentation, the airplane was rapidly divergent and was considered unsatisfactory in the simulator study. When a pitch damper was employed as a stability augmenter, the short-period mode became overdamped, and the airplane was easily controlled on the simulator. A steady-rolling analysis showed that the airplane can be made free of rolling divergence for all roll rates with an appropriate damper gain.

INTRODUCTION

Aircraft performance at supersonic and hypersonic speeds is sometimes penalized by high trim drag. This unfavorable condition results from the rearward shift of the neutral point (stability increase) for supersonic speeds and the requirement of static stability at subsonic speeds. The possibility occurs of improving the performance of aircraft which are operated predominantly at high speeds by permitting some static instability at subsonic speeds.

As the center of gravity is moved rearward, the occurrence of problems in both longitudinal stability and roll coupling can be anticipated at subsonic speeds. In the longitudinal mode, a pitch divergence is encountered when the center of gravity is located rearward of the maneuver point. In the rolling mode, it has been shown analytically (ref. 1) and demonstrated in flight (unintentionally in most instances) that roll coupling and a severe divergence may be encountered when the rolling velocity exceeds a certain critical value dependent on the pitching-moment-curve slope (static margin) or the yawing-moment-curve slope. Both of these stability problems must be investigated in any consideration of flight at small or negative static margins.

A preliminary analysis has been made of the stability both in pitch and steady roll (inertia coupling) of a hypersonic glider configuration for a subsonic-speed condition in which the airplane center of gravity is rearward of the neutral point. Since most high-speed aircraft are equipped with pitch dampers, the effect of auxiliary pitch damping in stabilizing such a system is considered.

Results of response calculations and a simulator study for a velocity of 700 ft/sec and an altitude of 40,000 feet are included.

SYMBOLS

a_n	normal acceleration, g units
A_1, B_1, C_1	coefficients of critical-rolling-velocity equation
b	wing span, ft
B, C	coefficients of longitudinal characteristic equation
\bar{c}	wing mean aerodynamic chord, ft
C_L	lift coefficient, $\frac{\text{Lift}}{qS}$
C_m	pitching-moment coefficient, $\frac{\text{Pitching moment}}{qS\bar{c}}$
C_n	yawing-moment coefficient, $\frac{\text{Yawing moment}}{qSb}$
C_Y	lateral-force coefficient, $\frac{\text{Lateral force}}{qS}$

g	acceleration due to gravity, ft/sec ²
h	altitude, ft
I_X	moment of inertia of airplane about principal X-axis, slug-ft ²
I_Y	moment of inertia of airplane about principal Y-axis, slug-ft ²
I_Z	moment of inertia of airplane about principal Z-axis, slug-ft ²
K	pitch-damper gain, deg/deg/sec
K_Y	nondimensional radius of gyration about Y-axis
m	airplane mass, slugs
p_0	steady rolling velocity, radians/sec
p_{cr}	critical rolling velocity, radians/sec
q	dynamic pressure, lb/sq ft
r	yawing velocity, radians/sec
S	wing area, sq ft
t	time, sec
$t_{1/2}$	time to damp to one-half amplitude, sec
V	airplane velocity, ft/sec
α	angle of attack of airplane principal axis, radians or deg
$\dot{\alpha}$	time rate of change of angle of attack, $\frac{d\alpha}{dt}$, radians/sec
α_0	initial angle of attack, deg
β	angle of sideslip, radians
δ_e	elevator deflection, radians or deg

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$\delta_{e,p}$ pilot-applied elevator deflection, radians or deg

θ angle of pitch, radian

$\dot{\theta}$ pitching angular velocity, $\frac{d\theta}{dt}$, radians/sec

$\ddot{\theta}$ pitching angular acceleration, $\frac{d^2\theta}{dt^2}$, radians/sec²

λ characteristic root of quadratic equation

$$\mu = \frac{m}{\rho S \bar{c}}$$

ρ atmospheric density, slugs/cu ft

$$\tau = \frac{m}{\rho S V}$$

dC_m/dC_L static margin

$$C_{L_\alpha} = \frac{\partial C_L}{\partial \alpha}$$

$$C_{m_\alpha} = \frac{\partial C_m}{\partial \alpha}$$

$$C_{m_q} = \frac{\partial C_m}{\partial \frac{\dot{\theta} \bar{c}}{2V}}$$

$$C_{n_\beta} = \frac{\partial C_n}{\partial \beta}$$

$$C_{n_r} = \frac{\partial C_n}{\partial \frac{rb}{2V}}$$

$$C_{Y_\beta} = \frac{\partial C_Y}{\partial \beta}$$

$$C_{m_{de}} = C_{m_q} + \frac{2V}{\bar{c}} KC_{m_{\delta_e}}$$

$$C_{m\delta_e} = \frac{\partial C_m}{\partial \delta_e}$$

$$L_\alpha = qS\bar{c}L_{\alpha}$$

$$M_\alpha = qS\bar{c}^2 C_{m\alpha}$$

$$M_q = \frac{qS\bar{c}^2}{2V} C_{mq}$$

$$N_\beta = qSbC_{n\beta}$$

$$N_r = \frac{qSb^2}{2V} C_{nr}$$

$$Y_\beta = qSY_\beta$$

$$M_{q_e} = M_q + KM_{\delta_e}$$

$$M_{\delta_e} = qS\bar{c}^2 C_{m\delta_e}$$

ANALYSIS

An airplane having a center-of-gravity position at or rearward of the neutral point and no stability augmentation may possess undesirable stability characteristics in both the pitch and roll modes. The effect of artificial pitch damping on the stability of these two modes will be considered.

Longitudinal Stability

As the short-period mode is the longitudinal mode of primary concern to pilots, this analysis has been restricted to a consideration of this mode. The airplane characteristics which influence pilots' opinions are period and damping of the oscillation and stick force per g. This investigation is limited to a consideration of the dynamic characteristics for variations in the aerodynamic parameters, pitching-moment-curve slope and damping in pitch. In the simulator study, satisfactory stick forces were mechanized so that pilots' opinions were dependent

only on the airplane dynamic characteristics simulated. The dynamic characteristics are analyzed for zero and unstable (positive) values of the pitching-moment-curve slope with and without a pitch damper included as a stability augments.

Equations of motion and system transfer function.- The longitudinal short-period characteristics are determined from the pitching-moment and lift equations of motion. These equations are written as

$$\left. \begin{aligned} I_Y \ddot{\theta} &= M_q \dot{\theta} + M_{\alpha} \alpha + M_{\delta_e} \delta_e \\ mV \dot{\alpha} - mV \dot{\theta} &= -L_{\alpha} \alpha \end{aligned} \right\} \quad (1)$$

when weight components, lift due to elevator deflection, pitching moment due to $\dot{\alpha}$, and variations in forward speed are neglected.

To increase pitch damping, an autopilot introducing an elevator deflection proportional to pitching velocity ($\delta_e = K\dot{\theta}$) is considered. Preliminary calculations and results of reference 2 showed that small gyro time lags have a negligible effect on airplane response. Consequently the effects of time lag are omitted in this analysis. The equations of motion with artificial pitch damping are

$$\left. \begin{aligned} I_Y \ddot{\theta} &= M_{q_e} \dot{\theta} + M_{\alpha} \alpha + M_{\delta_e} \delta_{e,p} \\ mV \dot{\alpha} - mV \dot{\theta} &= -L_{\alpha} \alpha \end{aligned} \right\} \quad (1a)$$

where

$$M_{q_e} = M_q + KM_{\delta_e} \quad (2)$$

and $\delta_{e,p}$ is a pilot-applied elevator deflection. The airplane transfer functions $\alpha/\delta_{e,p}$ and $\dot{\theta}/\delta_{e,p}$ are

$$\left. \begin{aligned} \frac{\alpha}{\delta_{e,p}} &= \frac{M_{\delta_e}/I_Y}{\lambda^2 + \left(\frac{L_{\alpha}}{mV} - \frac{M_{q_e}}{I_Y}\right)\lambda - \left(\frac{M_{\dot{\alpha}}}{I_Y} + \frac{L_{\alpha}}{mV} \frac{M_{q_e}}{I_Y}\right)} \\ \frac{\dot{\theta}}{\delta_{e,p}} &= \frac{\frac{M_{\delta_e}}{I_Y} \lambda + \frac{L_{\alpha}}{mV} \frac{M_{\delta_e}}{I_Y}}{\lambda^2 + \left(\frac{L_{\alpha}}{mV} - \frac{M_{q_e}}{I_Y}\right)\lambda - \left(\frac{M_{\dot{\alpha}}}{I_Y} + \frac{L_{\alpha}}{mV} \frac{M_{q_e}}{I_Y}\right)} \end{aligned} \right\} \quad (3)$$

Characteristic equation and stability.- The characteristic equation (denominator of the transfer function) is written generally as

$$\lambda^2 + B\lambda + C = 0 \quad (4)$$

and has the roots

$$\lambda = -\frac{B}{2} \pm \sqrt{\left(\frac{B}{2}\right)^2 - C}$$

For stability, the coefficients of a quadratic characteristic equation must be positive. Now, the coefficient B is always positive for normal values of $C_{L\alpha}$ and $C_{m_{qe}}$, that is, positive $C_{L\alpha}$ and negative $C_{m_{qe}}$ values. The coefficient C is normally positive (for negative pitching-moment-curve slopes) but may become negative for positive values of $C_{m\alpha}$. The condition of interest for this investigation is that of positive $C_{m\alpha}$ values, for which artificial pitch damping is employed to maintain positive values of coefficient C. This condition corresponds to a center-of-gravity location between the neutral point and maneuver point. The maneuver point is the center-of-gravity position for neutral maneuvering stability and is defined by $C = 0$.

The nature of the characteristic modes is determined by the sign of the discriminant $(B/2)^2 - C$. Substituting for B and C gives

$$\begin{aligned} \left(\frac{B}{2}\right)^2 - C &= \frac{\left(\frac{L_{\alpha}}{mV} - \frac{M_{qe}}{I_Y}\right)^2}{4} + \frac{M_{\alpha}}{I_Y} + \frac{L_{\alpha}}{mV} \frac{M_{qe}}{I_Y} \\ &= \frac{\left(\frac{L_{\alpha}}{mV} + \frac{M_{qe}}{I_Y}\right)^2}{4} + \frac{M_{\alpha}}{I_Y} \end{aligned} \quad (5)$$

The term $\left(\frac{L_{\alpha}}{mV} + \frac{M_{qe}}{I_Y}\right)^2$ must be positive or zero. Then for $M_{\alpha} > 0$, $(B/2)^2 - C$ is positive and the characteristic roots are real; and two aperiodic modes (which are stable for $C > 0$) rather than the usual short-period oscillation are obtained. These conditions ($M_{\alpha} > 0$ and $C > 0$) hold whenever the center of gravity is between the neutral and maneuver points and are the primary conditions of concern in this analysis.

Stability in Steady Roll

Preliminary analyses of roll-coupling problems are frequently made by the use of the equations of motion for steady rolling and employ the concepts of critical roll rates, rolling divergence boundaries, and stability and airplane responses in steady rolls. (See refs. 1, 3, and 4.) In reference 1 it was shown that the rolling velocity, above which a divergence can be expected with zero values of damping, is given

by the smaller value of $\sqrt{-\frac{M_\alpha}{I_Z - I_X}}$ and $\sqrt{\frac{N_\beta}{I_Y - I_X}}$. For the conditions of

interest in this paper, the longitudinal response consists of two aperiodic modes by virtue of artificial pitch damping, and it is of interest to investigate the effect of rolling velocity on the stability of these configurations.

The following constant coefficient of the characteristic equation for steady rolling (including M_{q_e} , N_r , L_α , Y_β) is from reference 4:

$$\begin{aligned}
 E = & \left(\frac{I_Y - I_X}{I_Z} \right) \left(\frac{I_Z - I_X}{I_Y} \right) p_o^4 - \frac{M_\alpha}{I_Y} \left(\frac{I_X - I_Y}{I_Z} \right) p_o^2 - \frac{N_\beta}{I_Z} \left(\frac{I_Z - I_X}{I_Y} \right) p_o^2 + \\
 & \frac{M_{q_e}}{I_Y} \frac{N_r}{I_Z} p_o^2 + \frac{L_\alpha}{mV} \frac{Y_\beta}{mV} \left(\frac{I_X - I_Y}{I_Z} \right) \left(\frac{I_Z - I_X}{I_Y} \right) p_o^2 - \frac{M_\alpha}{I_Y} \frac{N_\beta}{I_Z} - \\
 & \frac{M_{q_e}}{I_Y} \frac{N_\beta}{I_Z} \frac{L_\alpha}{mV} - \frac{M_\alpha}{I_Y} \frac{N_r}{I_Z} \frac{Y_\beta}{mV} - \frac{M_{q_e}}{I_Y} \frac{N_r}{I_Z} \frac{L_\alpha}{mV} \frac{Y_\beta}{mV}
 \end{aligned} \tag{6}$$

The coefficient E equated to zero will yield critical rolling velocities as functions of aerodynamics and inertia characteristics. In general form, the equation is

$$A_1 p_{cr}^4 + B_1 p_{cr}^2 + C_1 = 0$$

from which

$$p_{cr}^2 = -\frac{B_1}{2A_1} \pm \sqrt{\left(\frac{B_1}{2A_1}\right)^2 - \frac{C_1}{A_1}} \tag{7}$$

Positive values of p_{cr}^2 are indicative of regions of rolling instability, whereas negative or complex values of p_{cr}^2 (p_{cr} imaginary) are indicative of configurations free of rolling divergence for all rolling velocities.

For stability-boundary considerations, there are two conditions of interest:

(1) For negative B_1 , the boundary between real and imaginary roll rates is defined by the discriminant equated to zero. Thus,

$$\frac{C_1}{A_1} = \left(\frac{B_1}{2A_1} \right)^2 \quad (8)$$

(2) For positive B_1 , the boundary between real and imaginary roll rates takes a different form; that is, $p_{cr}^2 = 0$, from which the result is obtained

$$\frac{C_1}{A_1} = 0 \quad (9)$$

With equations (8) and (9), stability boundaries in terms of selected aerodynamic derivatives as variables may be constructed. An application of these results will be given subsequently.

RESULTS

Calculations of longitudinal and rolling stability were made for a high-speed aircraft configuration for a velocity of 700 ft/sec and an altitude of 40,000 feet. Physical and aerodynamic characteristics assumed for this investigation are given in table I. Results are presented for a range of static margins from 0.02 to -0.02 and pitch-damper gains from 0 to 2.

Longitudinal Stability

For positive damping conditions, there are three possible types of short-period modes: a damped oscillation, an overdamped response (two stable aperiodic modes), or a divergent and convergent mode, depending on the center-of-gravity position. In this investigation the center-of-gravity positions of interest are those falling up to $0.02\bar{c}$ rearward of the neutral and unaugmented maneuver points, for which the characteristic motion with the addition of artificial pitch damping is an overdamped response.

Constant $t_{1/2}$ curves.— Time to damp to one-half amplitude is the characteristic used to define overdamped systems. An equation relating

$t_{1/2}$ to $C_{m_{q_e}}$ and $C_{m_{\alpha}}$ is determined by substituting $\lambda = -\frac{0.693}{t_{1/2}}$ into the characteristic equation (eq. (4)). The result is

$$C_{m_{q_e}} = \left(\frac{2\mu t_{1/2}}{0.693\tau - \frac{1}{2} C_{L_{\alpha}} t_{1/2}} \right) C_{m_{\alpha}} + \frac{1.386\tau t_{1/2} K_Y^2 C_{L_{\alpha}} - 1.92K_Y^2 \tau^2}{0.693\tau t_{1/2} - \frac{1}{2} C_{L_{\alpha}} t_{1/2}^2} \quad (10)$$

Lines of constant $t_{1/2}$ are presented in figure 1 with $C_{m_{q_e}}$ as the ordinate and $C_{m_{\alpha}}$ as the abscissa. There are two characteristic damping times associated with each point $C_{m_{q_e}}, C_{m_{\alpha}}$ as indicated by the intersecting lines.

Within the parabola in the right-hand quadrant are contained the conditions for an oscillatory short-period response. The divergence boundary, or maneuver-point location, appears in the left-hand quadrant. Between the oscillatory and divergence boundaries is the region for stable overdamped responses. Stability deteriorates as static instability (positive $C_{m_{\alpha}}$ values) increases or $C_{m_{q_e}}$ decreases, as evidenced by the larger $t_{1/2}$ values.

Effect of damper gain on $t_{1/2}$.- In figure 2 is presented the variation of $t_{1/2}$ with damper gain K for values of $\frac{dC_m}{dC_L}$ of 0, 0.01, and 0.02. The upper set of curves for $\frac{dC_m}{dC_L}$ corresponds to the dominant mode in the longitudinal response, and it is desirable to keep $t_{1/2}$ small. As gain approaches infinity, the upper set of curves becomes asymptotic to the value $0.693 \frac{mV}{L_{\alpha}}$. For this configuration, there is little improvement in damping for increases in gain above 3.

The effect of artificial damping on the controllability of this particular configuration is indicated in the figure by the curves of constant α/δ_e . Both pitch damping and $\frac{dC_m}{dC_L}$ contribute to the airplane stiffness (coefficient C) so that the magnitude of α/δ_e decreases with an increase in K or a decrease in $\frac{dC_m}{dC_L}$. The usual range of α/δ_e for airplanes is about -1 to -3. Thus, figure 2 is useful for selecting pitch-damper gains in accordance with any selected stability and control requirements.

Response to unit-step elevator deflection.- The dynamics of several configurations are illustrated in figure 3 by means of angle-of-attack responses to unit-step elevator deflections for several static margins and pitch-damper gains. With no artificial damping, the maneuver point and neutral point are nearly coincident. Hence, the response for $K = 0$ is divergent for $\frac{dC_m}{dC_L} = 0.02$, about neutrally stable for $\frac{dC_m}{dC_L} = 0$, and stable and aerodynamically damped for $\frac{dC_m}{dC_L} = -0.02$. With artificial pitch damping added to the divergent configurations, the responses become stable and overdamped, as shown in figure 1, and would be expected to be tolerable to pilots.

Simulator results.- In order to determine a pilot's opinion of negative static-margin configurations both with and without artificial damping a simple closed-loop simulator consisting of an analog computer, chair, center control stick, and displays of normal acceleration and angle of attack was utilized. The stick was mechanized to provide a force per g of $4\frac{1}{2}$ pounds and a deflection per g of 1 inch for stable configurations. For the cases of no artificial damping and $\frac{dC_m}{dC_L} = 0$ and 0.02, the stick force per g was about 0 and -3 pounds (push), respectively. The pilot's task was to pull up from 1 g to 3g and trim the airplane. This task and the simulation of the airplane by the two-degree-of-freedom equations of motion are consistent and adequate for an analysis of short-period dynamics.

The simulator results, as time histories of normal acceleration and pilot's input to the elevator, are presented in figure 4 for two static margins, $\frac{dC_m}{dC_L} = 0$ and 0.02. In figure 4(a), the results for the basic airplane with no artificial damping are shown. With $\frac{dC_m}{dC_L} = 0$ the airplane is about neutrally stable, and with $\frac{dC_m}{dC_L} = 0.02$, the airplane is rapidly divergent. For both configurations the pilot was able to pull up to 3g and steady out.

With a static margin of 0, no pitching moment is produced by angle of attack. Consequently, the stick was first pulled back to pitch the airplane and then almost neutralized when the desired normal acceleration was obtained. The pilot was able to perform this task without difficulty although the control motions and forces are different from those required for maneuvering an airplane with positive static margins.

With negative static margin the task was difficult and the record was taken only after several practice runs. After initiating the motion with the control, the pilot had to rapidly reverse the elevator deflection in order to halt the divergence and trim the nose-up pitching moment. Although he could control the motion, the pilot considered this configuration unflyable for operational conditions because the control task required about 90 percent of his time.

With artificial pitch damping added, the unit-step responses for $\frac{dC_m}{dC_L} = 0$ and 0.02 are shown in figure 3 and the simulator results are shown in figure 4(b). The pilot found that these systems were easy to control. Since these configurations were overdamped, the pilot could pull up rapidly by initially overcontrolling and then easing off on the control as the desired normal-acceleration level was approached. In figure 4(b), the response for $\frac{dC_m}{dC_L} = 0.02$ is slower because the pilot did not overcontrol as much as for the $\frac{dC_m}{dC_L} = 0$ condition. In this application of a pitch damper, customary washout circuits must be omitted in order that the damper remain effective in the steady state. As a result of this simulator study, it appears that hypersonic glider configurations may be provided with satisfactory short-period characteristics by a pitch damper for center-of-gravity positions behind the neutral point.

Roll Coupling

Stability in steady roll.- Conditions were given previously for avoiding rolling divergence over the entire range of rolling velocities. (See eqs. (8) and (9).) With the given relations, boundary curves have been determined as a function of pitch-damper gain and pitching-moment-curve slope, the two significant parameters in this investigation, and are presented in figure 5. For values of K above the boundary curve for rolling stability, the particular configuration will be free of rolling divergence. The corresponding boundary for neutral longitudinal stability (lower curve) is also shown. At small values of $C_{m\alpha}$, rolling stability requires a somewhat higher pitch-damper gain than does longitudinal stability.

The region between the curves corresponds to conditions of $C_{m\alpha}$ and K for which rolling divergence would be expected for some finite range of roll rates. Rolling instabilities of this kind are character-

istic of many supersonic fighter aircraft and analyses are usually made of specific conditions by means of rolling divergence boundary plots. For examples, see references 1, 3, and 4.

To further illustrate the characteristics of the region between the two boundary curves of figure 5, critical rolling velocities are presented in figure 6 as a function of a damper gain for $C_{m\alpha} = 0$, 0.015, and 0.030. The curves originate at the $p_{cr} = 0$ axis at the minimum K value for longitudinal stability given in figure 5. Intersections of a given $C_{m\alpha}$ curve with a particular value of K define the roll-rate range for which a specific configuration would be divergent. The upper portions of the curves extend to K values less than the minimum for longitudinal stability and indicate the condition of spin stabilization of longitudinally divergent configurations.

The characteristics roots for roll rates of 0.5, 1.0, and 2.0 radians/sec are given in table II for four conditions of $C_{m\alpha}$ and K .

The configurations having $C_{m\alpha} = 0.04$, $K = 1.88$ and $C_{m\alpha} = 0$, $K = 0.48$ would appear in the completely stable zone of figure 5, and the characteristic roots given in the table are all stable. The other two configurations having $C_{m\alpha} = 0.04$, $K = 0.83$ and $C_{m\alpha} = 0$, $K = 0.15$ would appear in the area between the boundaries in figure 5 and each would have a divergent mode for a range of roll rates. In table II a divergent mode is indicated for each of these configurations at a particular roll rate.

Transient responses.- Two examples of angle-of-attack responses during steady rolling are presented in figure 7 for a roll rate of 0.5 radian/sec and an initial angle of attack of 5° . The configurations selected were $C_{m\alpha} = 0.04$, $K = 0.83$ and $C_{m\alpha} = 0.04$, $K = 1.88$ for which the characteristic roots are given in table II. For $K = 1.88$, the response is stable with small amplitudes. For $K = 0.83$, the response is slowly divergent but may not be objectionable to pilots.

Phugoid Characteristics

The short-period oscillation is usually the longitudinal mode of primary concern to pilots and the purpose of this investigation was to investigate a method of improving its characteristics. Although pitch damping stabilized the short-period mode, examination of the quartic longitudinal stability equation, obtained when perturbations in forward

speed are permitted, indicates a divergent condition for unstable pitching-moment curves and a negligible effect of pitch damping on this instability. This divergence is associated with the phugoid characteristics and flight tests of such configurations are required to determine the significance of this divergence. If the phugoid characteristics prove to be undesirable, an additional feedback of a quantity significant in the phugoid mode, such as forward speed, is required. An alternate control system, employing normal acceleration or angle-of-attack feedback, may be employed to improve simultaneously both short-period and phugoid characteristics.

SUMMARY OF RESULTS

A brief analysis, by means of stability calculations and a simulator study, has been made of the stability and controllability of a hypersonic glider configuration for center-of-gravity locations rearward of the neutral point. When a pitch damper was employed as a stability augmenter, the following results were obtained for a velocity of 700 ft/sec and an altitude of 40,000 feet:

1. Without augmentation, the airplane was rapidly divergent for negative static margins and was considered unsatisfactory in the simulator study.

2. With artificial pitch damping, the short-period response is stable and overdamped and the configuration was easily controlled on the simulator. An instability of the phugoid mode exists, however, for negative static margins and investigation by simulator or flight studies is required to determine the effect of this mode on the controllability of the airplane.

3. A roll-coupling analysis showed that the airplane can be made stable for all roll rates with an appropriate damper gain.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Field, Va., February 5, 1959.

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TABLE I
AIRCRAFT PHYSICAL AND AERODYNAMIC
CHARACTERISTICS

m , slugs	585
I_X , slug-ft ²	11,000
I_Y , slug-ft ²	126,000
I_Z , slug-ft ²	136,000
b , ft	35
\bar{c} , ft	25
S , sq ft	750
h , ft	40,000
V , ft/sec	700
C_{L_α} , per radian	2
C_{m_q} , per radian	-0.6
$C_{m_{\delta_e}}$, per radian	-0.08
C_{Y_β} , per radian	-0.286
C_{n_β} , per radian	0.057
C_{n_r} , per radian	-0.31

TABLE II
 CHARACTERISTIC ROOTS IN STEADY ROLLING

$C_{m\alpha}$	$K, \frac{\text{deg}}{\text{deg/sec}}$	$P_0, \text{radians/sec}$	Roots
0.04	1.88	0.5	-0.224, -3.645, -0.186 ± 1.358i
		1.0	-0.236, -3.489, -0.260 ± 1.640i
		2.0	-0.931, -2.591, -0.361 ± 2.438i
0.04	0.83	0.5	+0.013, -2.095, -0.184 ± 1.379i
		1.0	-0.072, -1.874, -0.253 ± 1.695i
		2.0	-0.902 ± 0.836i, -0.323 ± 2.513i
0	0.48	0.5	-0.245, -1.151, -0.232 ± 1.437i
		1.0	-0.103, -1.124, -0.316 ± 1.854i
		2.0	-0.555 ± 0.913i, -0.375 ± 2.799i
0	0.15	0.5	-0.054, -0.840, -0.201 ± 1.464i
		1.0	+0.058, -0.858, -0.249 ± 1.887i
		2.0	-0.366 ± 0.936i, -0.282 ± 2.826i

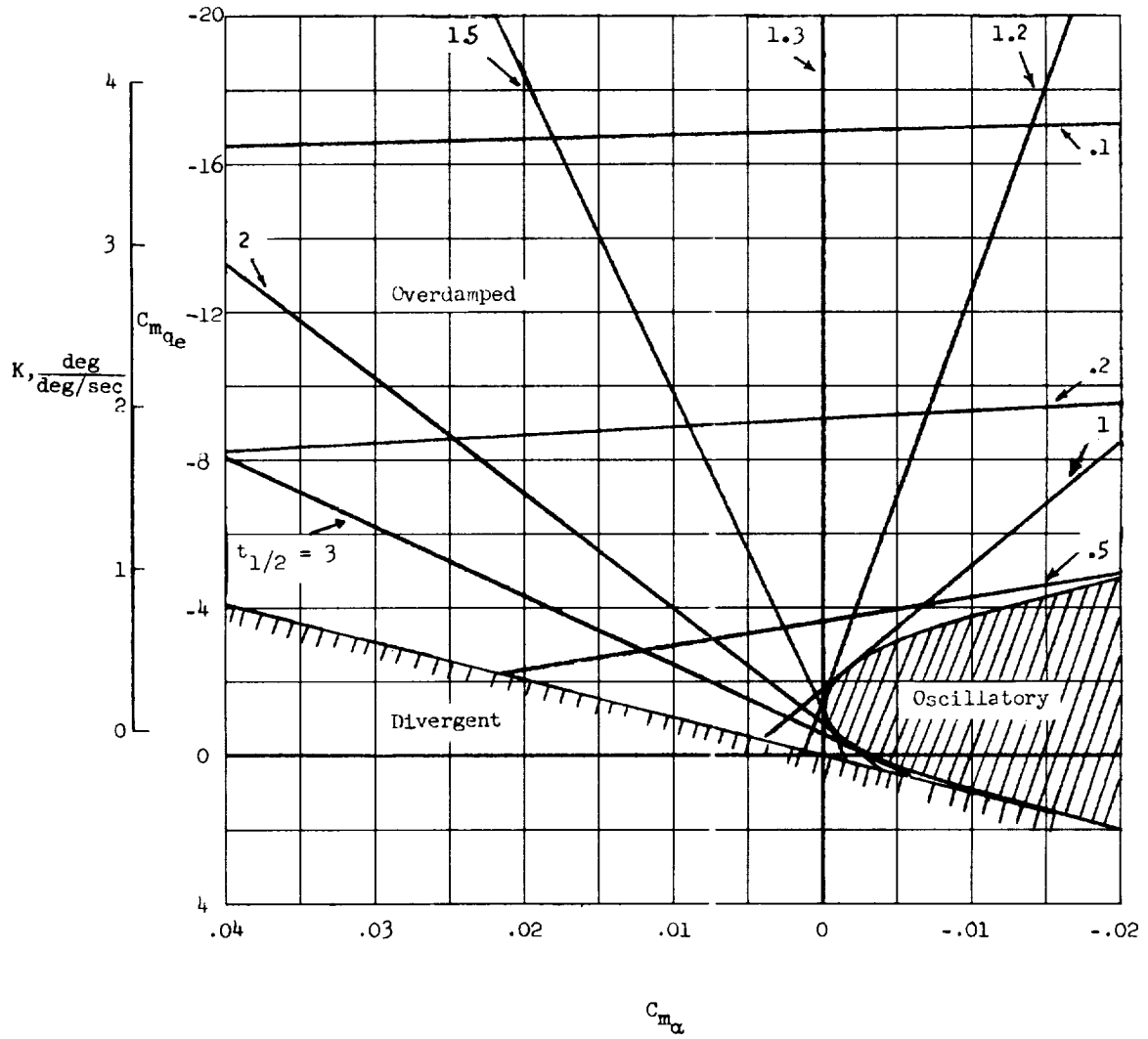


Figure 1.- Longitudinal characteristics of overdamped airplane.

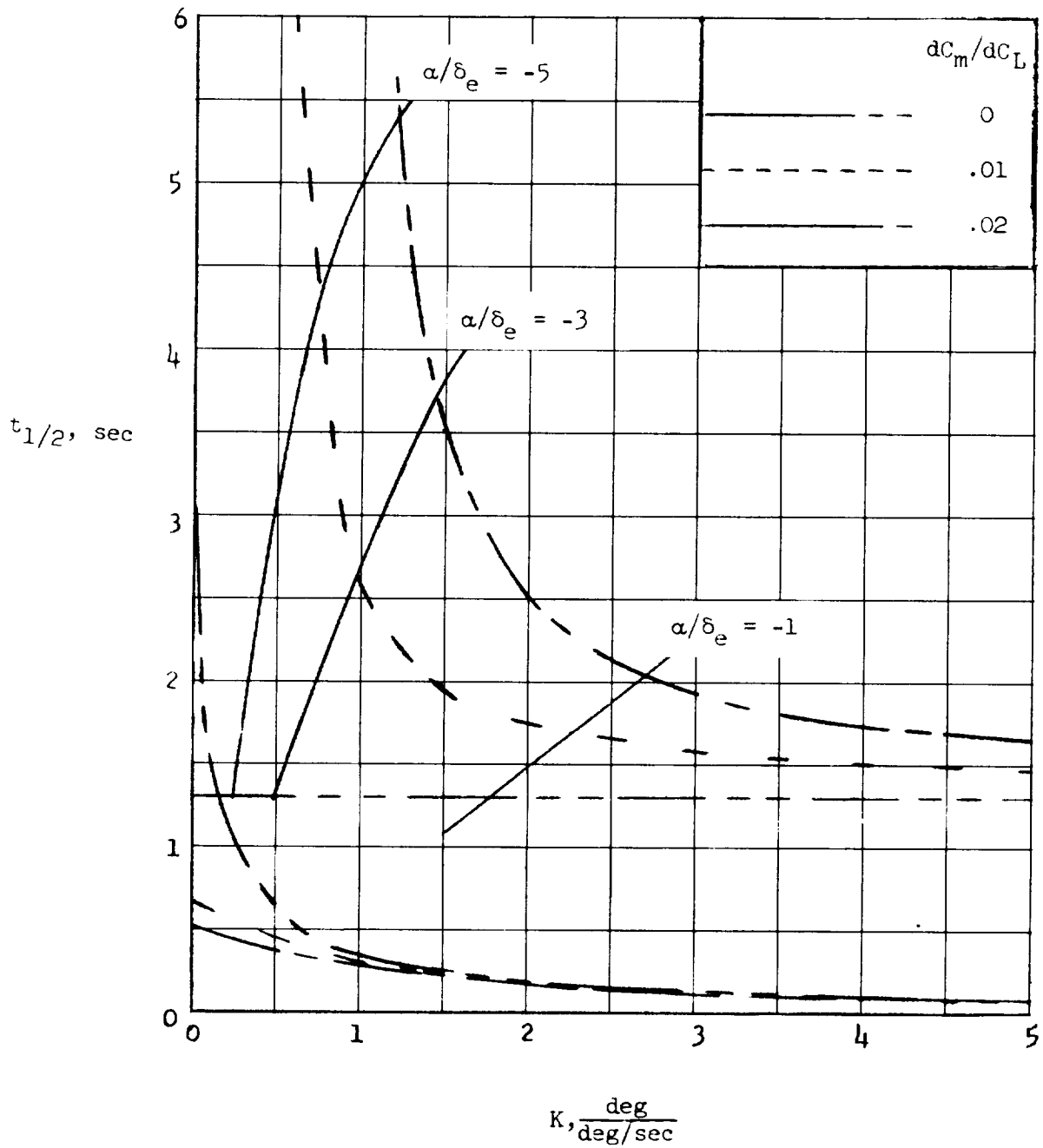


Figure 2.- Effect of pitch damping on longitudinal damping and α/δ_e .

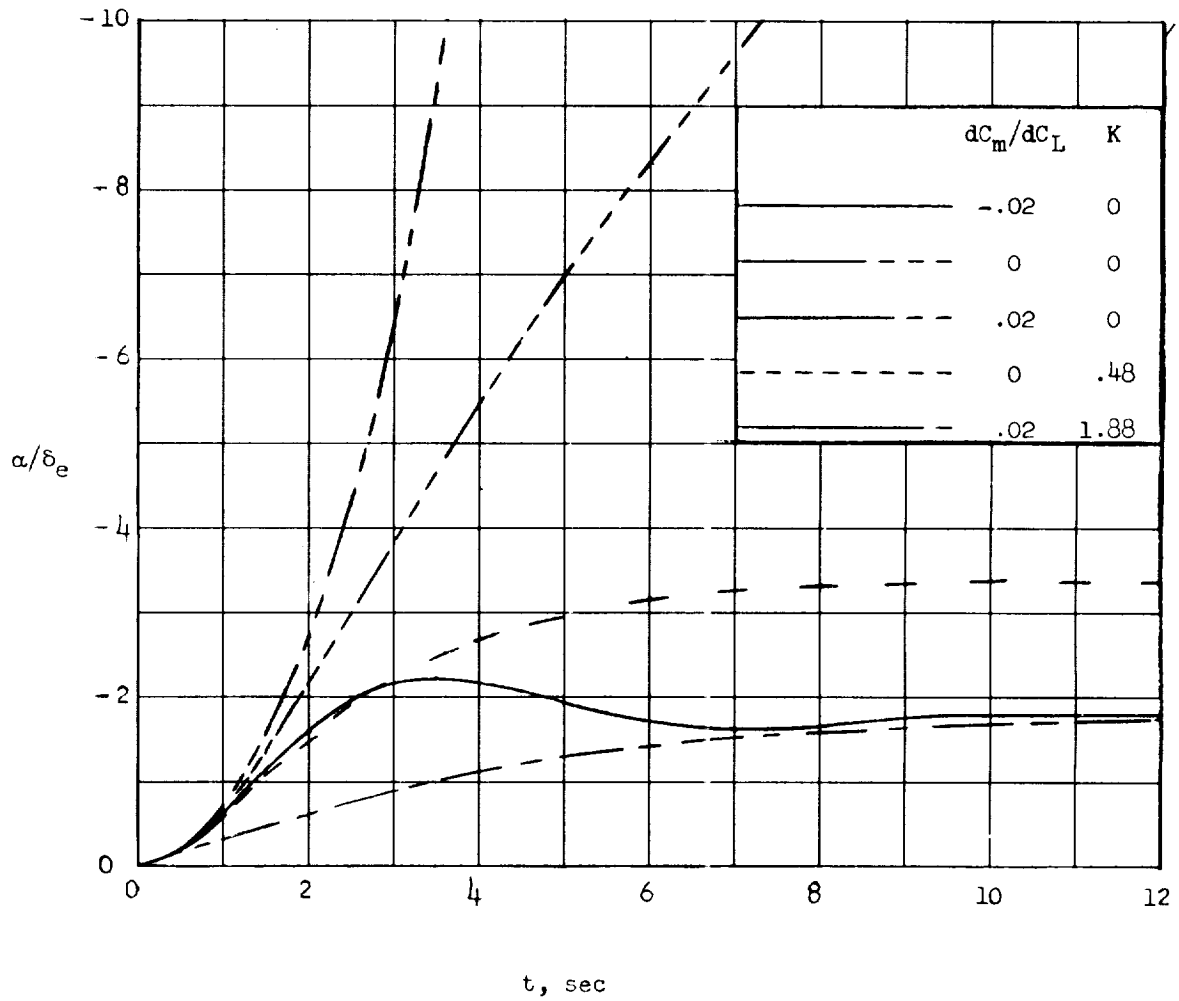
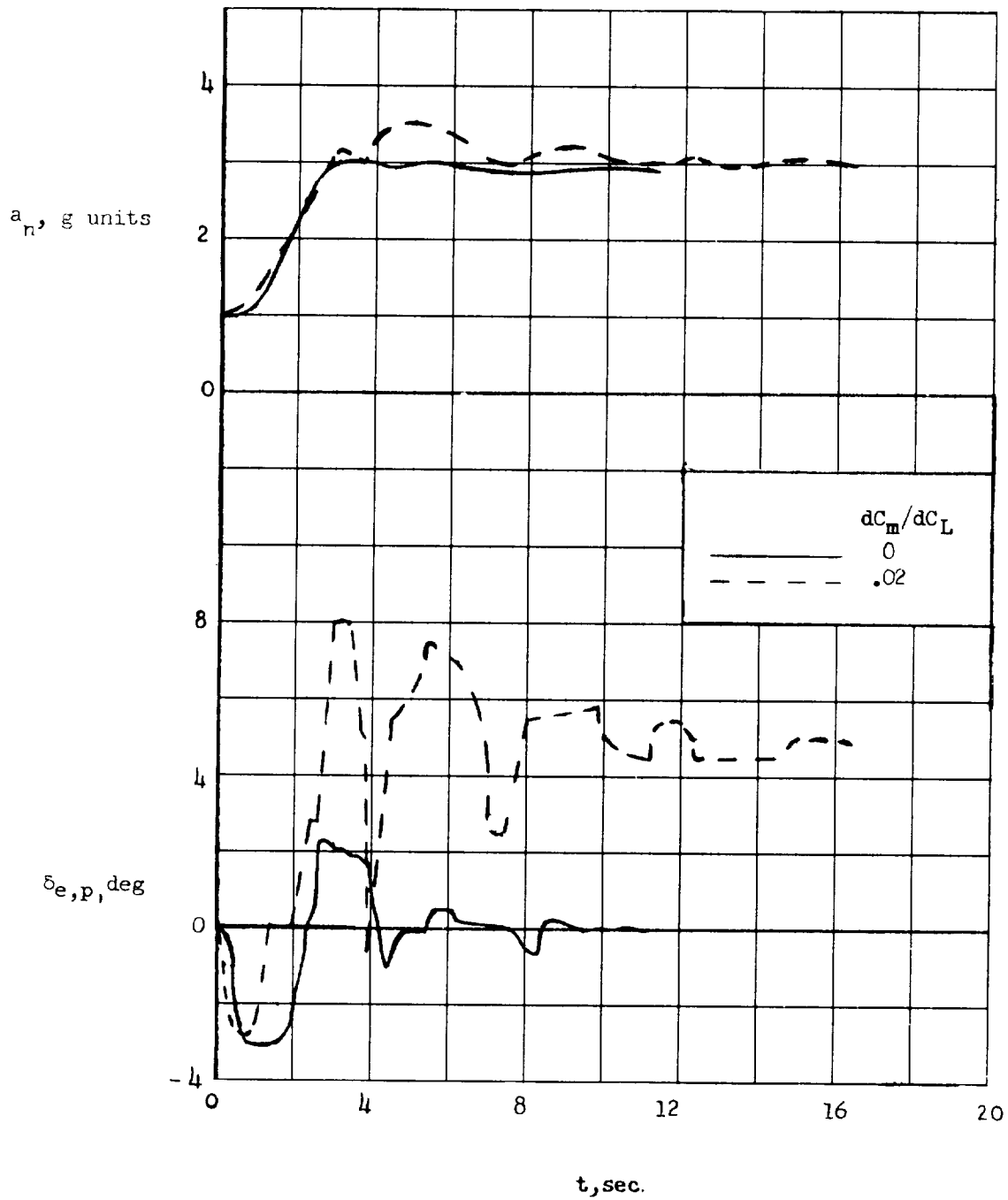


Figure 3.- Angle-of-attack response to unit-step elevator deflection.



(a) No artificial damping.

Figure 4.- Simulator results for a prescribed control task with rearward center-of-gravity positions.

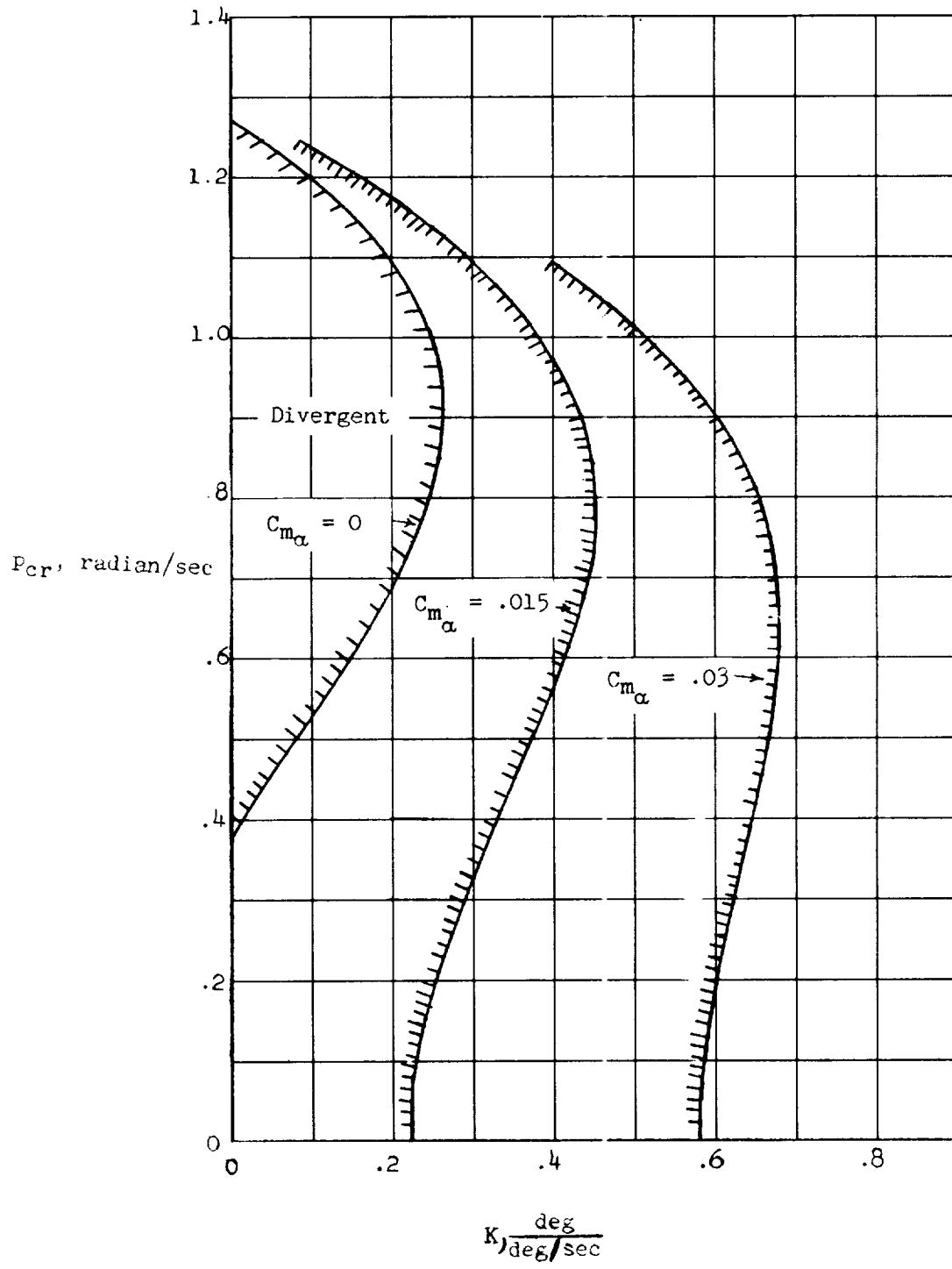


Figure 6.- Effect of pitch-damper gain K on critical rolling velocity.

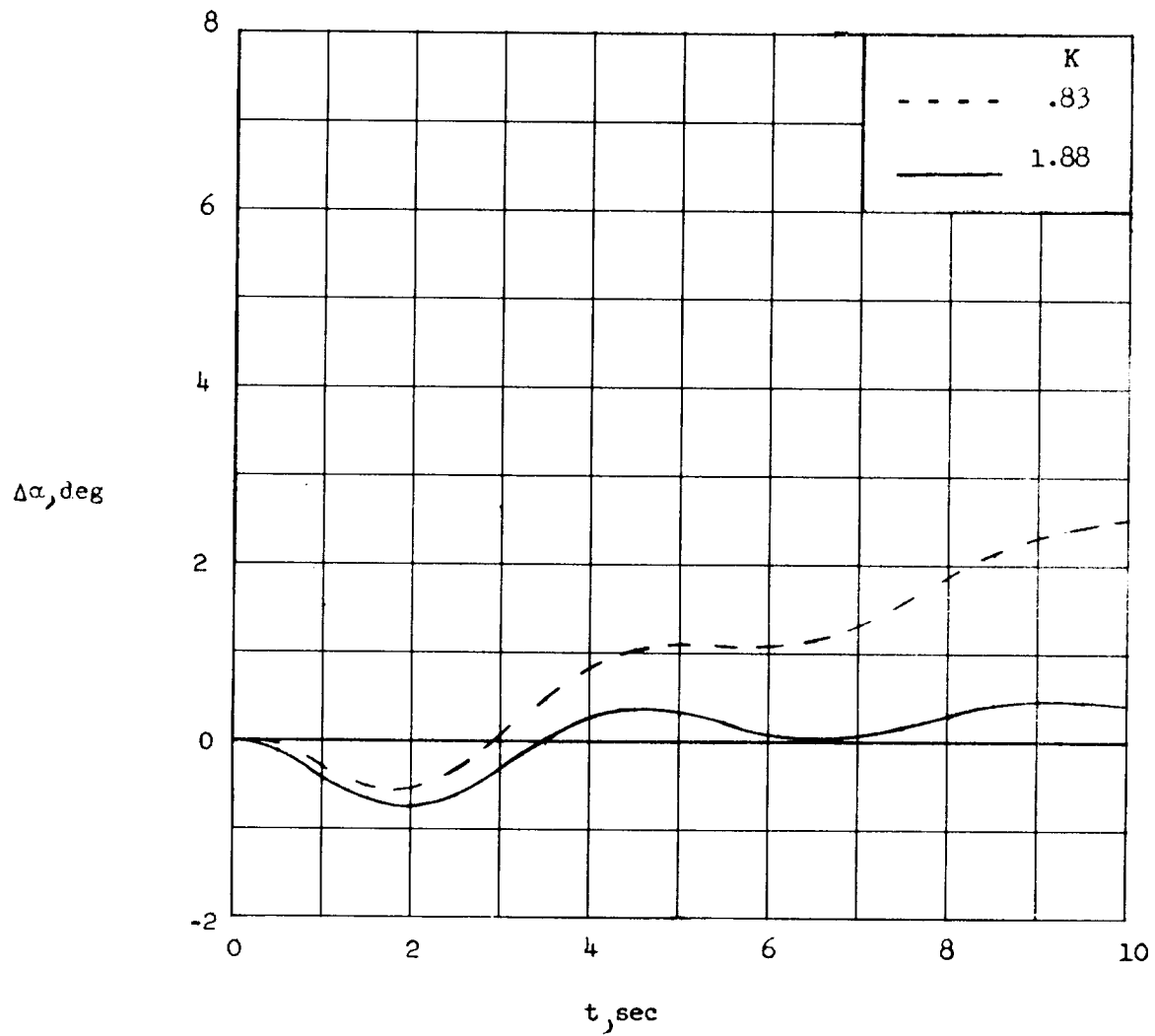


Figure 7.- Angle-of-attack response with a steady-rolling velocity.

$$\alpha_0 = 5^\circ; p_0 = 0.5 \text{ radian/sec}; \frac{dC_m}{dC_L} = 0.02.$$

